Inductive Pulsed Plasma Thruster Model with Time-Evolution of Energy and State Properties

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Inductive Pulsed Plasma Thrusters

- Energy stored in capacitor banks
- High current switch(es) permit discharge through an inductive coil
- Fast-rising current ionizes/electromagnetically accelerates gas

- Pulsed Inductive Thruster (planar IPPT)
  - Since 1980 – 1-m diameter, Exclusively pulsed gas injection
  - Since 1990s – Marx generator spiral configuration, current IPPT SOA
Model / Motivation

- No self-consistent, time-dependent $T_e$
- No gas-dependent properties

Modified Model

Circuit Equations (No Change)

\[
\frac{dI_1}{dt} = \frac{V L_C + (MI_1 + I_2 L_C) (dM/dt)}{L_C (L_0 + L_C) - M^2} - I_2 M R_p - I_1 R_e L_C
\]
\[
\frac{dI_2}{dt} = \frac{M (dI_1/dt) + I_1 (dM/dt) - I_2 R_p}{L_C}
\]
\[
\frac{dV}{dt} = - \frac{I_1}{C}
\]
\[
\frac{dM}{dt} = - \frac{L_C}{2 \delta_0} \exp \left( - \frac{z}{\frac{\delta_0}{2}} \right) \frac{dz}{dt}
\]

Continuity (No Change)

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0
\]
\[
\frac{dm}{dt} = \rho_A(z) v_z, \quad \text{where} \quad \rho_A(z) = \begin{cases} 
\rho_0 \left(1 - \frac{z}{\delta_m}\right) & \\
0 & 
\end{cases}
\]

Momentum

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \mathbf{j} \times \mathbf{B}
\]
\[
\frac{dv_z}{dt} = \left[ \frac{L_C I_1^2}{2 \delta_0} \exp \left( - \frac{z}{\frac{\delta_0}{2}} \right) - \rho_A(z) v_z^2 - p_A \pi \left( b^2 - a^2 \right) \right] / m(t)
\]
Adding Energy Equation

\[
\frac{\partial \varepsilon}{\partial t} + \nabla \cdot \left[ (\varepsilon + p) \mathbf{v} - \bar{B}_M \cdot \mathbf{v} \right] = \nabla \cdot \left( \frac{-\mathbf{E}' \times \mathbf{B}}{\mu_0} \right)
\]

\[
\varepsilon = \frac{p}{\gamma - 1} + \frac{1}{2} \rho v^2 + \frac{B^2}{2\mu_0}
\]

Energy

\[
V_p = R_p I_1 + \phi(t) = R'_p I_1 + L_p \frac{dI_1}{dt} + I_1 \frac{dL_p}{dt}
\]

Voltage across plasma

\[
P = \frac{d}{dt} \left( \frac{L_p I_1^2}{2} \right) + \frac{I_1^2}{2} \frac{dL_p}{dt} + I_2^2 R_p
\]

Net electrical power into plasma

\[
\frac{dL_p}{dt} = L_c \exp \left( -\frac{z}{z_0} \right) \frac{dz}{dt}
\]

Plasma inductance evolution
Adding Energy Equation (cont)

\[ \frac{\partial \varepsilon}{\partial t} + \nabla \cdot \left[ (\varepsilon + p) \mathbf{v} - \tilde{B}_M \cdot \mathbf{v} \right] = \nabla \cdot \left( -\frac{\mathbf{E}' \times \mathbf{B}}{\mu_0} \right) \]

\[ \varepsilon = \frac{p}{\gamma - 1} + \frac{1}{2} \rho v^2 + \frac{B^2}{2 \mu_0} \]

\[ \frac{dE}{dt} = \frac{L_c I_1^2}{2z_0} \exp \left( -\frac{z}{z_0} \right) v_z + I_2^2 R_p + \frac{d}{dt} \left( \frac{L_p I_1^2}{2} \right) - \left[ \frac{1}{2} \rho_A v_z^2 + \frac{\gamma_A p_A}{\gamma_A - 1} \pi (b^2 - a^2) \right] v_z \]

\[ E = \frac{p_{cs}}{\gamma_{cs} - 1} \delta_\alpha \pi \left( b^2 - a^2 \right) + \frac{1}{2} \rho v_z^2 + \frac{L_p I_1^2}{2} \]

1. Electromagnetic work
2. Ohmic Heating
3. Rate of change of electromagnetic field energy
4. Power lost accelerating newly-entrained gas
5. Net internal power convecting into the current sheet + work performed by ambient pressure against the current sheet face
Plasma Model

Ionization (Equilibrium – Saha)

\[
\frac{n_i n_e}{n_{i-1}} = \frac{2 \left(2\pi m_e k_B T\right)^{3/2}}{h^3} \frac{\sum g_i^i \exp \left(-\epsilon_i/k_B T\right)}{\sum g_i^{-1} \exp \left(-\epsilon_i^{-1}/k_B T\right)} = K_i \quad n_e^{N+1} + \sum_{i=1}^{N} \left[ n_e^{N-i} (n_e - n_o) \prod_{i=1}^{l} K_i \right] = 0
\]

Transport (resistivity & collisionality)

\[
\eta = \frac{m_e \sum_s \nu_{es}}{n_e e^2} \quad \nu_{es} = n_s Q_{es} \sqrt{\frac{8 k_B T_e}{\pi m_e}}
\]

- $p_e=10^9$ Pa
- $p_e=10^1$ Pa
- $p_e=10^2$ Pa
- $p_e=10^3$ Pa
- $p_e=10^4$ Pa
- $p_e=10^5$ Pa
Plasma Model

Non-Ideal Equation of State

Sheet Thickness

$$\delta_a = \sqrt{\delta_s^2 + \frac{\eta}{\mu_0}} t$$

Plasma Resistance

$$R_p \approx \frac{\pi \eta (b + a)}{\delta_a (b - a)}$$
Results I

From top to bottom

- Kinetic and thermal energy increase early and level off (decoupling)
- Magnetic field energy oscillates, returning to zero at the end of the first half-cycle
- Shorter half-cycle in the poorer-matched case
- Energy shifts from kinetic to thermal/internal mode as mass bit increases
Results II

- Temperature, plasma density, and pressure
  - Increase early (Ohmic heating, multiple ionization, entrainment of gas)
- Temperature & plasma density
  - Stabilize in late-time (decoupled)
- Pressure
  - Decreases in late-time (sheet expansion)
- Current decreases in peak value, increases in ‘half-cycle’ duration
  - Asymmetric nature consistent with greater electromagnetic work
Results III

- $I_{sp}$ – good quantitative agreement
- $\eta_t$ – good agreement in form, magnitude difference suspected due to modeling assumptions

- New model
  - Shifts optimum $\alpha$, lowers max achievable $\eta_t$ relative to no E-eqn / plasma model simulation
Conclusions

• Efficiency has a maximum value as a function of $\alpha$, consistent with previous, more simplified IPPT modeling

• For same $E_0$, higher efficiency achieved when the peak coil current was lower and more asymmetric, leading to significantly more energy being deposited in directed kinetic energy

• For argon, the plasma properties vary early in the discharge. As the sheet decouples, density and temperature reach approximately constant values while the pressure decreases through thermal sheet expansion

• Qualitatively and quantitatively the results from the model generally compare favorably with performance measurements. Disagreements can be attributed to the simplifying assumptions
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