On the scaling laws and similarity spectra for jet noise in subsonic and supersonic flow

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ABSTRACT

The scaling laws for the simulation of noise from subsonic and ideally expanded supersonic jets are reviewed with regard to their applicability to deduce full-scale conditions from small-scale model testing. Important parameters of scale model testing for the simulation of jet noise are identified, and the methods of estimating full- scale noise levels from simulated scale model data are addressed. The limitations of cold-jet data in estimating high-temperature supersonic jet noise levels are discussed. New results are presented showing the dependence of overall sound power level on the jet temperature ratio at various jet Mach numbers. A generalized similarity spectrum is also proposed, which accounts for convective Mach number and angle to the jet axis.

NOMENCLATURE

A_j - jet cross sectional area

c - sound velocity

d_j - jet exit diameter, characteristic length

f - frequency

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F_t - thrust

I - sound intensity ($I = \overline{p^2} / \rho c$)

I' - normalized acoustic far field intensity

L - characteristic length scale of eddies

m - mass flow rate

M - Mach number

p - pressure

P - sound power ($P = 4\pi r^2 I$)

P' - sound power per unit volume

r - distance from the sound source

R - gas constant

Re - Reynolds number ($\mathrm{Re} = \rho_j u_j d_j / \mu_j$)

s - entropy

St - Strouhal number ($St = f d_j / u_j$)

T - temperature

u - velocity

u_c - convective velocity

x - axial distance from the nozzle exit plane

y - radial distance from the jet axis

v_i - turbulent velocity fluctuation

W_m - mechanical power

δ_{ij} - Kronecker delta

μ - dynamic viscosity

ρ - density

γ - isentropic exponent

θ - angle from the jet axis (downstream)

η - acoustic efficiency

ω_f - characteristic circular frequency of the eddies

SUBSCRIPTS

av - average

c - chamber condition, convective

j - jet

p - peak

ref - reference condition

∞ - ambient fluid

1. INTRODUCTION

Noise from subsonic jets is mainly due to turbulent mixing, according to the early (original) theoretical model of Sir James Lighthill.[1-2] The turbulent mixing noise is primarily broadband. In perfectly expanded supersonic jets (nozzle exit plane pressure equals the ambient pressure), the large-scale mixing noise manifests itself primarily as Mach wave radiation[3] caused by the supersonic convection of turbulent eddies with respect to the ambient fluid. In imperfectly expanded supersonic jets, additional noise is generated on account of broadband shock noise emanating from shock-turbulence interaction[4] and screech tones[5], with the tonal amplitude likely occasioned by shock-acoustic wave interaction.[6]

Scale models are often used in early design stage as a means of predicting the acoustic environment associated with flight vehicles. A detailed knowledge of the mechanisms of noise generation and noise radiation by jets is essential in designing a scale model of the noise source.[7] In order to ensure complete similarity between model and full scale, we need to satisfy similarity of flow, noise generation, and noise propagation. For a fuller discussion of the underlying physical mechanisms of jet noise, especially of sound generation, the following references may be consulted: Crighton[8,9], Howe[10], Dowling[11], and Ribner[12]. For recent work on the sources of jet noise, Bogey and Bailly[13], and Tam et al.[14] may be consulted.

In practice, it is generally difficult to duplicate (simulate) *all* the characteristic parameters in the scale model. Model testing with even smaller rocket engines requires extensive safety precautions. Heated jet facilities also involve considerable complexity and cost. The use of less expensive facilities or lower gas temperatures, for example, would considerably simplify model testing.[7] The ability to conduct a scale model test with a substitute gas (air, nitrogen, helium, etc.) results in substantial savings (reduced costs of test facilities, test time) and advantages. These substitute gas tests entail some compromise of the actual physics of the hot jet.

In view of the difficulties associated with the matching of the dimensionless parameters of the scale model and the full scale, an understanding of the functional relationship respecting the various parameters is requisite for the interpretation of scale model data to predict the full-scale environment. Scaling laws for jet noise thus represent a topic of great practical interest. The purpose of this paper is to examine the scaling laws for simulating noise from both subsonic jets and ideally expanded supersonic jets in both cold and hot flow on the basis of both theoretical considerations and experimental facts. More general results will be presented for the effect of jet Mach number and jet temperature and on the overall sound pressure level, and for the similarity spectrum of jet noise. A significant portion of this work is derived from Ref. 15.

This investigation is concerned with hot and high speed jets from the point of view of noise generation only. Flow inhomogeneities (temperature, composition) could deform (refract and scatter) any sound wave passing through the jet. A discussion of the effects of refraction by the mean flow in the scaling laws is thus beyond the scope of the present article, and thus excluded from consideration here. It must be emphasized however that the refraction effects, exhibiting a dip in the overall sound pressure level (OASPL) near the jet axis, are important for high speed and hot jets.

2. DYNAMIC SIMILARITY

A schematic of the jet configuration (with an ambient medium at rest) is shown in Fig. 1. In general the sound pressure is a function of several variables

$$p = p\left(u_j, c_j, \rho_j, T_j, d_j, \mu_j, u_\infty, c_\infty, \rho_\infty, T_\infty, f, r, \theta\right) \tag{1}$$

From dynamic similarity considerations, the far-field mean square sound pressure can be expressed in a dimensionless form as

$$\frac{\overline{p^2}}{\left(\rho_j u_j^2\right)^2} = \phi\left[M_j, \frac{c_j}{c_\infty}, \frac{\rho_j}{\rho_\infty}, \frac{T_j}{T_\infty}, \frac{d_j}{r}, St, \theta, \mathrm{Re}, \frac{u_\infty}{c_\infty}\right] \tag{2}$$

In the above, the jet Mach number M_j, the Strouhal number St, and the jet Reynolds number Re are defined by

$$M_j = \frac{u_j}{c_j}, \quad St = \frac{fd_j}{u_j}, \qquad \mathrm{Re} = \frac{\rho_j u_j d_j}{\mu_j} \tag{3a}$$

where sound speeds c_j and c_∞ in the jet and the ambient are defined by

$$c_j = \sqrt{\gamma_j R_j T_j}, \qquad c_\infty = \sqrt{\gamma_\infty R_\infty T_\infty} \tag{3b}$$

In view of Eq. (4) Eq. (2) can be expressed as

$$\frac{\overline{p^2}}{\left(\rho_j u_j^2\right)^2} = \phi\left[M_j, \frac{c_j}{c_\infty}, \frac{\rho_j}{\rho_\infty}, \frac{d_j}{r}, St, \theta, \mathrm{Re}, \frac{u_\infty}{c_\infty}\right] \tag{4}$$

In the present article, the jet Reynolds number is assumed sufficiently high, so that the effects of boundary layer thickness at the nozzle exit on the radiated sound field are considered unimportant, and thus the Reynolds number effects will be left out of account. Also, this work is principally concerned with a stationary ambient, so that the parameter u_∞ / c_∞ representing flight effects does not enter into further consideration.

3. MECHANISMS OF NOISE GENERATION

3.1 Isothermal Jets

3.1.1 *Lighthill's theory for subsonic jets*

Sir James Lighthill [1,2] has shown by an acoustic analogy that aerodynamic sound is a consequence of turbulence, which provides a quadrupole source distribution for noise radiation in an ideal gas at rest. The dominant effect of steady low-speed solenoidal convection has been accordingly developed in terms of an inhomogeneous wave equation (derived on the basis of continuity and momentum equations) of the form[16]

$$\frac{\partial^2 \rho}{\partial t^2} - c_\infty^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \, \partial x_j} \tag{5a}$$

where the LHS represents the acoustic wave propagation, and the RHS (involving two space derivatives) contains the quadrupole sources that generate the noise field. The quantity T_{ij} is the Lighthillian acoustic tensor

$$\mathrm{T}_{ij} = \rho v_i v_j + \left(p - c_\infty^2 \rho\right)\delta_{ij} + \tau_{ij} \tag{5b}$$

where v_i is the velocity (turbulent velocity fluctuation), p the local pressure, and τ_{ij} the viscous stress tensor.

Here the first term, representing the contribution of Reynolds stress (or convective momentum flux) models the generation of sound by turbulence. The second term in general models the generation of sound by fluid inhomogeneities (such as of temperature), and has a dipole character. The temperature inhomogeneities are important as sound sources (also for refraction). If temperatures in the flow are not very different from those outside, the differences between c and c_∞ will be small, and the second term can be neglected[8]. This is so since $c = \left(\partial p / \partial \rho\right)_s$ and thus $\nabla p - c_\infty^2 \rho = 0$. For high speed jets (high jet Mach number) compressibility effects involving the mean density gradients lead to the appearance of monopole (volume) sources. The last term, modeling the viscous dissipation of sound, is represented by[8]

$$\tau_{ij} = -2\mu\left(e_{ij} - \frac{1}{3} e_{kk}\delta_{ij}\right) \tag{5c}$$

where $$e_{ij} = \left(\partial u_i / \partial x_j + \partial u_j / \partial x_i\right)/2, \quad e_{kk} = \partial u_k / \partial x_k \tag{5d}$$

In the above expression, the quantity e_{ij} stands for the rate of strain tensor, and e_{kk} the divergence of velocity, and μ the dynamic viscosity. If the jet Reynolds number is very high (usually the case, as considered here), the viscous contribution to T_{ij} becomes samll[8].

Generally speaking, only the first term in Eq. (5b) is thus dominant in cold flow (no marked temperature differences exist) and thereby retained:

$$T_{ij} = \rho v_i v_j \tag{5c}$$

If the flow Mach number is small (low subsonic range), the quantity ρ may be replaced by the mean density of the jet[8]. Lighthill[2] obtained a formal solution of Eqs. (5a) and (5c) with the aid of Green's functions. By the application of dimensional analysis to the formal solution, the acoustic power from isothermal subsonic jets is theoretically shown to be

$$P = K\rho_\infty u_j^8 c_\infty^{-5} d_j^2 \tag{6}$$

where K is a proportionality constant, called the *acoustic power coefficient.*[2] This relation is the celebrated Lighthill's u_j^8 law for subsonic jets. Subsonic cold-jet data confirm the u_j^8 dependence (with $K \approx 3\text{x}10^{-5}$), as seen in Fig. 2, which is adapted from Ffowcs Williams,[3] as reproduced from Chobotov and Powell.[17] The acoustic efficiency for subsonic cold jets is thus expressed as

$$\eta = \frac{P}{W_m} = \frac{\text{acoustic power}}{\text{jet mechanical power}} \propto u_j^5 \tag{7}$$

where the jet mechanical power W_m can be expressed in terms of the thrust F_t as

$$W_m = 0.5F_t u_j, \quad F_t = m_j u_j = \rho_j u_j^2 A_j = \rho_j u_j^2 \left(\frac{\pi}{4} d_j^2\right) \tag{8}$$

with the expression for the thrust applicable for perfectly expanded jets.

By Lighthill's theory, a dipole source, such as a temperature inhomogeneity, radiates as the sixth power. An approach similar to Lighthill's theory shows that a monopole source radiates as the fourth power.

3.1.2 Effect of source convection

The above theory holds only for stationary sources. Since quadrupoles are convecting downstream, the effect of moving sources on the direction of noise radiation becomes important and is accounted for by a convection factor, as first shown by Ffowcs Williams[3] and Ribner[18]

$$P(\theta) = K\rho_\infty u_j^{7.5} c_\infty^{-5} d_j^2 C^{-5} \tag{9}$$

where

$$C(M_c, \theta) = \left[(1 - M_c \cos\theta)^2 + \alpha^2 M_c^2\right]^{1/2} \tag{10}$$

with the factor C referring to a generalized Doppler factor for a source of finite length. Here θ is the angle from the jet axis, M_c the convection Mach number, and α accounts for finite decay time of the eddies. The convection Mach number M_c is defined as the effective Mach number of the convecting turbulent eddies in the mixing regions, and is defined by $M_c = u_c / c_\infty$, where u_c is the convective velocity of the eddies. For a stationary ambient, it can be expressed as:

$$M_c = M_c(M_j, c_j / c_\infty) \tag{11a}$$

The quantity α is defined as

$$\alpha^2 = \omega_f^2 L^2 / (\pi c_\infty^2) \approx const \tag{11b}$$

where ω_f and L represent the characteristic frequency and length scale of the eddies, respectively. The quantity $M_c \cos\theta$ represents the component of convective Mach number in the radiation direction[8]. The Doppler factor $(1 - M_c \cos\theta)$ for a point source predicts zero wavelength at the Mach angle (Crighton[8]). Lighthill[19] originally suggested a form of Eq. (10) with $\alpha = 0$, which corresponds to the limiting case of very large decay times for the eddies (or frequency approaching zero).

An integration of the sound power over all solid angles yields that[12]

$$P = \frac{1}{4\pi}\int_0^{\pi} P(\theta)\, 2\pi \sin\theta \, d\theta \tag{12}$$

so that

$$P = K\rho_\infty u_j^{7.5} c_\infty^{-5} d_j^2 \left(C^{-5}\right)_{av} \tag{13}$$

where

$$\left(C^{-5}\right)_{av} = \frac{1}{2}\int_0^{\pi} C^{-5} \sin\theta \, d\theta = \frac{1}{3M_c^5}\left[\psi(z_1) - \psi(z_2)\right] \tag{14}$$

represents the mean amplification factor[12]. In Eq. (14), we define

$$z_1 = 1 - M_c, \quad z_2 = 1 + M_c$$

and

$$\psi(z_i) = \frac{z_i\left(2z_i^2 + 3\alpha^2 M_c^2\right)}{\left(z_i^2 + \alpha^2 M_c^2\right)^{3/2}}, \quad i = 1,2$$

Fig. 3 shows the variation of the mean convective amplification factor with M_c for a typical value of $\alpha = 0.4$, as furnished by Eq. (14). This amplification factor is seen to slowly increase with M_c in the subsonic range, providing a u_j^8 dependence in the low speed region.[12] It ultimately approaches M_c^{-5} dependence at high Mach numbers.

A polar plot of the variation of directivity of jet noise (relative to $\theta = 90$ deg where convection and refraction effects are absent) at various convective Mach numbers is exhibited in Fig. 4a, as obtained from Eq. (10). Fig. 4b presents a linear plot of the directivity of the jet noise considered in Fig. 4a. The directivity at increased Mach numbers is clearly evident. Experimental data suggest the existence of a refractive dip close to the downstream jet axis as a result of refraction by the mean flow, which can be important for high speed and hot jets. Directivity (and spectral) effects of shear-noise (due to joint contribution of turbulence and mean flow) are also not considered here, and only self-noise due to turbulence is accounted for. The shear noise arises from the interaction between the mean shear and the transverse velocity fluctuations, and the self-noise arises solely from the turbulent fluctuations (Crighton[8]). Ribner[12] accounted for the shear noise directivity through a factor $\left(1+\cos^4\theta\right)$, which represents a relatively small contribution (maximum of 3 dB) to the overall directivity.

3.1.3 *Supersonic jets*

An examination of Fig. 2 suggests that the $u_j^8 d_j^2$ law of Lighthill for subsonic flow breaks down at high exhaust velocities, where the convection velocities of the eddies in the turbulent mixing region approach supersonic values. At the high exhaust velocities of present day rocket engines, this law predicts a physically unrealistic result that over 100% of the jet propulsive power is converted to noise.[20] Experimental data reveal a u^6 dependence of the sound power level for supersonic jets of low Mach numbers ($M_j = 1$ to 1.5). At higher Mach numbers, a u_j^4 dependence is noted by the measurements at Mach 2.5 (Ref. 21), as reviewed by Sutherland.[22] These trends are consistent with the measurements by Cole et al.,[23] as reviewed by McInerny.[24] The sixth-power (dipole character) and fourth-power (monopole character) laws correspond to temperature and volume sources (e.g., compressibility effects, combustion) respectively. At still larger Mach number (in excess of about 2.5), a u_j^3 dependence of sound power level is observed[3]. The u_j^3 dependence of sound power implies a constant acoustic efficiency independent of jet velocity. Some recent data by Tam et al.[14] suggest that the u_j^3 law for supersonic jets is not fully supported by the measurements.

Ffowcs Williams[3] extended Lighthill's theory for high-speed solenoidal convection and predicted a u^3 dependence of jet noise at high supersonic Mach numbers. A u_j^3 dependence is also indicated by Tam.[25] These predictions are in qualitative agreement with the data at high supersonic flow. The departure from the u_j^8 dependence at supersonic speeds may be partly attributed to compressibility effects (monopole sources), causing a reduction in source strength.[26] At increased Mach numbers, there is a reduction of transverse velocity fluctuations in the mixing layer, as indicated by the data of Goebel and Dutton[27] and predicted in Kandula and Wilcox[28].

Phillips[29] proposed an asymptotic theory for very high values of u_j / c_∞, according to which the acoustic efficiency must ultimately diminish like $u_j^{-3/2}$ or the sound power as $u_j^{3/2}$. There is very limited data at very high Mach numbers to provide a validation of this theory. Also at very high Mach numbers (characteristic of hypersonic regime), real gas effects and property variations can become significant such that the accuracy of the theory is rendered questionable.

Based on the foregoing discussion, we may roughly summarize the sound power level dependence on velocity in supersonic flow as follows:

$$P \propto \begin{cases} u_j^6 & 1.0 < M_j < 1.5 \\ u_j^4 & 1.5 < M_j < 2.5 \\ u_j^3 & 2.5 < M_j < 5 \end{cases} \tag{15}$$

Sutherland[22,30] proposed the following expression based on a physical model for the OAPWL from supersonic jets as

$$P = 90.8 + 10\log(GW_m) \tag{16a}$$

where G is a flow dependent function and W_m is the mechanical power (see Eq. 8), with G given as

$$G = \mathrm{T}\left(c^* / u_j\right)^2 \tag{16b}$$

Here T represents the ratio of isentropic exponents $\gamma_{\mathrm{tip}} / \gamma_\infty$ (the subscript tip refers to supersonic tip core). The quantity c^* is the critical sound velocity (corresponding to $M = 1$), which is the flow velocity at the end of the supersonic tip, defined by

$$\frac{c^*}{u_j} = \left[\left(\frac{\gamma+1}{\gamma-1}\right)\left\{1 - \left(\frac{p_\infty}{p_t}\right)^{(\gamma-1)/\gamma}\right\}\right]^{-1/2} \left(C_v\right)^{-1} \tag{16c}$$

where C_v denotes the velocity coefficient of nozzle (typically about 0.98). The model is based on the conception that the dominant sound source for supersonic jet flow is close to and downstream of the supersonic tip. The acoustic efficiency is shown to be proportional to G, with 0.5 percent representative of present day rocket engines. Excellent agreement is achieved between the measured acoustic power from supersonic jets and rockets (GW_m in the range of 10^4 to 10^{11} W).

3.1.4 *Spectral distribution*

Powell[31] first derived a similarity law for sound power spectrum of the form

$$\begin{aligned} P(f) &\sim \frac{\bar{\rho}^2 u_j^8 d_j^2}{\rho_\infty c_\infty^5}\left(f / f_p\right)^2 \quad & 0 \le f \le f_p \\ &\sim \frac{\bar{\rho}^2 u_j^8 d_j^2}{\rho_\infty c_\infty^5}\left(f / f_p\right)^{-2} \quad & f_p \le f \le \infty \end{aligned} \tag{17}$$

where f_p represents the peak frequency value, $\bar{\rho}$ the local time-averaged density. The f^{-2} law is connected with the initial shear layer region (dominated by high frequencies) between the nozzle exit and somewhat upstream of the end of the potential core ($x / d_j \approx 4$), while the f^2 law corresponds to the region beyond about $x / d_j \approx 8$, where there is fully developed turbulent flow in the jet.

In the transition zone centered around the end of the potential core, none of the laws is considered to apply[8].

Ribner[14] proposed for the self noise a semi-empirical spectrum of the form (ignoring convection effects)

$$P(f) = \frac{\bar{\rho}^2 u_j^8 d_j^2}{\rho_\infty c_\infty^5} H(\nu) \tag{18a}$$

where
$$H(\nu) = \frac{\nu^2}{\left(1+\nu^2\right)^2}, \qquad \nu = f / f_p \tag{18b}$$

which provides the asymptotic behavior according to Eq. (17).

On the basis of a detailed study of jet noise data from NASA Langley for sound power spectra for both hot and cold jets[32], Tam et al.[33] identified two distinct components of jet mixing noise (due to fine scale and large scale structures) from supersonic jets. Accordingly they proposed the existence of two universal (but empirical) similarity spectrum functions F and G, such that the overall jet noise spectrum is expressed as

$$S = \left[AF(f / f_L) + BG(f / f_f)\right] \left(d_j / r\right)^2 \tag{19}$$

where $F(f / f_L)$ is a spectrum for the large-scale turbulence/instability waves (characteristic of Mach wave radiation), and $G(f / f_f)$ is the spectrum for the fine-scale turbulence. The frequencies f_L and f_f correspond respectively to the peaks of the large-scale turbulence and fine-scale turbulence. These spectrum functions are normalized such that $F(1) = G(1) = 1$. Empirical correlations are proposed for the amplitudes A and B, and the peak frequencies of the two independent spectra as a function of the jet operating parameters $u_j / c_\infty, T_c / T_\infty$ and the inlet angle θ (based on the data of Seiner et al.[32] for M_j=2 for hot and cold jets). It is shown that the noise due to large-scale structure is dominant at small angles to the jet axis and that the fine-scale structure is dominant in the forward quadrant.

Fig. 5 displays the two similarity spectra of Tam for large-scale turbulence noise and fine-scale turbulence noise. For comparison purposes, the empirical spectrum due to Ribner[14] is also presented. It is interesting to note that the Ribner spectrum matches well with the large-scale turbulence noise spectrum due to Tam for frequencies below the peak frequency, whereas it compares better with Tam's fine-scale turbulence noise spectrum beyond the peak frequency except at very large frequencies. The intersection of Tam's large scale and small scale spectra at high frequencies points to a difficulty from the point of view of sound generation. It is now known that the relatively rapid decay of the small scale spectrum at high frequencies is connected with the atmospheric attenuation effects embodied in the original data from which the spectra are generated[34,35].

Sutherland[7] proposed a best-fit prediction model for the octave band sound power spectrum applicable to actual rocket exhausts, including those of clustered nozzles where neighboring jets interfere with each other.

3.2 Heated Jets

3.2.1 *Experimental considerations*

Recently there has been a surge of interest in obtaining acoustic data on hot jets, demonstrating the importance of entropy noise (dipole noise associated with fluid inhomogeneities), though there exists considerable debate on several aspects with regard to the differences between cold and hot jets; this is so in spite of the existence for two or three decades of theories of jet noise accounting for temperature effects, which can be compared or improved with reference to the more recent experimental data. The development of CAA (Computational Aeroacoustics), perhaps in conjunction with large eddy simulation (LES) or direct numerical simulation (DNS), can be a further tool in understanding the differences between cold and hot jets.

In recent times, data on hot jets are reported in Refs. 32, 35 and 37. While in commercial transport applications (turbojets), the jet static temperature is of the order of 800 K (M_j=0.6 to 0.9), the static temperatures in rocket exhausts are considerably higher and are of the order of 1500 K (M_j=2.5 to 3.5). Although cold air jets can be used to determine differences in a noise field due to geometric changes, the use of cold air jets to establish absolute values of a full-scale noise field is considered not feasible.[7] Cold air tests are thus good to indicate qualitative differences in the acoustic field but are only indicative of the order of magnitude of the actual phenomena of noise reduction.

Data on scale models generally suggest a 5-10 dB difference between cold- and hot-jet tests. According to a review by Fisher et al.,[38] for a constant mean velocity of the jet, the acoustic levels increase with an increase in mean temperature of the jet for $M_j < 0.7$ and decrease with an increasing temperature if $M_j > 0.7$. According to the data of Narayanan et al.,[39] for subsonic jets ($0 < M_j < 0.9$) at a fixed jet velocity, an increase in jet temperature diminishes the sound power level (Fig. 6). Thus there appears to be some uncertainty in the trends of temperature effect on noise at a constant jet velocity. On the other hand, at a given jet Mach number M_j, an increase in jet temperature enhances the sound pressure level as shown by Morgan et al. [7] (see Fig. 7). Kinzie and McLaughlin[40] note that significant differences exist between noise from moderately heated supersonic jets and unheated supersonic jets.

Heated air data of Seiner et al.[32] at M_j=2 suggest that the peak OASPL is higher at higher temperatures (6 dB increase as T_j increases from 313 to 1534 K), as demonstrated in Fig. 8. The peak angle of emission (angle to the inlet axis) decreases with an increase in jet temperature, as a result of an increase in convective Mach number. Experiments by Tanna[41] for cold and hot subsonic and supersonic jets show that the spectral content of noise from hot jets is fundamentally different from that of cold jets. As indicated by the data of Fortune and Gervais[36], there is a significant variation in peak frequency and amplitude as the jet temperature increases. The peak frequency diminishes as the jet temperature rises.[35,36] Viswanathan[35] reported jet noise data for a range of jet diameters, jet Mach number and jet total temperature ratio, and observed important differences in noise characteristics from hot and cold jets.

3.2.2 *Analyses and correlations*

In the presence of density differences between the jet fluid and the ambient fluid (such as helium jets in air), the corresponding acoustic power is proposed by Lighthill[2] as

$$P = K\rho_j^2 \rho_\infty^{-1} u_j^8 c_\infty^{-5} d_j^2 \tag{20}$$

since the Lighthill's stress tensor contains a factor ρ_j. With regard to the role of jet temperature, Lighthill points out that inhomogeneities in temperature amplify the sound due to turbulence, just as shear affects high-frequency components of the jet noise. According to Lighthill, the effects of velocity and temperature cannot be separated.

Mani[42,43] has shown, with the aid of a slug flow approximation, that mean density gradients act to generate dipole and monopole source terms, which produce respectively M^6 and M^4 dependence at high jet temperatures for constant value of $T_j - T_\infty$, where M denotes the ratio of jet velocity to ambient sound speed. The sixth and fourth power laws respectively correspond to temperature and volume sources (e.g. combustion), as indicated earlier.

Morfey et al.[26] developed scaling laws for both quadrupole and dipole components of turbulent mixing. They proposed an additional mixing noise due to dipole source at high jet temperatures and suggested the following relation for the normalized acoustic far-field intensity I':

$$I' = K_1\left(\frac{\rho_j}{\rho_\infty}\right)\left(\frac{c_j}{c_\infty}\right)^{-4}\left(\frac{u_j}{c_\infty}\right)^8 + K_2\left(\frac{\rho_j}{\rho_\infty}\right)\left(\frac{c_j}{c_\infty}\right)^{-2}\left(\frac{\Delta T}{T_j}\right)\left(\frac{u_j}{c_\infty}\right)^6 \tag{21a}$$

where $$I' = \frac{\overline{p^2}}{\rho_\infty^2 c_\infty^4}\left(\frac{r}{d_j}\right)^2, \quad \Delta T = T_j - T_\infty \tag{21b}$$

The dipole term is based on theoretical considerations of sound generation by convected density in-homogeneities. It is suggested that, in order to generalize the prediction scheme, the temperature ratio T_j / T_∞ be replaced by $(\rho_j / \rho_\infty)^{-1}$, the density ratio being the dynamically significant quantity.

On similar grounds, Liley[44] proposed the existence of an additional dipole source term arising from density fluctuations (due to temperature fluctuations), and suggested that the sound power per unit volume of turbulence can be expressed as

$$P'' = \alpha_p \frac{\rho_j^2}{\rho_\infty}\frac{u_j^8}{c_\infty^5 d_j} + \alpha_H \frac{\rho_j^2}{\rho_\infty}\frac{u_j^6}{c_\infty^3 d_j} \tag{22}$$

Tam et al.[33] proposed correlations for the peak sound pressure level (at 90 deg to the jet axis) for the large-scale turbulence and fine-scale turbulence as follows. For the large-scale turbulence, the correlation for the amplitude A (in dB/Hz) in Eq. (19) is proposed as

$$10\log\left(A / p_{ref}^2\right) = 75 + \frac{46}{(T_c / T_\infty)^{0.3}} + 10\log\left(u_j / c_\infty\right)^n \tag{23}$$

where $$n = 10.06 - 0.495\left(T_c / T_\infty\right).$$

The amplitude for the fine-scale turbulence, B (in dB/Hz) is recommended as

$$10\log\left(B / p_{ref}^2\right) = 83.2 + \frac{19.3}{\left(T_c / T_\infty\right)^{0.62}} + 10\log\left(u_j / c_\infty\right)^n \qquad (24)$$

where $n = 6.4 + 1.2/\left(T_c / T_\infty\right)^{1.4}$.

It is seen from the above relations, namely Eqs. (23) and (24), that the jet temperature has strong effect on the velocity component. In the case of large-scale turbulence, the velocity exponent n for cold jets ($T_c / T_\infty = 1$) is approximately equal to 9.5, which is somewhat larger than 8, as predicted by the Lighthill's acoustic analogy. In the case of fine-scale turbulence, the velocity exponent n reduces to 7.6 for cold jets ($T_c / T_\infty = 1$), in very close agreement with the subsonic jet value of 8, as predicted by Lighthill's theory. At a jet temperature ratio of 2, the value of the exponent reduces to 6.85. Recent data by Tam et al.[14] also suggest that the u_j^8 law is true only for cold subsonic jets, and is not valid for hot jets.

Massey et al.[45] suggested a correction factor for the temperature effects, as based on their data over a range of M_j =0.6 to 1.2 for rectangular jets issuing from converging nozzles:

$$OASPL - 10\log\left[\frac{\rho_j \rho_\infty}{\rho_{STP}}\right]\left(\frac{T_\infty}{T_{STP}}\right)^2\left(\frac{A_j}{r^2}\right) = f_1\left[\log\left(\frac{u_j - u_\infty}{c_\infty}\right)\right] \qquad (25)$$

where A_j represents the jet cross sectional area, and STP refers to the standard conditions.

Tam[46] proposed a jet noise scaling formula developed on the basis of dimensional analysis and the Buckingham π theorem. The formula is applied to data over a wide range of temperature and jet acoustic Mach number. The velocity exponent for the sound power is correlated with the jet total temperature ratio T_t / T_∞ and the angle from the jet axis, i.e., $n = n(T_t / T_\infty, \phi)$, where $\phi = 180 - \theta$. Viswanathan[47] proposed some new scaling laws for hot and cold jets. The scaling laws are based on the consideration that the velocity exponent for the sound power depends on the jet total temperature ratio and the angle ϕ.

Calculations by the author, using OVERFLOW Navier-Stokes CFD code,[48] have shown that the length of the supersonic core decreases with an increase in jet temperature, at a constant jet Mach number of 2 and constant ambient temperature (Fig. 9). This suggests that an increase in jet temperature not only introduces dipole sources but also alters the quadrupole source distribution by shortening the core length. Recent data on hot jets[37] reveal that although the potential core length is shortened in heated jets, only marginal changes relative to cold jets are manifested with regard to turbulence intensity, length and time scales of turbulence.

According to Dowling et al.[11], if the jet density is much lower than that that of the ambient the mean flow-acoustic interaction effects become important resulting in considerable amplification of the quadrupole field, and the sound intensity can scale to a lower power of the jet speed.

4. PROPOSED SCALING LAWS

4.1 Sound power

Based on a detailed study of the above considerations concerning experimental data and theoretical analyses, refinements to the scaling laws for jet noise are proposed as follows. In accordance with Lighthill-Ffowcs Williams-Ribner formulations, the following expression for the mean square sound pressure is proposed for both subsonic and supersonic flow within the framework of Eq. (4):

$$\frac{\overline{p^2}(\theta,f)}{\left(\rho_j u_j^2\right)^2} = K_1\left(\frac{\rho_\infty}{\rho_j}\right)\left(\frac{u_j}{c_j}\right)^{3.5}\left(\frac{c_j}{c_\infty}\right)^{3.5}\left(\frac{d_j}{r}\right)^2 G_1(M_c,\theta)G_2\left(\frac{f}{f_p},M_c,\theta\right) \tag{26}$$

where K_1 is a proportionality constant, G_1 is the directivity factor (owing to source convection), and G_2 accounts for the spectral distribution of the sound power. The directivity factor is essentially the same as given by Eq. (10) in accordance with Lighthill-Ffowcs Williams-Ribner formulation:

$$G_1 = C^{-5} = \left[(1 - M_c \cos\theta)^2 + \alpha^2 M_c^2\right]^{-5/2} \tag{27}$$

where a value of $\alpha = 0.4$ is considered here.

The convective Mach number M_c may be related to the jet Mach number as

$$\begin{aligned} M_c &= M_j / (1 + c_\infty / c_j) \\ &= M_j / \left[1 + \sqrt{(\gamma_\infty R_\infty T_\infty)/(\gamma_j R_j T_j)}\right] \end{aligned} \tag{28}$$

Eq. (28) is based on the result for symmetric convective Mach number corresponding to the case of supersonic instability waves in the shear layer[49]. This symmetric Mach number is a measure of the overall compressibility of the jet[49]. It differs somewhat from the expression for M_c in the case of Kelvin-Helmholtz instability waves.[32] An examination of Eq. (28) suggests that at increased jet temperatures (typical of practical applications), M_c depends primarily on M_j rather than the parameter u_j / c_∞, which is commonly chosen in the presentation of data on jet noise. A similar view is also expressed in Ref. 49. The choice of M_j as an appropriate scaling parameter is also consistent with its gas dynamic significance and its important role in imperfectly expanded supersonic jets.

Eq. (26) can also be recast alternatively as

$$\frac{\overline{p^2}(\theta, f)}{p_{ref}^2} = K_1 \left(\frac{c_j^4 \rho_j^2}{p_{ref}^2}\right) \left(\frac{\rho_\infty}{\rho_j}\right) \left(\frac{u_j}{c_j}\right)^{7.5} \left(\frac{c_j}{c_\infty}\right)^{3.5} \left(\frac{d_j}{r}\right)^2 G_1(M_c, \theta)\, G_2\left(\frac{f}{f_p}, M_c, \theta\right) \tag{29}$$

For an ideal gas, the density ratio is related to the temperature ratio by

$$\rho_j / \rho_\infty = T_\infty / T_j \tag{30}$$

Thus for an ideal gas, the temperature dependence of mean square sound pressure at a constant value of jet Mach number can be characterized as

$$\left(\overline{p_1^2} / \overline{p_2^2}\right) = \left(T_{j1} / T_{j2}\right)^{2.75} \frac{\left(C_1^{-5}\right)_{av}}{\left(C_2^{-5}\right)_{av}}, \quad M_j = const. \tag{31a}$$

where the subscripts 1 and 2 relate to flow conditions 1 and 2 respectively. On the other hand, for a constant jet velocity, the ratio of mean square sound pressure becomes

$$\left(\overline{p_1^2} / \overline{p_2^2}\right) \approx \left(T_{j2} / T_{j1}\right) \frac{\left(C_1^{-5}\right)_{av}}{\left(C_2^{-5}\right)_{av}}, \quad \frac{u_j}{c_\infty} = const. \tag{31b}$$

In Eqs. (31a) and (31b), the quantity $\left(C^{-5}\right)_{av}$ is obtained from Eq. (14), and any differences in the spectral distribution through the factor G_2 are ignored. This ignored difference does not seem to be appreciable (of the order of about 2 dB) in view of the fact that the shape of the spectrum is not significantly altered by the temperature ratio[36]. Eqs. (31a) and (31b) illuminate the fundamental difference in temperature scaling of mean square sound pressure. Generally speaking, Eq. (31a) suggests an amplification of sound pressure with increased jet temperature for a fixed value of M_j, while Eq. (31b) suggests a decreased value of sound pressure with an increase in jet temperature for a constant value of the u_j / c_∞.

No direct (separate) effect of temperature on the directivity and spectral factors G_1 and G_2 are envisaged by the formulation in Eqs. (26) or (29), and the effect of temperature is only indirectly present in the convective Mach number according to Eq. (28).

4.2 Similarity spectrum

A single similarity spectrum is also proposed here to apply to both subsonic and supersonic flow and to account for both the fine-scale turbulence and turbulence structure associated with Mach wave radiation. A semi-empirical spectrum is proposed here as

$$G_2 = \frac{\left(f / f_p\right)^{5/4a}}{\left[1+\left(f / f_p\right)^2\right]^{9/6a}} \tag{32a}$$

where

$$a = \left[0.2 + \exp\left(-2M_c / \sin(\theta/2)\right)\right]^{0.35} \tag{32b}$$

and the quantity M_c is given by Eq. (28). The factor a is designed to accommodate the physical effects of the convective Mach number M_c and the angle θ on the similarity spectrum, as demanded by the dimensional arguments. It also closely satisfies the asymptotic limit provided by Kolmogorov's -5/3 law[50] for turbulence energy spectrum at large frequencies in the incompressible limit limit of $M_c \to 0$, where noise directionality effect is absent.

The proposed expression is based on Von Karman-type interpolation formula for isotropic turbulence as suggested by Saffman. Von Karman[51] originally proposed a spectrum for turbulence of the form

$$G_2 = \frac{\left(f / f_p\right)^4}{\left[1+\left(f / f_p\right)^2\right]^{17/6}} \tag{33a}$$

which covers the range between the permanent largest eddies of f^4-law (as $f \to 0$) and the Kolmogorov inertial subrange characterized by the $f^{-5/3}$ law at very large values of f. Following Saffman, considering a f^{-2} law at $k \to 0$, Hinze[52] suggests a modified Von Karman spectrum of the form

$$G_2 = \frac{(f/f_p)^2}{\left[1+(f/f_p)^2\right]^{11/6}} \tag{33b}$$

valid for the turbulent transport of both a vector (velocity and momentum) and a scalar quantity. In reality, the jet turbulence is not isoptropic[53]. However, the general agreement of the asymptotic forms of sound power spectrum of Powell[31](Eq. 17) based on the jet structure with the measured noise spectra suggests that the form of the spectrum presented by Eq. (32) on the basis of isotropic turbulence provides an approximate representation of the jet noise behavior.

The proposed spectrum for jet noise, given by Eq. (32), is of more general form, capable of describing the spectrum in terms of convective Mach number and accounting for the directivity effects. The import of the proposed spectrum is that at high convective Mach numbers the broadband noise spectrum degenerates to the relatively sharply-peaked spectrum characteristic of large-scale turbulence noise governing Mach wave radiation. It is seen that the present spectrum reduces to the following limiting forms:

$$\begin{aligned} G_2 &\sim \begin{cases} (f/f_p)^{1.174} & f \to 0 \\ (f/f_p)^{-1.643} & f \to \infty \end{cases} \qquad M_c \to 0 \\ G_2 &\sim \begin{cases} (f/f_p)^{2.197} & f \to 0 \\ (f/f_p)^{-3.074} & f \to \infty \end{cases} \qquad M_c \to \infty \end{aligned} \tag{34}$$

In the incompressible limit, the present spectrum compares to the well-known –5/3 law of Kolmogorov[50] at large frequencies. Recent Direct Numerical Simulation (DNS) data by Bodony and Lele[54] suggest that, at $M_j = 1.2$, the turbulence energy spectrum is roughly of the form $f^{-3.33}$ at large values of wave number.

With regard to the broadband noise spectrum due to fine scale turbulence, Morris and Farassat[55] recently presented a detailed comparison of predictions from acoustic analogy theories and by the method of Tam and Auriault[56] involving adjoint Green's function for the linearized Euler equation.

5. RESULTS AND COMPARISONS

5.1 Overall sound power level

The variation of OAPWL with the jet Mach number according to the proposed scaling is portrayed in Fig. 10 with the jet temperature ratio as a parameter. For convenience, the data are plotted with reference to the OAPWL value at $M_j = 1$ and $T_j / T_\infty = 1$ (isothermal jet). The isothermal result is obtained from Eq. (13), and the temperature effects are evaluated from Eq. (31a). The calculations correspond to the perfectly expanded jet. Fig. 10 suggests that, at a given M_j, OAPWL depends only on the temperature ratio, as indicated by Eq. (31a). As is to be expected, the OAPWL transitions from a u_j^8 dependence in subsonic flow to u_j^3 at large supersonic Mach numbers. Referring to the isothermal jet, an increase of sound power of about 56 dB is predicted as the jet Mach number increases from 0.2 to 1.0. Although the calculations are shown for jet Mach numbers of 10, they should be used with caution for jet Mach numbers in excess of about 5 (hypersonic flow) where real gas effects (including dissociation) can become significant.

Fig. 11 compares the predicted OAPWL dependence on u_j / c_∞ with the best fit data (curve) shown in Fig. 2, comprising scale models, jet engines and rocket engines. Although the data are scattered in view of the various jet temperature levels, the comparison is never the less regarded useful, as it is occasionally reported in the literature (e.g., Ref. 35). In the calculations, the reference value of OAPWL is taken as the measured data at $u_j = 61$ m/s (200 ft/s). Here $M_c = \beta u_j / c_\infty$, and c_∞ is taken as 351 m/s (1150 ft/s), and β is a constant. Results are shown for $\beta = 0.75$ and 0.85. It is seen that the prediction is not sensitive to the value of β in the subsonic range, and that β =0.85 matches satisfactorily in both subsonic and supersonic range. The value of $\beta = 0.85$ can be shown to correspond to $T_j / T_\infty = 2.07$ according to Eq. (28). Excellent agreement is observed between the data and the present model for u_j / c_∞ up to 0.6. A maximum error of 10 dB is noted in the transition region. This figure illustrates the fact (as indicated by Crighton[8]) that if the exhaust speed of a turbojet engine is merely doubled, the noise power emitted is increased by 24 dB.

A comparison of the OAPWL variation with the jet Mach number with the data of Viswanathan[35] is shown in Fig. 12. The data shown are taken in the range of M_j=0.5 to 1.24, and correspond to a total jet temperature ratio of 3.2. In this comparison, the present result is referred to the test data at M_j=0.5. The increasing departure between the theory and the data as M_j increases is related to the transition region that is under consideration. A maximum error of about 10 dB is noted at M_j=1.4.

In an effort to make a formal comparison of the Sutherland model[22] (see Eq. 16a) with the present scaling formula, the expression for GW_m has been transformed to the following form in terms of M_j and T_j / T_∞ as follows:

$$GW_m = \phi \frac{\left(1 + \frac{\gamma - 1}{2} M_j^2\right)^{5/2}}{M_j^2} \left(\frac{T_j}{T_\infty}\right)^2 \tag{35a}$$

where

$$\phi = \left(\frac{\pi}{4} d_j^2\right) \mathrm{T} \frac{2^{3/2}}{(\gamma + 1)^{5/2}} (C_v)^{-2} \rho_\infty c_\infty^3 \tag{35b}$$

Eq. (35a) suggests that for large values of M_j (say $M_j > 2$) provides that

$$GW_m \sim M_j^3 (T_j / T_\infty)^2 \qquad (35c)$$

The Mach number dependence conforms to the well-known trends in the supersonic regime, as the data and the present model suggest. The temperature exponent of 2 is interestingly close to the value of 2.75 indicated by the present model (Eq. 31a). An examination of Eq. (16a) suggests that in the case of subsonic jets it does not fully recover the u_j^8 relationship according to the Lighthill theory. This circumstance is perhaps connected with the fact that the Sutherland model is formulated with a physical relevance to the supersonic tip region.

5.2 Directivity

Fig. 13 exhibits a comparison of the theoretical directivity with the test data at M_c=0.42 and M_c=0.78. The data for M_c=0.42 is taken from Pinker and Bryce[57], and those for M_c=0.78 are taken form Pietrasanta[58]. The theory seems to satisfactorily agree with the data, except for jet angles less than about 40 deg, where there is substantial deviation on account of the effect of refraction by the mean flow inhomogeneities. The effects of refraction seem to be more pronounced at larger values of M_c, as indicated by the steepness of the OASPL data at shallow angles.

5.3 Overall sound pressure level

Comparison of the present theory with the OASPL data of Morgan et al.[7] for various jet Mach numbers at θ= 60 deg for both cold and hot jets is presented in Fig. 14. Here the data for the cold jet at M_j=1 is considered as the reference point for the theoretical prediction (see Fig. 7 for the original data). A reasonable agreement between the theory and the data is apparent for both the cold and the hot jets over the jet Mach number range considered. It is seen that, at M_j = 1 and a temperature ratio of 3 (jet total temperature increased from 289 K to 878 K), the observed increase in OASPL is about 13 dB, while the present scaling law provides a value of 15.1 dB. Overall, the maximum error in the theory is seen to be about 5 dB. Both the theory and the data show increased OASPL at higher jet temperatures at a constant jet Mach number. Referring to the subsonic data of Narayanan,[39] as seen in Fig. 6, at a constant value of $u_j / c_\infty = 0.891$, a drop of OASPL of about 4 dB is noted as the jet static temperature is increased form300 K to 811 K. This compares favorably with a predicted value of about 5.3 dB according to the present scaling law, Eq. (31b).

Fig. 15 illustrates a comparison of OASPL with jet temperature at Mach number $M_j = 2$. The data are taken from Seiner et al.[32] Comparisons are made at $\theta = 92$ deg and $\theta = 78$ deg. In assessing these comparisons, the data and theory are made to identical at a temperature ratio near $T_j / T_\infty = 1$ (isothermal). Good agreement is noted between the scaling law and the data over a wide range of temperature especially at $\theta = 92$ deg.

5.4 Similarity Spectrum

The variation of similarity spectrum (Eq. 32) with convective Mach number at a constant value of $\theta = 90$ deg is provided in Fig. 16. The results indicate that at large values of convective Mach number, the spectrum becomes closer to the characteristic (similarity) spectrum for large-scale turbulence. As the jet convective Mach number is reduced, the proposed spectrum becomes progressively broader and approaches the similarity spectrum for fine-scale turbulence. Spectral data of Massey et al.[45] at $\theta = 40$ deg for a Mach number range of 0.6 to 1.2 qualitatively support this trend.

A comparison of the dependence of the similarity spectra at $M_c = 0.5$ for various angles to the jet axis is shown in Fig. 17a. We see that as the angle is increased from 30 deg to 150 deg, the proposed spectrum shifts from a sharply-peaked profile to a spectrum with a relatively broader peak. The spectra are generally bounded by the two limiting (characteristic) similarity spectra. A similar comparison is presented for $M_c = 1.0$ in Fig. 17b. An examination of Figs. 17a and 17b reveals that the SPL height between $\theta = 30$ deg and 150 deg is narrower for $M_c = 1.0$ relative to that seen for $M_c = 0.5$.

The comparisons suggest that the narrowband spectrum characterizing the large-scale turbulence noise of Mach wave radiation is likely a perturbation from the fine-scale spectrum. Evidence to this effect is also found from the recent experimental data of Hileman and Samimy[60], which suggest a rather continuous transition from the narrowband spectrum to a broadband spectrum as the angle of a Mach 1.3 jet (convective Mach number of 0.6) is increased from 20 to 90 deg. This character of rather gradual transition of the noise spectrum with jet angle has also been remarked by other investigators[35].

Fig. 18 shows a comparison of the proposed spectra with the large scale and small scale similarity spectra of Tam[33]. The limiting case of $M_c = 0$ indicated by the present model agrees with the spectrum of Tam for small scale turbulence except at high frequencies, where Tam' spectrum is affected by atmospheric absorption, as indicated earlier. On the other hand, the proposed model for $M_c \to 0$ agrees well with Kolmogorov's -5/3 law for turbulence energy spectrum at large frequencies. The limiting case of large value of M_c ($M_c \approx 2$) compares well Tam's large scale turbulence spectrum over the entire frequency range. The intersection of the fine-scale spectrum with the large-scale spectrum at high frequencies, as observed in the similarity spectra of Tam, is absent in the present predictions.

In what follows comparisons will be presented between the predicted similarity spectra with the observed spectra to provide a validation of the proposed spectra. Fig. 19 shows a comparison of the predicted spectra with the spectral data of Norum and Brown[59] at M_j=0.5 and $T_t / T_\infty = 1$ (subsonic cold jet) at θ= 30, 90 and 130 deg. The data are taken with a 1.91 cm diameter jet. The data are shown only at some selective frequencies, since the tabulated narrow band data are not available to the author at the present time. It should be pointed out that the constant 2 multiplying M_c in Eq. (32b) is based on correlation of the present model with the data of Norum and Brown[61] at θ= 30 deg. Satisfactory concurrence between the predictions and the data is demonstrated. Both the data and the model suggest that the spectra for θ= 90 deg and 130 deg are distinctively separated from that at θ=30 deg. The relatively closely spaced character of the 90 deg and 130 deg spectra indicated by both the data and the present model supports the general validity of the predictions.

Fig. 20 displays a comparison of the present model with the spectral data of Viswanathan[35] for a high temperature M_j=1 jet with T_t / T_∞=3.2. As in Fig. 19, the data are shown only at some selective frequencies. The data are obtained from 6.22 mm diameter jet. Good agreement is noted for θ=30 deg and 130 deg, but the comparison at θ=90 deg is less satisfactory. Again the relatively closely spaced nature of the 90 deg and 130 deg spectra is evident even in high temperature jets.

From the foregoing comparisons, it appears that the proposed scaling laws provide a reasonable framework for jet noise in hot and cold jets in both subsonic and supersonic flow. Application of these scaling laws to practical rocket exhausts involving multiple nozzles, jet interaction and ground reflection effects[61] and passage through exhaust ducts[62] require caution and further refinements.

6. DISCUSSION

Generally speaking, refraction of sound in a turbulent shear layer constitutes an important effect, although refraction effects are mostly confined to angles close to the jet axis, as evident from Fig. 13. Thus jet noise models based solely on sound sources are limited. Spectral and directional broadening can alter substantially the spectrum and directionality of noise source between the interior and the exterior of the jet. In this connection, Campos[63,64] investigated sound transmission through turbulent shear layers with regard to the prediction of both the spectrum and directivity of sound (spectral and directional broadening).

With regard to the experimental data presented here for comparison purposes, this article presents only a partial review of existing data, and a comprehensive review of all jet noise data is outside the scope of the present work. It does however accommodate the inclusion of both old and new jet noise data with which to compare the proposed scaling relations.

7. CONCLUSION

In the scaling laws for jet noise proposed here, the effect of jet temperature is accounted for by Lighthill's suggestion through the changes in the density factor in the quadrupole field. New results are presented demonstrating the effect of jet temperature on the overall sound power level at various jet Mach numbers. A continuous similarity spectrum is also proposed that is generally bounded by the discrete (characteristic) similarity spectra for large-scale and fine-scale turbulence. Effects of jet convective Mach number and angle from the jet axis are taken into account in the directivity factor and in the similarity spectra. The resulting predictions for the overall sound power levels, directivity and spectra are in general agreement with the available data.

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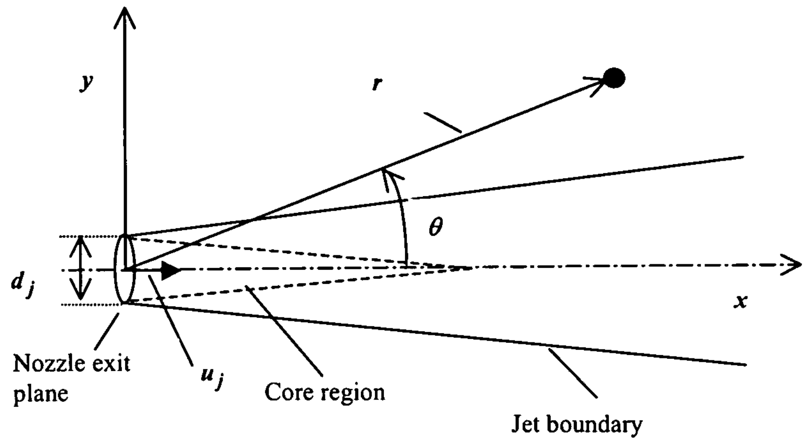
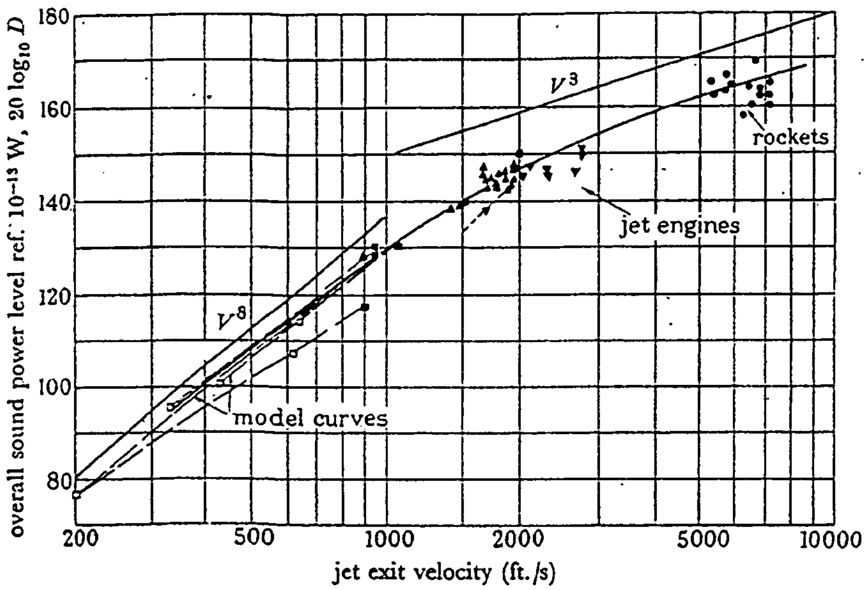


Figure1. Schematic of the jet configuration.



Variation of acoustic power levels from Chobotov & Powell (1957). ●, Rocket; ▼, turbojet (afterburning); ▲, turbojet (military power); ■, exit velocity > $M = 0{\cdot}8$; □, air model (exit velocity < $M = 0{\cdot}8$). D is the exit diameter in inches.

Figure 2 Variation of acoustic power levels and jet velocity, from Ffowcs Williams.[3]

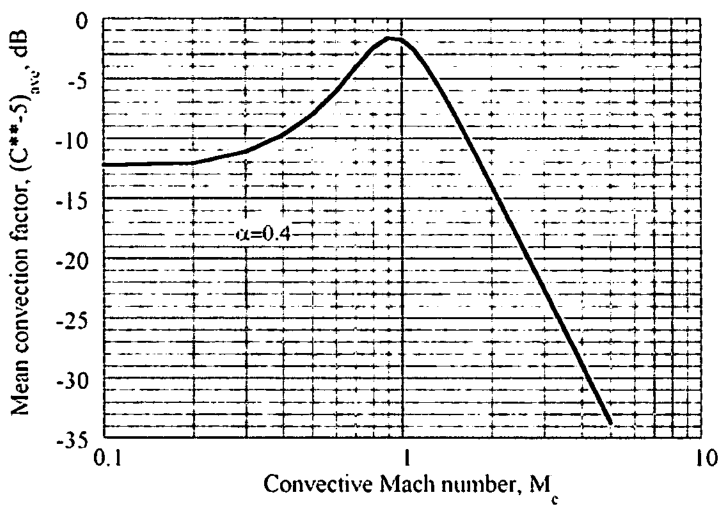


Figure 3. Variation of mean convective amplification factor with convective Mach number[3,12].

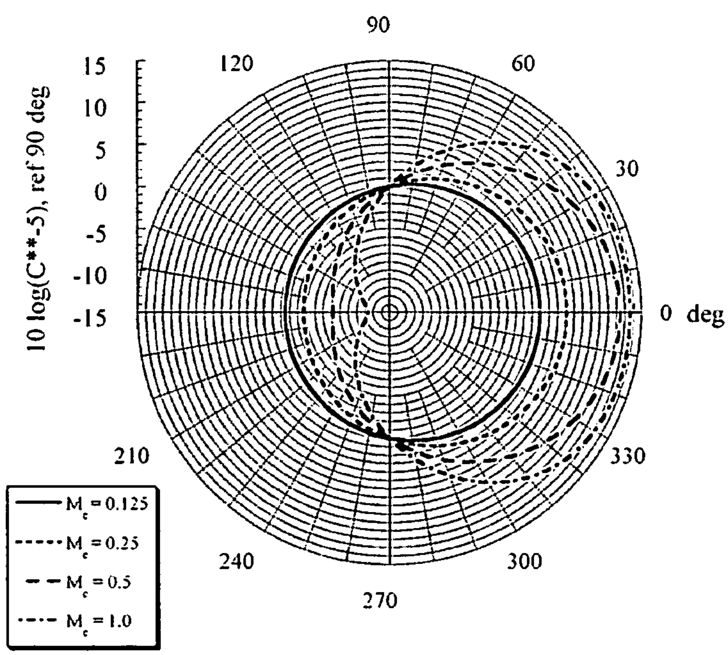


Figure 4a. Dependence of directivity of jet noise with convective Mach number[3,12].

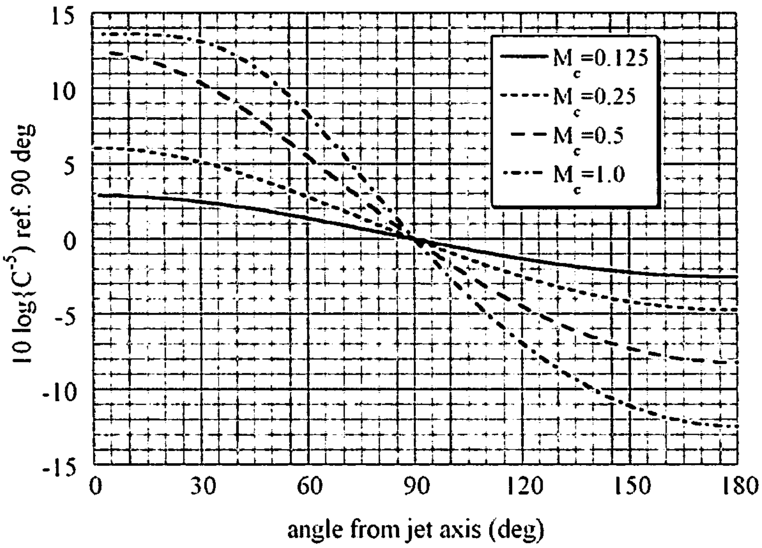


Figure 4b. Directivity as a function of convective Mach number[3,12].

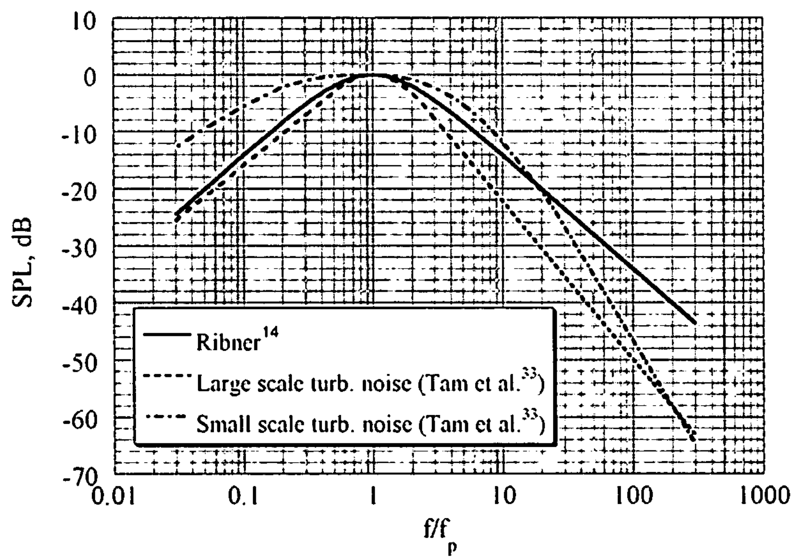


Figure 5. Similarity spectra due to Ribner[12] and Tam.[33]

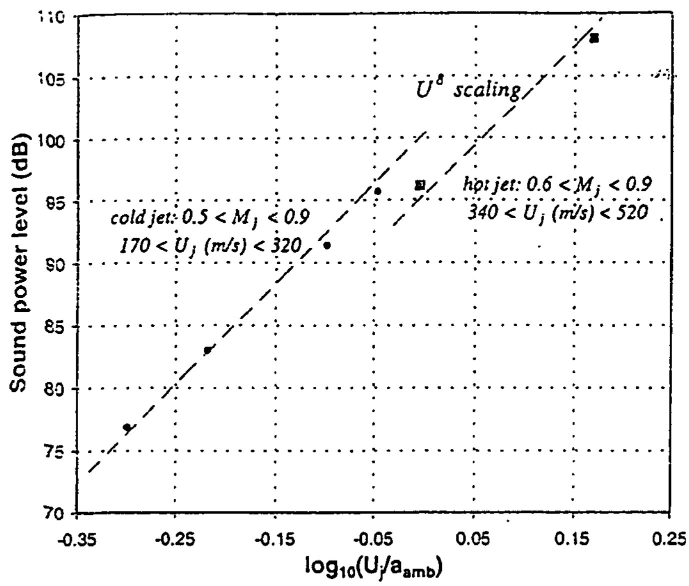


Figure 6. Dependence of sound power level with jet velocity for cold and hot jets, from Narayanan et al.[39]

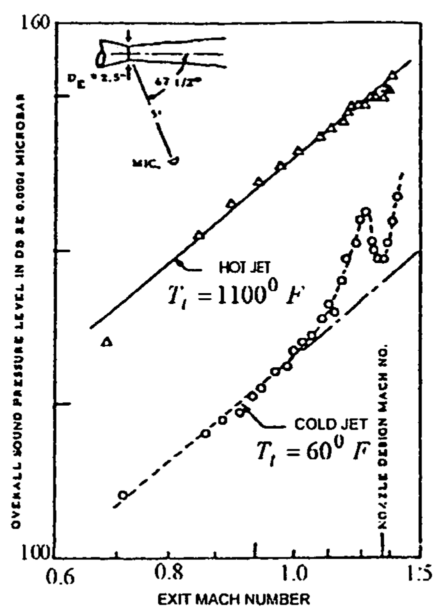


Figure 7. Variation of overall sound pressure level with jet Mach number for cold and hot jets, from Morgan et al.[7]

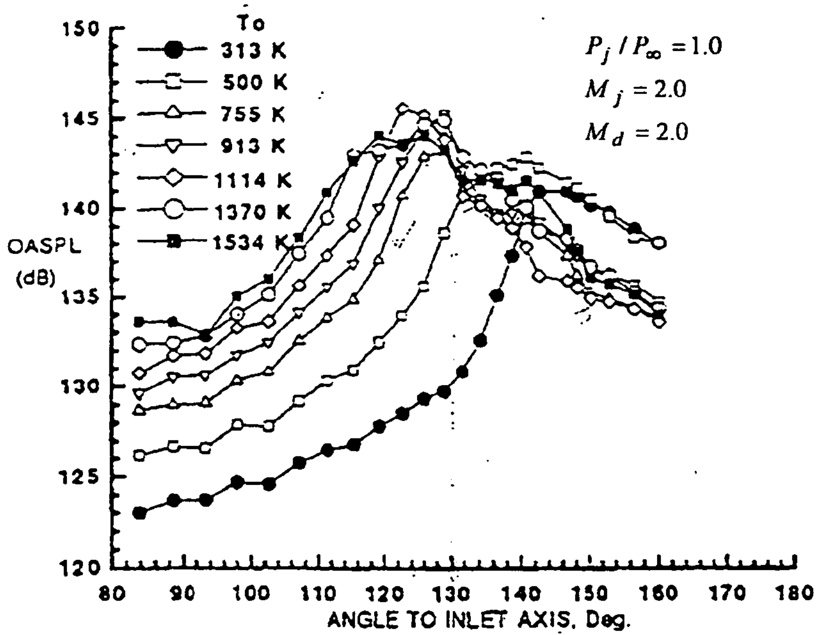


Figure 8. Dependence of overall sound pressure level on temperature as a function of the angle to the jet axis in supersonic flow, from Seiner et al.[32]

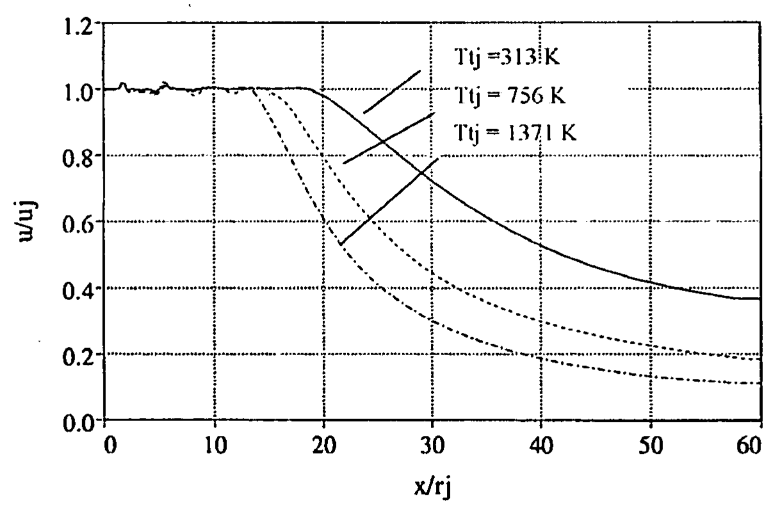


Figure 9. Variation of computed jet centerline velocity with jet temperature at Mach 2 and an ambient temperature of 540 R, illustrating the temperature dependence of supersonic core length[48].

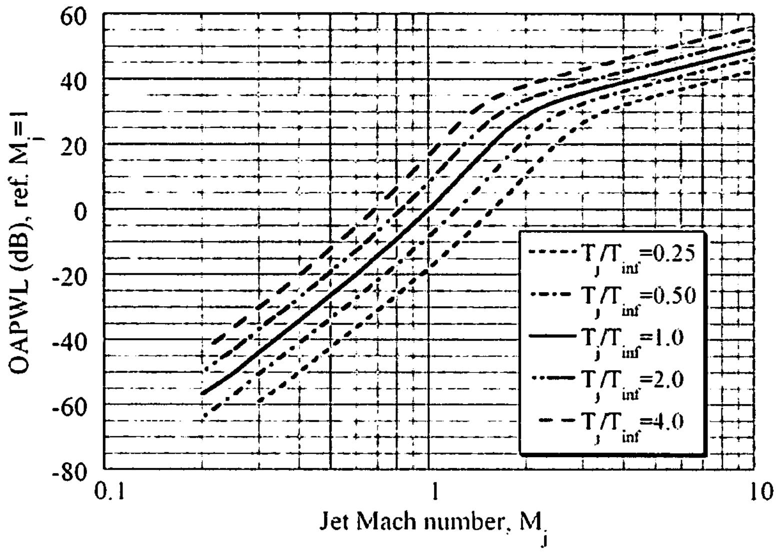


Figure 10. Effect of temperature ratio on overall sound power level.

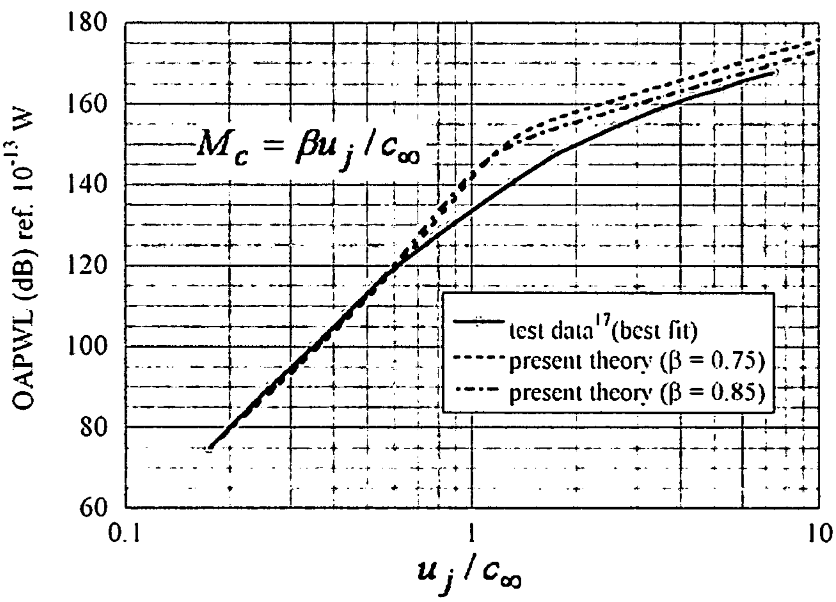


Figure 11. Comparison of OAPWL with test data.

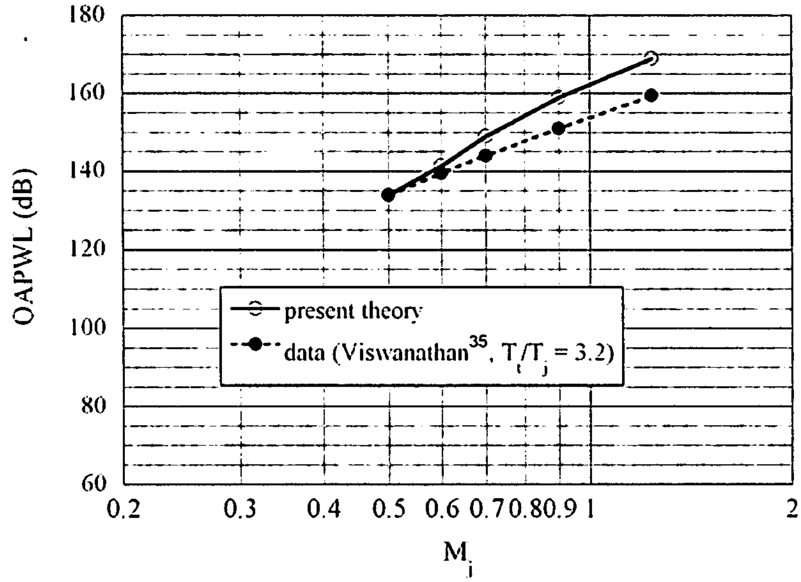


Figure 12. Comparison of OAPWL with test data of Viswanathan.[35]

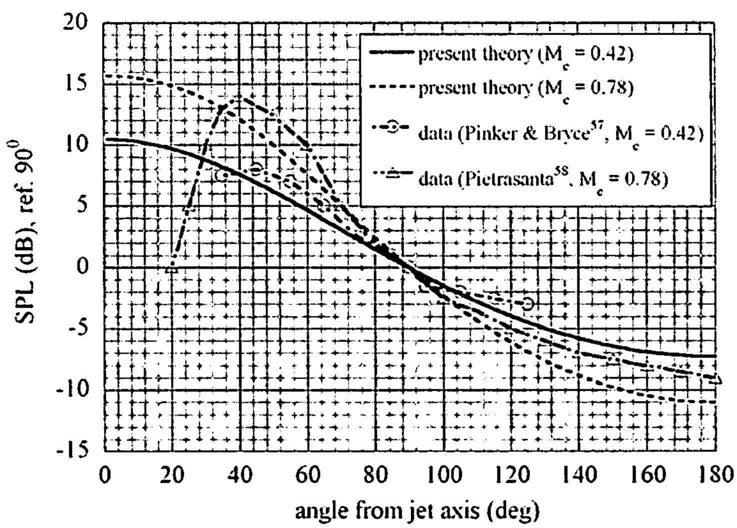


Figure 13. Comparison of directivity of jet noise.

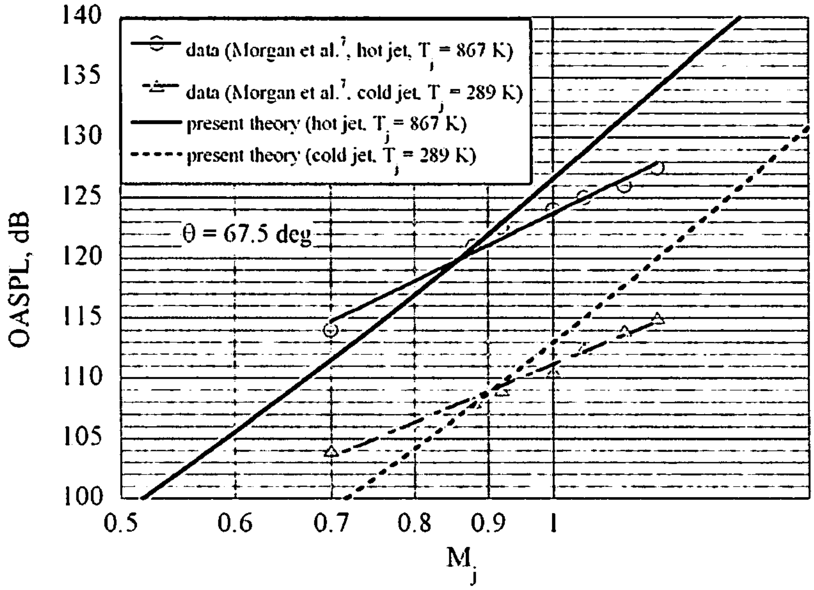


Figure14. Comparison of variation of OASPL with jet Mach number.

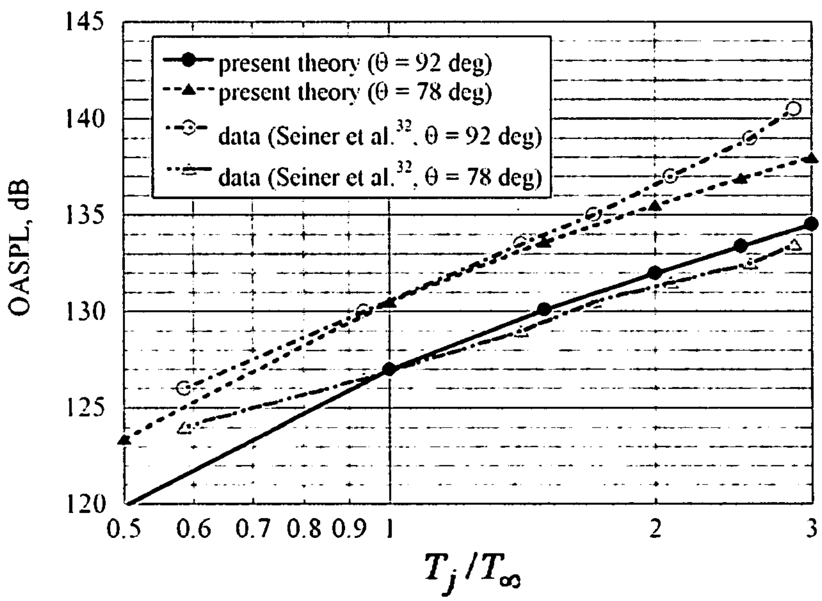


Figure 15. Comparison of variation of OASPL with jet temperature at $M_j = 2$.

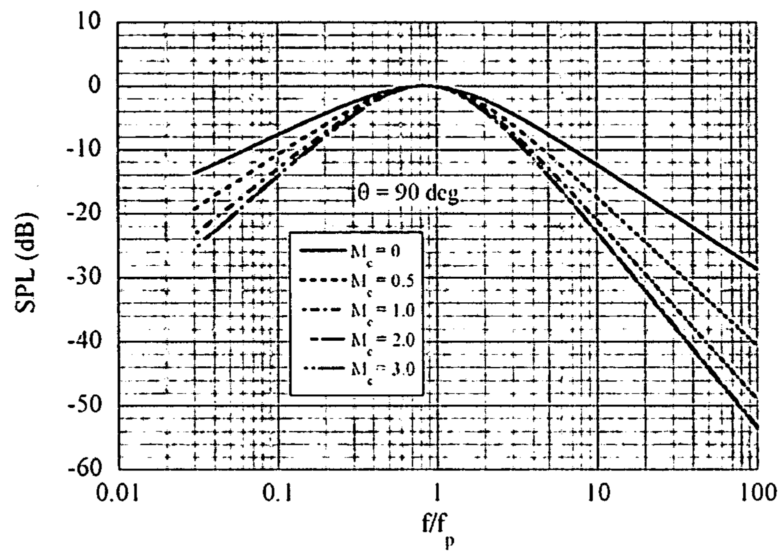


Figure16. Similarity spectra for jet noise at $\theta = 90$ deg.

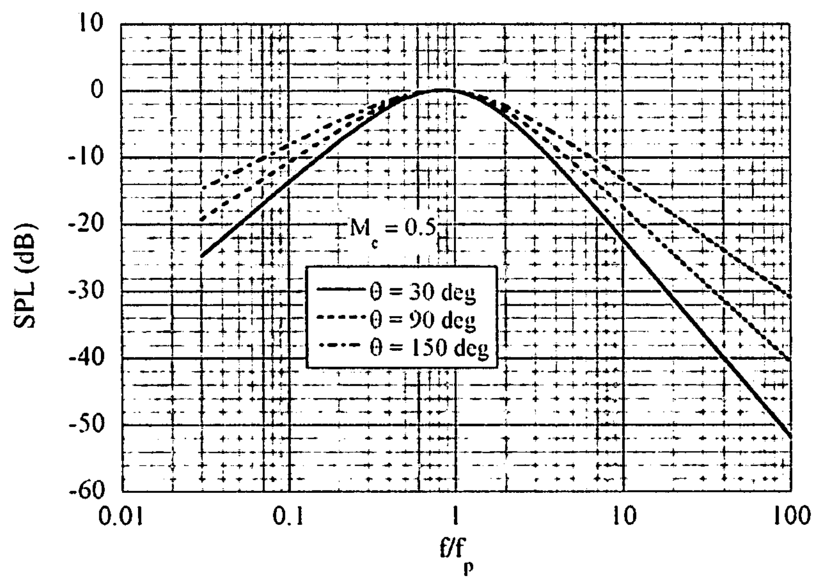


Figure 17a. Similarity spectra for jet noise for M_c=0.5.

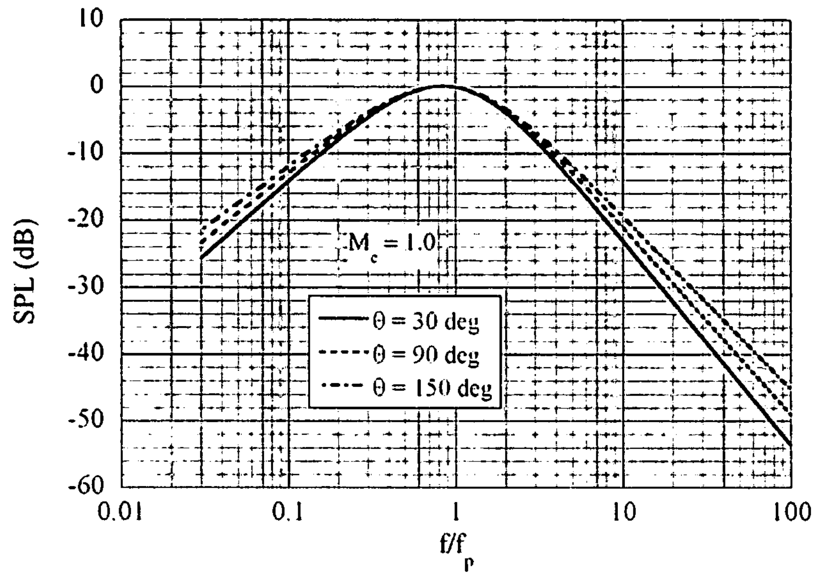


Figure 17b. Similarity spectra for jet noise for M_c=0.5.

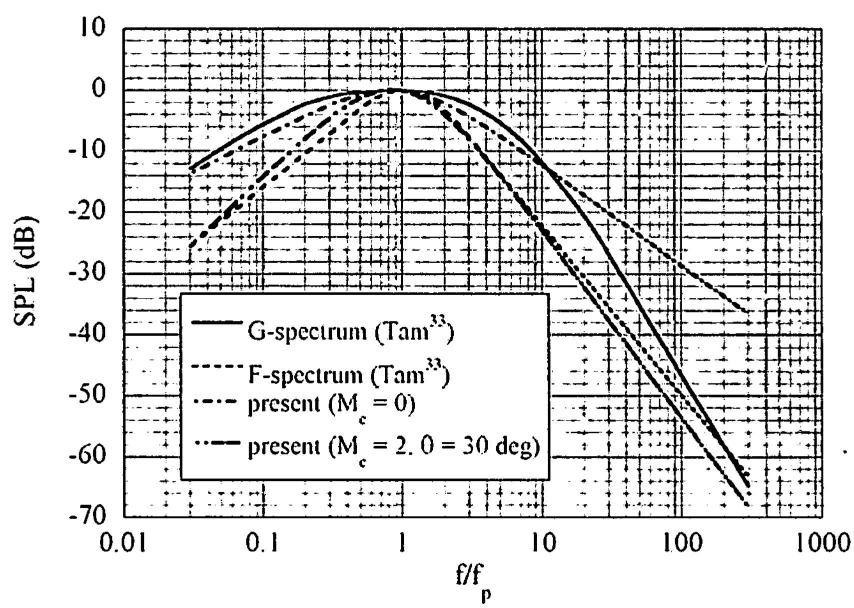


Figure 18. Comparison of similarity spectra with those of Tam[33].

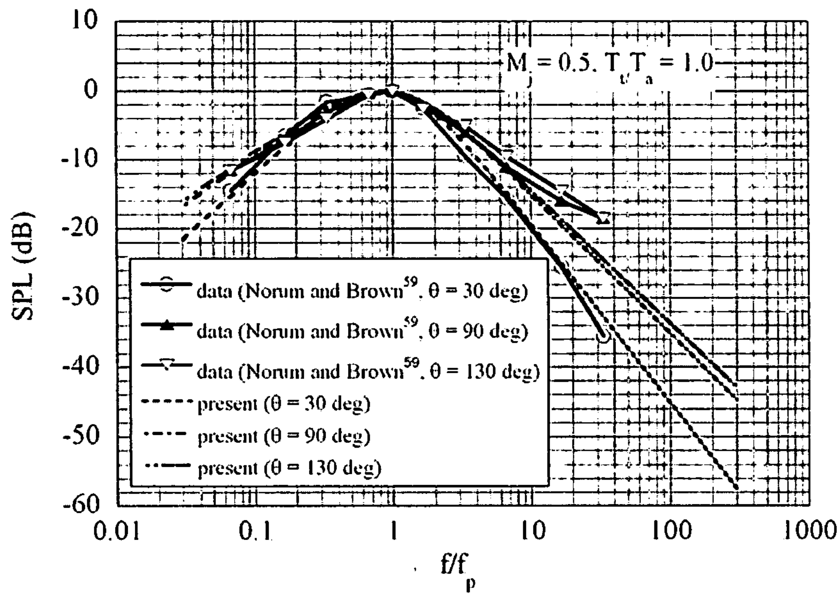


Figure 19. Comparison of similarity spectra with cold subsonic data of Norum and Brown[61].

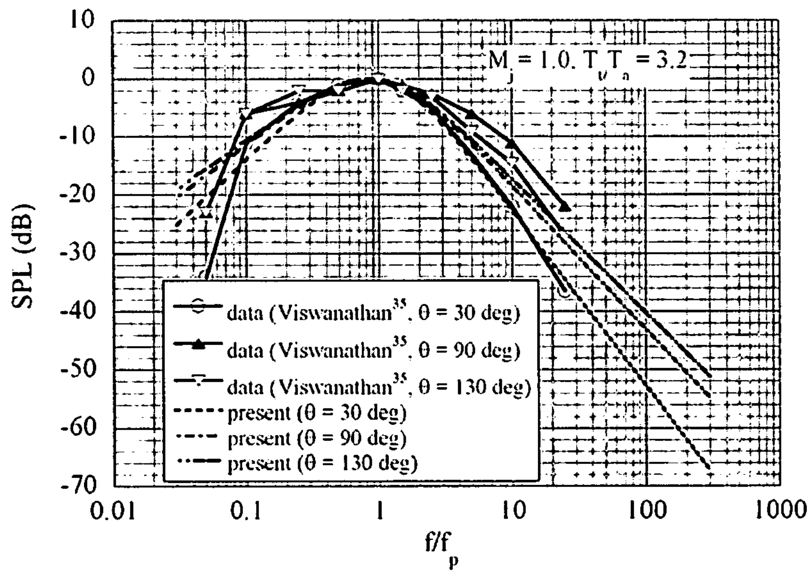


Figure 20. Comparison of similarity spectra with hot jet data of Viswanathan[35] for $M_j = 1$.

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14. ABSTRACT
The scaling laws for the simulation of noise from subsonic and ideally expanded supersonic jets are reviewed with regard to their applicability to deduce full-scale conditions from small-scale model testing. Important parameters of scale model testing for the simulation of jet noise are identified, and the methods of estmating full-scale noise levels are addressed. The limitations of cold-jet data in estimating high-temperature supersonic jet noise levels are discussed. New results are presented showing the dependence of overall sound power level on the jet temperature ratio at various jet Mach number and a generalized similarity spectrum is proposed.

15. SUBJECT TERMS
supersonic jets, scale-model testing, jet noise, sound power levels, simulation scaling laws

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