Droplet Deformation Prediction with the Droplet Deformation and Breakup Model (DDB)

Mario Vargas

NASA Glenn Research Center
Outline

• Motivation

• Objectives

• Selection of Droplet Deformation and Breakup (DDB) Model

• DDB Model Derivation and Assumptions

• Initial Value Problem and the Numerical Solution

• Experimental Data used to Test the Model

• Approach and Assumptions to apply the Model

• Results

• Conclusions
Motivation

Rotating Rig Experiments

Physics

Theoretical Modeling

CFD
Selection of DDB Model

- The DDB Model was proposed by Ibrahim, Yang and Przekwas (1993)
- The DDB is based on the type of droplet deformation observed in droplet breakup studies near the leading edge of an airfoil
  - “the liquid droplet is deformed from an initial spherical shape of radius R into an oblate spheroid of an ellipsoidal cross section with major semi-axis a and minor semi-axis b"
- Model governed by a second order ODE
  - Well tried numerical schemes available to do the integration
- Model prediction of displacement of the center of mass can be compared to experimental results
DDB Model

Assumptions

- No exchange of heat with surroundings
- Only forces involved: pressure, viscous and surface tension

\[
\frac{dE}{dt} = - \frac{dW}{dt}
\]
DDB Model

\[ \frac{dE}{dt} = - \frac{dW}{dt} \]

\[ \frac{dE_{\text{kinetic}}}{dt} + \frac{dE_{\text{potential}}}{dt} = \frac{dW_{\text{pressure}}}{dt} + \Phi \]


**DDB Model**

**Kinetic Energy Term**

\[
\frac{dE_{\text{kinetic}}}{dt} = \frac{d\left(\frac{1}{2}mv^2\right)}{dt}
\]

\[
\frac{dE_{\text{kinetic}}}{dt} = m \cdot \left(\frac{dv}{dt}\right) \cdot v = \frac{2}{3} \pi \cdot R^3 \cdot \rho \cdot v \cdot \frac{dv}{dt}
\]

\[
\frac{dE_{\text{kinetic}}}{dt} = \frac{2}{3} \pi \cdot R^3 \cdot \rho \cdot \frac{dy}{dt} \left(\frac{d^2 y}{dt^2}\right)
\]
DDB Model

Potential Energy Term

\[
\frac{dE_{potential}}{dt} = \frac{1}{2} \sigma \cdot \left( \frac{dA_s}{dt} \right)
\]

\[
A_{s-exact} = 2\pi a^2 + 2\pi b^2 \cdot \phi \\
\phi = \frac{1}{2\varepsilon} \cdot \ln \left( \frac{1 + \varepsilon}{1 - \varepsilon} \right) \\
\varepsilon = \sqrt{1 - \left( \frac{a}{b} \right)^2}
\]

Approximation to the surface area

\[
A_s = 2\pi (a^2 + b^2)
\]
DDB Model

Potential Energy Term

\[ A_s = 2\pi(a^2 + b^2) \]

\[ \frac{4}{3} a^2 \cdot b = \frac{4}{3} \pi \cdot R^3 \implies b = \frac{R^3}{a^2} \]

\[ y = \frac{a}{c} \quad c = \frac{8}{3} \]

\[ \frac{dA_s}{dt} = \frac{9\pi^3}{4} y \left[ 1 - 2 \cdot \left( \frac{c \cdot y}{R} \right)^{-6} \right] \cdot \frac{dy}{dt} \]

\[ \frac{dE_{\text{potential}}}{dt} = \frac{9\pi^3}{8} \sigma \cdot y \left[ 1 - 2 \cdot \left( \frac{c \cdot y}{R} \right)^{-6} \right] \cdot \frac{dy}{dt} \]
DDB Model

Work Done by the Pressure

\[
\frac{dW_{pressure}}{dt} = -\frac{1}{2} \cdot p \cdot A_p \cdot \left( \frac{dy}{dt} \right)
\]

Approximation to the projected area

\[A_p \approx \pi \cdot R^2\]

Approximation to the average pressure

\[p = \frac{1}{2} \cdot \rho_g \cdot U_{rel}^2\]

\[
\frac{dW_{pressure}}{dt} = -\pi \cdot R^2 \cdot \rho_g \cdot U_{rel}^2 \left( \frac{dy}{dt} \right)
\]
DDB Model

\[ \frac{dE}{dt} = - \frac{dW}{dt} \]

\[ \frac{dE_{\text{kinetic}}}{dt} + \frac{dE_{\text{potential}}}{dt} = \frac{dW_{\text{pressure}}}{dt} + \Phi \]

\[ \frac{2}{3} \pi R^3 \cdot \rho_i \cdot \frac{dy}{dt} \left( \frac{d^2 y}{dt^2} \right) \]

\[ \frac{9\pi^3}{8} \sigma \cdot y \left[ 1 - 2 \left( \frac{c \cdot y}{R} \right)^6 \right] \cdot \left( \frac{dy}{dt} \right) \]

\[ -\frac{\pi}{4} \cdot R^2 \cdot \rho_g \cdot U_{\text{rel}}^2 \left( \frac{dy}{dt} \right) \]

\[ \frac{8}{3} \pi R^3 \cdot \mu_l \cdot \left( \frac{1}{y} \frac{dy}{dt} \right)^2 \]
### DDB Model

**Pressure Force Term**

\[
\frac{2}{3} \pi R^3 \rho I \left( \frac{d^2 y}{dt^2} \right) = \frac{\pi}{4} R^2 \rho_g U_{rel}^2 - \frac{8}{3} \pi R^3 \mu_r \frac{d}{dy} \left( \frac{1}{y^2} \frac{dy}{dt} \right) - \frac{9\pi^3}{8} \sigma y \left[ 1 - 2 \left( \frac{c \cdot y}{R} \right)^{-6} \right]
\]

**Inertia Forces**

\[
ma \propto \rho L^3 \frac{dy}{ds} \frac{ds}{dt} \propto \rho L^3 V \frac{V}{L} \propto \rho V^2 L^2 \propto \rho \rho_2 L^2
\]
DDB Model

\[
\frac{2}{3} \pi \cdot R^3 \rho_l \left( \frac{d^2 y}{dt^2} \right) = \frac{\pi}{4} \cdot R^2 \cdot \rho_g \cdot U_{rel}^2 - \frac{8}{3} \pi \cdot R^3 \cdot \mu_l \left( \frac{1}{y^2} \frac{dy}{dt} \right) - \frac{9 \pi^3}{8} \sigma \cdot y \left[ 1 - 2 \left( \frac{c \cdot y}{R} \right)^6 \right]
\]

pressure force \hspace{1cm} \text{viscous force} \hspace{1cm} \text{surface tension force}

Non-dimensionalization of the equation:

\[ y^* = \frac{y}{R} \quad t^* = t \cdot \left( \frac{U_{rel}}{R} \right) \quad \text{Re} = \frac{\rho_g U_{rel} R}{\mu_g} \quad \text{We} = \frac{\rho_g U_{rel}^2 R}{\sigma} \quad K = \frac{\rho_l}{\rho_g} \]

\[
K \left( \frac{d^2 y^*}{dt^*^2} \right) + \frac{4N}{\text{Re}} \frac{1}{y^{*2}} \frac{dy^*}{dt^*} + \frac{27\pi^2}{16 \cdot \text{We}} y^* \left[ 1 - 2 \left( \frac{c \cdot y^*}{R} \right)^6 \right] = \frac{3}{8}
\]
**Initial Value Problem**

\[
K \left( \frac{d^2 y}{dt^2} \right) + \frac{4N}{Re} \frac{1}{y^2} \frac{dy}{dt} + \frac{27\pi^2}{16 \cdot We} y \left[ 1 - 2(c \cdot y)^{-6} \right] = \frac{3}{8}
\]

\[
y(0) = \frac{4}{3\pi} \quad \frac{dy}{dt}(0) = 0
\]

\[\frac{dy}{dz} = z\]

\[\frac{dz}{dt} = -\frac{4N}{K \cdot Re} \frac{1}{y^2} z - \frac{27\pi^2}{16K \cdot We} \left[ 1 - 2 \cdot \left( \frac{3\pi y}{4} \right)^{-6} \right] + \frac{3}{8K}\]

**IC:** \( y(0) = \frac{4}{3\pi} \)

**IC:** \( z(0) = 0 \)
**Input Parameters**

\[
\frac{dy}{dz} = z
\]

\[
\frac{dz}{dt} = -\frac{4N}{K \cdot \text{Re}} \frac{1}{y^2} z - \frac{27\pi^2}{16K \cdot \text{We}} \left[ 1 - 2 \left( \frac{3\pi y}{4} \right)^6 \right] + \frac{3}{8K}
\]

\[IC : y(0) = \frac{4}{3\pi}\]

\[IC : z(0) = 0\]

**Parameters for DDB Model**

- Density of air; viscosity of air
- Density of water; viscosity of water
- Surface tension of water
- Diameter of the droplet
- Slip Velocity

**Parameters for the Numerical Solution**

- Number of first order ODE
- Initial and final value of the independent variable in the interval where the solution is evaluated
- Number of steps
- Step size
- Error Tolerance
Experimental Data Rotating Arm
Experimental Data
Data Analysis

- MATLAB with Digital Imaging Processing tool box was used in the data analysis.

- Droplet movement in the horizontal and vertical directions tracked frame by frame.

- Program tracks width and height of droplet:
  - Ellipse superimposed on the deformed droplet
  - Major and minor semi-axis of the superimposed ellipse

- Knowing the major semi-axis allows calculation of the center of mass vertical displacement for half-droplet.
Experimental Data

droplet radius = 516 µm, airfoil velocity = 90 m/sec, airfoil chord = 0.710 m,

### Table 1

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<tr>
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<th>20</th>
<th>30</th>
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<th>50</th>
<th>60</th>
<th>70</th>
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<td>Time (µsec)</td>
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<td>253</td>
<td>387</td>
<td>520</td>
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### Table 2

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<td>x-Distance (mm)</td>
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<td>$U_{rel}$ (m/sec)</td>
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</table>
Experimental Data

droplet radius = 516 μm, airfoil velocity = 90 m/sec, airfoil chord = 0.710 m,
Approach

- Assume droplet is in quasi-steady equilibrium at each location along the trajectory
- Solve the model at each location
  - Use experimentally measured slip velocity at each location as model input
- Compare model prediction to experimental data
Results

Center of Mass Oscillation

location 100, distance from the leading edge of the airfoil = -120.5 millimeters
droplet radius = 516 µm, airfoil chord = 0.710 m, airfoil velocity = 90 m/sec.
Results

Center of Mass Oscillation

location 170, distance from the leading edge of the airfoil = -37.3 millimeters
droplet radius = 516 µm, airfoil chord = 0.710 m, airfoil velocity = 90 m/sec.
Results

Model Prediction Compared to Experimental Data

droplet radius = 199 µm, airfoil chord = 0.710 m, airfoil velocity = 50 m/sec.
Results

Model Prediction Compared to Experimental Data

droplet radius = 287 µm, airfoil chord = 0.710 m, airfoil velocity = 90 m/sec.
Results

Model Prediction Compared to Experimental Data

droplet radius = 439 µm, airfoil chord = 0.710 m, airfoil velocity = 90 m/sec.
Results

Model Prediction Compared to Experimental Data

droplet radius = 516 µm, airfoil chord = 0.710 m, airfoil velocity = 90 m/sec.
Results

Model Prediction Compared to Experimental Data

droplet radius = 685 µm, airfoil chord = 0.710 m, airfoil velocity = 90 m/sec.
Conclusions

• For small and medium size droplets (radius between 200 and 500 µm) the model prediction agrees with experimental data.

• For large droplets (radius larger than 500 µm) the model over-predicts displacement of the center of mass by a large margin.

• The increasing deviation between model prediction and experimental data as droplet size increases indicates that one or more model assumptions are invalid for large droplet sizes.

• The quasi-steady assumption needed to apply the DDB model works well for small and medium size droplets.

• The model can be used in the analysis of deformation of small and medium droplets from previous experiments.
END OF PRESENTATION