Chaos and beyond in a water filled ultrasonic resonance system

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Abstract. Finite amplitude ultrasonic wave resonances in a one-dimensional liquid-filled cavity, formed by a narrow band transducer and a plane reflector, are reported. The resonances are observed to include not only the expected harmonic and subharmonic signals (1,2) but chaotic signals as well. The generation mechanism requires attaining a threshold value of the driving amplitude that the liquid-filled cavity system becomes sufficiently nonlinear in response. The nonlinear features of the system were recently investigated via the construction of an ultrasonic interferometer having optical precision. The transducers were compressional, undamped quartz and lithium niobate crystals having the frequency range 1-10 MHz, driven by a high power amplifier. Both an optical diffraction system to characterize the diffraction pattern of laser light normally incident to the cavity and a receiving transducer attached to an aligned reflector with lapped flat and parallel surfaces were used to assess the generated resonance response in the cavity. At least 5 regions of excitation are identified:

1. *Linear* region: at low intensity of the ultrasonic wave only the driving frequency component is present. The diffraction pattern of a light beam, normal to the sound field, is symmetric.
2. *Nonlinear* region: with increased sound amplitude the diffraction pattern becomes asymmetrical indicating the generation of the harmonics.
3. *Subharmonic* region: further increase of the amplitude above a threshold value (sensitive to the alignment of the transmitter and the reflector) leads to the generation of subharmonics, as indicated by the occurrence of additional orders in the diffraction pattern.
4. *Chaos*: increasing the drive amplitude to a second threshold level results in the transition from a region of oscillation stability to an unstable region characterized by a cascade of subharmonic bifurcations culminating in chaotic oscillations. The diffraction pattern is smeared out and time-chaotic in this region.
5. *Beyond chaos*: further increase of the amplitude results in a transition of the chaotic state into a second more stable region. The diffraction pattern is stable and nearly continuous, indicating the presence of many low frequency components.

INTRODUCTION. While studying finite amplitude ultrasonic wave resonance in a one-dimensional liquid-filled cavity formed by a narrow band transducer and a plane reflector, subharmonics of the driver's frequency were observed [1,2] in addition to the expected harmonic structure.

A model was developed [3] which assumes that the subharmonics are parametrically excited waves produced by instabilities introduced through the vibration of the transducer face. The vibration of the transducer periodically alters the cavity length and therefore the resonant frequencies. The amplitude threshold for subharmonic generation is found to depend on the wave attenuation. Parametric oscillation in a liquid filled cavity is given by the Mathieu-type expression

$$\frac{d^2 g}{dz^2} + \frac{a\alpha c}{\omega}\frac{dg}{dz} + \left(a - 2q\cos z\right)g = 0 \qquad (1)$$

where g is the cavity displacement, α is the wave attenuation coefficient, c is the sound velocity, ω is the angular frequency, $a = (\omega_n/\omega)^2$ where ω is the nth resonance frequency, q = a(A/*l*) where A is the transducer drive displacement amplitude, *l* is the cavity length, and z = ωt where t is time. There are two conditions that must be satisfied for subharmonic excitation:

1. The resonant frequency of the lowest subharmonic mode must be nearly equal to one half the drive frequency;
2. The amplitude A of the driver must be large enough to satisfy the threshold condition given as

$$A > l\alpha c / \omega . \qquad (2)$$

The solutions of the Mathieu's equation occur in the alternating regions or bands of stability and instability (defined by the factors "a" and "q" in Eq.(2)) with increasing values of the factor "a" (see Figure1).

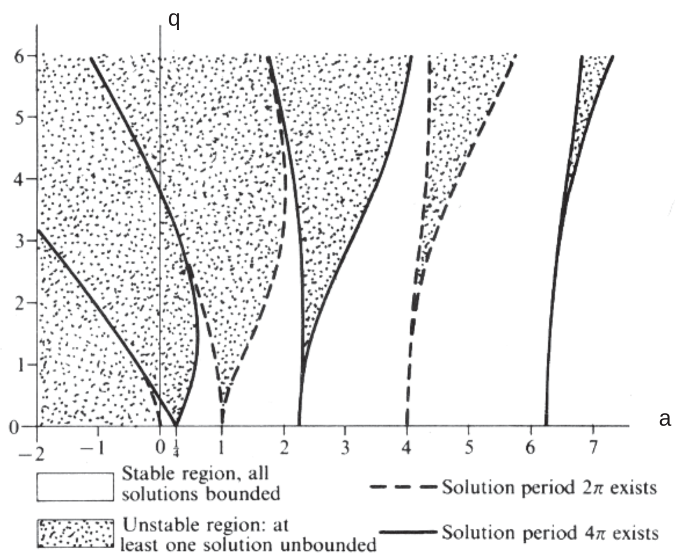


FIGURE 1. Regions of solution for the Mathieu's Function [6]

Both a and q are functions of the transducer drive amplitude. A more complete accounting of the phenomenon may be described by expanding Eq.(2) to include nonlinear contributions. The more complete equation is given as [3]

$$\frac{d^2 g}{dz^2} + \frac{a\alpha c}{\omega}\frac{dg}{dz} + (a - 2q\cos z)g + \frac{\omega^2}{2c^2}(\frac{B}{A}+2)(\frac{B}{A}+1)(\frac{a}{2} - 2q\cos 2z)g^3 = 0 \qquad (3)$$

where B/A is the nonlinearity parameter of the liquid. A first order approximation to the solution of Eq.(4) indicates that the threshold condition of parametric excitation is the same as that given by Eq. (2). Other theoretical studies to address the nonlinear features of this parametric system have been reported recently [4,5].

REGIONS OF ULTRASONIC WAVE GENERATION IN THE LIQUID FILLED CAVITY. The experimental system shown in Fig.2 consists of an interferometer with optical precision controls used to adjust the positions of the piezo-electric transducer (1MHz-10MHz; driven by a powerful amplifier) and a receiving transducer attached to an aligned reflector with lapped flat and parallel surfaces used to measure the generated frequency components in the cavity.

A visual assessment of the phenomena is obtained by passing laser light through the ultrasonic beam as indicated in Fig.3. The laser light is diffracted into various orders n at angles γ_n given by

$$\sin\left(\gamma_n\right) = n\lambda\omega / c \quad (4)$$

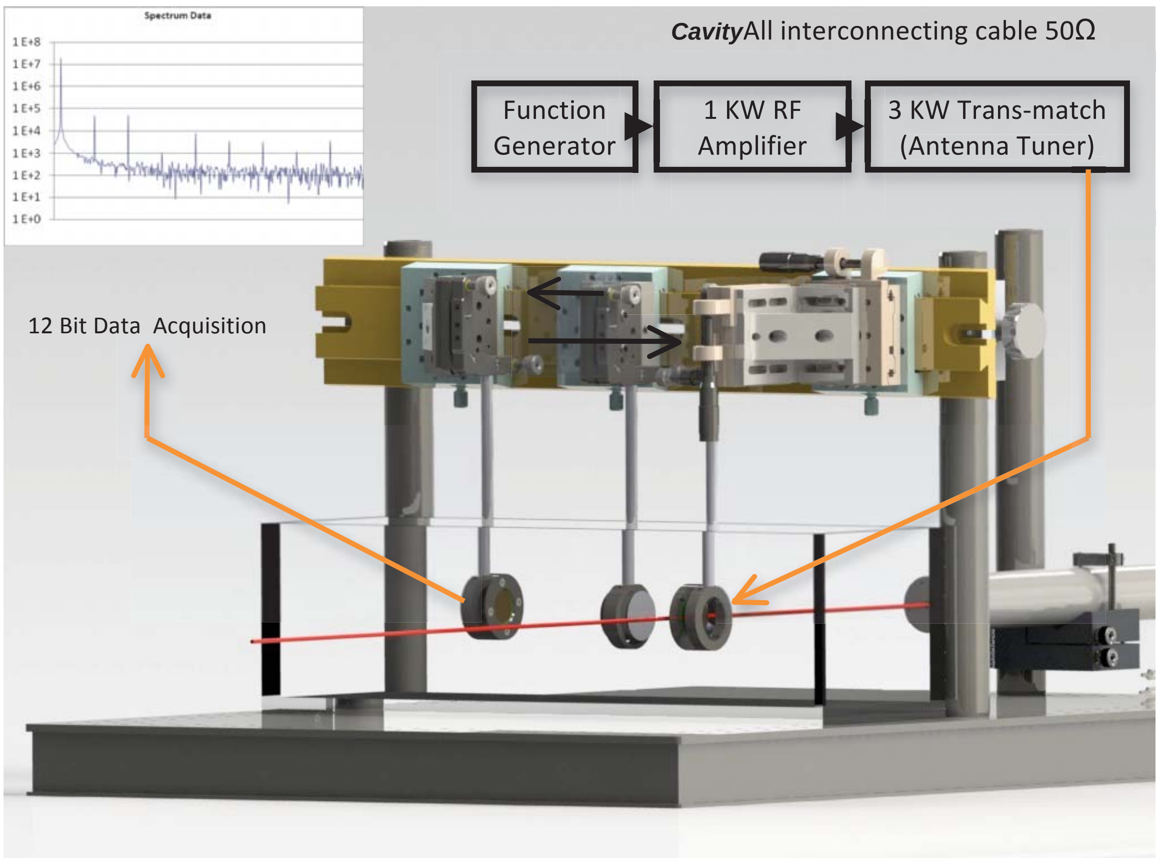


FIGURE 2. Experimental system

where λ is the wavelength of the light. Figure 3 shows the diffraction patterns obtained for various transducer drive amplitudes (voltages): (a) low amplitude ultrasonic waves (5 V); (b) finite amplitude waves (50V) resulting in an asymmetric diffraction pattern; and (c) parametric resonance (150V) producing extra diffraction orders due to the generation of subharmonics. The received frequency spectrum corresponding to the parametric resonance region is shown in Fig.4. In addition to the driver transducer frequency at 5MHz, the second and third harmonics at 10MHz and 15MHz, as well as subharmonics at 2.5 MHz are displayed in Fig.4.

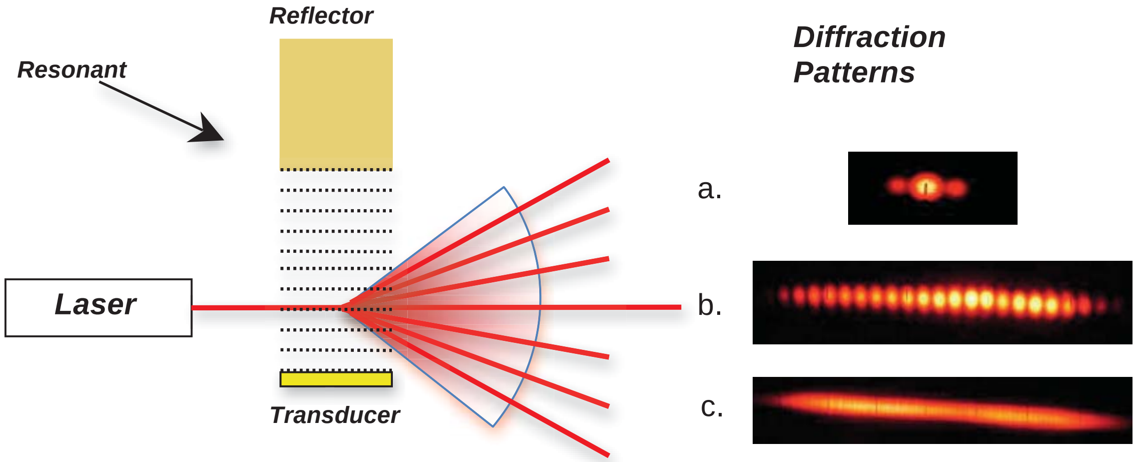


FIGURE 3. Laser beam diffraction by ultrasonic wave: a Linear region, b. Nonlinear region, c.Subharmonic region

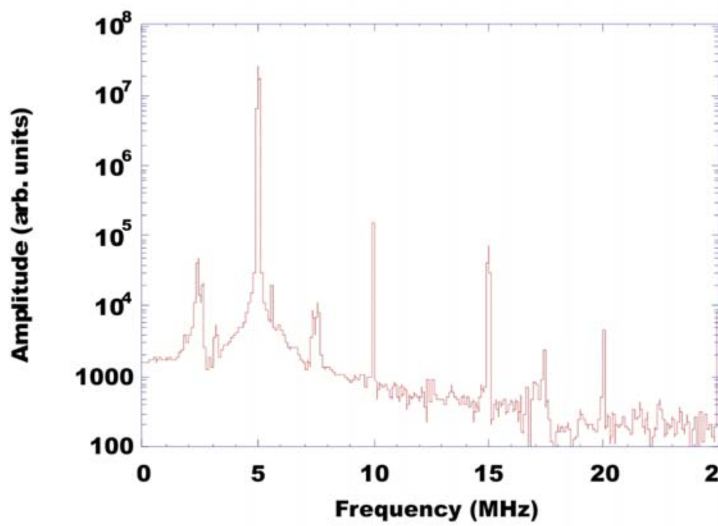
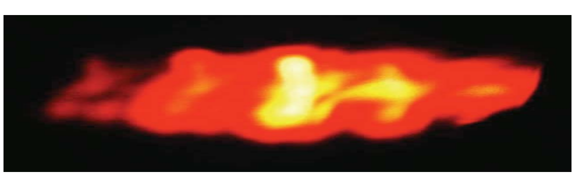
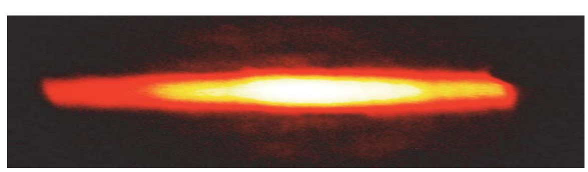


FIGURE 4. A plot of wave amplitude vs. frequency components including cavity generated fractional harmonics

PATH TO CHAOS. A significantly higher transducer drive voltage (450V) in the parametric resonance region leads to a cascade of bifurcations with increasing drive amplitudes that culminates in the generation of the chaotic pattern shown in Fig.5a. Instead of distinct diffraction orders, the laser produces a smeared out image due to the chaotic oscillations. Further increases in the transducer drive voltage (to 500 V) leads to a second region of stability following the region of chaotic instability. The diffraction pattern in the second region of stability is shown on Fig.5b. The pattern is similar to that of Fig.3c, indicating the presence of stable subharmonics.



a. Chaotic region



b. Stable subharmonic region beyond chaos

FIGURE 5. Laser beam diffraction of ultrasonic waves: a. In the chaotic region (450 V) b. Stable subharmonic region beyond the chaotic region (500 V).

CONCLUSION. In an ultrasonic parametric system, increasing acoustic drive amplitudes from a region of oscillation stability into an unstable region leads to a cascade of bifurcations (subharmonics) culminating in chaotic oscillations. A further increase in the amplitude results in a reversion of the chaos into a second region of stability.

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