Droplet Deformation Prediction with the Droplet Deformation and Breakup Model (DDB)

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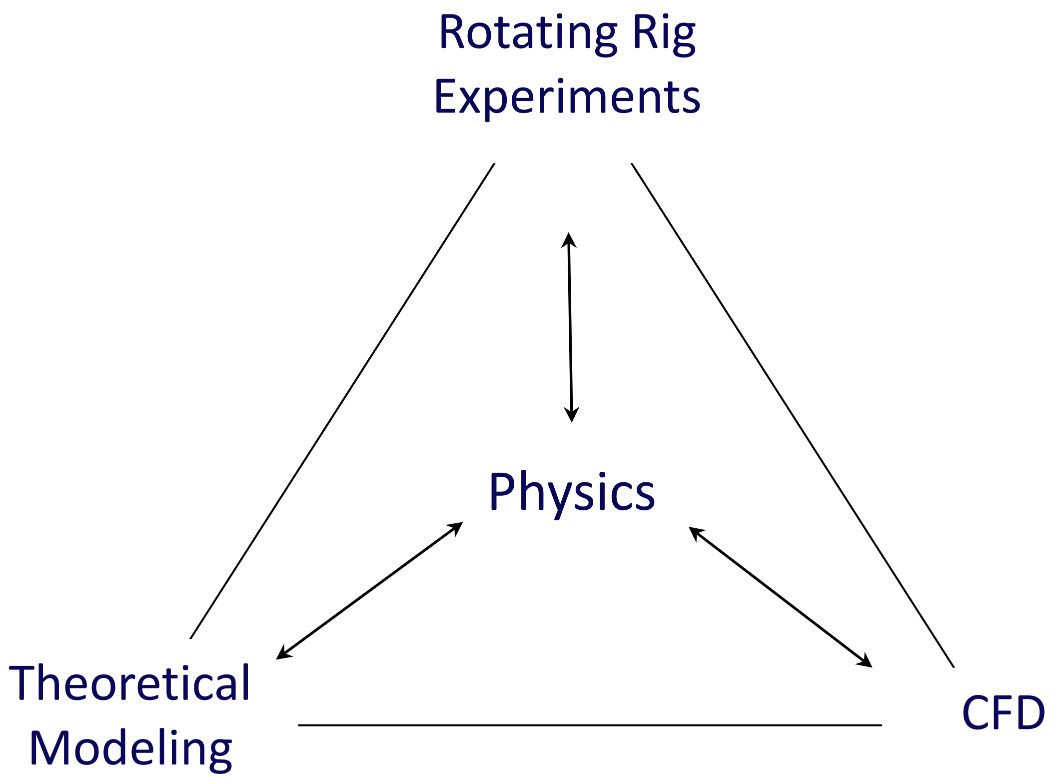
Outline

- Motivation
- Objectives
- Selection of Droplet Deformation and Breakup (DDB) Model
- DDB Model Derivation and Assumptions
- Initial Value Problem and the Numerical Solution
- Experimental Data used to Test the Model
- Approach and Assumptions to apply the Model
- Results
- Conclusions

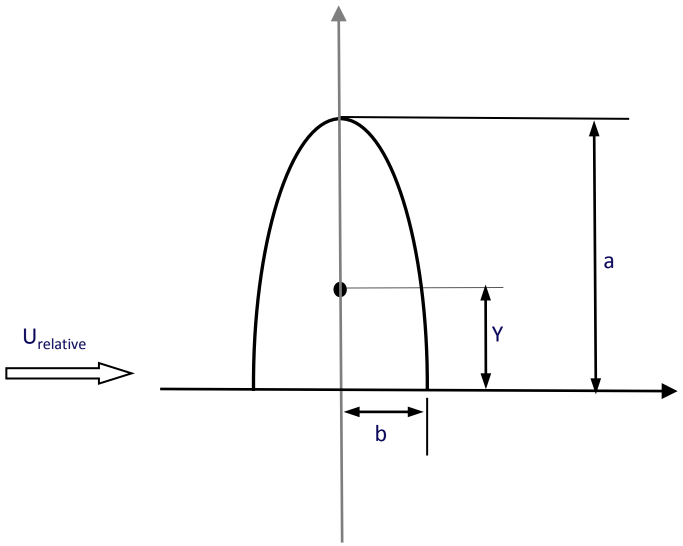
Motivation


Rotating Rig Experiments
Physics
Theoretical Modeling
CFD

Selection of DDB Model

- The DDB Model was proposed by Ibrahim, Yang and Przekwas (1993)
- The DDB is based on the type of droplet deformation observed in droplet breakup studies near the leading edge of an airfoil
  - "the liquid droplet is deformed from an initial spherical shape of radius R into an oblate spheroid of an ellipsoidal cross section with major semi-axis a and minor semi-axis b"
- Model governed by a second order ODE
  - Well tried numerical schemes available to do the integration
- Model prediction of displacement of the center of mass can be compared to experimental results

DDB Model



$$\frac{dE}{dt} = -\frac{dW}{dt}$$

Assumptions

- No exchange of heat with surroundings
- Only forces involved: pressure, viscous and surface tension

DDB Model

$$\frac{dE}{dt} = -\frac{dW}{dt}$$

$$\frac{dE_{kinetic}}{dt} + \frac{dE_{potential}}{dt} = \frac{dW_{pressure}}{dt} + \Phi$$

DDB Model

Kinetic Energy Term

$$\frac{dE_{kinetic}}{dt} = \frac{d(\frac{1}{2}mv^2)}{dt}$$

$$\frac{dE_{kinetic}}{dt} = m \cdot \left(\frac{dv}{dt}\right) \cdot v = \frac{2}{3}\pi \cdot R^3 \cdot \rho_l \cdot v \cdot \frac{dv}{dt}$$

$$\frac{dE_{kinetic}}{dt} = \frac{2}{3}\pi \cdot R^3 \cdot \rho_l \cdot \frac{dy}{dt}\left(\frac{d^2y}{dt^2}\right)$$

DDB Model

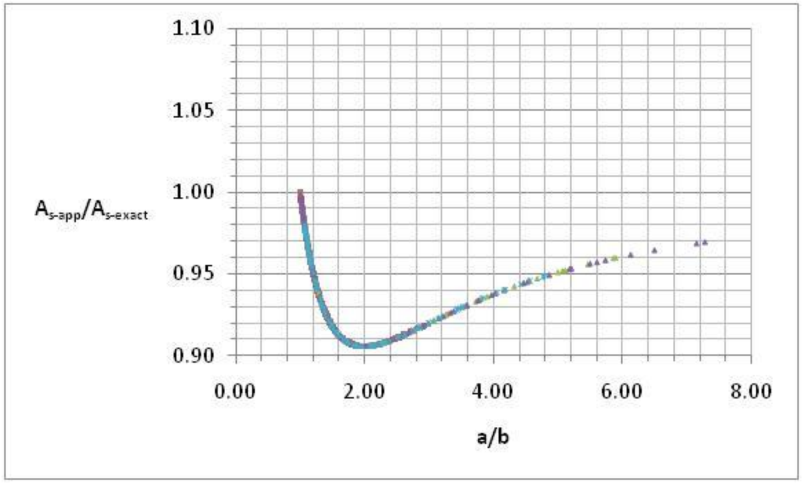
Potential Energy Term

$$\frac{dE_{potential}}{dt} = \frac{1}{2}\sigma \cdot \left(\frac{dA_s}{dt}\right)$$

$$A_{s-exact} = 2\pi a^2 + 2\pi b^2 \cdot \phi \qquad \phi = \frac{1}{2\varepsilon} \cdot \ln\left(\frac{1+\varepsilon}{1-\varepsilon}\right) \qquad \varepsilon = \sqrt{1-\left(\frac{a}{b}\right)^{-2}}$$

Approximation to the surface area

$$A_s = 2\pi(a^2 + b^2)$$



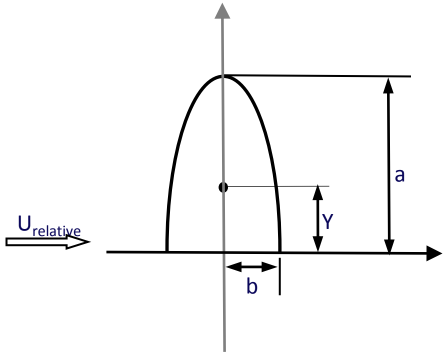
DDB Model

Potential Energy Term

$$A_s = 2\pi(a^2 + b^2)$$

$$\frac{4}{3}a^2 \cdot b = \frac{4}{3}\pi \cdot R^3 \Rightarrow b = \frac{R^3}{a^2}$$

$$y = \frac{a}{c} \qquad c = \frac{8}{3}$$



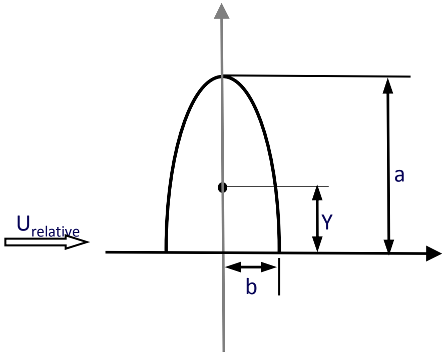
$$\frac{dA_s}{dt} = \frac{9\pi^3}{4} y\left[1 - 2\cdot\left(\frac{c\cdot y}{R}\right)^{-6}\right]\cdot\left(\frac{dy}{dt}\right)$$

$$\frac{dE_{potential}}{dt} = \frac{9\pi^3}{8}\sigma \cdot y\left[1 - 2\cdot\left(\frac{c\cdot y}{R}\right)^{-6}\right]\cdot\left(\frac{dy}{dt}\right)$$

DDB Model

Work Done by the Pressure

$$\frac{dW_{pressure}}{dt} = -\frac{1}{2} \cdot p \cdot A_p \cdot \left(\frac{dy}{dt} \right)$$



Approximation to the projected area

$$A_p \approx \pi \cdot R^2$$

Approximation to the average pressure

$$p = \frac{1}{2} \cdot \rho_g \cdot U^2{}_{rel}$$

$$\frac{dW_{pressure}}{dt} = -\frac{\pi}{4} \cdot R^2 \cdot \rho_g \cdot U_{rel}^2 \left(\frac{dy}{dt} \right)$$

DDB Model

$$\frac{dE}{dt} = -\frac{dW}{dt}$$

$$\frac{dE_{kinetic}}{dt} + \frac{dE_{potential}}{dt} = \frac{dW_{pressure}}{dt} + \Phi$$

$$\frac{2}{3}\pi \cdot R^3 \cdot \rho_l \cdot \frac{dy}{dt}\left(\frac{d^2 y}{dt^2}\right)$$

$$\frac{9\pi^3}{8}\sigma \cdot y\left[1 - 2\cdot\left(\frac{c \cdot y}{R}\right)^{-6}\right]\cdot\left(\frac{dy}{dt}\right)$$

$$-\frac{\pi}{4}\cdot R^2 \cdot \rho_g \cdot U_{rel}^2\left(\frac{dy}{dt}\right)$$

$$\frac{8}{3}\pi \cdot R^3 \cdot \mu_l \cdot \left(\frac{1}{y}\frac{dy}{dt}\right)^2$$

DDB Model
Pressure Force Term

$$\frac{2}{3}\pi \cdot R^3 \rho_l \left(\frac{d^2 y}{dt^2} \right) = \frac{\pi}{4} \cdot R^2 \cdot \rho_g \cdot U_{rel}^2 - \frac{8}{3}\pi \cdot R^3 \cdot \mu_l \cdot \left(\frac{1}{y^2} \frac{dy}{dt} \right) - \frac{9\pi^3}{8} \sigma \cdot y \left[1 - 2 \cdot \left(\frac{c \cdot y}{R} \right)^{-6} \right]$$

pressure force

$$Inertia\ Forces = ma \propto \rho L^3 \frac{dv}{ds}\frac{ds}{dt} \propto \rho L^3 V \frac{V}{L} \propto \rho V^2 L^2 \propto p_s L^2$$

DDB Model

$$\frac{2}{3}\pi \cdot R^3 \rho_l \left(\frac{d^2 y}{dt^2}\right) = \frac{\pi}{4} \cdot R^2 \cdot \rho_g \cdot U_{rel}^2 - \frac{8}{3}\pi \cdot R^3 \cdot \mu_l \cdot \left(\frac{1}{y^2}\frac{dy}{dt}\right) - \frac{9\pi^3}{8}\sigma \cdot y\left[1 - 2 \cdot \left(\frac{c \cdot y}{R}\right)^{-6}\right]$$

pressure force   viscous force   surface tension force

Non-dimensionalization of the equation:

$$y^* = \frac{y}{R} \qquad t^* = t \cdot \left(\frac{U_{rel}}{R}\right) \qquad \mathrm{Re} = \frac{\rho_g U_{rel} R}{\mu_g} \qquad We = \frac{\rho_g U^2{}_{rel} R}{\sigma} \qquad K = \frac{\rho_l}{\rho_g}$$

$$K\left(\frac{d^2 y^*}{dt^{*2}}\right) + \frac{4N}{\mathrm{Re}}\frac{1}{y^{*2}}\frac{dy^*}{dt^*} + \frac{27\pi^2}{16 \cdot We} y^*\left[1 - 2\left(c \cdot y^*\right)^{-6}\right] = \frac{3}{8}$$

Initial Value Problem

$$K\left(\frac{d^2y}{dt^2}\right)+\frac{4N}{\mathrm{Re}}\frac{1}{y^{*2}}\frac{dy}{dt}+\frac{27\pi^2}{16\cdot We}y\left[1-2(c\cdot y)^{-6}\right]=\frac{3}{8}$$

$$y(0)=\frac{4}{3\pi} \qquad \frac{dy}{dt}(0)=0$$

$$\frac{dy}{dz}=z \qquad IC: y(0)=\frac{4}{3\pi}$$

$$\frac{dz}{dt}=-\frac{4N}{K\cdot\mathrm{Re}}\frac{1}{y^2}z-\frac{27\pi^2}{16K\cdot We}\left[1-2\cdot\left(\frac{3\pi y}{4}\right)^{-6}\right]+\frac{3}{8K} \qquad IC: z(0)=0$$

Input Parameters

$$\frac{dy}{dz} = z \qquad IC: y(0) = \frac{4}{3\pi}$$

$$\frac{dz}{dt} = -\frac{4N}{K \cdot \mathrm{Re}} \frac{1}{y^2} z - \frac{27\pi^2}{16K \cdot We}\left[1 - 2 \cdot \left(\frac{3\pi y}{4}\right)^{-6}\right] + \frac{3}{8K} \qquad IC: z(0) = 0$$

Parameters for DDB Model

- Density of air; viscosity of air
- Density of water; viscosity of water
- Surface tension of water
- Diameter of the droplet
- Slip Velocity

Parameters for the Numerical Solution

- Number of first order ODE
- Initial and final value of the independent variable in the interval where the solution is evaluated
- Number of steps
- Step size
- Error Tolerance

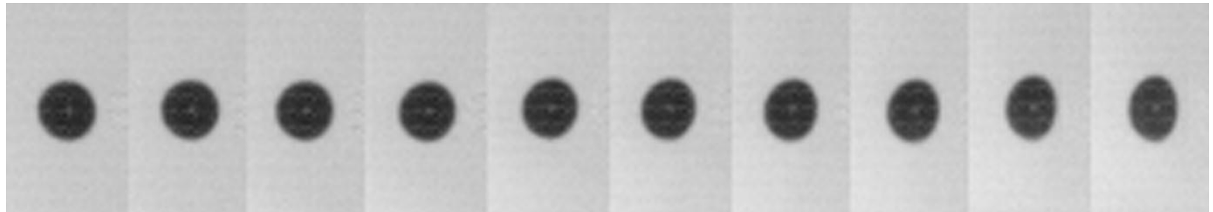
Experimental Data
Rotating Arm

Experimental Data
Data Analysis

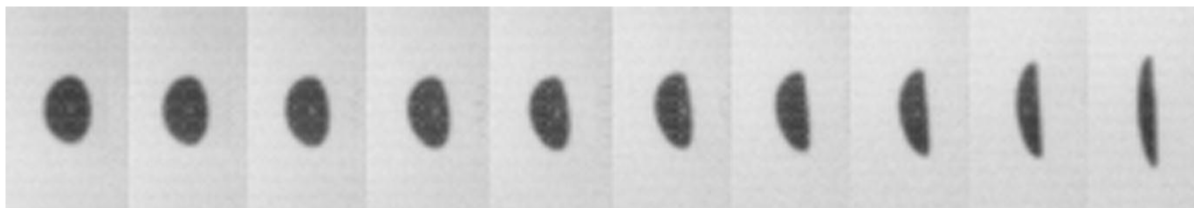
- MATLAB with Digital Imaging Processing tool box was used in the data analysis
- Droplet movement in the horizontal and vertical directions tracked frame by frame
- Program tracks width and height of droplet
  - Ellipse superimposed on the deformed droplet
  - Major and minor semi-axis of the superimposed ellipse
- Knowing the major semi-axis allows calculation of the center of mass vertical displacement for half-droplet

Experimental Data

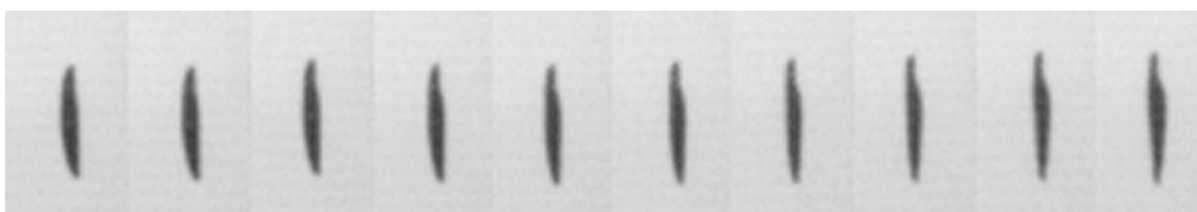
droplet radius = 516 μm, airfoil velocity = 90 m/sec, airfoil chord = 0.710 m,



Frame #	1	10	20	30	40	50	60	70	80	90
Time (μsec)	0	120	253	387	520	653	787	920	1053	1187
x-Distance (mm)	-238.6	-227.9	-215.9	-204.0	-192.1	-180.1	-168.2	-156.3	-144.3	-132.4
U_{rel} (m/sec)	14.5	15.3	16.2	17.3	18.4	19.6	20.9	22.3	23.8	25.5



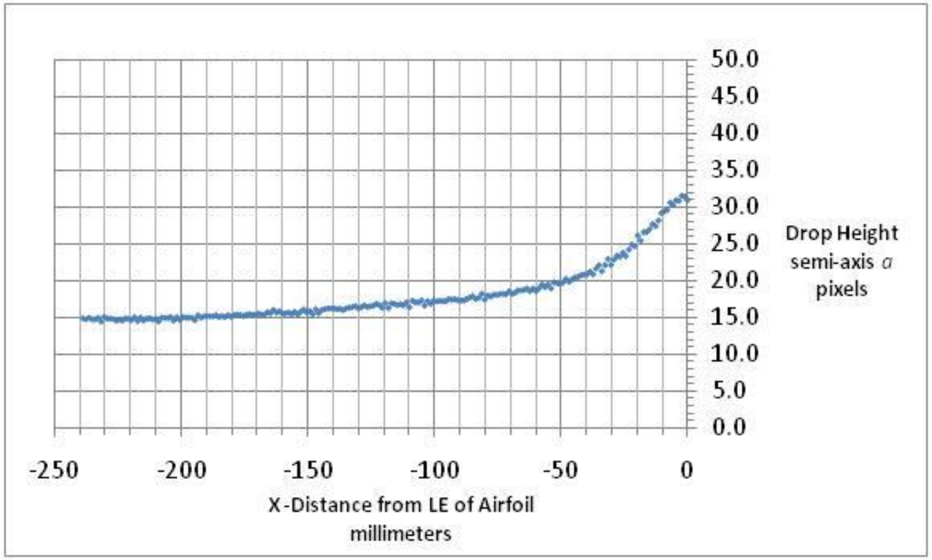
Frame #	100	110	120	130	140	150	160	170	180	190
Time (μsec)	1320	1453	1587	1720	1853	1987	2120	2253	2387	2520
x-Distance (mm)	-120.5	-108.6	-96.7	-84.8	-72.9	-61.0	-49.1	-37.3	-25.5	-13.8
U_{rel} (m/sec)	27.4	29.5	31.9	34.7	38.0	41.9	46.7	52.7	60.3	70.0



Frame #	191	192	193	194	195	196	197	198	199	200
Time (μsec)	2533	2547	2560	2573	2587	2600	2612	2627	2640	2653
x-Distance (mm)	-12.6	-11.5	-10.3	-9.2	-8.0	-6.8	-5.7	-4.5	-3.4	-2.3
U_{rel} (m/sec)	71.1	72.3	73.5	74.7	75.9	77.2	78.5	79.8	81.2	82.6

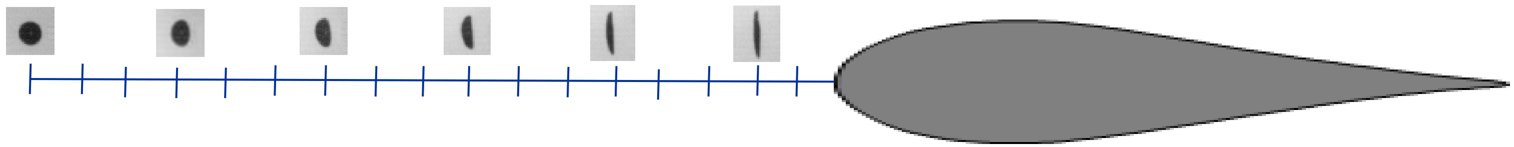
Experimental Data

droplet radius = 516 μm, airfoil velocity = 90 m/sec, airfoil chord = 0.710 m,



Approach

- Assume droplet is in quasi-steady equilibrium at each location along the trajectory
- Solve the model at each location
  - Use experimentally measured slip velocity at each location as model input
- Compare model prediction to experimental data

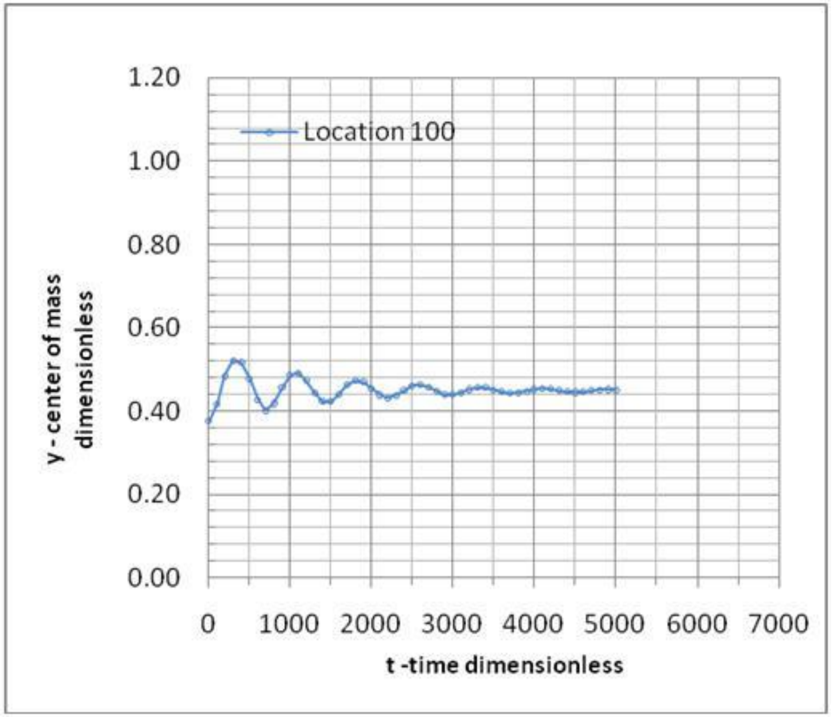


Results

Center of Mass Oscillation

location 100, distance from the leading edge of the airfoil = -120.5 millimeters

droplet radius = 516 µm, airfoil chord = 0.710 m, airfoil velocity = 90 m/sec.

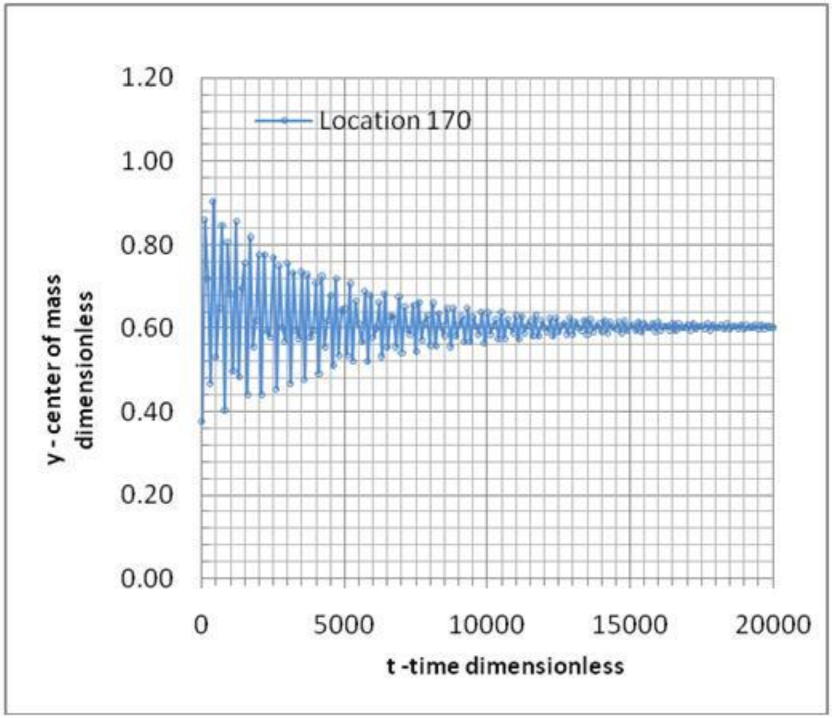


Results

Center of Mass Oscillation

location 170, distance from the leading edge of the airfoil = -37.3 millimeters

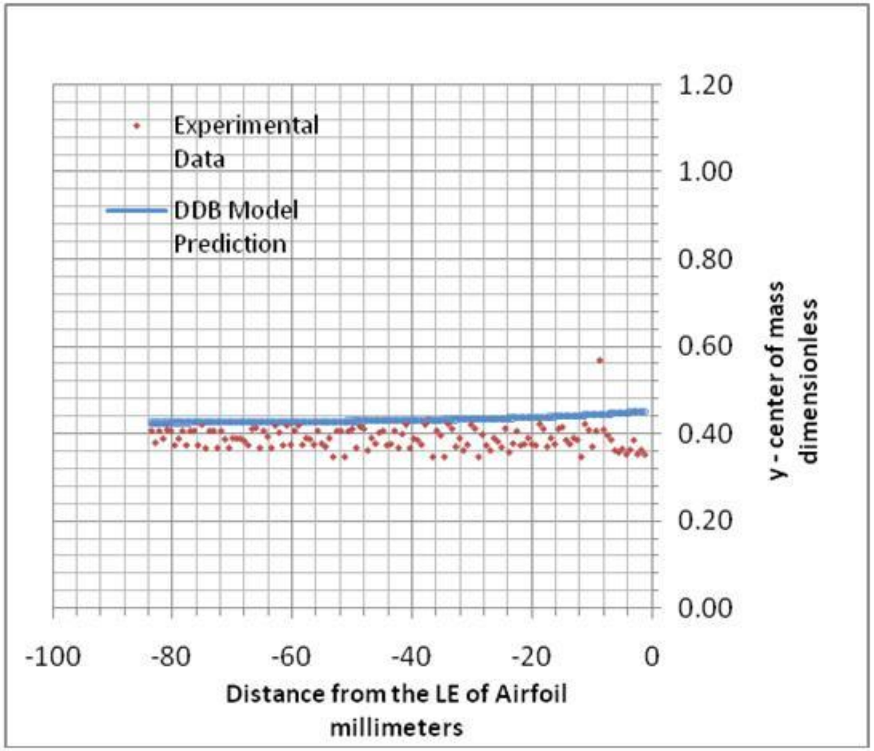
droplet radius = 516 µm, airfoil chord = 0.710 m, airfoil velocity = 90 m/sec.



Results

Model Prediction Compared to Experimental Data

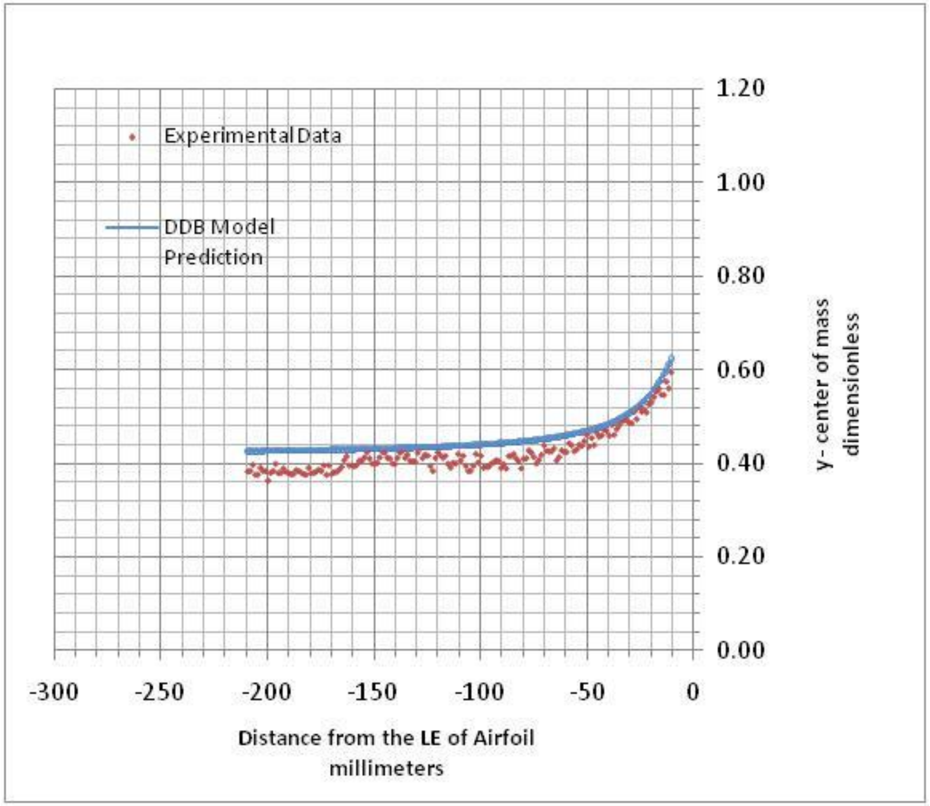
droplet radius = 199 µm, airfoil chord = 0.710 m, airfoil velocity = 50 m/sec.



Results

Model Prediction Compared to Experimental Data

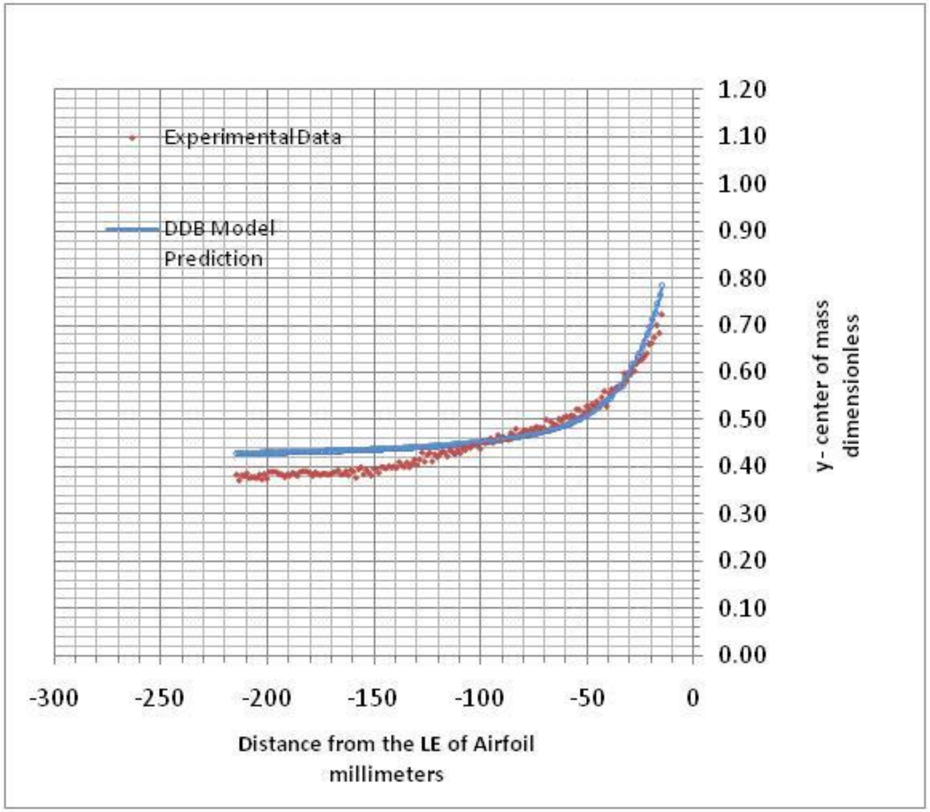
droplet radius = 287 µm, airfoil chord = 0.710 m, airfoil velocity = 90 m/sec.



Results

Model Prediction Compared to Experimental Data

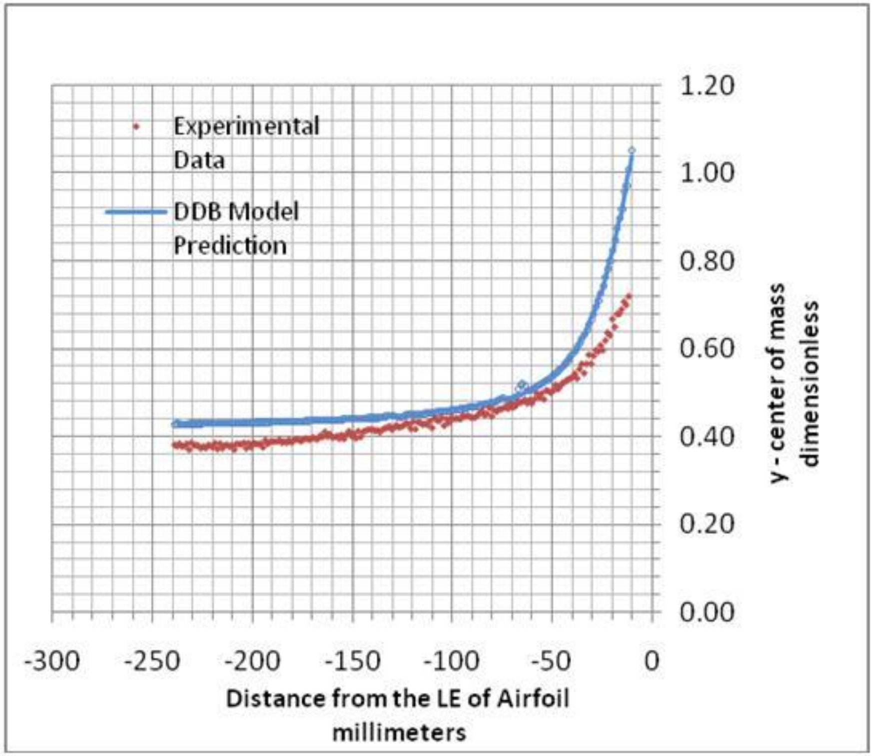
droplet radius = 439 µm, airfoil chord = 0.710 m, airfoil velocity = 90 m/sec.



Results

Model Prediction Compared to Experimental Data

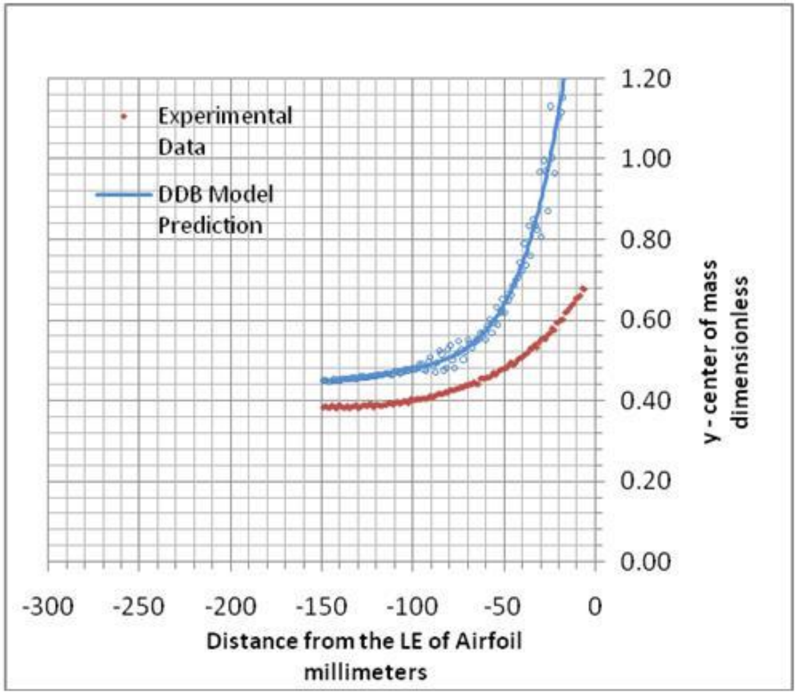
droplet radius = 516 μm, airfoil chord = 0.710 m, airfoil velocity = 90 m/sec.



Results

Model Prediction Compared to Experimental Data

droplet radius = 685 μm, airfoil chord = 0.710 m, airfoil velocity = 90 m/sec.



Conclusions

- For small and medium size droplets (radius between 200 and 500 µm) the model prediction agrees with experimental data.
- For large droplets (radius larger than 500 µm) the model over-predicts displacement of the center of mass by a large margin
- The increasing deviation between model prediction and experimental data as droplet size increases indicates that one or more model assumptions are invalid for large droplet sizes
- The quasi-steady assumption needed to apply the DDB model works well for small and medium size droplets
- The model can be used in the analysis of deformation of small and medium droplets from previous experiments

END OF PRESENTATION