

Deleterious Thermal Effects Due To Randomized Flow Paths in Pebble Bed, and Particle Bed Style Reactors.
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ABSTRACT:

A review of literature associated with Pebble Bed and Particle Bed reactor core research has revealed a systemic problem inherent to reactor core concepts which utilize randomized rather than structured coolant channel flow paths. For both the Pebble Bed and Particle Bed Reactor designs; case studies reveal that for indeterminate reasons, regions within the core would suffer from excessive heating leading to thermal runaway and localized fuel melting. A thermal Computational Fluid Dynamics model was utilized to verify that In both the Pebble Bed and Particle Bed Reactor concepts randomized coolant channel pathways combined with localized high temperature regions would work together to resist the flow of coolant diverting it away from where it is needed the most to cooler less resistive pathways where it is needed the least. In other words given the choice via randomized coolant pathways the reactor coolant will take the path of least resistance, and hot zones offer the highest resistance. Having identified the relationship between randomized coolant channel pathways and localized fuel melting it is now safe to assume that other reactor concepts that utilize randomized coolant pathways such as the foam core reactor are also susceptible to this phenomenon.

I. Introduction:

Few rocket propulsion concepts offer the combination of high thrust and reasonable efficiency that can be obtained from a Nuclear Thermal Rocket (NTR). Long considered one of the most basic forms of Nuclear Propulsion, the solid-core nuclear thermal rocket engine concept typically employs a uranium fueled nuclear reactor core and hydrogen (H_2) gas working fluid. The H_2 gas acts first as fuel element coolant as it passes through the nuclear reactor core followed by rocket working fluid when the then super heated hydrogen is expanded out of a nozzle in order to produce thrust.

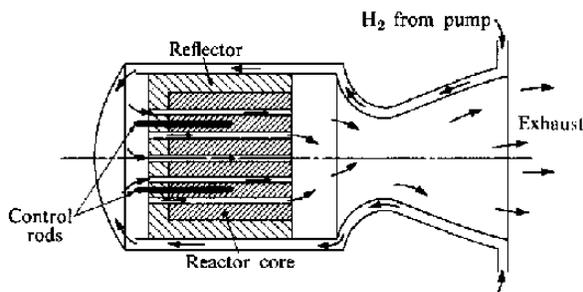


Fig 1. Solid-Core Nuclear Thermal Rocket Engine⁽¹⁾

Famously the Nuclear Engine for Rocket Vehicle Application (NERVA) test program of the 1960's extensively demonstrated the feasibility of the nuclear thermal rocket design concept testing nearly 30 nuclear thermal rocket engines such as the KIWI-B4 which produced approximately 75,000 pounds of thrust with an efficiency (isp) of 825 seconds. While the engineering advancements achieved by first the NERVA and latter the ROVER programs is still to this day particularly impressive, the fundamental reactor core concept investigated during those programs suffered from significant mass and volume shortcomings. This comes as no surprise when one considers that the NTR concept expects to operate a thermal nuclear reactor at pressures as high as 1000 psi or greater and temperatures as high as 4000 °F or greater. Frankly the NERVA/ROVER designs were very large and very heavy, and that does not lend itself well to the typically low mass requirements, and limited available space of today's space launch vehicles. In order to support the future design and development of a potentially more lightweight, and compact nuclear thermal rocket engine a review of the, Pebble Bed, Particle Bed, and Foam Core reactor concepts was performed because of various positive traits that each concept offers in addressing these NTR requirements. The function of the review was to identify both the strong and weak points of each fuel element concept, hoping to glean the ideal fuel rod characteristics for optimum thermal heat transfer to the coolant/working fluid while still maintaining structural integrity. I.E. a large fuel rod surface area ideally shaped to promote heat transfer to the hydrogen working fluid while simultaneously avoiding structural degradation due to the high operating pressures and temperatures that it would be exposed to. A brief overview of each fuel element concept follows.

Expected Advantages of the Pebble Bed Reactor:

Pebble Bed Reactors are helium cooled graphite moderated high temperature reactors ⁽²⁾. The pebble bed reactors use thousands of marble to softball sized ceramic coated uranium fuel pebbles⁽³⁾;

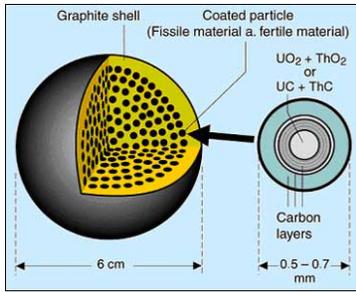


Fig 2. Typical Pebble Cross Sectional View

that when stacked like cannon balls have significant volume between them⁽⁴⁾; massively increasing the total volume requirements of this type of core design.⁽³⁾



Fig 3. Pebble Stacking

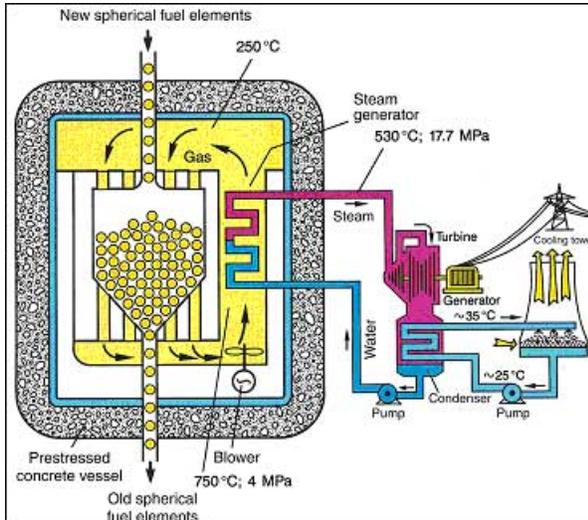


Fig 4. Pebble Bed Thermal Electric Reactor

The sheer size and encapsulating style of the pebbles can also lead to structural issues when built up fission product gases are trapped within the pebble by the outer coating layers, forcing the uranium fuel to migrate away from the center of the pebble. Migration is a key problem in certain types of pebbles, placing undue stress on the outer coating layers, and possibly affecting localized reactivity levels leading to thermal

instabilities. The pebble bed reactor has largely been relegated to the production of electricity due primarily to its sizable volume requirements. However the design has demonstrated reactivity stability significantly superior to traditional close packed hexagonal designs; pebble bed reactors essentially cannot melt down because any liquefied uranium would be safely contained inside the pebble. This is certainly a trait that high temperature NTR's could benefit from.

A 5.1×10^7 BTU/hr (15 MW) electric, Helium cooled pebble bed plant was successfully demonstrated in Germany⁽⁵⁾; operating for 21 years. The plant demonstrated helium exit temperatures of 1700 degrees Fahrenheit and while this is not the highly desirable 5000 degrees Fahrenheit or greater that NTR's would like to achieve it is still extremely good when compared to most other typical land based nuclear power plants. However the German plant did also suffer from temperature instabilities which at one time resulted in localized thermal runaway that heavily contaminated the pressure vessel with Cs-137 and Sr-90.

The Chinese HTR-10⁽⁶⁾, a 3.4×10^7 BTU/hr (10 MW) thermal, pebble bed reactor test facility is currently the only operating pebble bed reactor worldwide. The Chinese are planning to manufacture 6.8×10^8 BTU/hr (200 MW) electric modular pebble bed reactors based on a scaled up HTR-10 design.

A Similar South African⁽⁷⁾ 5.6×10^8 BTU/hr (165 MW) electric modular pebble bed reactor design effort which cost 9.244 billion Rand over a ten year period was cancelled in 2010. This may be an indicator that the thermal instabilities first encountered in the German plant are a larger issue than originally anticipated as further indicated by delays in the cash rich Chinese effort. The resulting radiation that the German plant suffered due to thermal instabilities has resulted in a cleanup effort that is currently planned to last beyond the year 2070 at a cost that will massively exceed the original cost of manufacturing the plant. This is probably not a desirable characteristic for a modular reactor designed for developing nations. The combination of reactive stability and high operating temperature may make the pebble bed reactor a sound starting point for a nuclear thermal rocket, but should be weighed against the designs sizeable volume, and therefore associated mass requirements due to the reactor core. In addition the designs revealed thermal instabilities which initially looked overcome-able but have now gone on to plague it through several decades and major design efforts should not be ignored.

Expected Advantages of the Particle Bed Reactor:

While Nuclear Thermal Rockets (NTR) generally offer a far superior efficiency (isp) when compared to chemical rockets much of that gain is then lost due to their poor thrust to weight ratios (T/W), making many solar system missions where time of travel is not a factor untenable. For example with the total mass and volume determined by the launch vehicle many robotic missions will trade a couple of extra years of travel time by sacrificing the high mass (reactor core weight) and large volume requirements (due to H₂ propellant) of a typical NTR in order to carry far more scientific equipment at a slower speed to the desired objective. Fortunately Particle Bed Reactors developed in the late 1980's and early 90's offer a sizable improvement in reactor core mass. The Particle Bed Reactors (PBR) use 400 micron sized particles that when stacked have little to no volume between them; massively decreasing the volume requirements of this type of core design. By far the greatest expected advantage of a particle bed reactor engine is its estimated thrust to weight ratio of 35:1. This is far superior to the typical NERVA program Nuclear Thermal Propulsion (NTP) system which averaged a thrust to weight ratio of about 4:1. However these results should be contrasted against modern chemical liquid oxygen- liquid hydrogen (LOX/LH₂) engines which routinely achieve 50:1 and higher, for example the Space Shuttle Main Engine T/W is about 65:1 in vacuum. The modular design of the close packed hexagonal fuel element style core allows for variable thrust dependant on the number of fuel elements incorporated into the reactor core design. For example a manned Mars mission study involving particle bed fuel elements proposed engine thrusts of either 25 k lbf, 40 k lbf, or 75 k lbf, each achieved via an identical geometry of individual fuel elements, but with a differing number of them utilized. An estimated efficiency of 1000 sec isp was also provided in that study. This increased expected efficiency over NERVA style reactor cores is most likely due to the unique ceramic coated particle stacking which offers a large surface area for heat transfer to the core cooling/working fluid ⁽⁸⁾⁽⁹⁾⁽¹⁰⁾.

Particle Bed Reactor Underlying Theory: The Particle Bed technology was first developed at Brookhaven National Laboratory, located in Upton, New York. The technology was at one time classified as a special access only program named Timber Wind. If budget is any indicator of the level of development, the Timber Wind program received about \$139 million between the years 1987 and 1991. Some Timber Wind program documentation was made available to the public after the termination of the programs special access only status ⁽¹¹⁾.

Particle Bed reactors achieve their high thrust to weight ratios due to their unique compactness of the reactor core. This is accomplished using 400 micron sized uranium fuel particles which easily stack together with little volume between them and yet still allow for the flow of hydrogen coolant past each particle, thus creating an incredibly large total surface area for heat transfer to the hydrogen coolant in a very small volume. Power densities as high as 1 GW per cubic foot were estimated, resulting in an overall reactor core about the size of a small filing cabinet.

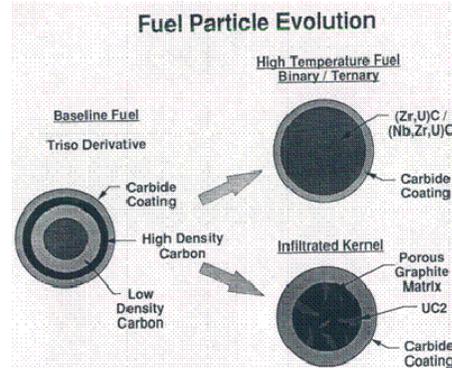


Fig 5. Typical Particle Cross Sectional View

The nuclear fuel rod and reactor core system is comprised of millions of fuel particles contained inside a pair of concentric tubes named "Frit's". The outer "Cold Frit" was made of a porous aluminum material, while the inner "Hot Frit" was made of a porous tapered cylinder comprised of either a carbide-coated carbon-carbon or graphite material. Each frit varied in porosity longitudinally, increasing from top to bottom. The frit tubes were positioned inside a hexagonal block of neutron moderating material, capped at the top and bottom with beryllium-alumina end caps which both completed the particle bed enclosure, and assisted with fuel rod assembly within the reactor core.

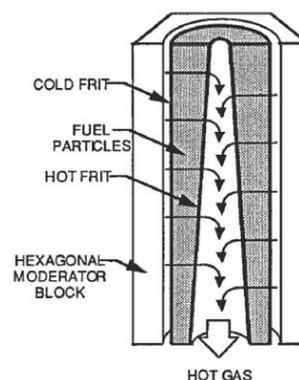


Fig 6. Particle Stacking

Hydrogen gas would first act as the reactor core coolant traveling down passages between the moderator and the cold frit, and through the fuel particles picking up heat as it went along. The hydrogen would then travel down the hot gas path located along the center of the fuel rod where it would combine with similar H₂ gas from the other fuel rods in the engine plenum chamber. The then superheated hydrogen gas would be expanded out of a nozzle as the working fluid producing rocket engine thrust.

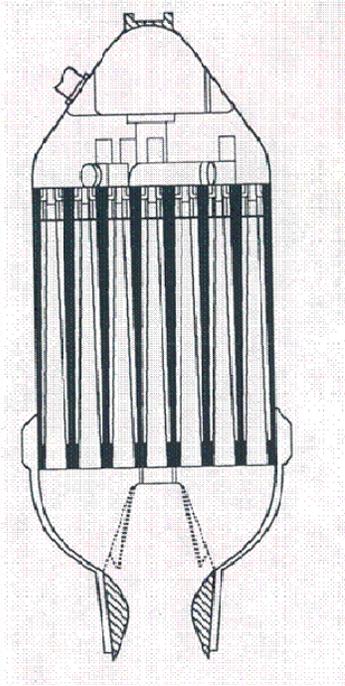


Fig 7. Particle Bed Thermal Nuclear Reactor Propulsion Concept.

The limited documentation made available from the Timber Wind Program reveals that the very public issues encountered by the Pebble Bed design were exacerbated by the smaller particle size of the particle bed reactor. For example it appears that testing here again revealed localized high temperature regions that failed to receive an adequate flow of hydrogen coolant. These marble sized regions would then suffer from excessive heating, subsequent thermal runaway and localized fuel melting. The small particle size also created additional design issues, for example reactor operation would result in permanent deformation i.e. bowing of the specially manufactured porous material frits due to thermal expansion leading to settling of the fuel particles. Fuel settling would then affect both coolant flow and fission reactivity. Fuel particles were also known to block the porous passages of the frit inhibiting the flow of hydrogen coolant. The consen-

sus of internet speculation is that the Particle Bed design was initially conceived for an extremely short duration (with approximately a 2 min runtime), high power need i.e. a throwaway reactor type mission and that the significant shortcomings of the design would simply be overcome by the limited run time, throwaway nature of its use^{(12) (13) (14)}.

Expected Advantages of the Foam Fuel Element Reactor:

The Foam Fuel Element Reactor (FFE) (or Porous Nuclear Fuel Element Reactor) attempts to overcome some of the negative aspects of the particle bed reactor while still capitalizing on its phenomenal thrust to weight ratio (T/W) which can be as high as 35 to 1. Recall that the Timber Wind program revealed that particle bed fuel elements are prone to permanent deformation of the particle (frit) container and that the loose fuel particles were known to block the porous passages of the frit inhibiting the flow of hydrogen coolant. The Foam Fuel element eliminates both of these issues by depositing Uranium based bi-carbide, tri-carbide or carbonitride fuels onto porous carbon foam. This concept offers an expected efficiency (isp) of 925 seconds. In theory a foam with the correct combination of porosity and structural integrity should retain both the phenomenal surface area to volume ratio first realized by the Particle bed concept, and yet would neither clog up the frit with loose particles nor deform.

Foam Fuel Element Reactor Underlying Theory:

The Foam Fuel Element design patent was applied for in June of 2004, and was awarded on March of 2011 to the Sandia Corporation, located in Albuquerque, New Mexico. Patent inventors are listed as Dennis L. Youchison, Brian E. Williams, and Robert E. Benander⁽¹⁵⁾.

Foam Fuel Element reactors achieve their high thrust to weight ratios due to their unique compactness of the reactor core. This is accomplished using a highly porous foam made of for example UC, ZrC, and NbC. The foam fuel element is manufactured via a Chemical Vapor Deposition (CVD)/ Chemical Vapor Infiltration (CVI) process of refractory metal carbides onto reticulated vitreous carbon foam that has a porosity of as high as 90%. Coolant flow could also easily be controlled by varying the porosity of the foam in multiple layers. The CVD/CVI process has been utilized for the manufacture of materials that are used throughout industry so a host of material options have been demonstrated such as Zr, Nb, Mo, Hf, Ta,

W, Re, TiC, TaC, ZrC, SiC, HfC, BeC₂, B₄C, GdC, HfB₂, ZrB₂, SiN₄, TiO₂, BeO, SiO₂, ZrO₂, HfO₂, Y₂O₃, Al₂O₃, Sc₂O₃, and Ta₂O₅.⁽¹⁵⁾

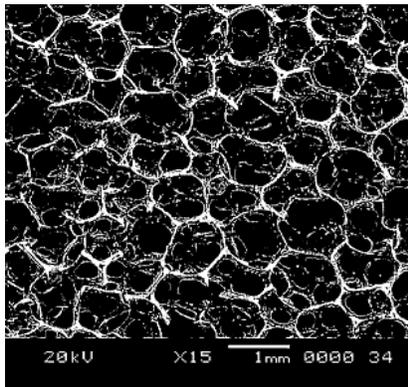


Fig 8. Foam Cross Section

The structural integrity of the vitreous foam lends itself to either traditional or particle bed style fuel elements as the excerpt from the original patent application demonstrates as follows⁽¹⁵⁾.

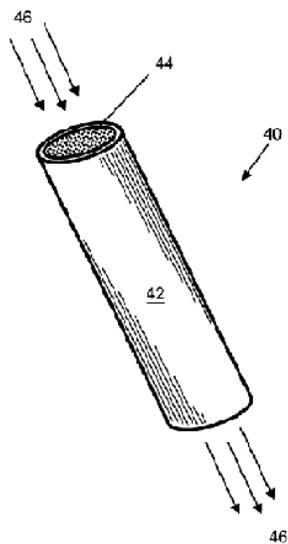


FIG. illustrates a schematic isometric view of an example of a nuclear fuel element 40 comprising porous nuclear fuel 44 encased within metal cladding 42, according to the present invention. Gas coolant 46 flows through the gas-permeable, porous nuclear fuel 44, exchanging heat with a high heat transfer efficiency from the high-porosity nuclear fuel, due to the large extended surface area of the porous fuel, and at a high temperature due to the thinness of the nuclear fuel itself.

Fig 9. Potential Foam Fuel Configuration

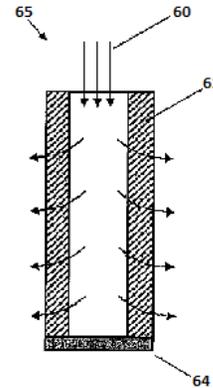


FIG. illustrates a schematic cross-section view of an example of an annular porous nuclear fuel element 60 in the shape of a thick-walled cylinder 62, having a closed end 64, where gas cooling 65 enters through the open central core and exits by flowing radially outwards through the porous fuel 62.

Fig 10. Potential Foam Fuel Configuration

A typical reactor core achieves its maximum temperature only at its center but this design allows for increased Uranium enriched fuel loading at the top and bottom of the core in order to achieve maximum temperatures throughout the bulk of the core. Of course other unique power distributions could also easily be created with this concept. This fact combined with the ability to control the level of porosity throughout the stack may provide a solution to thermal instabilities encountered by other similar concepts.

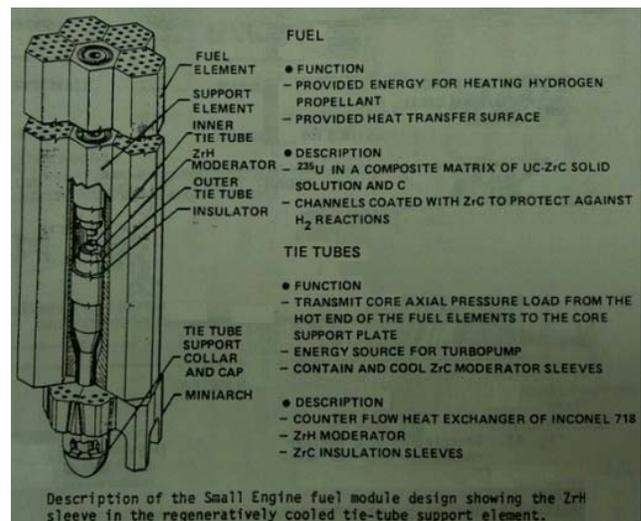


Fig 11. The Small Engine

While the Foam Core concept is clearly an improvement on the particle bed design it remains to be seen via extensive testing just how much of an improvement.

II RESULTS AND DISCUSSION:

It was during the review of these fuel element concepts that a deleterious thermal effect systemic to each of these designs was identified. First it was determined that a small localized hot spot could first occur for any of a host of reasons unrelated to coolant flow, such as from an outright design flaw, fuel migration within the pellet/particles affecting fission reactivity or even small instabilities that are inherent to a large scale fission reaction. Then it was hypothesized that randomized coolant channel pathways combined with localized high temperature regions would work together to resist the flow of coolant diverting it away from where it is needed the most to cooler less resistive pathways where it is needed the least. In other words given the choice via many optional flow paths the reactor coolant will take the path of least resistance, and hot zones offer the highest resistance. Small regions that did not receive adequate coolant would then suffer from excessive heating, which would further compound the problem resulting in subsequent thermal runaway and localized fuel melting.

Structural Requirements: Subsequently a thermal fluid analysis was performed in order to demonstrate the observations gleaned from the reactor core concept review. An integral portion of that thermal fluid analysis entailed the modeling and simulation of the 2 dimensional “Thermal Effects Demonstrator” model; which consisted of a series of randomized coolant channels and their accompanying heated (fuel element) half circle pellets. The diameter of both the pellets and the channels was 1/8 inches. The center pellet was varied in temperature between 2000 and 5000 degrees Fahrenheit in 500 degree increments while all other pellets were kept at a constant 2000 °F. Analysis was performed with both constant temperature and volumetric heat generation pellet models. Inlet boundary conditions were set at 2000 °F, Hydrogen gas, via a 0.1 Ft/s velocity inlet. Mass flow rate Exit boundary conditions were set using a 1000 psi pressure outlet. Mass flow rate was recorded in 5 of the channels. Overall model dimensions are shown as follows. Frankly it is amazing that such a simple analysis could provide such striking insight into the behavior of multi million and even billion dollar reactors that failed to operate as they were designed too.

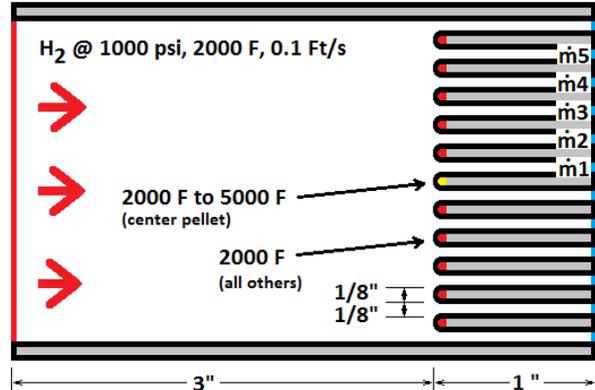


Fig 12. Thermal Effects Demonstrator

FLUENT the Computational Fluid Dynamics (CFD) program was utilized extensively for the thermal fluid analysis of the “Thermal Effects Demonstrator”. FLUENT’s unique capability to combine the analysis of thermal fluid and structural thermal conductivity proved valuable during this review. This solution combination provided a method for rapidly demonstrating the deleterious thermal coupling that can occur in volumetrically heated bodies that are cooled by randomized flow paths. The Air Force computational model generator PILGRIM was utilized for computational grid construction. PILGRIM produces block to block structured grids in the plot 3-d formatted list (P3dF) format. PILGRIM proved ideal for constructing simplified 2-d models and allowed for easy grid modification. The combination of a thermal fluid and structural thermal conductivity FLUENT analysis with rapid grid manufacturing via PILGRIM allowed for a sizable number of solutions to be produced in a relatively short period of time.

Thermal Fluid Inputs: FLUENT is a Computational Fluid Dynamics program capable of modeling flow, heat transfer, turbulence, and reactions. FLUENT’s extensive capabilities are tailored for the modeling and simulation of a diverse range of physical phenomenon ranging as far as the internal combustion engine with sliding or deforming mesh to the flow over a high speed aircraft. FLUENT has also managed to combine that diversity with a user friendly graphical user interface. For example Fluent easily modeled the physics of the Thermal Effects Demonstrator; which consisted of an H₂ velocity inlet, multiple pressure boundary exits, and heat conduction across material boundary conditions of Molybdenum-rhenium. FLUENT also supported temperature dependent fluid, and material properties such as piecewise linear inputs for Density, Specific Heat, Thermal conductivity, and Viscosity. The graphical user interface offers a host of fluid, and material properties to support a quick set up, and

FLUENT also supports a variety of grid topologies including the Plot-3d Formatted List (P3dF) format employed for this analysis. However 2-dimensional rather than 3-d grids were employed for this analysis in order to reduce run times. By taking into account hydraulic diameter during calculation the diameter of the coolant channels can be simulated thus eliminating the need for a 3-dimensional analysis.

General problem setup information	
Solver Type	Pressure Based
Velocity Formulation	Absolute
Time	Steady State Solution
Space	2-dimensional Planar
Turbulence Model	Spalart-Allmaras (1 eqn)

Solution Method	
Simple Scheme with Pressure-Velocity Coupling	

Spatial Discretization	
Gradient	Least Squares Cell Based
Pressure	Standard
Density	First Order Upwind
Momentum	First Order Upwind
Modified Turb Vis	First Order Upwind
N2	First Order Upwind
H2	First Order Upwind
Energy	First Order Upwind

This was a very straight forward analysis, solutions would typically reach convergence in less than 200 iterations, and under 5 minutes of runtime without any ramping of the residuals.

Computational Model: The United States Air Force’s computational model construction program, or grid builder, named PILGRIM was used for grid construction and editing. PILGRIM is a block to block structured grid builder. It uses a classic point, line, surface, volume construction approach that is well suited for cases such as this one where model construction began from blue prints. PILGRIM has a user friendly menu based Graphical User Interface (GUI), with simultaneous scripting that can easily be edited. This is accomplished by modifying the text based script, and then re-running the script. For example the size or number of pellets and coolant channels in a given test case could easily be modified using a text editor by adjusting the position of the points that the grid was originally built off of in the script. Using the PILGRIM GUI to select and run the edited script would then produce the new grid.

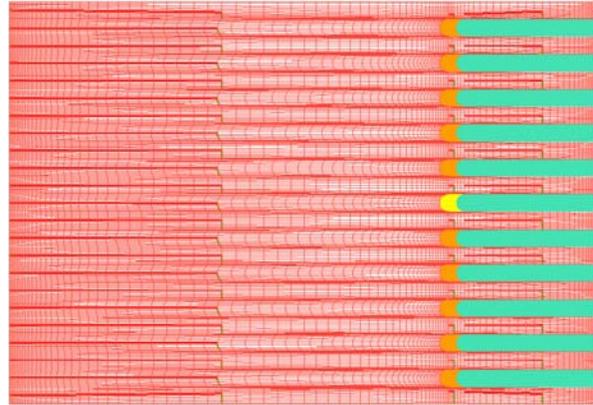


Fig 13. Typical Computational Grid Model

This 2-d grid is typical of those tested. All coolant channels were comprised of at least 15 cells across their width, and all remaining walls have wall spacing of 0.01 inches. The grid is comprised of a generous, and yet very small number 28,336 cells.

Thermal Fluid Results:

Case 22, runs 1 through 7

This case analysis was performed with a constant temperature assigned to the center pellet beginning with 2000 °F in run number 1 and then ramped up in 500 degree increments to 5000 °F in run number 7. All other pellets were kept at a constant temperature of 2000 °F. Results are shown at every 1000 °F.

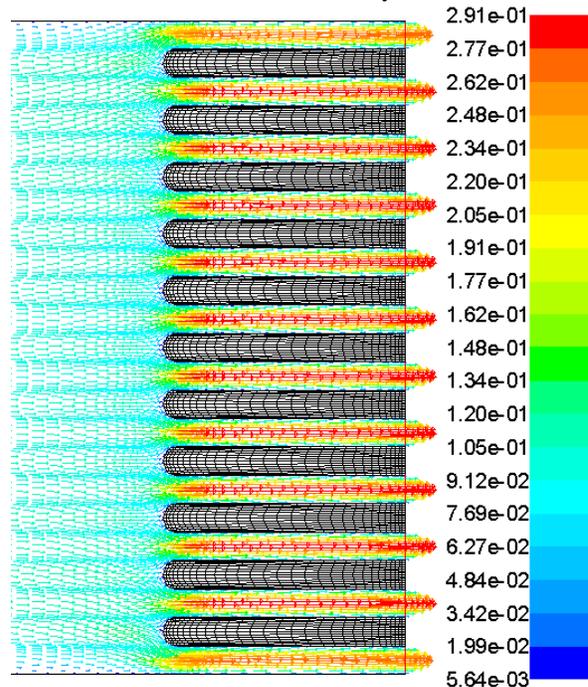


Fig 14. Velocity Vectors (Ft/s), Run 1

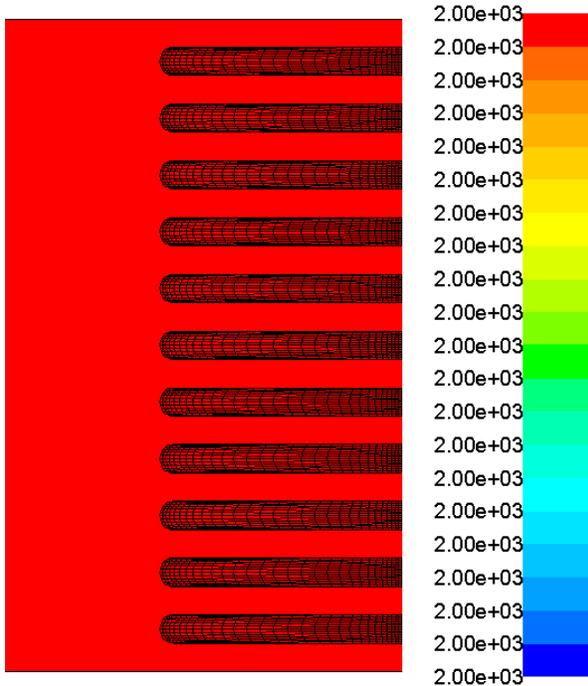


Fig 15. Filled Temperature Contours (°F), Run 1

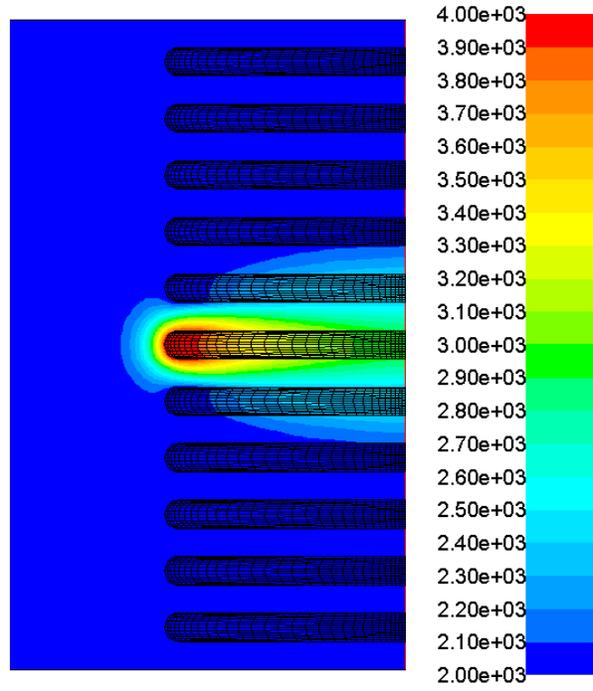


Fig 17. Filled Temperature Contours (°F), Run 5

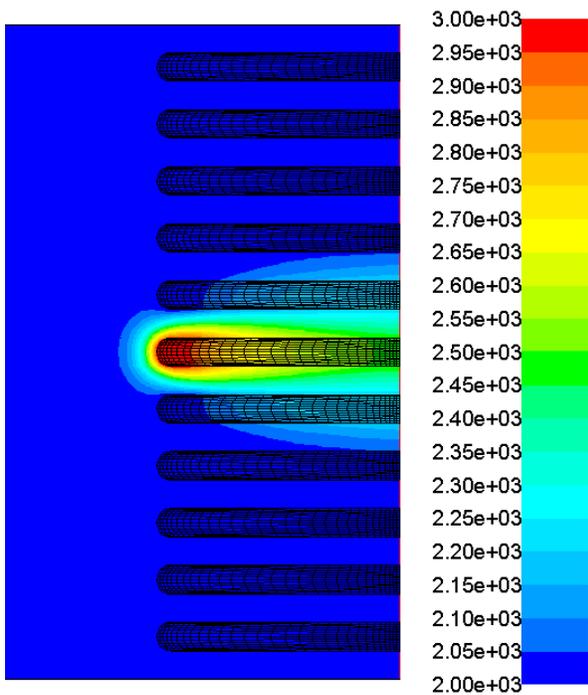


Fig 16. Filled Temperature Contours (°F), Run 3

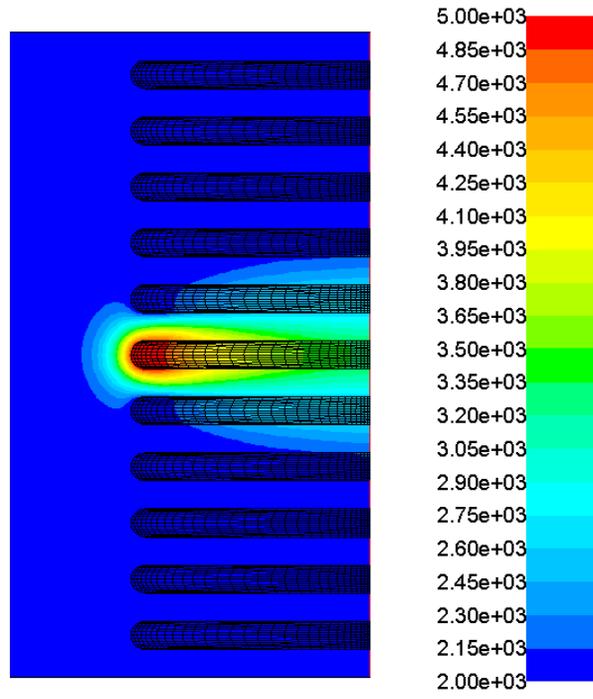


Fig 18. Filled Temperature Contours (°F), Run 7

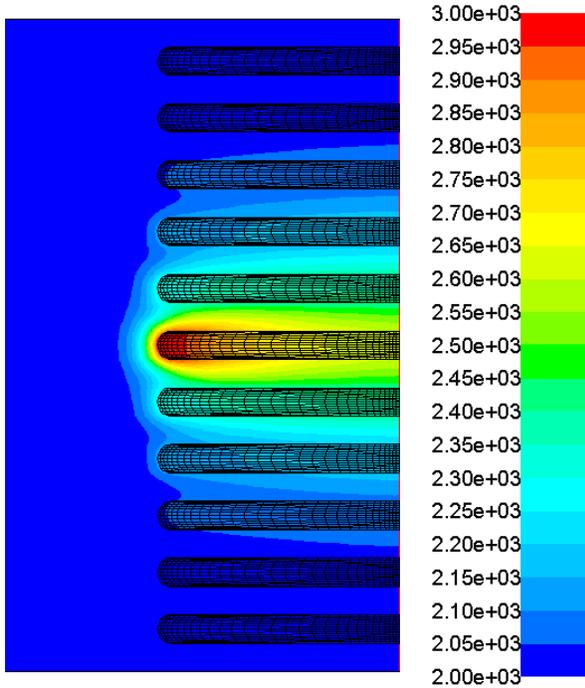


Fig 22. Filled Temperature Contours (°F), Run 3

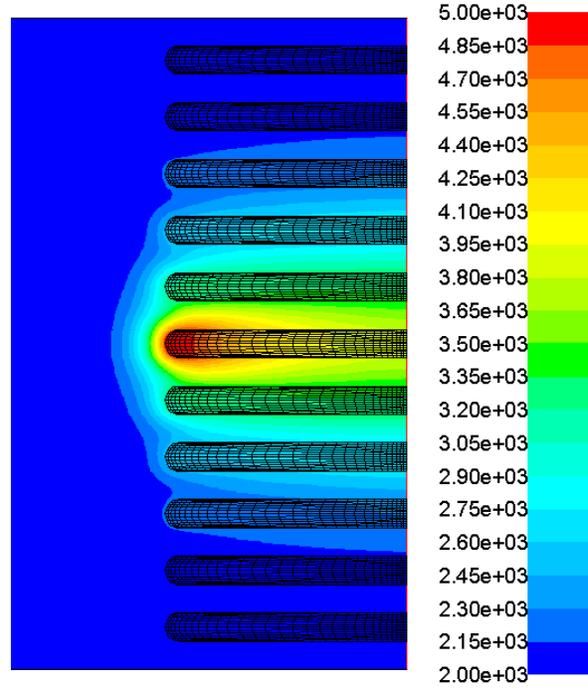


Fig 24. Filled Temperature Contours (°F), Run 7

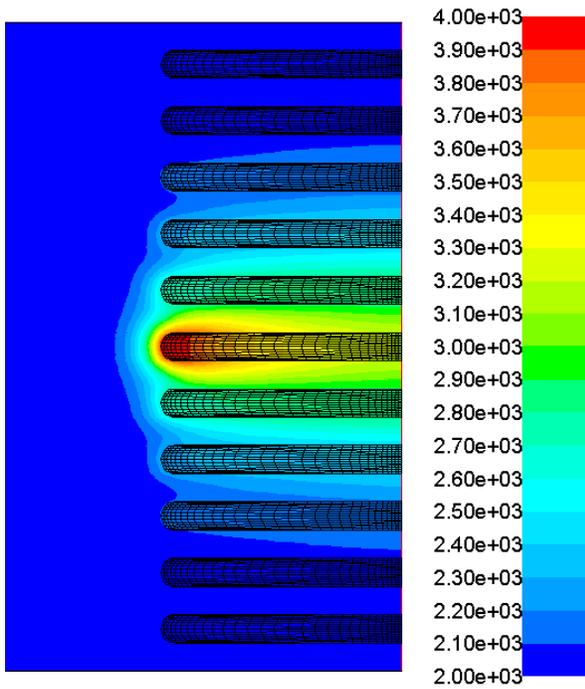


Fig 23. Filled Temperature Contours (°F), Run 5

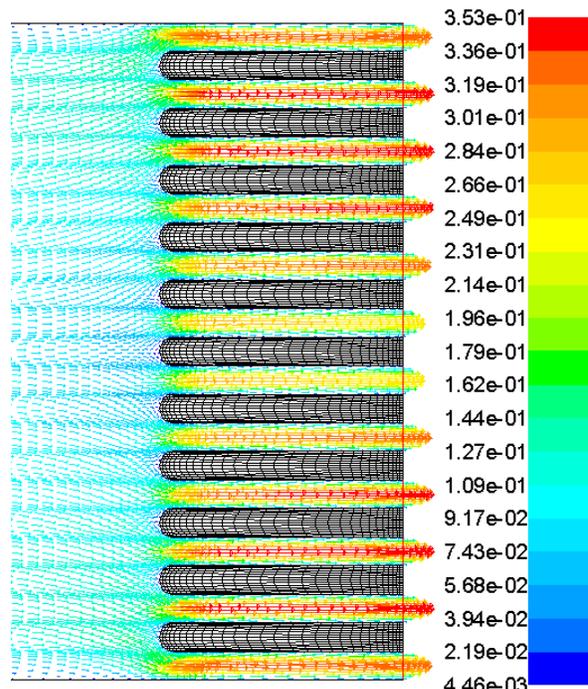
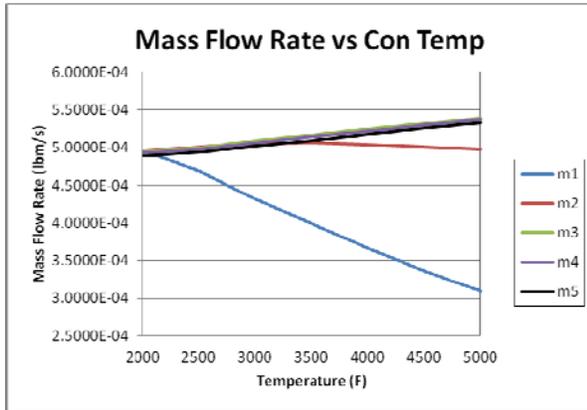


Fig 25. Velocity Vectors (Ft/s), Run 7

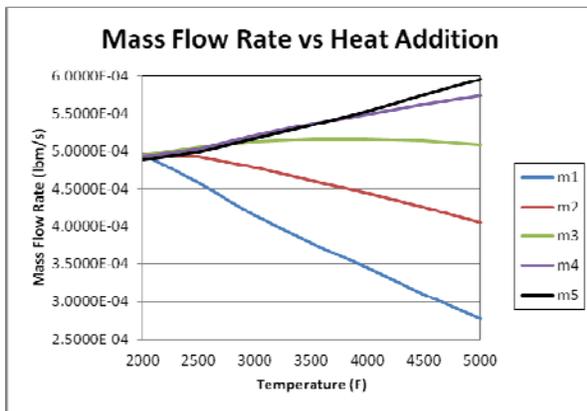


Fig 26. Mass flow channels

A study of the mass flow rate through channels 1 through 5 of case 22, the Constant Temperature model, reveals that the nearest channel, m1, suffers a significant drop in coolant as it heats up.



However case 23, the volumetric Heat Addition model, reveals an even more striking reduction in mass flow rate away from the areas that need it the most. This is caused by thermal coupling of the simulated malfunctioning pellet m1, with its nearest neighbor pellet m2. This thermal bootstrapping effect is a function of temperature and distance between a pellets nearest neighbors. Thus it would be particularly pronounced in a stack comprised of thousands of pebbles or in a densely packed particle bed. This simple solution mathematically proves that “one bad apple really can spoil the whole barrel”.



Having identified a common cause for localized excessive heating, thermal instabilities, and subsequent thermal runaway in the Pebble Bed and Particle Bed reactors it is now also possible to address potential solutions. When assessing advanced propulsion Nuclear Thermal Rockets a great deal of emphasis is often placed on the surface area to volume ratio of a given concept. However from a thermodynamic standpoint more is not always necessarily better. The model created operates perfectly mass balanced with all mass entering on the left and leaving on the right. The difference in pressure i.e. the ΔP between the inlet and exit ensures that the mass flow rate does not change either in the main large channel nor does it change in the smaller sub channels once the flow has entered them. But when the flow encounters surface area at the tips of the particles that is perpendicular to the ΔP it stagnates and in even this models modest stack of 11 pellets it has little to no ΔP to direct it in the cross wise direction of the model. Then having no ΔP to drive it, the coolant is instead directed by what begins as relatively small thermal effects that can then go on to rapidly build up out of control as a pellet is literally starved for coolant.

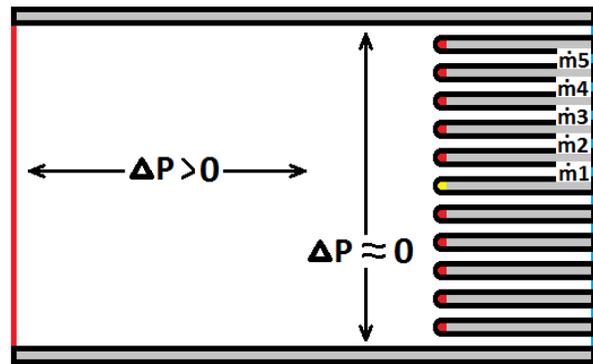


Fig 27. Parallel ΔP versus Perpendicular ΔP

Thus the question should not be “what is the surface area to volume ratio (S/V)”, of a given design but should instead be “what is the surface area that is parallel to the ΔP to volume ratio ($=S/V$)”, and the accompanying question of “what is the surface area that is perpendicular to the ΔP to volume ratio ($+S/V$)”, of a given design. While the former is beneficial, the latter can have significant deleterious effects on the operation of the design. For example a quick fix for the Pebble Bed concept exists by simply removing the perpendicular surface areas from the design. This is accomplished by installing a set of cross flow pumps to establish just enough ΔP in what is currently the perpendicular direction, in order to drive the flow and overcome the relatively small thermal effects before

they have a chance to build up and create a localized thermal runaway. Unfortunately it is far more challenging to identify a quick fix for the Particle Bed concept because of the particularly random surface area directions of the millions of irregularly shaped particles that exist within the stack.

III SUMMARY AND CONCLUSIONS:

A thermodynamic cause for Pebble Bed and Particle Bed localized thermal runaway was identified. For example purposes only, a potential solution to prevent localized thermal runaway in Pebble Bed reactors was also provided. A method for determining the resilience to localized thermal instabilities of similar reactor concepts such as Foam Core designs was proposed. Research could continue with simplified models to determine the ratio of cross flow ΔP to parallel ΔP that is required to avoid the onset of thermal runaway in designs with a positive $+S/V$ ratio. Additional research could be performed with more advanced models to determine the maximum allowable $(+S/V)/(-S/V)$ ratio required to avoid the onset of thermal runaway.

IV REFERENCES:

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