PRECISION MASS PROPERTY MEASUREMENTS USING A FIVE-WIRE TORSION PENDULUM

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INTRODUCTION

A method for measuring the moment of inertia of an object using a five-wire torsion pendulum design is described here. Typical moment of inertia measurement devices are capable of 1 part in 10^3 accuracy and current state of the art techniques have capabilities of about one part in 10^4 [1]. The five-wire apparatus design, Figure 1, shows the prospect of improving on current state of the art. Current measurements using a laboratory prototype indicate a moment of inertia measurement precision better than a part in 10^4. In addition, the apparatus is shown to be capable of measuring the mass center offset from the geometric center. Typical mass center measurement devices exhibit a measurement precision up to approximately 1 μm. Although the five-wire pendulum was not originally designed for mass center measurements, preliminary results indicate an apparatus with a similar design may have the potential of achieving state of the art precision.

MEASUREMENT APPARATUS

An apparatus designed to generate a rotation about an axis provides the ability to measure an objects mass center and moment of inertia. The measurement apparatus typically attempts to produce a pure rotation about a single degree of freedom. When designing an apparatus to measure the moment of inertia to a high precision, care must be taken to minimize the extra degrees of freedom in the system. The measurements of rotation will have uncertainties when there are significant other degrees of freedom. Bifilar and trifilar pendulums do not constrain the swinging or lateral translation modes. To improve the accuracy of a standard trifilar pendulum, the lateral pendulum modes need to be constrained. Five support wires are sufficient to constrain all but one degree of freedom. In a five-wire pendulum, two additional wires are arranged as shown in Figure 1 to minimize rotations about the other two rotational axes. The design reduces errors due to tilt and horizontal translational degrees of freedom. The three attach points on the platform supporting the inertia to be measured are positioned equidistant from the center of rotation. At one attach point, a single vertical wire is used and the other two attach points consist of two wires. The horizontal components of the wires which are splayed out from a single attach point provide horizontal stiffness.

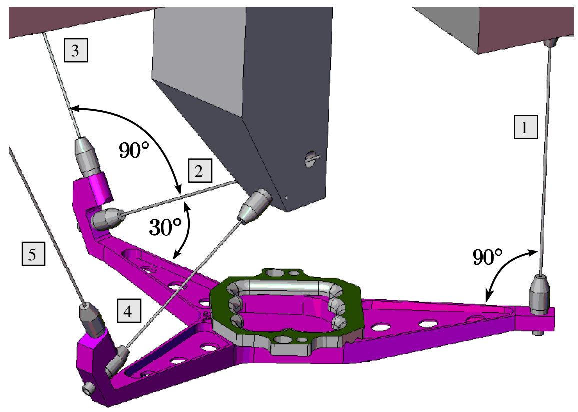


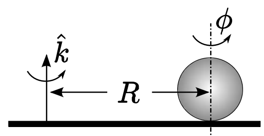
FIGURE 1. Five-wire torsion pendulum.

Moment of Inertia Measurement

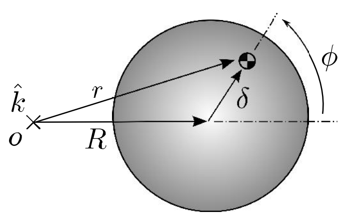
In order to measure the moment of inertia, I, the object is rotated about an axis and the pendulum natural frequency, ω, is used to determine the radius of gyration, R_g, about that rotational axis. The relationship between the torsion pendulum rotational frequency and the radius of gyration is given by: $R_g^2 = {}^{I}\!/_{m} = {}^{k}\!/_{\omega^2}$, where k is the torsion coefficient or stiffness constant of the pendulum and m is the total mass. The full moment of inertia tensor contains six independent terms. As such, a moment of inertia measurement device must be capable of determining the radius of gyration about at least six different axes of rotation.

Mass Center Measurement

By measuring the pendulum natural frequency of rotation, the mass center offset from the geometric center of an object is also obtained. The instantaneous moment of inertia consists of the moment of inertia about the objects mass center plus the parallel axis theorem components of the objects mass times the square of the distance



(a) Side View



(b) Top View

FIGURE 2. Mass center measurement configuration using a torsion pendulum.

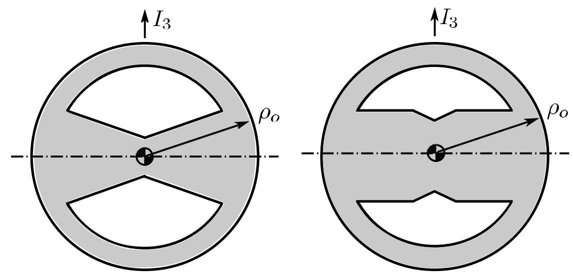
from the rotation center to the mass center, r^2. Placing an object on the pendulum far from the rotation axis amplifies the contribution due to the mass center offset from the geometric center, due to the quadratic dependence on r. By changing the orientation of the object with a fixed geometric location relative to the pendulum rotation center, the mass center offset is determined by measuring the change in the natural frequency.

Consider for example an object with a mass center offset from the geometric center of magnitude δ. The object's geometric center is placed on the pendulum at a location of R from the pendulum rotation axis, $\hat{k}$. Refer to Figure 2 for a graphical depiction. The mass center offset within a plane can be determined by rotating the object about an axis parallel to the pendulum rotation axis. The offset is determined by measuring the pendulum oscillation frequency for different rotation angles of ϕ. For each rotation angle ϕ, a corresponding change in the pendulum oscillation frequency will be observed.

MEASUREMENT OBJECT

Two different types of test objects are used to demonstrate mass property measurements with the five-wire pendulum. One object is specifically designed for evaluating moment of inertia measurements, while the other is intended for demonstrating mass center measurements.

For moment of inertia measurements, a preferred principal axis of inertia sphere is used as the test object. The spherical objects are designed such that the principal moment of inertia difference ratio, $(I_{p_{33}} - I_{p_{22}})/I_{p_{11}}$, is greater than 10 %. A



(a) 2-Part Sphere (b) 3-Part Sphere

FIGURE 3. Preferred principal axis of inertia sphere simplified cross-section. The cross section view for the 2-part sphere (a) is in the plane of the seam between the two hemispherical shells. The cross section view for the 3-part sphere (b) is in a plane perpendicular to the plane of the two seams for each hemispherical shell.

set of spheres were fabricated as part of a student project in the Precision Engineering course taught at Stanford University by Professors Dan DeBra and Dave Beach [2]. The preferred principal axis of inertia spheres are constructed of either two or three brass parts and then brazed together to create a complete sphere. The moment of inertia difference ratio for the spherical geometry is designed to be on the order of 10 %. Figure 3 shows a simplified cross-sectional view of the two designs.

For mass center measurements, a sphere with a fixed mass center offset is used as the test object [3]. A 2 inch (50.8 mm) diameter BeCu sphere was drilled with two holes forming a "V" to produce a mass center offset with respect to the geometric center. The sphere is designed to have a theoretical mass center offset of approximately δ = 40 μm.

Prior to the measurement process, the spheres must be marked to define a reference frame. Two perpendicular great circles are drawn onto the surface of the spheres to be used as a coordinate reference. On each of the fabricated preferred principal axis of inertia spheres, the seam is faintly visible as a silver line resulting from the brazing material. One great circle is drawn in a plane parallel to the visible brazed seam. The other two measurement planes are located mutually perpendicular to the first great circle. For the fixed mass center offset sphere, mutually perpendicular great circles are also drawn on the surface. The first great circle is drawn to intersect the two drilled holes comprising the "V". Again, the other two measurement planes are located mutually perpendicular to the first great circle.

MEASUREMENT RESULTS

The torsion pendulum is designed to rotate about a single axis. As such the measurement object, i.e. the sphere, must be rotated to different orientations with respect to the rotation axis. The choice of these orientations must adequately map out the inertia ellipsoid. For the preferred principal axis of inertia spheres, a set of nine measurement configurations was performed, with rotation angles at $\pi/4$ radian increments. For the fixed mass center offset sphere, measurements are also performed at $\pi/4$ radian increments, for eight measurements in each plane. Measurement configurations are chosen to adequately capture the minimum and maximum location of the mass center relative to the pendulum rotation axis. At the time of this publication, mass center measurements have been conducted in two of the three independent planes and the data analysis has been completed on the first measurement plane.

Moment of Inertia

In a five-wire torsion pendulum shown in Figure 1, five wires are arranged to minimize translational degrees of freedom, thereby reducing errors due to tilt and translation. The desired measurement parameter is the pendulum natural frequency of oscillation. The oscillation frequency is extracted from the recorded pendulum position time response data. By reducing the errors associated with generating a pure rotation, the five-wire pendulum has demonstrated a consistent frequency measurement to better than a part in 10^5. The oscillation frequencies for the the five-wire pendulum are on the order of 2.5 Hz to 4.0 Hz for the unloaded and loaded pendulum configurations respectively.

For the preferred principal axis of inertia sphere measurements, the frequency of oscillation was measured from about one part in 10^5 to a few parts in 10^6 for the various measurement orientations. Table 1 shows the resulting moment of inertia tensor measurement values for the preferred principal axis of inertia spheres. The results indicate a repeatable moment of inertia measurement on the order of a part in 10^4 or better.

For the moment of inertia measurements, the five-wire pendulum was calibrated with the two-part sphere on the platform. The calibration procedure was not repeated for the 3-part sphere. As a result, the indicated measurement results for the 3-part sphere may contain additional systematic errors as compared to the 2-part sphere measurement results. A different loading of the pendulum platform may result in subtle changes to the pendulum stiffness coefficient, which is estimated by the calibration process. Still, the repeatability of the measurements is the true indication of the precision associated with the measurement apparatus.

Mass Center

Although the constructed five-wire torsion pendulum was not originally designed to perform mass center measurements, the torsion pendulum has demonstrated precision measurement results. As already indicated, the measurement object for the mass center measurements is a specially designed fixed mass center offset spherical mass. By assuming a homogeneous density and an ideal geometry of a perfect sphere with the removed material, one can estimate a theoretical mass center offset value. Furthermore, by measuring the drilled hole locations and geometry of the actual fabricated sphere, the expected mass center location can be further estimated. Yet, this object has been used previously in other mass center measurement experiments and therefore has historical measurement results for comparison. As a reference, this work uses the values obtained by Conklin [4]. Although this particular sphere has only had the mass center offset measured to approximately 1 μm, Conklin has shown that the applied velocity modulation measurement technique can be used to measure the mass center location of spheres to state of art precision on the order of $\sim 0.1\,\mu$m.

The five-wire pendulum has been used previously to demonstrate the feasibility of mass center measurements with the torsion pendulum apparatus. These previous efforts estimate the mass center offset using a two configuration measurement approach. These two configurations correspond to the minimum and maximum radial distances from the pendulum rotation axis. Clearly, the previous measurements are approximate and the result is expected to be an under estimate of the true value.

This work further investigates the use of the five-wire pendulum for mass center measurements, which utilizes a more complete set of measurement configurations. As already indicated, eight measurements are performed in each measure-

Sphere	m_o kg	$[I_p]$ kg·mm^2	$[\Sigma_p]$ kg·mm^2 $\times 10^{-2}$	σ_{p_i}/I_{p_i} $\times 10^{-4}$	$\frac{I_{p_{33}}-I_{p_{22}}}{I_{p_{11}}}$
2-Part	0.4489	$\begin{bmatrix} 112.67 & & \\ & 112.98 & \\ & & 126.41 \end{bmatrix}$	$\begin{bmatrix} 1.1 & & \\ & 0.2 & \\ & & 0.1 \end{bmatrix}$	$\begin{bmatrix} 0.94 & & \\ & 0.19 & \\ & & 0.05 \end{bmatrix}$	0.1192
3-Part	0.4551	$\begin{bmatrix} 113.15 & & \\ & 113.60 & \\ & & 126.50 \end{bmatrix}$	$\begin{bmatrix} 1.7 & & \\ & 0.1 & \\ & & 1.1 \end{bmatrix}$	$\begin{bmatrix} 1.50 & & \\ & 0.12 & \\ & & 0.89 \end{bmatrix}$	0.1141

TABLE 1. Measurement results for preferred principal axis of inertia spheres.

ment plane. The pendulum frequency of oscillation was measured to a couple parts in 10^5 or better for the various measurement orientations. Using the eight equally-spaced measurements for the measurement plane with the largest mass center offset component, the mass center offset was determined to be a magnitude of 41.89 μm within the rotation plane. Repeatable frequency measurements with the five-wire pendulum result in an estimated mass center offset measurement error of better than 0.8 μm. It should be noted that a full error analysis on the mass center measurements has not yet been completed. The mass center measurement results contain for example systematic errors associated with the measurement apparatus calibration procedure. The measurement apparatus calibration parameters utilized for the mass center measurements are those used for the moment of inertia measurements. A slight change in the calibration coefficients and the measurement results correspondingly is expected. Table 2 summarizes the current and historical measurements for the mass center offset from the geometric center for the fixed mass center offset sphere.

Source	MC Offset	Notes
(a)	40.0 μm	Theoretical, [3]
(b)	44.0 μm	$\sigma = 10\,\mu$m, [4]
(c)	41.17 μm	$\sigma = 1.5\,\mu$m, [4]
(d)	40.23 μm	Error: $\sim$1 μm, [5]
(e)	41.89 μm	Error: $\sim$0.8 μm

TABLE 2. Comparison of measurement results for the fixed mass center offset sphere. Measurement sources: (a) Design value, (b) Estimate from geometry, (c) Velocity modulation, (measured), (d) Torsion pendulum, (Single Plane, measured, two-configuration estimate). (e) Torsion pendulum, (Single Plane, measured).

SUMMARY

The five-wire pendulum is capable of determining moment of inertia properties to better than a part in 10^4. The prototype five-wire measurement apparatus shows the prospect of further improving on current state of the art precision. The apparatus is also capable of mass center offset measurements to better than $\sim$0.8 μm. Further work is necessary to completely characterize the error associated with the mass center measurements. For mass center measurements, an apparatus with a similar five-wire design shows the prospect of achieving state of the art. For the moment of inertia measurements, current limitations are associated with environmental disturbances, primarily vibrations induced by human motion. A full description of the associated pendulum measurement errors is available in [5].

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