Inductive Pulsed Plasma Thruster Model with Time-Evolution of Energy and State Properties

19th Advanced Space Propulsion Workshop

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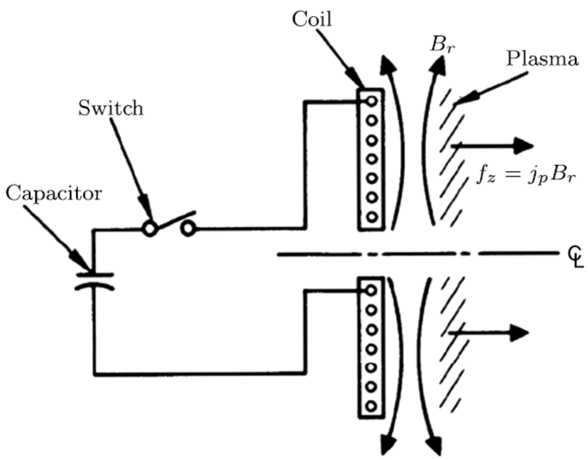
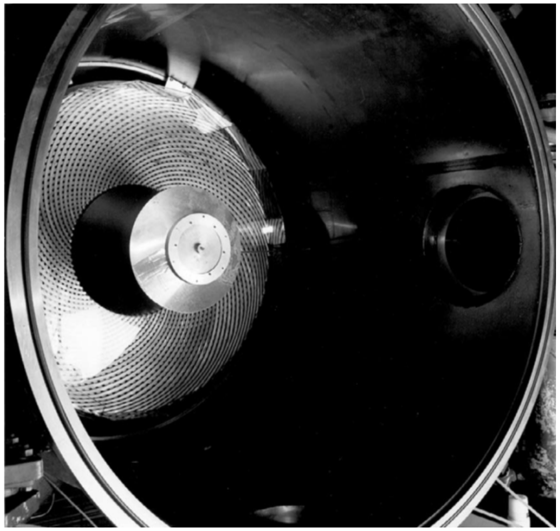
Inductive Pulsed Plasma Thrusters

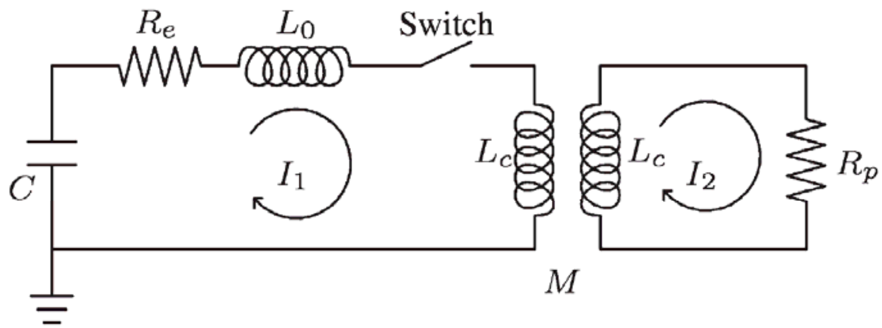
- Energy stored in capacitor banks
- High current switch(es) permit discharge through an inductive coil
- Fast-rising current ionizes/electromagnetically accelerates gas

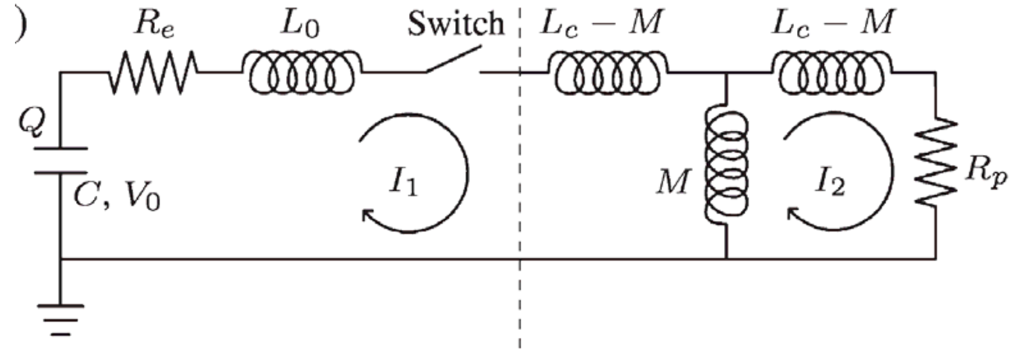




- Pulsed Inductive Thruster (planar IPPT)
  - Since 1980 – 1-m diameter, Exclusively pulsed gas injection
  - Since 1990s – Marx generator spiral configuration, current IPPT SOA

Model / Motivation





$$\frac{dI_1}{dt} = \frac{VL_C + (MI_1 + I_2L_C)\frac{dM}{dt} - I_2MR_p - I_1R_eL_C}{L_C(L_0 + L_C) - M^2}$$

$$\frac{dI_2}{dt} = \frac{M\frac{dI_1}{dt} + I_1\frac{dM}{dt} - I_2R_p}{L_C}$$

$$\frac{dV}{dt} = -\frac{I_1}{C}$$

$$\frac{dM}{dt} = -\frac{L_C}{2z_0}\exp\left(-\frac{z}{2z_0}\right)\frac{dz}{dt}$$

$$m(t) = m_0 + \int_{t=0}^{t} \rho_A v_z dt$$

$$\frac{L_C I_1^2}{2z_0}\exp(-z/z_0) = \rho_A v_z^2 + m(t)\frac{dv_z}{dt}$$

$$\eta = 6 \times 10^{-4} T_e^{-3/2} \text{ ohm-m} \qquad R_p \simeq \frac{\pi\,\eta\,(b+a)}{\delta_a(b-a)}$$

$$\delta_a(t) = \sqrt{\frac{\eta}{\mu_0}(t + t_0)} \qquad t_0 = \frac{\mu_0 \delta_s^2}{\eta}$$

- No self-consistent, time-dependent T_e
- No gas-dependent properties

Lovberg & Dailey, PIT Primer, 1994.

Modified Model

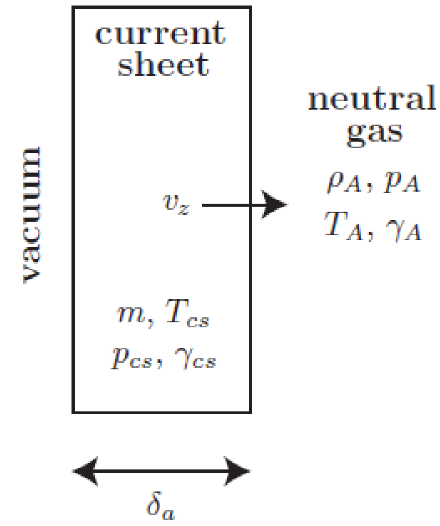
Circuit Equations (No Change)

$$\frac{dI_1}{dt} = \frac{V\,L_C + (MI_1 + I_2L_C)\,(dM/dt) - I_2MR_p - I_1R_eL_C}{L_C\,(L_0 + L_C) - M^2}$$

$$\frac{dI_2}{dt} = \frac{M\;(dI_1/dt) + I_1\;(dM/dt) - I_2R_p}{L_C},$$

$$\frac{dV}{dt} = -\frac{I_1}{C}$$

$$\frac{dM}{dt} = -\frac{L_C}{2z_0}\exp\left(-\frac{z}{2z_0}\right)\frac{dz}{dt}$$



Continuity (No Change)

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0 \;\longrightarrow\; \frac{dm}{dt} = \rho_A\,(z)\,v_z, \quad \text{where } \rho_A\,(z) = \begin{cases} \rho_0\,(1 - z/\delta_m) \\ 0 \end{cases}$$

Momentum

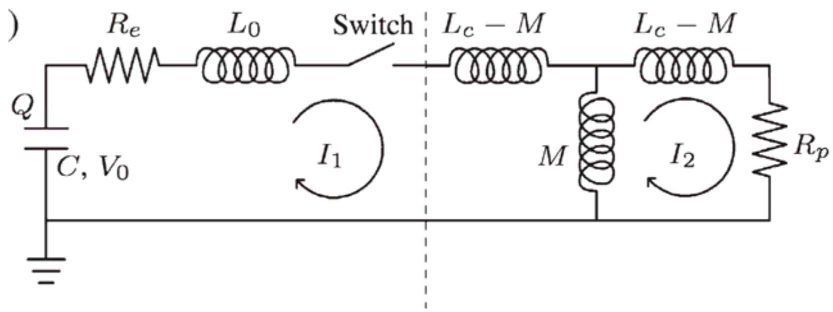
$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \mathbf{j}\times\mathbf{B}$$

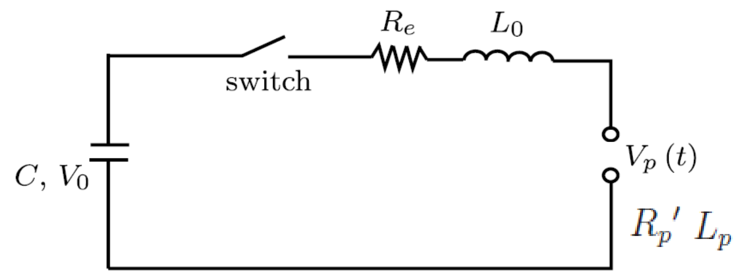
$$\longrightarrow\; \frac{dv_z}{dt} = \left[\frac{L_cI_1{}^2}{2z_0}\exp\left(-\frac{z}{z_0}\right) - \rho_A\,(z)\,v_z{}^2 - p_A\pi\,(b^2 - a^2)\right]/m\,(t)$$

Adding Energy Equation

Energy

$$\frac{\partial \varepsilon}{\partial t} + \nabla \cdot \left[(\varepsilon + p)\,\mathbf{v} - \bar{\bar{B}}_M \cdot \mathbf{v} \right] = \nabla \cdot \left(\frac{-\mathbf{E}' \times \mathbf{B}}{\mu_0} \right) \qquad \varepsilon = \frac{p}{\gamma - 1} + \frac{1}{2}\rho v^2 + \frac{B^2}{2\mu_0}$$





$$V_p = R_p I_1 + \dot{\phi}(t) = R_p{}' I_1 + L_p \frac{dI_1}{dt} + I_1 \frac{dL_p}{dt}$$

Voltage across plasma

$$P = \frac{d}{dt}\left(\frac{L_p I_1{}^2}{2}\right) + \frac{I_1{}^2}{2}\frac{dL_p}{dt} + I_2{}^2 R_p$$

Net electrical power into plasma

$$\frac{dL_p}{dt} = \frac{L_c}{z_0} \exp\left(-\frac{z}{z_0}\right) \frac{dz}{dt}$$

Plasma inductance evolution

Adding Energy Equation (cont)

Energy

$$\frac{\partial \varepsilon}{\partial t} + \nabla \cdot \left[(\varepsilon + p)\,\mathbf{v} - \bar{\bar{B}}_M \cdot \mathbf{v} \right] = \nabla \cdot \left(\frac{-\mathbf{E}' \times \mathbf{B}}{\mu_0} \right) \qquad \varepsilon = \frac{p}{\gamma - 1} + \frac{1}{2}\rho v^2 + \frac{B^2}{2\mu_0}$$

(1) (2) (3) (4) (5)

$$\frac{dE}{dt} = \frac{L_c {I_1}^2}{2 z_0} \exp\left(-\frac{z}{z_0} \right) v_z + I_2^2 R_p + \frac{d}{dt}\left(\frac{L_p {I_1}^2}{2} \right) - \left[\frac{1}{2}\rho_A {v_z}^2 + \frac{\gamma_A p_A}{\gamma_A - 1}\pi \left(b^2 - a^2 \right) \right] v_z$$

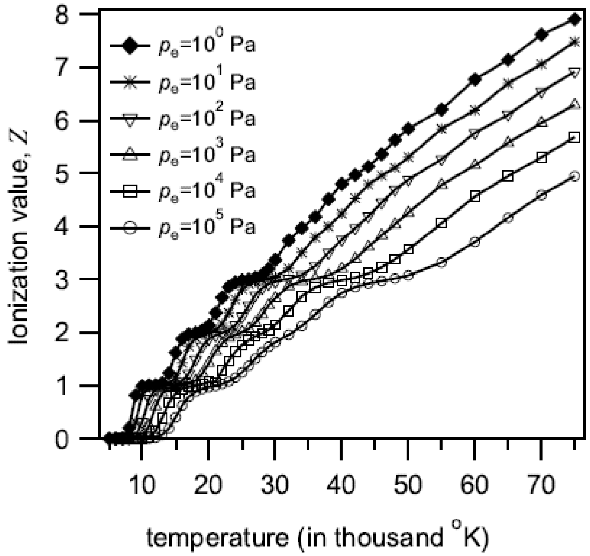
$$E = \frac{p_{cs}}{\gamma_{cs} - 1}\delta_a \pi \left(b^2 - a^2 \right) + \frac{1}{2} m {v_z}^2 + \frac{L_p {I_1}^2}{2}$$

1. Electromagnetic work
2. Ohmic Heating
3. Rate of change of electromagnetic field energy
4. Power lost accelerating newly-entrained gas
5. Net internal power convecting into the current sheet +
   work performed by ambient pressure against the current sheet face

Plasma Model

Ionization (Equilibrium – Saha)

$$\frac{n_i n_e}{n_{i-1}} = \frac{2\,(2\pi m_e k_B T)^{3/2}}{h^3}\,\frac{\sum_l g_l^i\ \exp\left(-\epsilon_l^i/k_B T\right)}{\sum_l g_l^{i-1}\ \exp\left(-\epsilon_l^{i-1}/k_B T\right)} = K_i \qquad n_e^{N+1} + \sum_{l=1}^{N}\left[n_e^{N-l}\left(n_e - l n_\circ\right)\prod_{i=1}^{l} K_i\right] = 0$$

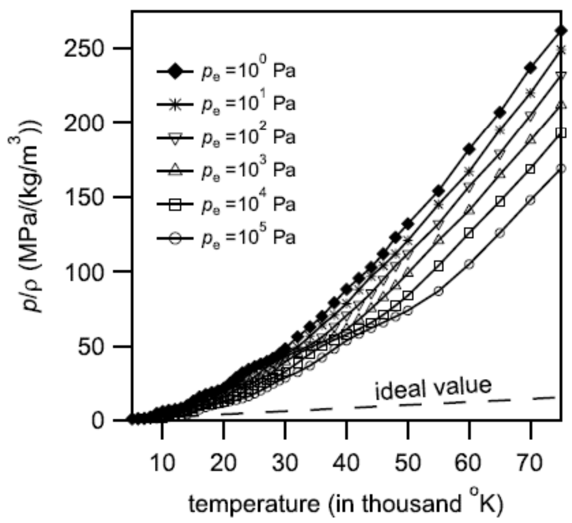


Transport (resistivity & collisionality)

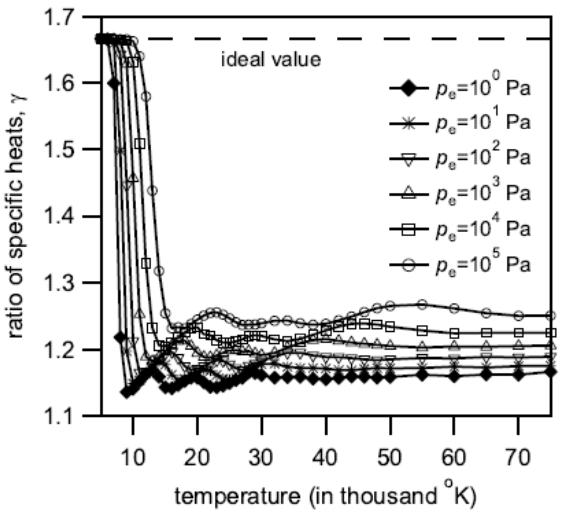
$$\eta = \frac{m_e \sum_s \nu_{es}}{n_e e^2} \qquad \nu_{es} = n_s Q_{es}\sqrt{\frac{8 k_B T_e}{\pi m_e}}$$

Plasma Model

Non-Ideal Equation of State



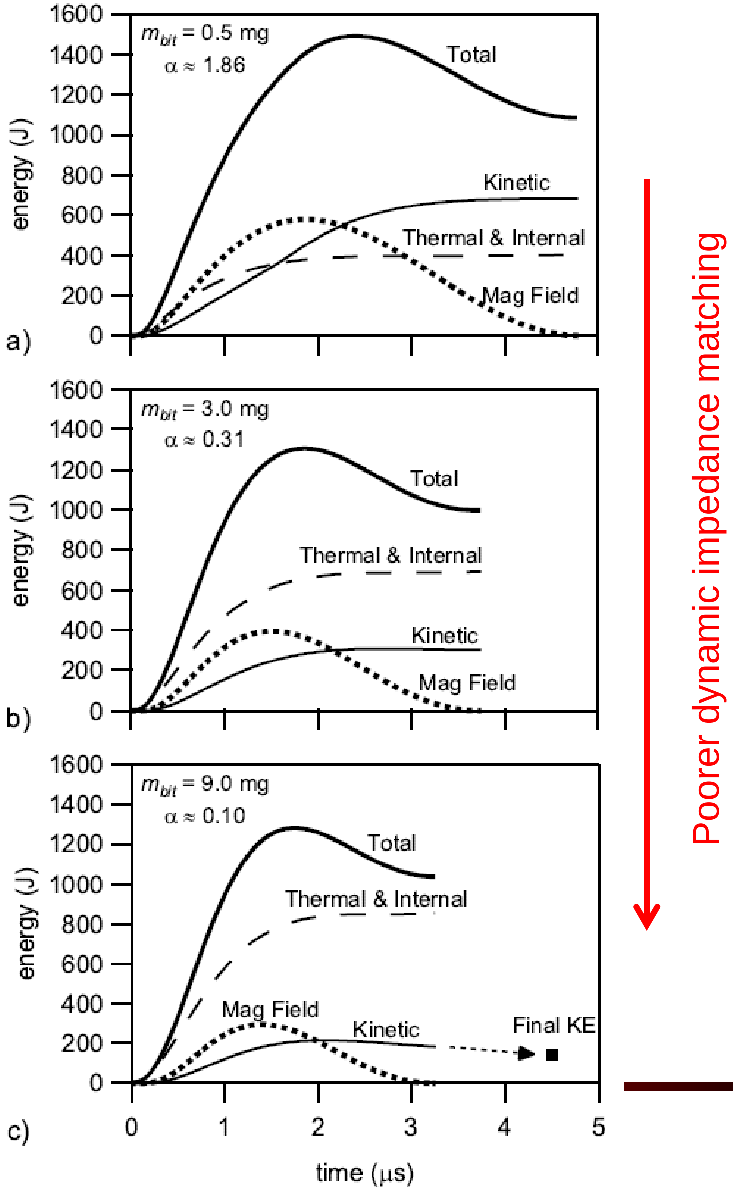
Specific Heat Ratio



$$\delta_a = \sqrt{\delta_s^2 + \frac{\eta}{\mu_\circ} t}$$ Sheet Thickness

$$R_p \approx \frac{\pi\eta(b+a)}{\delta_a(b-a)}$$ Plasma Resistance

Results I

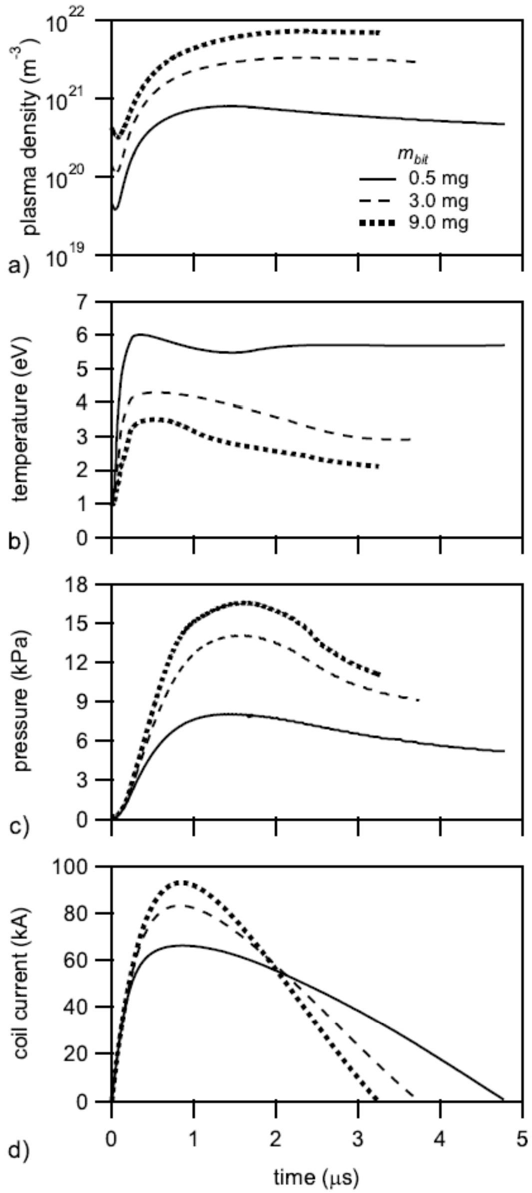


$$\alpha = \frac{C^2 {V_0}^2 L_C}{2 m_{\text{bit}} {z_0}^2}$$

From top to bottom

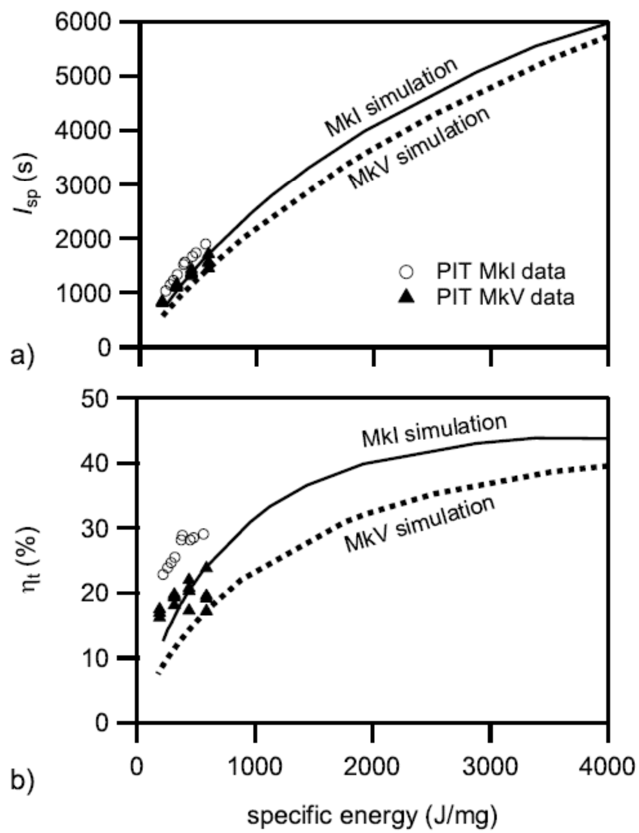
- Kinetic and thermal energy increase early and level off (decoupling)
- Magnetic field energy oscillates, returning to zero at the end of the first half-cycle
- Shorter half-cycle in the poorer-matched case
- Energy shifts from kinetic to thermal/internal mode as mass bit increases

Results II

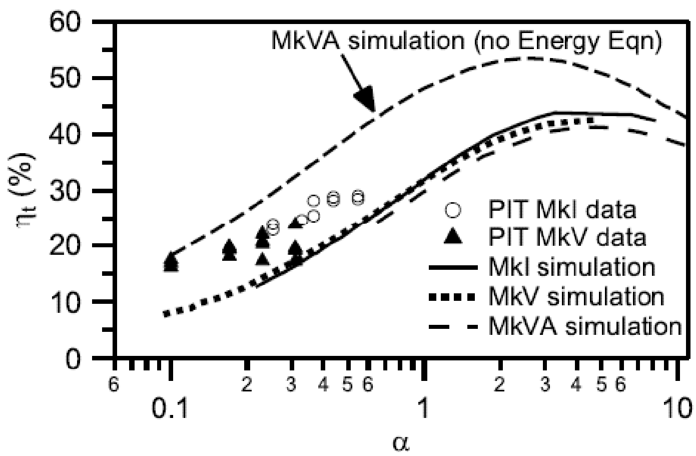


- Temperature, plasma density, and pressure
  - Increase early (Ohmic heating, multiple ionization, entrainment of gas
- Temperature & plasma density
  - Stabilize in late-time (decoupled)
- Pressure
  - Decreases in late-time (sheet expansion)
- Current decreases in peak value, increases in ‘half-cycle’ duration
  - Asymmetric nature consistent with greater electromagnetic work

Results III



$$\alpha = \frac{C^2 {V_0}^2 L_C}{2 m_{\text{bit}} {z_0}^2}$$



- I_{sp} – good quantitative agreement
- η_t – good agreement in form, magnitude difference suspected due to modeling assumptions

- New model
  - Shifts optimum α, lowers max achievable η_t relative to no E-eqn / plasma model simulation

Conclusions

- Efficiency has a maximum value as a function of α, consistent with previous, more simplified IPPT modeling
- For same E_0, higher efficiency achieved when the peak coil current was lower and more asymmetric, leading to significantly more energy being deposited in directed kinetic energy
- For argon, the plasma properties vary early in the discharge. As the sheet decouples, density and temperature reach approximately constant values while the pressure decreases through thermal sheet expansion
- Qualitatively and quantitatively the results from the model generally compare favorably with performance measurements. Disagreements can be attributed to the simplifying assumptions

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