

# Measurement Decision Risk – The Importance of Definitions<sup>1</sup>

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## Abstract

One of the more misunderstood areas of metrology is the Test Uncertainty Ratio (TUR) and the Test Accuracy Ratio (TAR). There have been many definitions over the years, but why are these definitions important to a discussion on measurement decision risk? The importance lies in the clarity of communication. Problems can immediately arise in the application (or misapplication) of the definition of these terms. In other words, while it is important to understand the definitions, it is more important to understand concepts behind the definitions and to be precise in how they are applied.

The objective of any measurement is a decision. Measurement Decision Risk is a way to look at the quality of a measurement, and although it is not a new concept, it has generated a lot of attention since its addition as a requirement in the new U.S. National Standard, ANSI/NCSL Z540.3-2006. In addition to Measurement Decision Risk as the prime method of managing measurement risk, Z540.3 has added, as a fall-back, an explicit definition for TUR. The impact these new requirements may have on calibration service providers has become the topic of much discussion and in some cases concern.

This paper will look at the concepts behind the definitions and how they relate to Measurement Decision Risk. Using common examples, this paper will also provide a comparison of various elements of risk related to measurement science using the concepts of TAR, TUR, accuracy ratios, and Consumer Risk (False Accept Risk). The goal of this paper is to provide a better understanding of their relevance to the measurement decision process.

## Introduction

It is well known that the international definition of metrology is the science of measurement [1], but, though metrology may be about measurements, the focus should be on what is being done with the information from the measurements. In the simplest terms, measurements are made to support decisions and/or establish facts. For example, measurements help make decisions:

- To continue or stop a process (including a space launch)
- To accept or reject a product
- To rework or complete a design

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<sup>1</sup> This is an adaptation of a paper with the same title presented at the 2007 NCSLI Conference in St. Paul, MN.

- To take corrective action or withhold it
- To establish scientific or legal fact.
- To establish research or investigative fact.

If the data from measurements are not being used in decision-making or in establishing facts (including scientific research), the measurement is unnecessary. *“The more critical the decision, the more critical the data. The more critical the data, the more critical the measurement”* [2].

Validating the suitability of measuring and test equipment (M&TE) is an essential component of the measurement process and calibration is one of the oldest methods of doing this. Calibration has been a part of the measurement process since the Egyptian Cubit Stick was used in the building of the pyramids [3]. As technology increased in the mid-20<sup>th</sup> century, the need for more precision in testing and calibration led to the development of new methods in determining the risks involved in measurement related decisions. The TUR and TAR developed out of these activities and have been used for over 50 years as tools for mitigating measurement decision risk. However, the problem with the TUR/TAR is the lack of a consensus on how to apply them or even how to define them.

ANSI/NCSL Z540.3-2006 [4] is the new calibration program consensus standard for the U.S. As with any new consensus standard, organizations owning, using, and/or calibrating M&TE will have to review the Z540.3 for potential impacts to their businesses. The review will have to consider impacts that may be either positive or negative. After the review, some organizations will have to make the decision whether or not to incorporate the standard, while other organizations may have to bid on contracts that include the standard. All affected organizations will need to understand the implications of measurement decision risk and the explicit definition of TUR. This paper begins to address the concepts behind measurement decision risk and the TUR as an aid in this review. The following areas will be discussed:

1. A look at the history and development of measurement decision risk and the TUR.
2. Discuss the current definitions of TUR/TAR.
3. Discuss how the Z540.3 definition of TUR is linked to measurement decision risk and the limitations of its use.
4. Review an example of the application of TUR in an off-nominal case and the corresponding calculation of measurement decision risk.

### **Development of Measurement Decision Risk**

A look at the origin of the TUR/TAR helps in understanding their relationship to measurement decision risk.

Measurement decision risk analysis traces its roots to the early work on consumer and producer risk analysis done by Alan Eagle, Frank Grubbs, and Helen Coon [5, 6] in the late 1940's and early 1950's. Eagle's 1954 paper describes the methods for calculating the consumer and producer risk and how to establish “test limits” in relation to design limits which have become known as guardbands today. The focus of the paper was to analyze and mitigate the “test errors” which are “inherent in the test equipment and/or test personnel” used in the inspection of manufactured complex electronic equipment [5]. The key point to Eagle's method was quantifying and using consumer/producer risk (the two components of measurement decision risk) as a part of the manufacturing process. This concept is applicable wherever decisions are based on measurements.

In 1955, the U.S. Navy recognized the need for improved measurement reliability in their guided missile program. Building upon Eagle's work [5], Jerry Hayes set out to establish a basis for accuracy ratios versus decision risks for application in the Navy's calibration program [7]. The practice at the time was to use a 10:1 ratio, but that value was considered unsupportable by the nation's calibration support and measurement traceability infrastructure. Using the relationship between the design specifications, testing limits, and instrument error, Hayes proposed using a "family of curves" to determine the specific testing risk or reliability. The problem with this method was a new family of curves had to be established each time a process or design tolerance changed. A change in a process or tolerance nullified the assumptions upon which the first set of curves was built [7, 8]. Some important aspects of the Hayes paper, still relevant today, are the need for calibrated equipment used in testing, establishment of reasonable testing risk levels, reasonable design tolerances, and adequate procedures for testing.

Hayes continued to work on methods of assuring measurement reliability based on consumer risk. In the mid-50's, computing consumer risk was a very arduous task (requiring use of a slide rule), which Hayes decided not to require U.S. Navy contractors to perform. With very specific assumptions on process, a consumer risk of 1% was selected, which calculated to be about a 3:1 accuracy ratio. Hayes, working with Stan Crandon, decided to pad the ratio to account for uncertainty in the reliability of the tolerances industry was using for the measurement standards. Thus the 4:1 ratio requirement was developed and established as Navy policy and subsequently adopted as a requirement in military procurement standards both here and abroad [8]. This ratio became what is known today as the TAR and later evolved into the TUR.

### A Look at Existing Definitions

Various definitions for TAR and TUR have appeared in many texts, papers and documents over the years, but there has never been a consensus standard available that provided a definition, much less specified how to apply the rule. The following are some current definitions.

- The American Society for Quality defines TAR and TUR in terms of calibration [3].

*Test accuracy ratio - (1) In a calibration procedure, the test accuracy ratio (TAR) is the ratio of the accuracy tolerance of the unit under calibration to the accuracy tolerance of the calibration standard used.*

$$TAR = \frac{UUT\_tolerance}{Std\_tolerance}$$

*Test uncertainty ratio - In a calibration procedure, the test uncertainty ratio (TUR) is the ratio of the accuracy tolerance of the unit under calibration to the uncertainty of the calibration standard used.*

$$TUR = \frac{UUT\_tolerance}{Std\_uncertainty}$$

Note: UUT is the Unit under Test and Std represents the calibration standard.

- Although a direct definition of TUR is avoided, NASA's Space Shuttle Program has ratio requirements that apply to calibration and article or material measurements [9].

**Paragraph 4: Article or Material Measurement Processes**

*The Expanded Uncertainty in any article or material measurement process shall not exceed ten percent of the tolerance of the article or material characteristic being measured.*

**Paragraph 5: Calibration Measurement Processes**

*... the Expanded Uncertainty in any calibration measurement process shall not exceed 25 percent of the tolerance of the parameter being measured.*

Note: Expanded Uncertainty is defined in the ANSI/NCSL Z540-2-1997 [10] which is the U.S. adoption of the International Organization for Standardization (ISO) *Guide to the Expression of Uncertainty in Measurement* (GUM).

- ANSI/NCSL Z540.1-1995 [11] is the predecessor to the Z540.3. The Z540.1 Handbook [12], through the use of a note, considered the TAR and TUR interchangeable, although the definition of the TAR in the guidance differs from traditional definitions.

**Interpretive Guidance for Section 10.2 of the Handbook [12]**

*As a default alternative to doing an uncertainty analysis, a laboratory may rely on a Test Accuracy Ratio (TAR) of 4:1. A TAR of 4:1 means that the tolerance of the parameter (specification) being tested is equal to or greater than four times the combination of the uncertainties of all the measurement standards employed in the test.*

*If it is determined that the TAR is less than 4:1, then one of the following methods may be used: uncertainty analysis as described above, guard-banding, widening the specification, or another appropriate method.*

*Note: Some refer to TARs as Test Uncertainty Ratios or TURs*

- The Z540.3 [4] provides an explicit definition of TUR, but does not address the TAR.

**3.11 Test uncertainty ratio**

*The ratio of the span of the tolerance of a measurement quantity subject to calibration, to twice the 95% expanded uncertainty of the measurement process used for calibration.*

*NOTE: This applies to two-sided tolerances.*

The definition uses the expanded uncertainty as defined in Z540-2 [10] where  $k$  is the coverage factor. The definition in equation form:

$$TUR = \frac{Upper - Lower}{2 \cdot U} \quad U = k \cdot u \quad k = 2$$

There are key differences between the Z540.3 and earlier TUR definitions.

1. The earlier TUR denominator is not well defined which leads to inconsistent applications.
2. The denominator for the Z540.3 TUR is explicitly defined, thus providing better uniformity in the application of the TUR.

The ASQ definition of the TAR [3] is one of the more popular applications of the risk rules-of-thumb, most likely due to the simplicity of its implementation. It is used in many labs and other applications and can be found in many papers and training guides for calibration and quality inspection. It must be cautioned however, as such, the ASQ definition of the TAR is non-

compliant with either Z540.1 or Z540.3 and can lead the user into a false sense of security, which is a main focus of this paper

As mentioned earlier, the objective of a measurement is a decision and Measurement Decision Risk is a tool to assess the suitability of the measurement process. The intent of a TAR or a TUR is to mitigate the risks in measurement, but their value is diminished without a concise definition. Problems can quickly arise when different meanings are applied to the same term. The devil is in the details.

### Linking the TUR to Measurement Decision Risk

It is from Eagle's work the TUR was first developed [7, 8]. The consumer risk equation associated with Eagle's work [5, 6] is presented here, but will not be discussed in detail.

$$CR = \frac{1}{\pi} \int_k^\infty \int_{-r \cdot (k+t)+b}^{r \cdot (k-t)-b} e^{-\frac{(t^2+s^2)}{2}} ds dt$$

To help explain the relationship of the TUR to Measurement Decision Risk, the functional relationship of the equation will be discussed using the three "influence" variables,  $b$ ,  $k$ , and  $r$ .

$$CR = CR(b, k, r)$$

The variable  $b$  is the deviation of the test limits from the specification limits [i.e.,  $\mu \pm (k\sigma_x - b\sigma_e)$ ]. This establishes the required "test limit," (also known as a guardband or acceptance limit) to achieve a desired consumer risk. For the purposes of this discussion, the specification limits will equal the test limits (i.e.,  $b = 0$ ), therefore leaving only two influence variables for this discussion.

The variable  $k$  is the number of standard deviations the performance specification limit is to the product mean, which is assumed to be centered (i.e.,  $\mu \pm k\sigma_x$ ). Figure 1 illustrates the relationship of the specification limits and production (or process) distribution with the limits falling at  $\pm 2\sigma$ .

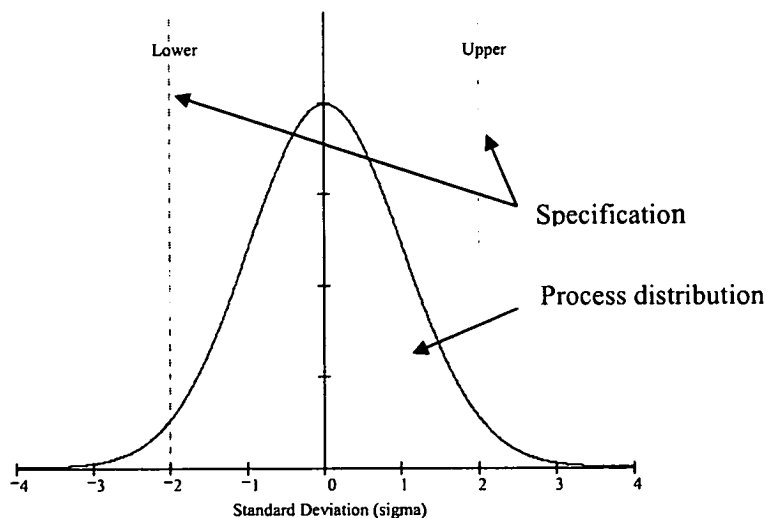


Figure 1: The relationship of the specification limits to the process distribution for  $\pm 2\sigma$ .

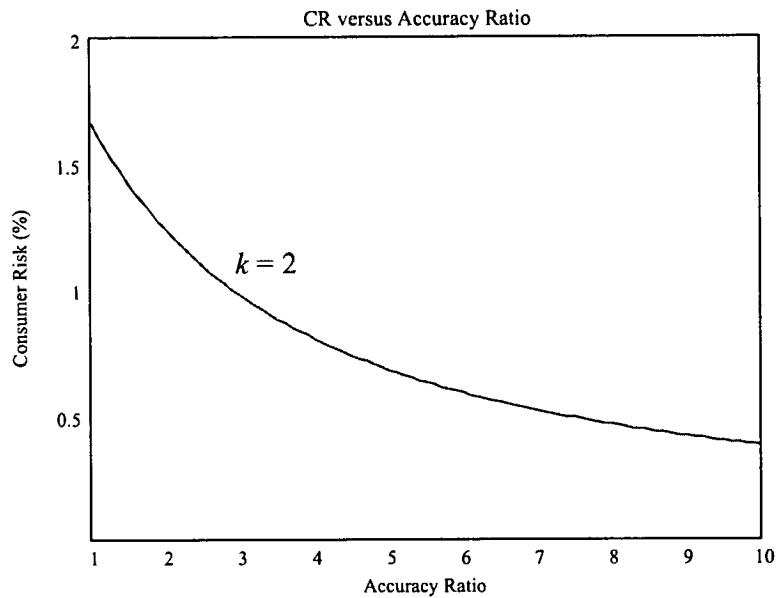
The last variable to consider is  $r$  which is a ratio of two standard deviations.

$$r = \frac{\sigma_x}{\sigma_e}$$

The numerator  $\sigma_x$  is what Eagle described as “the true standard deviation of the product distribution.” This represents the subject of interest. Eagle described the denominator  $\sigma_e$  as “the standard deviation of the errors of measurement” [5]. In metrology, the standard deviation is an indicator of uncertainty (i.e.,  $\pm k\sigma$  indicates  $\pm ku$ ). Although it is considered the origin of the TUR and is referred to as an “accuracy ratio,” without any confidence limits or coverage factors ( $\pm k$ ), in reality,  $r$  is a pure “uncertainty ratio.”

$$r = \frac{\sigma_x}{\sigma_e} = \frac{u_x}{u_e}$$

The relationship between  $r$  and  $k$  becomes more evident when graphically displayed. Figure 2 shows the “knee” of the curve intersecting the 1% risk level at approximately an  $r$ -value of 3:1. As 1% was the risk level Hayes was seeking, adding a little extra margin, 4:1 became the “rule of thumb” [7, 8].



**Figure 2: Consumer Risk versus the “accuracy ratio” for the specific case where the specification limits lie at 2 standard deviations from the mean of the production distribution (i.e.,  $k=2$  for  $\mu \pm 2\sigma_p$ ).**

The curve in Figure 2 represents the relationship between  $r$  and consumer risk where  $k = 2$ . Eagle noted in his original paper that if the “performance specification” falls at more than 2 standard deviations of the production distribution (i.e.,  $k > 2$ ), then mitigation efforts are not required [5]. His statement may not be true for all cases in today’s technology, but as discussed earlier, this was the assumption that Hayes and Crandon used to establish the 4:1 rule in the mid-1950s [8].

Hayes and Crandon used a chart similar to Figure 2 for their analysis as they searched for a method to improve measurements without levying a large amount of calculations on the contractors. When Hayes allowed the use of a ratio between the tolerances of the subject of interest and the measuring equipment, the idea was supposed to be temporary until better computing power became available or a better method could be developed [8]. As discussed earlier, Hayes settled on 4:1, but others went with a more conventional and conservative 10:1. NASA used the 10:1 for all calibration and article measurement requirements through the first moon landing in 1969. After that, calibration requirements were changed to 4:1 while test measurement requirements remained at 10:1 [9].

As time has gone by, many have tried to help the TUR/TAR become a tool that is more soundly founded in measurement science. The Z540.3 comes very close to accomplishing that goal, but as will be demonstrated, more information is required to use the TUR. To explain this, let's take another look at the ratio from the consumer risk equation, but now only extend the standard deviation to measurement process uncertainty.

$$r = \frac{\sigma_x}{\sigma_e} = \frac{\sigma_x}{u_e}$$

Remember that the relationship of the tolerance to the product mean is  $\mu \pm k\sigma_x$ . Combined with the value of  $r$  as stated above, the Z540.3 TUR can be transformed (slightly) where the relationship of  $\pm k\sigma_x$  becomes more evident.

$$TUR = \frac{Upper - Lower}{2 \cdot U_{95}} = \frac{Upper - Lower}{2 \cdot k_e \cdot u_e} = \frac{2 \cdot k_x \cdot \sigma_x}{2 \cdot k_e \cdot u_e} = \frac{k_x \cdot \sigma_x}{k_e \cdot u_e}$$

As shown here, the TUR can be represented as a ratio of intervals ( $\pm k\sigma$ ). For Z540.3, the coverage factor in the denominator is usually inferred to be  $k = 2$  (95% of normal). Although there is no definition for the value of  $k$  in the numerator, it might be assumed that it represents the  $k$ -factor in the consumer risk equation. It is this  $k$ -factor that becomes the link between the TUR and measurement decision risk.

The next step is to establish a method for obtaining the value of the  $k$ -factor for a calibration process. One method that can be used for calibration processes is to allow the End of Period Reliability (EOPR) for the Unit under Test (UUT) to represent the "product distribution" [13]. EOPR is the probability of a unit being in-tolerance when it is returned for routine calibration at the end of its normal interval. The EOPR is reported in percentages and if the EOPR is assumed to be normally distributed, this percentage can be related back to a confidence limit (or coverage factor). For example, an EOPR of 95.45% would then be represented by a value of  $k = 2.0$ . It is in this manner that the EOPR fits the relationship of specification limits to the process distribution as previously shown in Figure 1. For this type of relationship, the value of  $k$  can be obtained from statistic books or calculated.

Using EOPR to represent the UUT product distribution opens up many questions about the level of "drill down" required in obtaining this information (e.g., nomenclature, manufacture, model number, serial number, or parameter value). The answers to these questions are beyond the scope of this paper, but an excellent detailed discussion can be found in Section 6.3 of NASA Reference Publication 1342 [2].

### Applying Definitions – Example 1

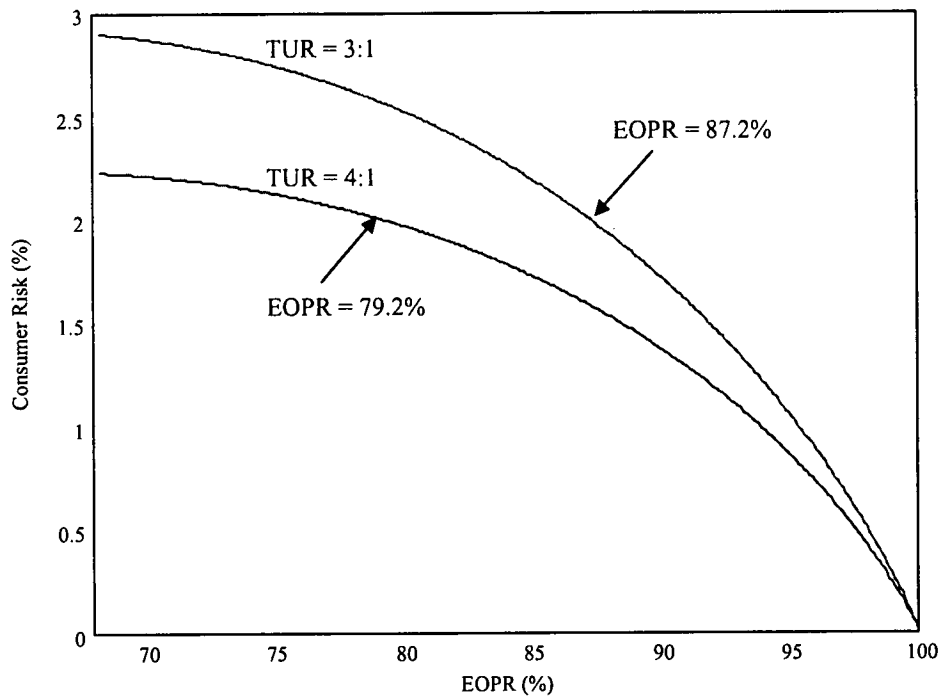
A generic example of a UUT and a working standard will be used to illustrate the concepts just discussed. The uncertainty is a generic estimate with no assumptions defined, as it only represents the combined standard uncertainty of the measurement process

Parameter	UUT	Working Standard
Tolerance	±4.0	±1.0
$u_c = 0.67$		
TAR = 4:1		
TUR = 3:1		

**Table 1: A generic calibration example.**

With a TUR of less than 4:1, if the Z540.3 is applicable, this calibration would be unacceptable for application of the TUR fall-back rule. But is the calibration “good enough” – is the consumer risk (false accept risk) acceptable (2% or less per the Z540.3)?

As discussed above, the assumption in the creation of the original 4:1 ratio, with respect to the desired level of measurement risk, was 1% [8]. The value of  $k$  was a pivotal assumption because when  $k = 2$  (in a normal distribution), more than 95% of the process would be contained within the specification limits. As stated earlier, the product distribution is to be represented by the EOPR of the UUT. Figure 3 illustrates the relationship between consumer risk and EOPR for TUR ratios of 4:1 and 3:1.



**Figure 3: Consumer Risk versus the EOPR.**



Figure 3 shows it is possible for a TUR below 4:1 to be acceptable (given 2% is acceptable), but it also demonstrates that 4:1 may not provide a desired level of risk. For example, as Figure 3 shows, the EOPR must be above 79% for a 4:1 to provide a consumer risk of 2%. The bottom-line is, as described, a TUR may not provide a proper or desired level of quality without additional information about the UUT error distribution. As stated earlier, using the TUR alone may lull an organization into a false sense of security by not quantifying a key component of risk.

### Example 2

All types of calibrations will be affected by Z540.3, but those that do not fit the general mold of the TUR or 2% rule are the calibrations that will create the most problems for quality managers and calibration service providers. This example will look at the calibration of a digital micrometer, an operation performed millions of times a year across our planet. Table 2 contains the specification information for the micrometer and calibration standard (Class 2 Gage Blocks).

Parameter	Digital Micrometer 0-1 inch (0-25.4 mm)	Gage Block, Class 2
Tolerance	± 0.0001 inch (± 2.54 µm)	± 0.000004 inches (± 0.1016 µm)
Resolution	± 0.00005 inch (± 1.27 µm)	N/A

**Table 2: The specification information for the digital micrometer and the calibration standard.**

A new or overhauled micrometer is calibrated using at least 10 points to cover its range, but only 4 points for routine calibrations (0, plus low, middle, and high). The purpose of this example is to examine the measurement risk using TAR and TUR, therefore only one point of the calibration will be discussed - the mid point of 0.50000 inches (12.700 mm).

Using the ASQ definition, the calibration TAR has a value of 25:1.

$$TAR = \frac{UUT\_tolerance}{Std\_tolerance} = \frac{0.0001}{0.000004} = 25$$

As previously stated, the purpose of this example (and paper) is the examination of usage of measurement risk tools. As such, the uncertainty analysis is presented in table form without the detailed calculations. The analysis follows the guidelines of the Z540-2 [10]. The specific methods followed were those recommended in *Uncertainty Analysis Principles and Methods* [14], developed for the Naval Air Warfare Center Aircraft Division (NAWCAD) and Air Force Flight Test Center (AFFTC). Readers desiring the detailed calculations may contact the author. Table 3 contains the analysis results for the digital micrometer.

Uncertainty Source	Standard Uncertainty inches ( $\mu\text{m}$ )	Confidence Level (%)	Type (A or B)	Distribution
Gage Blocks	0.000002 (0.0508)	95.00	B	Normal
Resolution	0.0000144 (0.3658)	100.00	B	Uniform
Environmental	0.0000001 (0.00254)	95.00	B	Normal
Random Error or Repeatability	0.0000083 (0.2108)	95.00	A	Student's t
Combined Uncertainty	0.0000168 (0.4267)			

**Table 3: Results of the uncertainty analysis for a digital micrometer.**

The combined standard uncertainty from Table 3 is calculated into the Z540.3's TUR for the following result. For the 95% expanded uncertainty, assume  $k = 2$ .

$$TUR = \frac{Upper - Lower}{2 \cdot U_{95}} = \frac{0.0002}{2 \cdot 2 \cdot 0.0000168} = 3$$

As can be seen, even with a very high TAR, when the calibration standard is passive (i.e., the UUT is the "reading" instrument), the TUR may fall below 4:1, in this case due to instrument resolution.

### Example 3

The subject of Example 2 was a digital micrometer with a resolution that is one half of the unit tolerance. Many times the micrometer resolution and tolerance are equal, as in the case for Example 3. In this example, the micrometer has a vernier scale in addition to the regular graduations, allowing the user to read more precisely. The TAR for this example is 25:1, the same as for Example 2. Table 4 contains the specification information for this example.

Parameter	Analog (Vernier) Micrometer 0-1 inch (0-25.4 mm)	Gage Block, Class 2
Tolerance	$\pm 0.0001$ inch ( $\pm 2.54 \mu\text{m}$ )	$\pm 0.000004$ inches ( $\pm 0.1016 \mu\text{m}$ )
Resolution	$\pm 0.0001$ inch ( $\pm 2.54 \mu\text{m}$ )	N/A

**Table 4: The specification information for the analog micrometer and the calibration standard.**

Uncertainty Source	Standard Uncertainty inches (μm)	Confidence Level (%)	Type (A or B)	Distribution
Gage Blocks	0.000002 (0.0508)	95.00	B	Normal
Resolution	0.0000255 (0.6477)	95.00	B	Normal
Environmental	0.0000001 (0.00254)	95.00	B	Normal
Random Error or Repeatability	0.0000201 (0.5105)	95.00	A	Student's t
Combined Uncertainty	0.0000326 (0.8280)			

Table 5: Results of the uncertainty analysis for the analog micrometer.

The results of the uncertainty analysis are taken from Table 5 and again calculated into the Z540.3's TUR for the following result.

$$TUR = \frac{Upper - Lower}{2 \cdot U_{95}} = \frac{0.0002}{2 \cdot 2 \cdot 0.0000326} = 1.5$$

As expected, the results are well below the 4:1 TUR required by the Z540.3.

### Discussion of Example Results

The examples above were chosen for a reason – they do not meet the requirements of the existing or new standards for calibration in terms of the TUR. But do they meet the Z540.3's 2% rule for false accept risk (consumer risk)? The best way to answer that question is to calculate the consumer risk for the calibration. For illustrative purposes, the consumer risk for the analog micrometer will be calculated as it is the “worst-case scenario” of the examples.

The consumer risk equation given earlier will be set as a function of the variables  $k$ ,  $r$ , and  $b$ .

$$CR(k, r, b) = \frac{1}{\pi} \int_k^{\infty} \int_{-r \cdot (k+t)+b}^{r \cdot (k-t)-b} \frac{e^{-\frac{(t^2+s^2)}{2}}}{2} ds dt \quad r = \frac{\sigma_x}{u_e}$$

It is important to note at this point, as mentioned earlier,  $\sigma_x$  is described as “the true standard deviation of the product distribution,” thus the  $k$ -factor for the  $\sigma_x$  must also be represented by the “true” EOPR. The “reported” or “observed” EOPR discussed earlier<sup>2</sup> is directly influenced by the measurement uncertainty associated with the calibration process (including the calibration standards) and by the “true” EOPR. Due to these influences, “true” EOPR will always be larger

<sup>2</sup> Figure 3 plots consumer risk versus “true” EOPR although the discussion centered on “observed” EOPR.

than “observed” EOPR [15], which creates errors in the risk calculations. The magnitude of these errors are relatively small when the Z540.3 TUR is above 4:1, but the magnitude begins to increase exponentially as the TUR decreases significantly below 4:1. The adjustment of “observed” EOPR to “true” EOPR can easily be handled with variance addition rules and applied to the particular risk algorithm being used. Assuming a normal distribution for the UUT and measurement process, the following equations easily adjust the EOPR for use in consumer risk equations similar to Eagle’s such as the function stated above.

$$\sigma_{obs} = \frac{(\pm)Tolerance}{k_{obs}} \quad \sigma_x = \sqrt{\sigma_{obs}^2 - \sigma_e^2} \quad k_{true} = \frac{(\pm)Tolerance}{\sigma_x}$$

At one of Kennedy Space Center’s calibration labs, the nomenclature “micrometer” has a reported or “observed” EOPR of just over 96%, but the model number of the analog micrometer has an observed EOPR of 97.3%. For an assumed normal distribution, this equals a coverage factor of 2.21 (i.e.,  $k = 2.21$ ). The observed EOPR data is used to solve for the value of  $\sigma_x$  and  $r$  as shown above, and then inserted into the function which yields the following results.

$$\sigma_{obs} = \frac{0.0001}{2.21} = 0.0000452 \quad \sigma_x = \sqrt{0.0000452^2 - 0.0000326^2} = 0.0000313$$

$$k_{true} = \frac{0.0001}{0.0000313} = 3.195 \quad r = \frac{0.0000313}{0.0000326} = 0.961 \quad CR(3.195, 0.961, 0) = 0.06\%$$

Although the analog micrometer has a very low TUR, calculating the consumer risk provides evidence that there is a high probability the calibration is “good.” If the model number specific EOPR information were not available, then the observed EOPR for overall nomenclature (96%) would have yielded a consumer risk for the calibration of approximately 0.22%, which again is much lower than Z540.3’s 2% requirement.

Figure 4 plots the consumer risk over a range of “observed” EOPR values for both micrometers

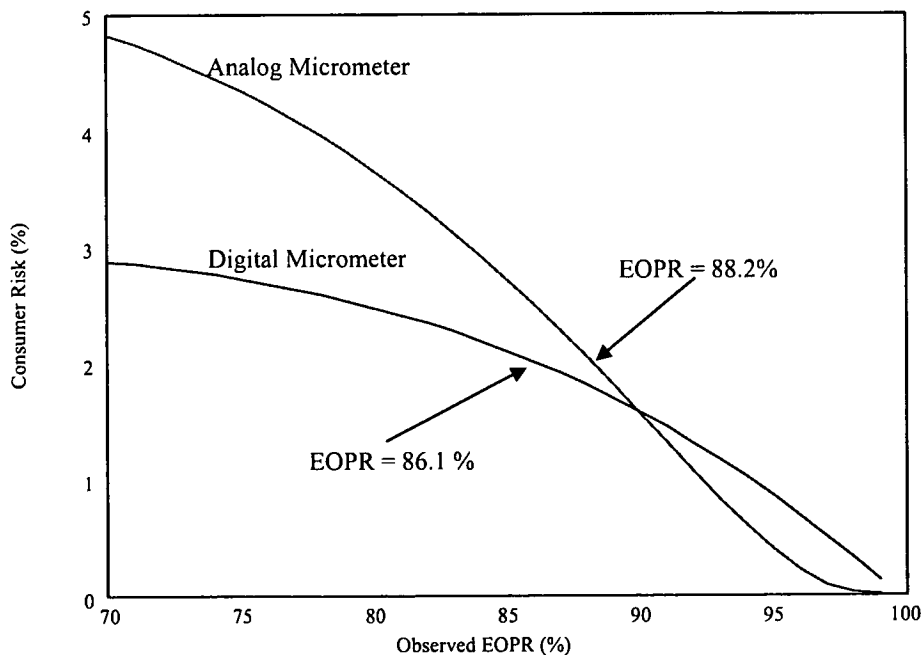


Figure 4: Consumer Risk versus the “observed” EOPR.

used in the examples. The analog micrometer requires at least an 88.2% “observed” EOPR to meet the Z540.3’s 2% rule. This translates to a 93.0% “true” EOPR when the measurement process uncertainty is taken into account. In the same fashion, the digital micrometer requires an “observed” EOPR of 86.1% which equals an 87.2% “true” EOPR. Note that for the analog micrometer, the difference between the observed and true EOPR is much greater than the digital micrometer due to the larger process uncertainty (i.e., smaller TUR).

## Summary and Conclusions

It was stated earlier “the devil is in the details.” When it comes to the application of requirements, this has never held more truth. As it has been shown, an improperly defined TAR does not have a direct quantitative relationship to measurement decision risk. It was also demonstrated that even an explicitly defined TUR does not fully assess the suitability of a measurement process without additional information. Concise definitions are needed to properly fulfill requirements because “the devil” loves ambiguity.

For over fifty years the TUR/TAR has been a part of metrology and specifically calibration processes. Its development was intended to be a temporary stop-gap due to a lack of mathematical computing power, which is a problem that does not exist today. The goal of this paper was to provide a better understanding of the concepts behind the definitions of TAR and TUR and their relevance to decision risk in measurement processes. With an understanding of these concepts, it is hoped the reader will have a better appreciation of the potential pitfalls of using general rules-of-thumb, especially when the rules are not adequately defined.

Although not discussed in this paper, there are many benefits in using risk analysis tools in lieu of generalized rules. The potential of increasing product or process quality is usually obvious, but there is also the potential for economic benefits. A case in point is the micrometer example; the TURs do not meet the requirements of Z540.3, but the risk analysis indicates the process has a low Consumer Risk (False Accept Risk). The alternative to risk analysis is to either develop a different calibration process or change the specification tolerances of the micrometer, both which have the potential for negative economic impact. These benefits extend far beyond the discussion of this paper and include other areas such as Producer Risk (False Reject Risk) which can have a large economic impact in the area of rework and product acceptance decisions. The benefits of risk analysis can far out weigh the costs and are limited only by the imagination of the user willing to apply the science.

The final closing thought concerns the actual value of false accept risk as specified in the Z540.3. The establishment and implementation of this requirement should be a business decision based on technical, cost, and impact estimates. Z540.3 specifies false accept risk “shall not exceed 2%,” which is probably appropriate for many applications, but not all. In some applications, 2% may be an excessive level of risk (e.g., space flight or nuclear weapons), but what of the cases where 2% risk is not warranted, or a more likely scenario, not obtainable? It is imperative upon users of M&TE to be clear on the requirements for the application of their equipment. The application requirements should establish the *required* false accept risk, in lieu of relying on a target or default value which may not be appropriate, or may not even be obtainable.

It cannot be overstated; the devil is in the details.

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