Effective Jet Properties for the Estimation of Turbulent Mixing Noise Reduction by Water Injection

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A one-dimensional control volume formulation is developed for the determination of jet mixing noise reduction due to water injection. The analysis starts from the conservation of mass, momentum and energy for the control volume, and introduces the concept of effective jet parameters (jet temperature, jet velocity and jet Mach number). It is shown that the water to jet mass flow rate ratio is an important parameter characterizing the jet noise reduction on account of gas-to-droplet momentum and heat transfer. Two independent dimensionless invariant groups are postulated, and provide the necessary relations for the droplet size and droplet Reynolds number. Results are presented illustrating the effect of mass flow rate ratio on the jet mixing noise reduction for a range of jet Mach number and jet Reynolds number. Predictions from the model show satisfactory comparison with available test data on supersonic jets. The results suggest that significant noise reductions can be achieved at increased flow rate ratios.

Nomenclature

A = cross sectional area of the jet

B = Spalding transport number, $h_{fg} / [c_{pj}(T_{j1} - T_{sat})]$

c = sound velocity

c_p = specific heat at constant pressure

c_1, c_2, c_3 = invariants (dimensionless)

C_D = drag coefficient, $2F_D / (\rho_j u_j^2)$

d = diameter

f = frequency

h_{fg} = latent heat of evaporation

k = thermal conductivity of gas

$\dot{m}_j$ = jet mass flow rate

$\dot{m}_w$ = component of mass flow rate of water in the jet axial direction

$\dot{m}_{wt}$ = total injected mass flow rate of water

M_j = jet Mach number

M_{je} = fully expanded jet Mach number

N_p = droplet number per unit volume of the mixture

Nu = droplet Nusselt number

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p	=	pressure
Pr	=	Prandtl number of gas, $c_{pg}\mu_g / k_g$
q_p	=	heat of evaporation of the droplet
R	=	gas constant
Re_j	=	jet exit Reynolds number, $\rho_{j1} u_{j1} d_{j1} / \mu_{j1}$
Re_p	=	droplet Reynolds number, $\rho_{j1}\left|u_{j1} - u_p\right| d_p / \mu_{j1}$
SPL	=	sound pressure level
T	=	temperature
T_{je}	=	equivalent gas temperature for heat transfer
u	=	velocity in the streamwise direction
x	=	axial distance from the nozzle exit plane
x_t	=	length of potential core in the jet

Greek Symbols

α	=	angle of water injection (measured from the downstream horizontal direction)
γ	=	isentropic exponent
μ	=	dynamic viscosity
ρ	=	density
η	=	fraction of injected water flow rate that is evaporated

Subscripts

a	=	ambient fluid
D	=	drag
g	=	gas
j	=	jet
l	=	liquid
p	=	droplet
sat	=	saturation
t	=	stagnation
w	=	injected water
1	=	jet exit
2	=	effective jet property

I. Introduction

Three distinct components of noise are present in supersonic jets: turbulent mixing noise, broadband shock associated noise and screech tones. While the latter two are associated with imperfectly expanded jets, the turbulent mixing noise generally represents dominant component of supersonic jet noise to the overall sound pressure level (OASPL), see Washington & Krothapalli[1]. The high noise levels (160 to 170 dB) radiated by launch vehicles at lift-off induce severe vibration on the launch vehicle structure and payload, and ground support equipment.

Water injection has been traditionally considered for the suppression of high noise levels from rocket exhausts in launch vehicle environments. For example, large amounts of water (of the order of 300,000 gpm) are used for the suppression of ignition overpressure (IOP) and lift-off noise during Space Shuttle launches. The water mass flow

rate to the SRB exhaust mass flow rate ratio is maintained around one to two in order to meet payload design requirements of 145 dB (Dougherty & Gust[2]; Jones[3]). Water injection could reduce noise by as much as 12 dB.

Water injection mitigates all the three components of jet noise: the turbulent mixing noise, the screech, and broadband shock noise. Two principal mechanisms leading to the diminution of jet noise by water injection are the reduction of jet velocity and jet temperature (Moriniere, Gervais & Puebe[4]). The decrease of jet velocity is occasioned through momentum transfer between the liquid and the gaseous phases, and the reduction of the jet temperature is achieved due to partial vaporization of the injected water (Zoppellaeri & Juve[5,6]). The effect of water may also be regarded as effectively increasing the effective jet density (Jones[3]). Important velocity reductions are achieved within a few diameters of the nozzle exit. Noise reductions of the order of 10 dB are realized for both cold and hot jets[5,6].

Several design parameters influence the effectiveness of noise reduction by water injection. These include water to jet mass flow rate ratio, axial injection location, water injection angle, number of injectors, method of injection (jet type or spray type), droplet size, water pressure, and water temperature. Optimal injection parameters need to be determined for the design of efficient water deluge system. Data of Zoppellari & Juve[5] and of Norum[7] suggest that best noise reductions of the order of 10 to 12 dB are obtained at injection angles of 45 to 60 deg., injection near the nozzle exit (especially for shock-containing jets), and high mass flow rates. Also the optimum number of injectors appears to be around eight. Experiments by Krothappalli et al.[8] and Greska & Krothapalli[9] at reduced water mass flow rate ratios (about 0.1) through the use of microjets show sizable noise reduction for application to aircraft jet engines.

Experiments with water injection suggest that the mass flow rate ratio appears to be an important parameter. Tests conducted with water to jet mass flow rate ratios up to four (Zoppellari & Juve[5]) reveal that significant noise reductions can be achieved at high water flow rate ratio. In the case of cold jets, beyond a critical mass flow rate ratio, the velocity reduction and thus the noise reduction is small. For hot jets, only a fraction of the liquid is effective in reducing the air jet velocity due to drop evaporation. At low water flow rates, it is possible to reduce the shock associated noise significantly. At higher mass flow rates, momentum transfer principally affects the mixing noise over a broad range of frequency.

At considerably high mass flow rates, the benefit of velocity reduction of the air jet by momentum transfer between the two phases is partly opposed by the emergence of new parasitic sources linked to water injection, which include the impact noise of air on the water jets, fragmentation of these water jets, and unsteady movement of the droplets. A compromise can be found between significant penetration of water jet into the air jet and low impact noise. A significant parameter is the velocity component of water jets that is perpendicular to the air jet. If this component is high, water penetrates deeply into the air jet and mixing takes place rapidly. If this component is small, water does not produce significant drag and impact noise.

On the theoretical front, studies on the effect of water droplets on noise reduction are very few, and these are concerned only (primarily) with plane waves. Sound attenuation and attenuation studies in a two-phase medium originated from the work of Sewell[10] on the assumption of immovable particles. Epstein and Carhart[11] considered sound attenuation by particles, allowing the particles to oscillate and considering both momentum and heat exchange. They compared the theory with the measurements with water droplets by Knudsen et al.[12] and Marble et al.[13,14] conducted theoretical investigations to determine the sound attenuation in vaporizing droplets, and in fans and ducts by vaporization of liquid droplets. For plane waves (Marble et al.[14], the attenuation magnitude is shown to exceed 5 dB/m at 25 C with a cloud of 0.7 μ m radius droplets constituting 1 % of the gas mixture (see, Krothapalli et al.[15]). Temkin and Dobbins[16] presented a classical study of both sound attenuation and dispersion by spherical droplets, considering the particulate relaxation processes of momentum and heat transport. Dupays and Vuillot[17,18] extended the analysis of Temkin and Dobbins[16] to accommodate the effects of droplet evaporation and combustion. Unfortunately, these results are not directly applicable to the study of turbulent mixing noise attenuation in jets.

In view of the importance of water injection in jet noise suppression, a theoretical understanding of the mechanism of noise reduction is useful in the design and optimization of water deluge systems for launch acoustics application. To the authors' knowledge, theoretical studies addressing the effect of water injection in mitigating jet mixing noise have not been previously reported. It is the purpose of this report to develop a simple one-dimensional

analytical model for estimating the jet mixing noise suppression due to water injection. The proposed model will be compared with existing data on jet noise.

II. ANALYSIS

A. Control Volume Formulation and Conservation Equations
The governing equations of continuity, momentum and energy for the control volume (**Fig. 1**) are expressed as

$$\rho_{j1} u_{j1} A_{j1} = \rho_{j2} u_{j2} A_{j2} - \eta \dot{m}_w \tag{1}$$

$$\rho_{j2} u_{j2}^2 A_{j2} - \rho_{j1} u_{j1}^2 A_{j1} = -F_d + \eta \dot{m}_w u_p \tag{2}$$

$$\dot{m}_{j2}\left(c_{pj} T_{j2} + \frac{u_{j2}^2}{2}\right) = \dot{m}_{j1}\left(c_{pj} T_{j1} + \frac{u_{j1}^2}{2}\right) - F_d u_p + \eta \dot{m}_w T_p c_{pl} \tag{3}$$

The equation of state for the gas is expressed by

$$\rho_{j1} R_{j1} T_{j1} = \rho_{j2} R_{j2} T_{j2} \tag{4}$$

In the above equations, $\dot{m}_w$ is the mass flow rate of water in the jet axial direction, η the fraction of the injected water flow rate that is in liquid state in the form of droplets, F_d the drag force, u_p the droplet velocity. The subscripts 1 and 2 refer to the inlet and exit states of the control volume under consideration.

The quantity $\dot{m}_w$ is related to the total injected water flow rate $\dot{m}_{wt}$ as

$$m_w = m_{wt} \cos \alpha \tag{5}$$

where α represents the angle of injection (measured from the downstream axis).

B. Physical Assumptions
The following physical assumptions are made in the analysis to facilitate a tractable solution.

a.   The pressure in the jet is constant. This approximation is appropriate for perfectly expanded supersonic jets, where turbulent mixing is a major noise component.

b.   The gas and gas-vapor mixture obey perfect gas law (calorically and thermally perfect).

c.   The gas constant for the gas and the gas-water vapor mixture is approximately the same.

d.   The droplets are uniformly distributed in the gas mixture in the control volume.

e.   The droplet size is uniform. In reality, the atomization produces a drop size distribution (such as lognormal), and an appropriate characteristic drop diameter (such as Sauter mean d_{32}) may represent drop drag and heat transfer more accurately. Also, the droplet size diminishes as it traverses downstream through the control volume due to evaporation.

f.   The drop temperature is uniform (assumption of infinite thermal diffusivity), and is at the saturation state. Thus

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$$T_p = T_{sat} \tag{6}$$

The uniform temperature model belongs to one of the limiting cases of transport processes (Faeth[19]) in evaporating and combusting sprays.

g. Thermal properties of the gas and water are independent of temperature.

h. The droplet drag and heat transfer are approximated by existing correlations.

i. The axial extent of the control volume is taken as two jet diameters ($n = L/d_j = 2$) in order to ensure that the effective jet properties due to water injection are representative. For comparison purposes, the length of the potential core x_t in chemical rockets follows closely the empirical relation of the form (Eldred[20])

$$\frac{x_t}{d_{j1}} = 3.45(1 + 0.38 M_{je})^2 \tag{7}$$

where M_{je} stands for the fully expanded jet Mach number.

j. Droplet evaporation is controlled by heat transfer from the gas to the droplets (thermal conductivity-controlled), as opposed to diffusion-controlled (Dupays and Vuillot[17]). This assumption appears reasonable for hot jets at relatively high temperature (typical of rocket exhausts) under investigation. For cold jets, mass transfer by diffusion can become important, with the driving potential represented by the difference between vapor pressure at the droplet surface and the partial pressure of the vapor in the bulk gas.

k. The droplets are assumed to be rigid (no surface tension effects; no deformation) and spherical.

l. Radiative heat transfer from the drop (drop to the surroundings) and the gas is neglected.

m. Parasitic noise generation due to water impact and breakup is not accounted for.

C Solution

With the above assumptions, eqs. (1) to (4) are now solved for the effective jet circumstances at the downstream section 2 of the control volume. A closed form solution is thereby obtained as follows.

1. Effective Jet Velocity

From the momentum equation in conjunction with the continuity equation, an expression of the jet velocity ratio can be obtained as:

$$\frac{u_{j2}}{u_{j1}} = \frac{1}{\left(1 + \eta \dfrac{\dot{m}_w}{\dot{m}_{j1}}\right)} \left[1 - \frac{F_d}{\rho_{j1} u_{j1}^2 A_{j1}} + \frac{\eta \dot{m}_w u_p}{\rho_{j1} u_{j1}^2 A_{j1}} \right] \tag{8}$$

The second term in the parenthesis on the RHS can be expressed as

$$\phi = \frac{F_d}{\rho_{j1} u_{j1}^2 A_{j1}} = N C_D \left(\frac{A_p}{A_{j1}} \right) \frac{1}{2} \left(1 - \frac{u_p}{u_{j1}} \right)^2 \tag{9}$$

where
$$N_p = \frac{3}{2} \left(\frac{\dot{m}_w}{\dot{m}_{j1}} \right) \left(\frac{\rho_{j1}}{\rho_p} \right) \left(\frac{d_{j1}}{d_p} \right)^3 n(1 - \eta) \tag{10}$$

denotes the number of droplets per unit volume of the gaseous mixture in the control volume. The quantity ϕ can be expressed as

$$\phi = \psi (\dot{m}_w / \dot{m}_{j1})(1 - \eta) \tag{11}$$

where
$$\psi = \frac{3}{2} \left(\frac{\rho_{j1}}{\rho_p} \right) \left(\frac{d_{j1}}{d_p} \right)^3 n \frac{1}{2} C_D \left(\frac{\mathrm{Re}_p}{\mathrm{Re}_{j1}} \right)^2 \tag{12}$$

The third term on the RHS can be written as

$$\frac{\eta \dot{m}_w u_p}{\rho_{j1} u_{j1}^2 A_{j1}} = \eta \left(\frac{\dot{m}_w}{\dot{m}_{j1}} \right) \left[1 - \frac{\mathrm{Re}_p}{\mathrm{Re}_{j1}} \frac{d_{j1}}{d_p} \right] \tag{13}$$

where Re_{j1} is the jet Reynolds number, and Re_p the droplet Reynolds number defined by

$$\mathrm{Re}_{j1} = \frac{\rho_{j1} u_{j1} d_{j1}}{\mu_{j1}}, \quad \mathrm{Re}_p = \frac{\rho_{j1}(u_{j1} - u_p) d_p}{\mu_{j1}} \tag{14}$$

and d_p is the droplet diameter.

2. Effective Jet Temperature

Consideration of the energy equation in conjunction with the equation of state furnishes the relation for the effective jet temperature

$$\frac{T_{j2}}{T_{j1}} = \frac{1}{\left(1 + \eta \dfrac{\dot{m}_w}{\dot{m}_{j1}} \right)} \left[1 + \frac{u_{j1}^2}{2 c_{pj} T_{j1}} - \frac{F_D u_p}{c_{pj} T_{j1}} \right] - \frac{u_{j2}^2}{2 c_{pj} T_{j1}} \tag{15}$$

where
$$\frac{u_{j1}^2}{2 c_{pj} T_{j1}} = \frac{\gamma - 1}{2} M_{j1}^2 \tag{16a}$$

and
$$\frac{u_{j2}^2}{2c_{pj}T_{j1}} = \left(\frac{u_{j2}}{u_{j1}}\right)^2 \left(\frac{\gamma-1}{2}\right) M_{j1}^2 \qquad (16b)$$

3. *Effective Jet Density*
The density ratio is evaluated from

$$\frac{\rho_{j2}}{\rho_{j1}} = \left(\frac{T_{j1}}{T_{j2}}\right) \qquad (17)$$

4. *Effective Jet Mach number*
An expression for the effective jet Mach number is given by

$$\frac{M_{j2}}{M_{j1}} = \left(\frac{u_{j2}}{u_{j1}}\right)\left(\frac{\rho_{j2}}{\rho_{j1}}\right)^{1/2} \qquad (18)$$

5. *Effective Jet Exit Area*
The effective jet exit cross sectional area becomes

$$\frac{A_{j2}}{A_{j1}} = \left(\frac{\rho_{j2}}{\rho_{j1}}\right)\left(\frac{u_{j1}}{u_{j2}}\right)\left(1+\eta\frac{m_w}{m_{j1}}\right) \qquad (19)$$

D. Droplet Drag and Heat Transfer

1. *Droplet Drag*
The droplet drag in the presence of evaporation is obtained from the relation (**ref**)

$$C_D = C_{D0}/(1+B) \qquad (20)$$

where
$$C_{D0} = \frac{24}{Re_p} + \frac{6}{1+Re_p^{0.5}} + 0.4 \qquad (21)$$

and B is the Spalding transport number defined by

$$B = \frac{h_{fg}}{c_{pj}\left(T_{je} - T_{sat}\right)} \qquad (22)$$

The correlation for the drag coefficient in the absence of evaporation, as given by eq. (21), is obtained from White[21]. The effect of Mach number on the droplet drag is not however accounted for. The first term in eq. (21) represents the Stokes drag[22] valid for Reynolds numbers less than one. The quantity T_{je} is called the effective gas temperature, which will be described in the next section (sec. **2.4.2**).

2. *Droplet Heat Transfer and Evaporation*
The fraction of the injected liquid that is evaporated is determined as follows. The heat transferred from the gas to the droplets is given by

$$q_g = N_p Nu \pi k d_p \left(T_{je} - T_p\right) \tag{23}$$

where N_p is obtained from eq. (10). The drop Nusselt number Nu for an evaporating droplet is computed from the relation (Kays & Crawford[23])

$$Nu = Nu_0(1 + B)/B \tag{24}$$

where Nu_0, denoting the drop Nusselt number in the absence of mass transfer, is expressed by the well-known Ranz-Marshall correlation[24]

$$Nu_0 = 2 + 0.6 \, Re_p^{0.5} \, Pr^{0.33} \tag{25}$$

The heat of evaporation for the droplets is provided from

$$q_p = \eta \dot{m}_w h_{fg} \tag{26}$$

From eqs. (23) and (24), we find an expression for the evaporation fraction η as

$$\frac{1}{\eta} = 1 + \frac{Pr}{Nu} \frac{Re_{j1}}{\left(\dfrac{d_{j1}}{d_p}\right)^2 \dfrac{3}{2}\left(\dfrac{\rho_{j1}}{\rho_p}\right)_n} \frac{1}{}\left[\frac{h_{fg}}{c_{pj}\left(T_{je} - T_{sat}\right)}\right] \tag{27}$$

where Pr is the Prandtl number of the gas (taken as 0.7). Eq. (27) suggests that the evaporation factor η is independent of the water to jet mass flow rate ratio.

The equivalent gas temperature T_{je} in eq. (27) is chosen to ensure that $T_{je} \gg T_{sat}$ so that the evaporation is heat transfer controlled. In the present work a value of T_{je} = 2500 R is considered, which compares with T_{sat} = 672 R (saturation temperature of water at atmospheric pressure). It is tacitly assumed that the chosen value of T_{je} partly accounts for vaporization by mass diffusion.

E. Invariant Groups
An examination of eqs. (12), (13) and (27) suggests that the following invariant groups appear in the analysis:

$$\frac{Re_{j1}}{\left(d_{j1}/d_p\right)}\frac{1}{\left(\rho_{j1}/\rho_p\right)} = c_1 \tag{28}$$

$$\frac{Re_p}{Re_{j1}}\left(\frac{d_{j1}}{d_p}\right) = c_2 \tag{29}$$

and

$$\left(\frac{\rho_{j1}}{\rho_p}\right)\left(\frac{d_{j1}}{d_p}\right)^3 \frac{\text{Re}_p}{\text{Re}_{j1}^2} = c_3 \tag{30}$$

Since

$$c_2 = c_1 c_3$$

it is evident that two independent relations exist for the estimation of Re_p and d_{j1}/d_p. These are now evaluated as

$$\frac{d_{j1}}{d_p} = \left[\frac{1}{c_1} \frac{\text{Re}_{j1}}{\left(\rho_{j1}/\rho_p\right)}\right]^{1/2} \tag{31a}$$

$$\text{Re}_p = c_2 \frac{\text{Re}_{j1}}{\left(d_{j1}/d_p\right)} \tag{31b}$$

Thus the unknown quantities Re_p and d_{j1}/d_p are determined in terms of the adjustable constants c_1 and c_2. In view of the complexity with regard to the knowledge of droplet diameter and drop Reynolds number, it is postulated here that the constants c_1 and c_2 are *invariant*. These constants may be established by a correlation with available test data.

F. Drop Characteristics
In view of the critical nature of the drop size in the present investigation, we briefly present a review of the physical processes that control the drop characteristics in systems involving sprays and atomization (Sirignano[25]).

1. Drop Deformation and Drop Breakup
The physical processes of drop deformation and breakup are extremely complex (Hsiang and Faeth[26]. According to the classical description of drop breakup, atomization occurs by primary breakup near the liquid surface followed by secondary breakup. Two types of drop breakup are generally observed: bag breakup (deformation initiated near the upstream end of the drop), and shear breakup (liquid shearing at the periphery of the drop. Multimode breakup is also present. The breakup processes are not instantaneous.

Hinze[27] demonstrated that the breakup regime transitions are primarily dependent on the *Weber number We* and the *Ohnesorge number Oh* defined by

$$We = \rho|u_d - u_g|^2 d_p /(2\sigma), \quad Oh = \mu_d /\left(\rho_d d_p \sigma\right)^{1/2} \tag{32}$$

The breakup regime map is thus presented in terms of these two dimensionless groups (Hsiang and Faeth[26]). At low values of Oh, drop deformation becomes significant at We near unity, and drop breakup becomes significant at We of about 10, with bag, multimode, and shear breakup regimes observed as We increases. With an increase in Oh, drop deformation and drop breakup occur at progressively larger values of We, because viscous forces inhibit drop deformation, which represents the first stage of breakup process.

The secondary breakup is correlated in terms of *Eotos number* E_0 (also known as Bond number Bo) defined by Hsiang & Faeth[26]

$$Eo = a_d \rho_d d_0^2 /\sigma \tag{33}$$

where a_d denotes the drop acceleration. Results show that the secondary breakup is not a localized process, but extends over a significant region of about 40 drop diameters.

Measurements suggest that the critical Weber number We_{crit} required for the onset of drop breakup (transition criterion for breakup under ste4ady disturbances) is given by (Faeth[19]; Hsiang and Faeth[26])

$$We_{crit} = \mathrm{Re}_{crit}^{1/2}/2 \qquad (34)$$

where Re_{crit} is the critical Reynolds number of the drop.

2. *Drop Size*
On account of the shearing action between the gas and the drop, drops breakup quickly from their initial size near the injector station to a few microns downstream of the injection station (Krothapalli et al.[8]). Turbulence in the gas phase could be influenced by the presence of the droplets. Turbulence in the gas phase is generally reduced by the injection of droplets. Measurements of Krothapalli et al.[8] suggest that the arithmetic mean diameter remain fairly uniform across the jet.

G. Jet Mixing Noise Reduction
Once the effective jet conditions (velocity, temperature and Mach number) are obtained, the reduction in the jet turbulent mixing noise is evaluated with the aid of the scaling laws, recently proposed by Kandula & Vu[28]. In this reference, the turbulent mixing noise reduction is presented as a function of the jet Mach number and the jet to ambient temperature ratio (see **Fig.** 2) in both subsonic and supersonic flow.

III. Results and Comparison

The above analysis suggest that the effective jet conditions in the presence of water injection are dependent primarily on the water-to-gas mass flow rate ratio, and independent of momentum flux ratio, which governs primarily the penetration depth of the injected water normal to the jet. A correlation of the analysis with the test data of Norum[7] for hot supersonic turbulent mixing noise reduction at $M_j = 1.45$ yields the values of the invariants c_1 and c_2 as

$$c_1 = 5, \quad c_2 = 0.05 \qquad (35)$$

Thus in all the results presented below, the above values of the invariants are considered.

A. Predictions for the Effective Jet Conditions
Fig. 3a displays the variation of fractional evaporation η as a function of the jet Reynolds number for various jet temperatures, as given by eq. (27). It is seen that evaporation increases with an increase in jet Reynolds number and an increase in jet temperature, as is to be expected. The variation of η with the jet temperature is illustrated in Fig. 3b.

Fig. 4a shows the variation of the effective jet velocity with the water mass flow rate ratio. The effective jet velocity decreases with an increase in water flow rate on account of the momentum transfer between the gas and the droplets. For mass low rate ratios beyond about four, the rate of decrease in the jet velocity is relatively small. This result seems to be consistent with the experimental observation of Zoppellari et al.[5], which suggests that beyond a mass flow rate ratio of about four, further reduction in jet noise are not appreciable. The theory suggests that the effective jet velocity is independent of jet exit temperature, exit Mach number and exit Reynolds number.

The variation of the effective jet temperature with the water flow rate is displayed in Fig. 4b for various jet exit Mach numbers. The effective jet temperature decreases with an increase in water flow rate due to heat transfer from the gas to the droplets and subsequent evaporation. The rate of decrease in the effective jet temperature begins to slow for water flow rate ratios beyond about four, as in the case of the effective jet velocity. At a given mass flow rate ratio, the effective jet temperature decreases with an increase in the jet exit Mach number. A crossover trend with the exit Mach number is noted at a flow rate ratio of about 0.6.

Illustrated in Fig. 4c is the dependence of the effective jet Mach number as a function of the water mass flow rate. The trend is similar to that indicated for the effective jet temperature. The results suggest that below a water flow rate ratio of one, the effective jet temperature is independent of the jet exit Mach number.

Fig. 4d depicts the distribution of the effective jet density with the water mass flow rate at various jet exit Mach numbers. It is seen that the effective jet density increases with the flow rate, and with an increase in the jet exit Mach number.

Calculations suggest that the jet cross sectional area increases with the water flow rate (Fig. 4e). At a fixed flow rate ratio, the effective jet area decreases with an increase in the jet exit Mach number.

The effect of jet exit Reynolds number on the distribution of effective jet velocity, jet temperature and jet Mach number are indicated in Figs. 5a-c. Fig. 5a suggests that the change in the effective jet velocity is relative small for a three orders of magnitude change in the jet exit Reynolds number. A similar trend is noted with regard to the dependence of jet exit temperature and jet Mach number on the jet exit Reynolds number.

These results for the variation of the effective jet conditions suggest that the theory of invariant groups formulated here (in terms of the constants c_1 and c_2) seems to provide a satisfactory first step in our understanding of the role of water injection in reducing the jet velocity and jet temperature.

It should be cautioned that the present results would entail error at relatively high water flow rate in view of the parasitic noise generation due to water impact.

B. Comparisons with Experimental Data

A comparison of the present theory with the test data of Norum[7], from which c_1 and c_2 are determined, is presented in Fig. 6. The data correspond to hot supersonic jet of air from a convergent-divergent (CD) nozzle operation at $T_t = 1560$ R, and $M_j = 1.45$. The jet exit Reynolds number Re_{j1} is about 1.3E6. At this condition, supersonic jet mixing noise dominates upstream noise radiation, and Mach wave radiation dominates the downstream noise radiation. In the data, water is injected at 45 deg., and the number of injectors includes 6, 12 and 18. The data shown correspond to maximum noise reductions with water injection.

For the test data considered above, the present theory suggests that $\eta = 0.49$, indicating that 49 percent of $\dot{m}_w$ is evaporated within the control volume chosen. Also, $\mathrm{Re}_p = 2.1$ and $d_{j1}/d_p = 3.13\mathrm{e}4$, as obtained form eqs. (31a) and (31b). The agreement between the present prediction and the data is seen to be favorable, considering the complexity of the problem, and the number of assumptions made in the analysis. Both the data and the theory suggest that the reduction in mixing noise increases with an increase in water flow rate. At the highest water flow rate considered in the data, a reduction of 3.8 dB in the mixing noise is achieved. Calculations show that the noise reduction is primarily achieved through jet velocity reduction through momentum transfer, while the jet temperature reduction is small enough to contribute appreciable noise reduction.

IV. Conclusion

The effect of water injection on the turbulent jet mixing noise reduction has been theoretically investigated for the first time on the basis of a one-dimensional control volume formulation for the effective jet exit conditions. The theory yields two dimensionless invariant groups, involving the ratio of droplet diameter to jet exit diameter and the ratio of droplet Reynolds number and jet Reynolds number. Correlation of the theory with available data for hot

supersonic jet mixing noise reduction provided a satisfactory agreement. The theory predicts the experimental trend that the jet mixing noise reduction increases with an increase with the water mass flow rate. It is demonstrated that the water mass flow rate is an important parameter characterizing the mixing noise reduction, and that the water to jet momentum flux ratio is unimportant. The conception of effective jet exit conditions, proposed here, appears to be a significant first step in our understanding the mechanisms of jet noise reduction due to water injection. Further comparisons with additional data are in progress.

Acknowledgments

References

[1]Washington, D., and Krothapalli, A., The role of water injection on the mixing noise supersonic jet, AIAA-98-2205, June 1998.

[2]Dougherty, N.S., and Guest, S.H., A correlation of scale model and flight aeroacoustic data for the Space Shuttle vehicle, AIAA-84-2351, AIAA/NASA 9th Aeroacoustics Conference, Williamsburg, Virginia, October 1984.

[3]Jones, J.H., Scaling of ignition startup pressure transients in rocket systems as applied to the space shuttle overpressure phenomenon, JANNAF 13th Plume Technology Meeting, CPIA-PUB-357, Vol. 1, 1982.

[4]Moriniere, V., Gervais, Y., and Peube, J.-L, Numerical calculation of spectrum emission and jet noise reduction by injection of mass, *Internoise 95*, Newport Beach, pp. 273-276, July 1995.

[5]Zoppellari, E., and Juve, D., Reduction of jet noise by water injection, AIAA-97-1622, 1997.

[6]Zoppellari, E., and Juve, D., Reduction of hot jet noise by water injection, AIAA-98-2204, 1998.

[7]Norum, T.D., Reductions in multi-component jet noise by water injection, AIAA-2004-2976, *10th AIAA/CEAS Aeroacoustics Conference, Manchester*, Great Britain, May 2004.

[8]Krothapalli, A., Venkatakrishnan, L., Lourenco, L., Greska, B., and Elavarasan, R., Turbulence and noise suppression of a high-speed jet by water injection, *J. Fluid Mechanics*, Vol. 491, 2003, pp. 131-159.

[9]Geska, B., and Krothapalli, A., Jet noise reduction using aqueous microjet injection, AIAA-2004-2971, *10th AIAA/CEAS Aeroacoustics Conference*, Manchester, Great Britain, May 2004.

[10]Sewell, C.J.T., On the extinction of sound in a viscous atmosphere by small obstacles of cylindrical and spherical form, *Phil. Trans. Roy. Soc.* (London), A210, 1910, pp. 239-270.

[11]Epstein, P.S., and Carhart, R.R., The absorption of sound in suspensions and emulsions, I. Water fog in air, *J. Acoust. Soc. Am.*, Vol. 25, 1953, pp. 553-565.

[12]Knudsen, V.O., Wilson, J.V., and Anderson, N.S., The attenuation of sound in fog and smoke, *J. Acoust. Soc. Am.*, Vol. 20, 1948, pp. 849-857.

[13]Marble, F.E., and Wooten, D.C., Sound attenuation in a condensing vapor, *Phy. Fluids*, Vol. 13, pp. 2657-2664.

[14]Marble, F.E., and Candel, S.M., Acoustic attenuation in fans and ducts by vaporization of liquid droplets, *AIAA Journal*, Vol. 13, 1975, pp. 634-639.

[15]Krothapalli, A., Venkatakrishnan, Elavarasan, R., and Lourenco, L., Supersonic jet noise suppression by water injection, AIAA-2000-2025, *6th AIAA/CEAS Aeroacoustics Conference*, Hlahaina, Hawaii, June 2000.

[16]Temkin, S., and Dobbins, R.A., Attenuation and dispersion of sound by particulate Relaxation Processes, *J. Acoust. Soc. Am.*, Vol. 40, No. 2, 1966, pp. 314-324.

[17]Dupays, J., and Vuillot, F., Propagation of Acoustic Waves in a two-phase vaporizing mixture, *J. Propulsion*, Vol. 18, No. 1, 2002, pp. 222-224.

[18]Dupays, J., and Vuillot, F., Propagation of an Acoustic Wave in a two-phase reactive medium, AIAA-98-3696, 1998.

[19]Faith, G.M., Evaporation and combustion of Sprays, *Prog. Energy Combust. Sci.*, Vol. 9, 1983, pp. 1-76.

[20]Eldred, K.M., Acoustic loads generated by the propulsion system, NASA SP-8072, 1971.

[21]White, F.M., *Viscous Fluid Flow*, 2nd ed., McGraw-Hill, New York, 1991, p. 182.

[22]Stokes, G.G., On the effect of internal friction of fluids in motion, *Cambridge Phil. Trans.*, Vol. 9, 1851, pp. 8-106.

[23]Kays, W.M., and Crawford, M.E., *Convective Heat and Mass Transfer*, 2nd ed., McGraw-Hill, New York, 1980.

[24]**Ranz**, W.E., and Marshall, W.P., *Chemical Eng. Prog.*, Vol. 48, No. 141, 1952, **p. 173. (title)**.

[25]Sirignano, W., Fluid Dynamics of Sprays- 1992 Freeman Scholar Lecture, J. Fluids Engineering, Vol. 115, pp. 345-378, September 1993.

[26]Hsiang, L.-P., and Faith, G.M., Drop deformation and breakup due to shock wave and steady disturbances, *Int. J. Multiphase Flow*, Vol. 21, No. 4, 1995, pp. 545-560.

[27]Hinze, J.O., Fundamentals of the hydrodynamical mechanism of splitting in dispersion processes, *AIChE Journal*, Vol. 1, 1955, pp. 289-295.

[28]Kandula, M., and Vu, B., On the scaling laws for jet noise in subsonic and supersonic flow, AIAA-2004-2976, *9th AIAA/CEAS Aeroacoustics Conference*, Hilton Head, South Carolina, May 2003.
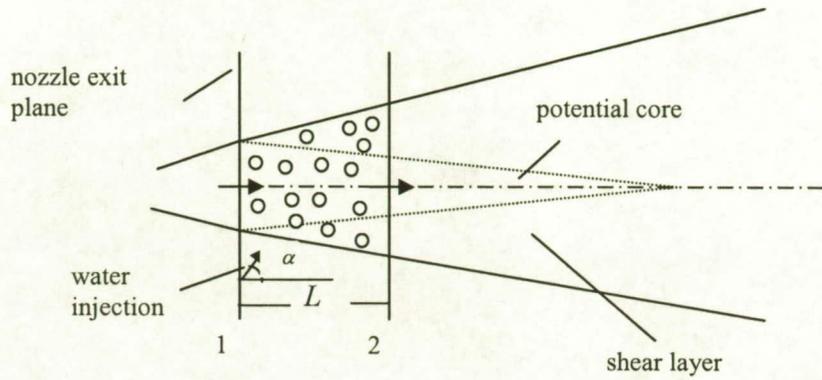
Fig. 1 Schematic of the jet configuration with water injection.
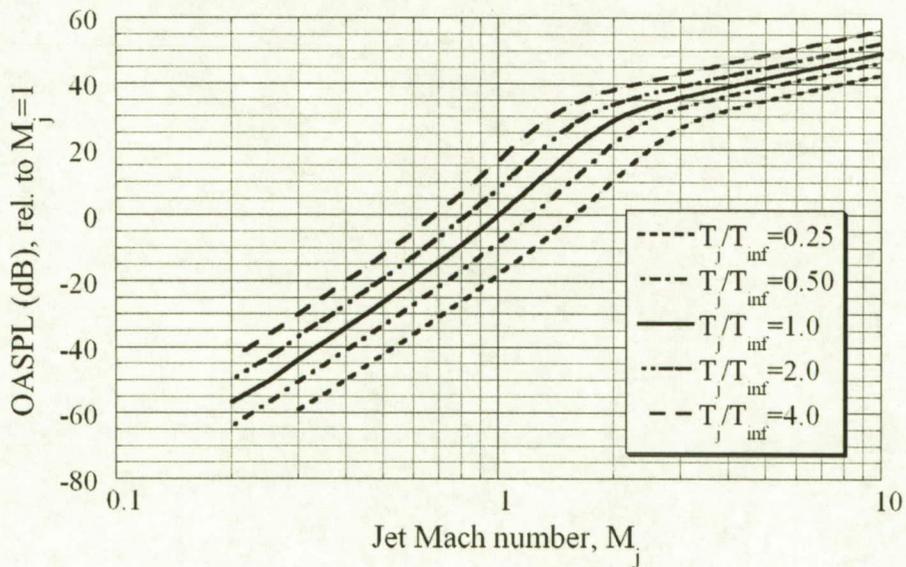
Fig. 2. Effect of jet Mach number and jet temperature on overall sound power, from Kandula & Vu[28].
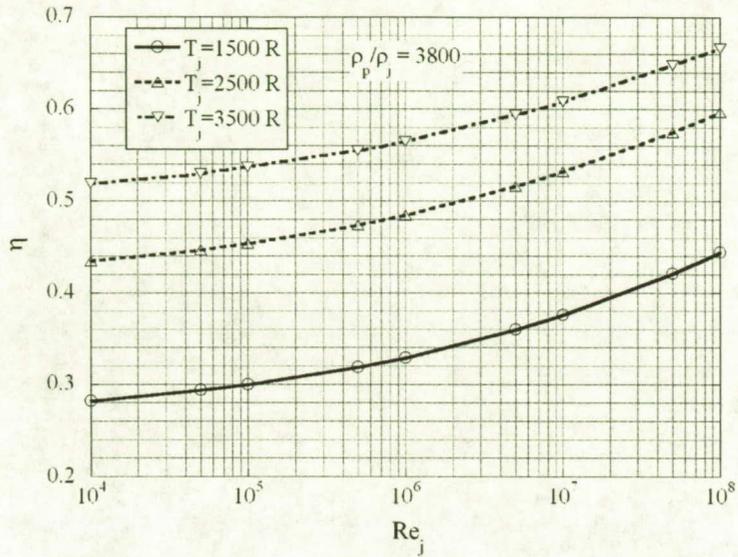
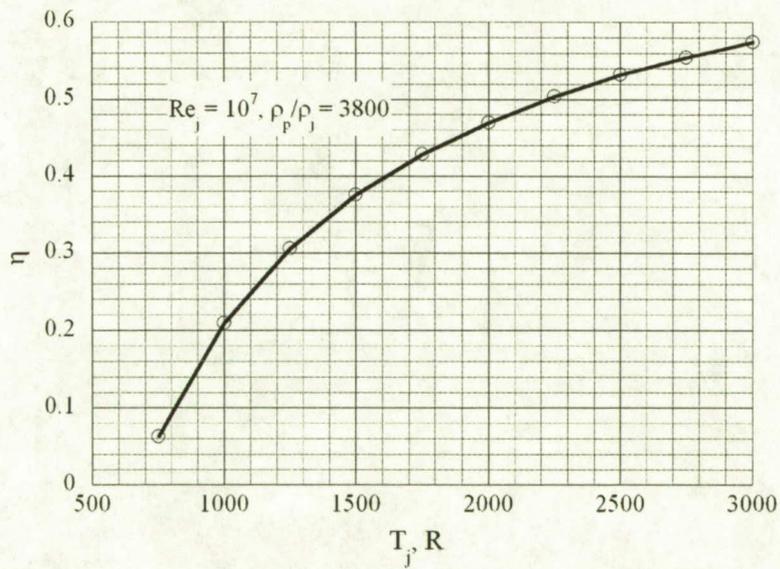Fig. 3a Variation of the fractional evaporation with the jet Reynolds number.

Fig. 3b Variation of fractional evaporation with jet temperature.
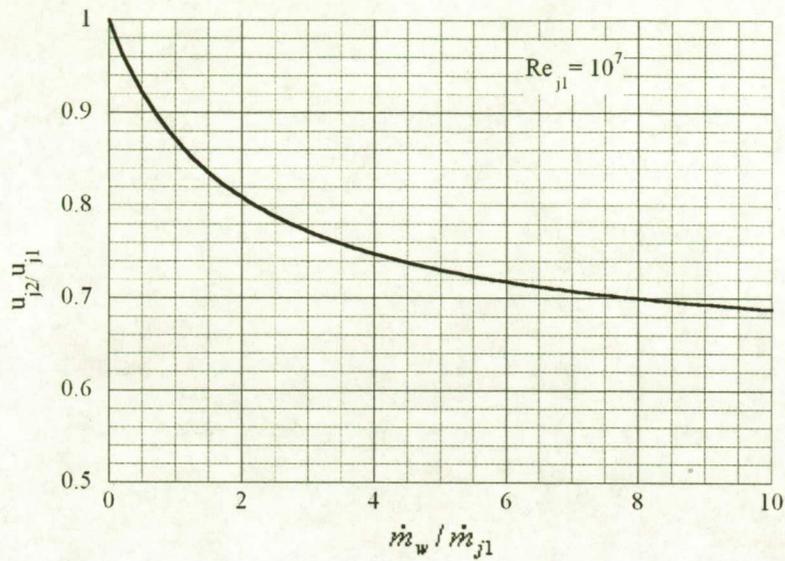
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Fig. 4a Variation of effective jet velocity with the water mass flow rate.
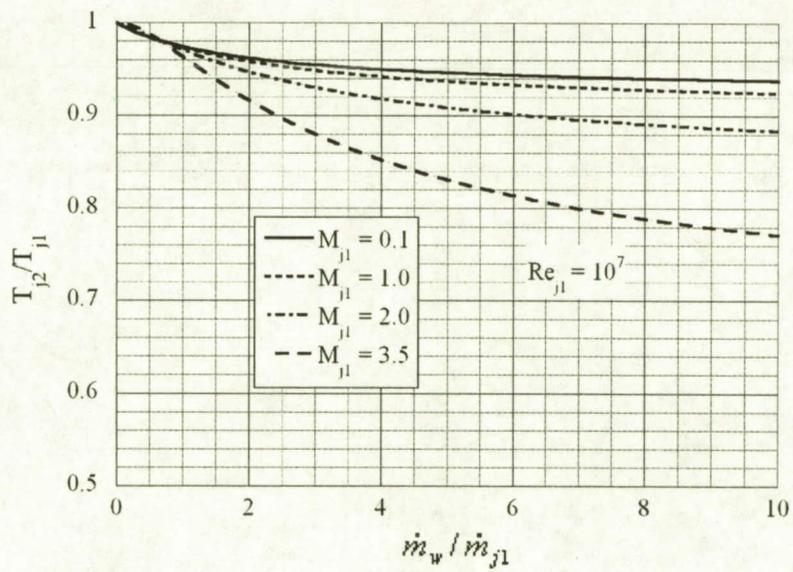
Fig. 4b Variation of effective jet temperature with the water mass flow rate.
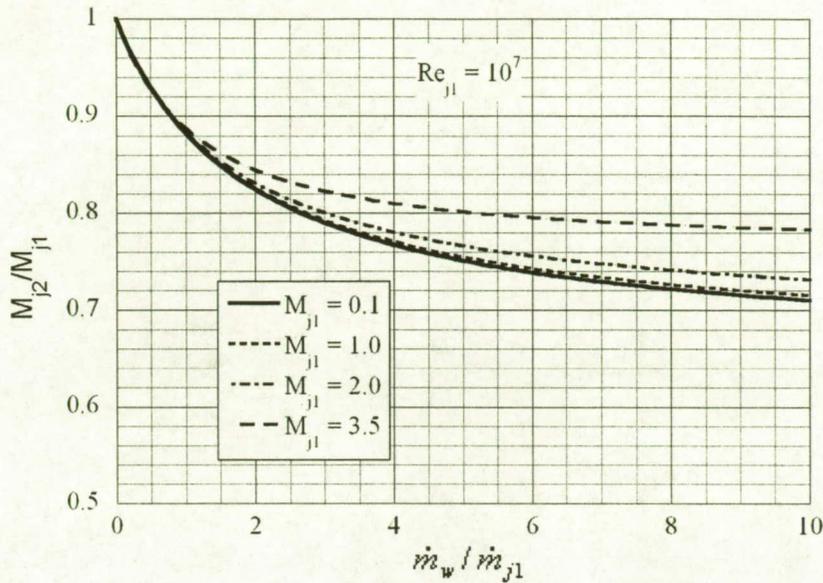
Fig. 4c Variation of effective jet Mach number with the water mass flow rate.
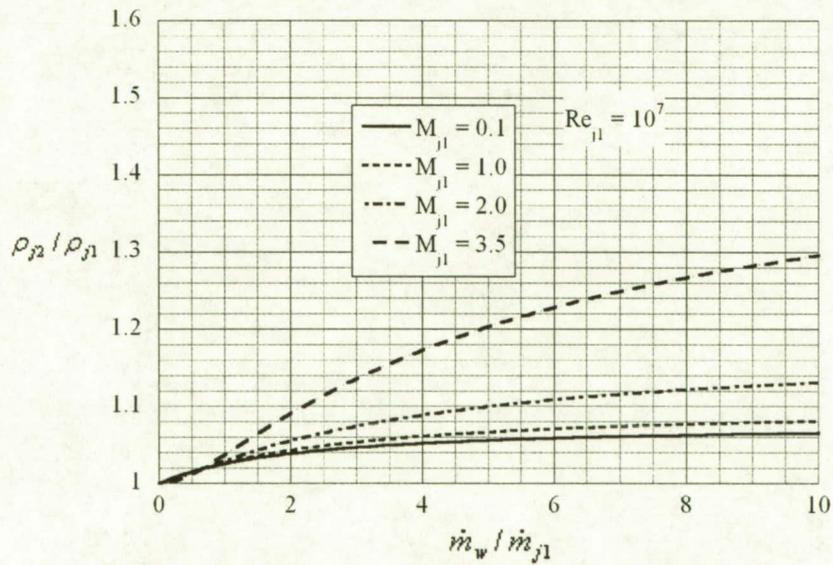
Fig. 4d Variation of effective jet density with the water mass flow rate.
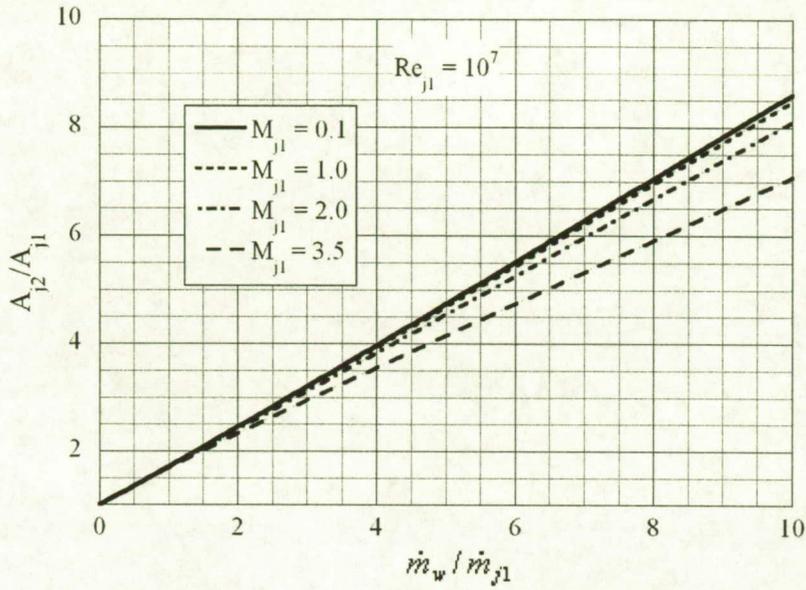
Fig. 4e Variation of effective jet cross sectional area with the water mass flow rate.
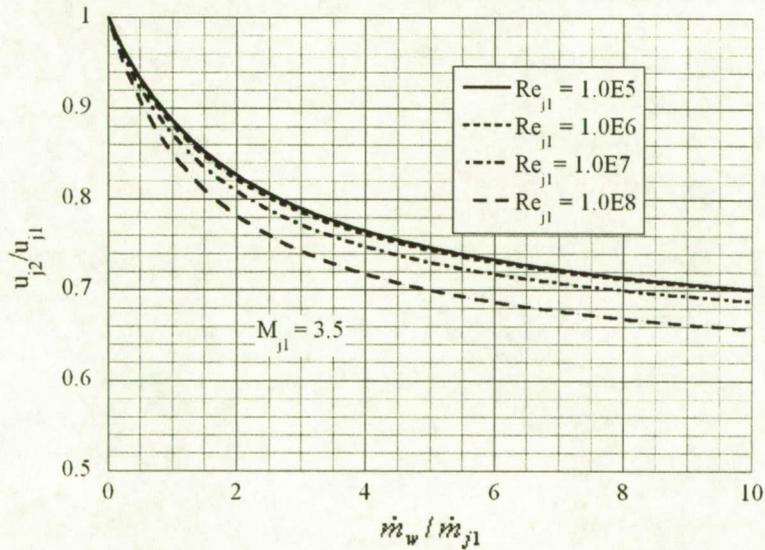
Fig. 5a Dependence of effective jet velocity with jet Reynolds number.
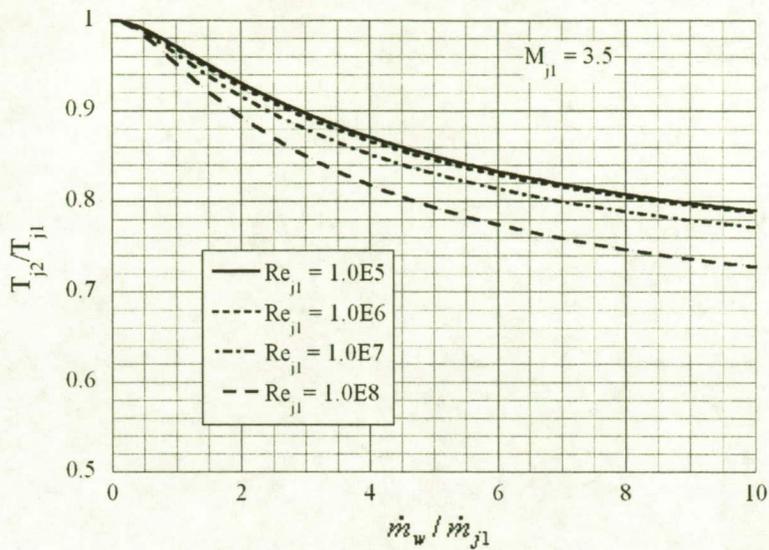
Fig. 5b Dependence of effective jet temperature on jet Reynolds number.
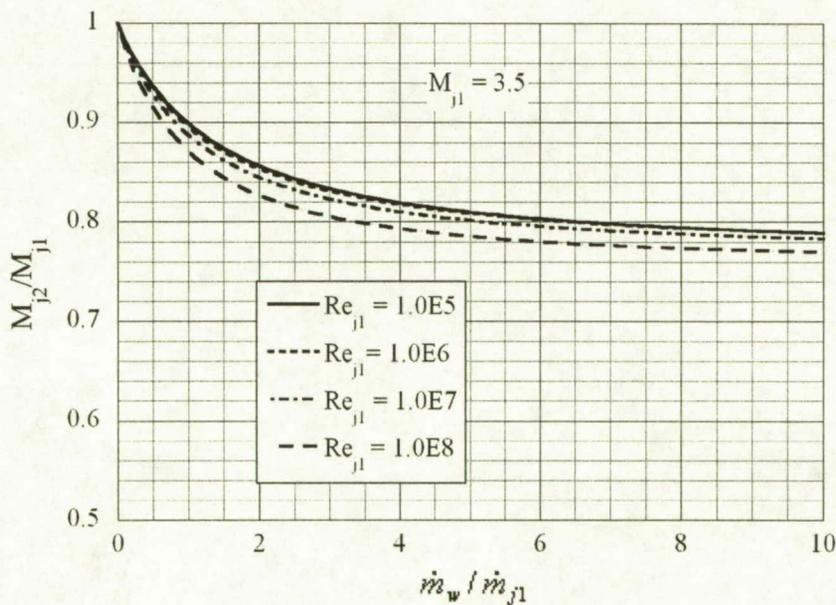
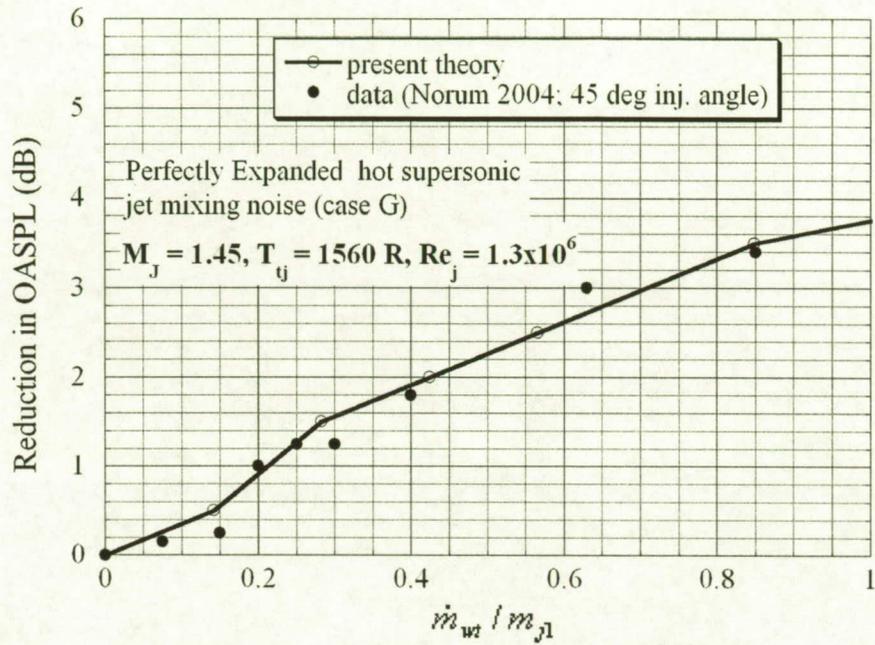Fig. 5c Dependence of effective jet Mach number on jet Reynolds number.

Fig. 6 Comparison of the predictions with data of Norum (2004) for turbulent mixing noise reduction due to water injection.

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14. ABSTRACT
A one-dimensional control volume formulation is developed for the determination of jet mixing noise reduction due to water injection. The analysis starts from the conservation of mass, momentum and energy for the control volume, and introduces the concept of effective jet parameters (jet temperature, jet velocity and jet Mach number). It is shown that the water to jet mass flow rate ratio is an important parameter characterizing the jet noise reduction on account of gas-to-droplet momentum and heat transfer. Two independent dimensionless invariant groups are postulated, and provide the necessary relations for the droplet size and droplet Reynolds number. Results are presented.

15. SUBJECT TERMS
jet mixing noise reduction, water injection, noise reduction, one-dimensional control volume formulation

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