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Kinetic model of electric potentials in localized collisionless plasma structures under steady quasi-gyrotropic conditions

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Localized plasma structures, such as thin current sheets, generally are associated with localized magnetic and electric fields. In space plasmas localized electric fields not only play an important role for particle dynamics and acceleration but may also have significant consequences on larger scales, e.g., through magnetic reconnection. Also, it has been suggested that localized electric fields generated in the magnetosphere are directly connected with quasi-steady auroral arcs. In this context, we present a two-dimensional model based on Vlasov theory that provides the electric potential for a large class of given magnetic field profiles. The model uses an expansion for small deviation from gyrotropy and besides quasineutrality it assumes that electrons and ions have the same number of particles with their generalized gyrocenter on any given magnetic field line. Specializing to one dimension, a detailed discussion concentrates on the electric potential shapes (such as “U” or “S” shapes) associated with magnetic dips, bumps, and steps. Then, it is investigated how the model responds to quasi-steady evolution of the plasma. Finally, the model proves useful in the interpretation of the electric potentials taken from two existing particle simulations. © 2012 American Institute of Physics. [<http://dx.doi.org/10.1063/1.4747162>]

I. INTRODUCTION

Thin current sheets with a thickness of the order of typical ion scales (gyroradius or ion inertial length) or smaller play a key role in the structure and dynamics of space plasmas. Ample evidence in support of this fact is available from *in-situ* magnetospheric observations^{1–11} and from numerical simulations and theory.^{12–27} Thin current sheets are thought to be relevant for magnetic reconnection^{5,7,9,10,12,14,15,18,20,22,24} and possibly as the magnetospheric cause of thin quasi-steady auroral arcs.^{21,28,29} It is characteristic of thin plasma structures that they contain significant electric fields. They are based on the difference between the properties of the relevant particle species, particularly, the small electron/ion mass ratio. They occur on scales on which the effects of finite gyroradii or inertial lengths cannot be ignored. Their full understanding requires kinetic considerations. Magnetohydrodynamics (MHD), which frequently provides reasonable results on larger scales, is widely unable to resolve thin magnetic structures, although often MHD indicates the presence of a thin current sheet by a singularity of the electric current density.²²

On scales exceeding the Debye-length, typical space plasmas are quasi-neutral with ion and electron densities being kept approximately equal by electric forces. Those tend to become larger for smaller length scales. As already indicated above, the electric signature associated with a localized magnetic structure is of particular interest for magnetosphere-aurora connection. Indeed, several regions in the magnetotail are possibly associated with quasi-steady (or “monoenergetic”) auroral arcs.^{21,28,30–35} More diffuse arcs presumably are associated with magnetospheric Alfvén waves. Details are given in a recent review.²⁹ In the present paper, the focus is on the former class. Typically, above the acceleration region of a quasi-steady arc the electric field is

directed perpendicular to the magnetic field. The corresponding electric potentials are often described to be of “U” shape or of “S” shape.^{36–41} In this paper, we develop a model that associates the electric potential profile directly with the magnetic field and density profiles of the local structure.

In the investigation of localized structures in plasmas, two main areas can be distinguished. The first is concerned with the question of how they are generated. In this area, most studies apply numerical simulations. For example, particle simulations have shown that the slow externally driven evolution of a wide plasma structure under a variety of circumstances leads to the formation of thin embedded current sheets.^{24,26} We will return to the electric signature of such sheets in Section VI.

The other area concentrates on the local structure of quasi-steady plasma conditions, using analytical methods where available. They are widely based on exploiting constants of the motion,^{21,28,42–45} particularly when the dependence on one or more coordinates can be ignored. Choosing this method, we assume a steady-state collisionless plasma that is translationally invariant with respect to the Cartesian y -coordinate. Then the canonical momentum P_y and the Hamiltonian H given by

$$P_y = mv_y + qA, \quad H = \frac{m}{2}(v_x^2 + v_z^2) + \psi \quad (1)$$

$$\text{with } \psi = \frac{1}{2m}(P_y - qA)^2 + q\phi$$

are constants of the motion. Here, m and q are particle mass and charge, v_x, v_z are x and z components of particle velocity, A and ϕ are functions of x, z and denote the magnetic flux function and the electric potential, respectively. The way in which Eq. (1) is written emphasizes that due to the y

invariance the motion can be understood as a planar motion in a force field described by the potential ψ . Throughout this paper, we assume that the electric and magnetic fields have no y components, such that they are given by

$$\vec{B} = \nabla A(x, z) \times \vec{e}_y, \quad \vec{E} = -\nabla \phi(x, z). \quad (2)$$

Under these conditions

$$f_s = F_s(P_y, H_s) \quad (3)$$

is a Vlasov equilibrium distribution function for arbitrary positive F_s . The subscript s denotes the particle species. We choose singly charged ions ($q_i = e$) and electrons ($q_e = -e$), e being the elementary charge. (P_y is used as an independent variable and therefore does not carry a species label.)

Various choices of F_s have been used to describe the properties of thin current sheets. Schindler and Birn²¹ and Birn *et al.*²⁸ used local Maxwellians in the v_x, v_z plane, such that

$$F_s = g_s(P_y) \exp(-H_s/T_s), \quad (4)$$

where T_s is the temperature (with Boltzmann's constant being absorbed into T_s). It was found that in the range $T_i > T_e$, typical potential differences of the order of T_i/e could be achieved for sufficiently different choices of $g_i(P_y)$ and $g_e(-P_y)$. For $g_i(P_y) = g_e(-P_y)$, the potentials typically were an order of magnitude smaller. The sensitivity of the potential to the choice of g_s suggests that it would be desirable to put the P_y dependence of F_s on a more physical basis. This paper makes a step in that direction. We assume a plasma structure in which the magnetic field strength remains large enough for justifying a power expansion in terms of the ratio of the ion gyroradius and the length scale of the structure, truncated after the second order. Adopting Eq. (4), we impose two conditions on the choice of g_s . The first is the familiar condition of quasi-neutrality. The second condition (addressed as “P-condition”) requires that there is a frame of reference where for every field line the number of particles that have their *generalized gyrocenters* on that field line is the same for electrons and ions. (In zeroth order, the generalized gyrocenter reduces to the ordinary gyrocenter, details are explained in the following section.) One may imagine that the plasma has been generated in that way by some kind of source process. As we will see, this model has a number of interesting properties that seem worthwhile exploring. It is favorably tested with earlier simulation results (Section VI).

II. THE MODEL

Particle distribution functions of the form (4), written explicitly, take the form

$$f_s(v_x, v_z, x, z, P) = g_s(P) \exp\left(-\frac{m_s(v_x^2 + v_z^2)}{2T_s} - \frac{(P - q_s A(x, z))^2}{2m_s T_s} - \frac{q_s \phi(x, z)}{T_s}\right). \quad (5)$$

The variable P_y was replaced by P , as there is no danger of confusion.

The first condition that we impose on g_s is the quasi-neutrality condition

$$n_e = n_i, \quad (6)$$

where $n_s = \int f_s dv_x dv_z dP$ is the density of species s . The second condition is the *P-condition*

$$u_e(A^*) = u_i(A^*), \quad (7)$$

where

$$u_s(A^*) = \int \delta(P - q_s A^*) f_s dv_x dv_z dP dx dz. \quad (8)$$

This condition is based on a generalization of the gyrocenter for gyrotropic motion, which under the present symmetry would be located on the field line where $v_y = 0$ or, using Eq. (1), where the condition is $P - q_s A = 0$ holds. We generalize this notion to include systems that admit deviations from gyrotropy. If $P - q_s A = 0$ holds we say that the particle has its *generalized gyrocenter* on the field line with flux value A . Thus, $u_s(A^*)$ is the number density of particles of species s that in the present frame of reference have their generalized gyrocenter located on the field line $A = A^*$. Then the P-condition (7) postulates that this number is the same for electrons and ions.

The only equation left to assure selfconsistency is Ampère's law

$$-\Delta A = \mu_0 J(A), \quad (9)$$

where $J(A)$ is the electric current density, after eliminating ϕ via Eq. (6).

III. THE EXPANSION

Here, we introduce the expansion for small gyroradii. The smallness parameter is the ratio of the ion gyroradius and the local scale length L , represented by $\varepsilon_s = \sqrt{m_s T_s} / (eBL)$, so that formally one expands in powers of $\sqrt{m_s}$. Further below, the electrons will be set to the gyrotropic limit $m_e \rightarrow 0$.

We start with density, which in an explicit form can be written as

$$n_s = \frac{2\pi}{\sqrt{m_s T_s}} \exp\left(-\frac{q_s \phi}{T_s}\right) \int G_s(P) \exp\left(-\frac{1}{2m_s T_s} (P - q_s A)^2\right) dP, \quad (10)$$

where g_s was replaced by $G_s = g_s T_s^{3/2} / m_s$ which is treated as finite for vanishing m_s to ensure finite density. The Gaussian kernel of the integral localizes the integrand near $P = q_s A$. This suggests substituting $P = q_s A + \delta P$ and expanding G_s in powers of δP , which gives after δP integration

$$n_s = (2\pi)^{3/2} \exp\left(-\frac{q_s \phi}{T_s}\right) \left(G_s(q_s A) + \frac{m_s T_s}{2e^2} G_s(q_s A)'' \right) + O(\varepsilon_s^4), \quad (11)$$

where the prime symbol denotes differentiation with respect to A . Since odd powers of ε_s do not contribute, truncation after the second order means that the error is of order ε_s^4 . The quasi-neutrality condition (6) can now be used to express ϕ in terms of A ,

$$\exp\left(e\phi(A)\left(\frac{1}{T_e} + \frac{1}{T_i}\right)\right) = U(A), \quad (12)$$

where

$$U(A) = \frac{G_i(eA) + \frac{m_i T_i}{2e^2} G_i(eA)''}{G_e(-eA) + \frac{m_e T_e}{2e^2} G_e(-eA)''} \quad (13)$$

and the error term was suppressed. We now make use of the smallness of the mass ratio m_e/m_i by taking the limit $m_e/m_i \rightarrow 0$, obtaining

$$U(A) = \frac{G_i(eA)}{G_e(-eA)} \left(1 + \frac{m_i T_i}{2e^2} \frac{G_i(eA)''}{G_i(eA)}\right). \quad (14)$$

From Eq. (12), we then find

$$e\phi = \frac{T_e T_i}{T} \ln U, \quad T = T_e + T_i. \quad (15)$$

Let us now turn to the P-condition (7). Explicitly, the function u_s reads

$$u_s(A^*) = \frac{2\pi}{\sqrt{m_s T_s}} G_s(q_s A^*) \times \int \exp\left(-\frac{e^2}{2m_s T_s} (A - A^*)^2 - \frac{q_s \phi(A)}{T_s}\right) V(A) dA, \quad (16)$$

where $V(A) = \int_A ds/B$ is the differential flux tube volume, ds being the arc length differential on field line A and B the magnetic field magnitude. An expansion analogous to that applied to density gives

$$u_s(A^*) = \frac{(2\pi)^{3/2}}{e} G_s(q_s A^*) \exp\left(-\frac{q_s \phi(A^*)}{T_s}\right) V(A^*) + \frac{(2\pi)^{3/2} m_s T_s}{e 2e^2} G_s(q_s A^*) \left(\exp\left(-\frac{q_s \phi(A^*)}{T_s}\right) V(A^*)\right)'', \quad (17)$$

where now the prime symbol denotes differentiation with respect to A^* .

After inserting ϕ from Eq. (15) into Eq. (17) and applying the limit of vanishing electron mass, we obtain an explicit expression for the P-condition (7). Analyzing that condition, we first find that in the zeroth order it is satisfied identically. This is intuitively clear, because to lowest order the gyrocenter and particle locations coincide so that the P-condition is an automatic consequence of quasi-neutrality. The next higher nonvanishing order terms are linear in m_i (order ε_i^2). In that order, the P-condition (7) gives

$$\frac{m_i T_i}{2e^2} \frac{G_i(eA)''}{G_i(eA)} = \frac{m_i T_i}{2e^2 V(A)} \left(V(A) \left(\frac{G_i(eA)}{G_e(-eA)} \right)^{-T_e/T} \right)'' \times \left(\frac{G_e(-eA)}{G_i(eA)} \right)^{T_e/T}. \quad (18)$$

As this equation holds for arbitrary A^* and there is no danger of confusion, we replaced A^* by A . After dividing by $m_i T_i/(2e^2)$ and defining

$$Y = V(A) \left(\frac{G_e(-eA)}{G_i(eA)} \right)^{T_e/T}, \quad (19)$$

Eq. (18) assumes the form

$$\frac{G_i(eA)''}{G_i(eA)} = \frac{Y''}{Y}. \quad (20)$$

For any Y , considered an arbitrary function of A , Eq. (20) is read as a differential equation for $G_i(A)$, which has the general solution

$$G_i(A) = Y(A) \left(c + c_0 \int_{A_0}^A \frac{d\tilde{A}}{Y(\tilde{A})^2} \right), \quad (21)$$

where c, c_0 are arbitrary constants for any fixed A_0 . From Eqs. (11), (12), and (19), we find

$$G_i(A) = \frac{n(A)V(A)}{(2\pi)^{3/2} Y(A)}. \quad (22)$$

Equating (21) and (22) gives

$$\frac{n_0 V_0}{Y_0^2} + c_0 \int_{A_0}^A \frac{d\tilde{A}}{Y(\tilde{A})^2} = \frac{n(A)V(A)}{Y(A)^2}, \quad (23)$$

where the factor $(2\pi)^{3/2}$ was absorbed in the arbitrary constants, and then the constant c was expressed by n_0, V_0, Y_0 , the values of n, V, Y at A_0 . Further, introducing non-dimensional quantities $\hat{n} = n/n_0$, $\hat{V} = V/V_0$, and $\hat{Y} = Y/Y_0$, Eq. (23) assumes the form

$$1 + d_0 \int_{A_0}^A \frac{dA'}{\hat{Y}(A')^2} = \frac{\hat{n}\hat{V}}{\hat{Y}^2}, \quad (24)$$

where d_0 is an arbitrary constant replacing c_0 .

Equation (24), read as an integral equation for \hat{Y} , has a unique positive solution (satisfying the boundary condition at $A = A_0$) determined by

$$\hat{Y}^2 = \hat{n}\hat{V} \exp\left(d_0 \int_{A_0}^A \frac{dA'}{\hat{n}(A')\hat{V}(A')}\right). \quad (25)$$

Using the definitions of U (lowest order) and Y given by Eqs. (14) and (19), we obtain the electric potential ϕ from Eq. (15)

$$e(\phi - \phi_0) = -\frac{T_i}{2} \ln \frac{\hat{n}}{\hat{V}} + \alpha \int_{A_0}^A \frac{dA'}{\hat{n}(A')V(A')}, \quad (26)$$

where α is an arbitrary constant replacing d_0 and $\phi_0 = \phi(A_0)$.

This equation holds in the frame of reference where the P-condition is satisfied. A transformation to a reference frame that moves with velocity η in the y direction gives in that frame

$$e(\phi - \phi_0) = -\frac{T_i}{2} \ln \frac{\hat{n}}{\hat{V}} + \alpha \int_{A_0}^A \frac{dA'}{\hat{n}(A')V(A')} - \eta(A - A_0). \quad (27)$$

Equation (27) is the desired result for two-dimensional configurations. It is comforting that the knowledge of the ion temperature, density, and flux tube volume suffices to find the electric potential as a function of A .

In the special case of one-dimensional fields with translational invariance with respect to x and y , the magnetic flux function A depends on z alone, such that the magnetic field has an x component only. In the derivation of the expression for ϕ , the spatial integrations in the x, z plane is replaced by z integrations with the flux tube volume V being replaced by $1/B$. Accordingly, one finds

$$e(\phi - \phi_0) = -\frac{T_i}{2} \ln(\hat{n}\hat{B}) + \alpha \int_{A_0}^A \frac{\hat{B}(A')}{\hat{n}(A')} dA' - \eta(A - A_0). \quad (28)$$

Another difference in comparison with the 2D case is the additional constant of the motion P_x , which in the absence of the x component of the vector potential can be replaced by v_x . Thus,

$$F_s = g_s(P_y) \exp\left(-\frac{m_s v_x^2}{2T_{1s}} - \frac{h_s}{T_{2s}}\right), \quad h_s = \frac{m_s}{2} v_z^2 + \psi \quad (29)$$

is a Vlasov equilibrium distribution function. It takes into account anisotropy by attributing different temperatures to the motion in x and in y, z . Carrying out the same expansion as above for that case, one recovers the expression (28) with the only modification that T_i is to be replaced by T_{2i} . The temperature associated with the x motion does not enter explicitly. Although T_{1s} appears in n_s and u_s , it disappears after defining $g_s = \sqrt{m_s}/(\sqrt{T_{1s}T_{2s}})G_s$, where G_s plays the same role as in the isotropic case.

The expressions (27) and (28) are the results of power expansions and break down when the formal values of ϕ become too large. This can pose a limitation to the choice of the free constants α and η . For instance, if one considers a one-dimensional plasma structure where n and B assume nonvanishing constant values for $z = \pm\infty$, the constants α and η have to vanish so that we find

$$e(\phi - \phi_0) = -\frac{T_i}{2} \ln(\hat{n}\hat{B}) = -\frac{T_i}{2} \ln(\hat{p}\hat{B}) \quad (30)$$

instead of Eq. (28). On the right hand side of Eq. (30), density n is replaced by pressure p associated with the x, z plane, such that $\hat{n} = \hat{p}$. In the 1D cases, one can then take into account Eq. (9) (in integrated form) by introducing the pressure balance

$$p + \frac{B^2}{2\mu_0} = p_c, \quad (31)$$

where p_c is a constant. Thus, for one-dimensional fields, ϕ can be expressed in terms of the magnetic field alone.

IV. EXAMPLES

Here, we apply the findings of the previous section to simple one-dimensional structures. Where appropriate, we use z as the independent variable instead of A ; note that $A(z)$ is monotonic as B does not vanish (the expansion breaks down if B becomes too small). The plasma is assumed to reach asymptotically homogeneous states for $z = \pm\infty$. We use quantities $\bar{B} = B/B_2$, $p = p/p_2$, etc., where the quantities labeled by the subscript “2” are taken at $z \rightarrow \infty$ (or at the right boundary for finite systems), so that $\bar{B}(\infty) = 1$, $\bar{p}(\infty) = 1$. The pressure balance (31) then gives

$$\bar{p} = 1 + \frac{1}{\beta_2} (1 - \bar{B}^2), \quad \beta_2 = \frac{B_2^2}{2\mu_0 p_2}. \quad (32)$$

Using Eq. (32) in Eq. (30) and setting $\phi(\infty) = 0$, we find for $\varphi = e\phi/T_i$ the expression

$$\varphi(z) = -\frac{1}{2} \ln \left(\bar{B}(z) + \frac{1}{\beta_2} (1 - \bar{B}(z)^2) \bar{B}(z) \right). \quad (33)$$

As the first group of examples, we consider magnetic dips and bumps, assuming that $\bar{B}(z)$ is symmetric with respect to $z = 0$ and has a single extremum (at $z = 0$). In that case, $\varphi(z)$ has either a single extremum or three extrema. The number of extrema, as well as the extreme values, are determined by the parameters $\bar{B}(0)$ and β_2 alone. Thus, for $\bar{B}(0)$ kept fixed, two otherwise different choices of $\bar{B}(z)$ provide different potentials $\varphi(z)$, but if β_2 is fixed also they have the same qualitative shape, as defined by the number of extrema and by the extreme values. This is illustrated by Figure 1.

Figure 2 gives the qualitatively different regimes of the $\varphi(z)$ profiles in $\bar{B}(0), \beta_2$ space. As the figure indicates, for suitable parameters a magnetic dip ($\bar{B}(0) < 1$) can be associated with either an electric dip (or “U-shape”) or an electric bump. More complex φ profiles with additional side minima also occur. The same applies to magnetic bumps ($\bar{B}(0) > 1$).

Next, we consider magnetic steps with monotonically increasing $\bar{B}(z)$ and again $\bar{B}(\infty) = 1$. Here, the parameters that determine the qualitative shape of the electric potential are $\bar{B}_1 = \bar{B}(-\infty)$ (or for finite systems the value at the left boundary) and β_2 . Correspondingly, the two different magnetic profiles of Figure 3 (with same parameters) have the same qualitative potential shape.

Figure 4 gives the different regions where qualitatively different φ shapes occur. The figure shows the presence of S shaped potentials of different orientation with and without

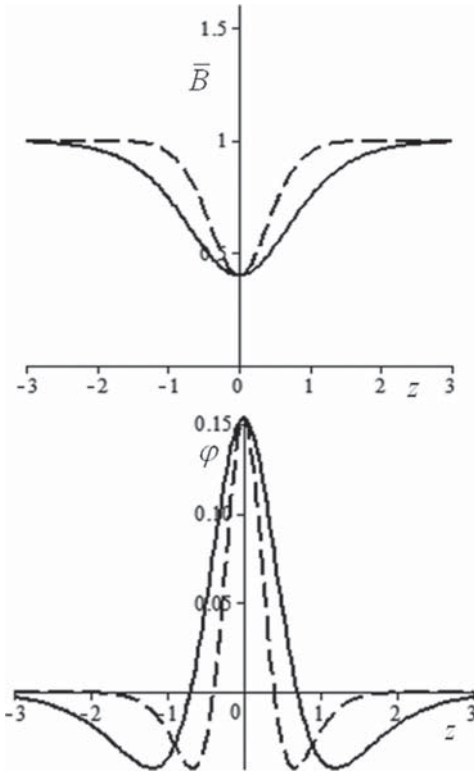


FIG. 1. Two examples for magnetic dips. The solid and dashed curves shown in the upper graph correspond to two different choices for $\bar{B}(z)$, picked arbitrarily except that they have the same parameter $\bar{B}(0)$ (in addition to symmetry, single extremum and $\bar{B}(\infty) = 1$). The lower graph shows the corresponding electric potentials ϕ given by Eq. (33) with β_2 fixed also.

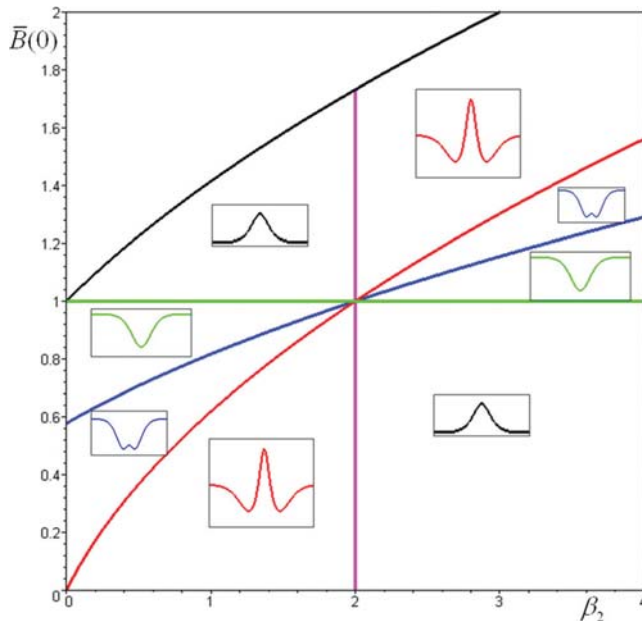


FIG. 2. Parameter space $\bar{B}(0)$ vs. β_2 for magnetic dips and bumps as defined in the text. Shown are the regions that correspond to different qualitative shapes of the electric potential $\phi(z)$, sketched inside the boxes. Same qualitative shapes are indicated by the same color. The region above the black curve is unphysical (negative plasma pressure), the green line separates magnetic dips ($\bar{B}(0) < 1$) from magnetic bumps ($\bar{B}(0) > 1$), on the red curve $\phi(0)$ changes sign, on the blue curve the two side minima on one side merge with the central extremum, and on the magenta line the side minima on one side disappear when their ϕ values reach 0.

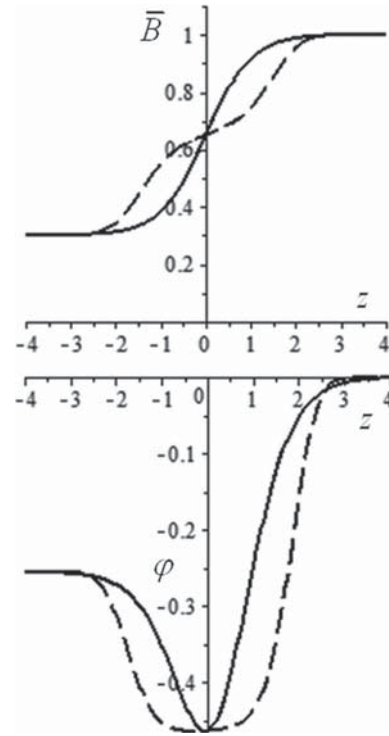


FIG. 3. Two examples for magnetic steps. The solid and dashed curves shown in the upper graph correspond to two different choices for $\bar{B}(z)$, picked arbitrarily except that they have the same parameter \bar{B}_1 (in addition to monotonic increase with z and $\bar{B}(\infty) = 1$). The lower graph shows the corresponding electric potentials ϕ given by Eq. (33) with a fixed value of β_2 .

modification involving a minimum. Note that on the red line, ϕ becomes U-shaped although the magnetic profile is S-shaped.

V. SLOW EVOLUTION

Steady state models are largely interpreted as snapshots of slowly evolving systems, driven by a suitable boundary condition, such as addition of magnetic flux. This raises the following question. Suppose our model applies to the electric potential of a given initial snapshot of such a slowly evolving system. Does it then remain valid during the subsequent evolution? We will answer this question for the cases of magnetic dips/bumps and magnetic steps as considered in the previous section. Conveniently, we refer to earlier analytical studies of quasistatic evolution of one- and two-dimensional systems in the gyrotropic limit.^{24,26} There, it was demonstrated that in that limit, the one-dimensional evolution is governed by simple similarity transformations and can be expressed through a single parameter, say $b(t)$, representing the external driving. We found that for the quantities that are relevant in the present context, the transformations are

$$\begin{aligned} B(z, t) &= b(t)B_0(b(t)z) \\ n(z, t) &= b(t)n_0(b(t)z) \\ p(z, t) &= b(t)^2p_0(b(t)z) \\ \phi(z, t) &= \phi_0(b(t)z), \end{aligned} \quad (34)$$

where p is the zz component of the pressure tensor, summed over species. The subscript 0 refers to the initial state.

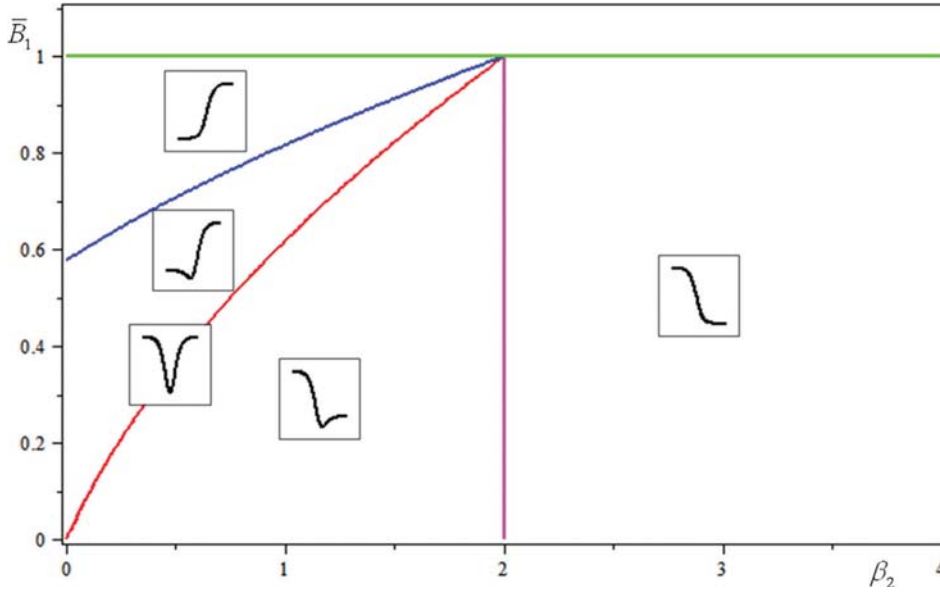


FIG. 4. Parameter space \bar{B}_1 vs. β_2 for magnetic steps. Shown are the regions that correspond to different qualitative shapes of the electric potential $\phi(z)$, indicated in the boxes. Steps where B decreases with z (i.e., $\bar{B}_1 > 1$) are left out, because these shapes can be reduced to corresponding shapes with $\bar{B}_1 < 1$ by reversal of the z axis and a renormalization.

Assuming constant asymptotic states (at $z = \pm\infty$), we can use Eq. (30) for the model potential, which we write as

$$\phi_m(z) = -\frac{T_i}{2e} \ln(\bar{n}(z)\bar{B}(z)), \quad (35)$$

where, as before, the bar symbol indicates normalization with respect to the value at $+\infty$, so that $\phi_m(\infty) = 0$. The time variable is suppressed.

Within our approximations and by our assumption that the model holds initially, the initial potential is given by

$$\phi_0(z) = \phi_{m0}(z) = -\frac{T_{i0}}{2e} \ln(\bar{n}_0(z)\bar{B}_0(z)). \quad (36)$$

According to Eq. (34) at some later time t , the potential is

$$\phi(z) = \phi_0(bz) = -\frac{T_{i0}}{2e} \ln(\bar{n}_0(bz)\bar{B}_0(bz)). \quad (37)$$

Again using Eq. (34) we find $\bar{n}(z) = \bar{n}_0(bz)$, $\bar{B}(z) = \bar{B}_0(bz)$ such that the model at time t given by Eq. (35) becomes

$$\phi_m(z) = -\frac{T_i}{2e} \ln(\bar{n}(z)\bar{B}(z)) = b\phi(z), \quad (38)$$

where we used that T scales as b .

Thus,

$$\phi(z) = \frac{1}{b} \phi_m(z). \quad (39)$$

So, strictly speaking, the model is not conserved during the evolution. However, the violation consists only in the presence of the constant factor $1/b$. The *adjusted model potential*

$$\phi_a = \frac{1}{b} \phi_m \quad (40)$$

would provide correct modeling (within the present simplifications).

Thus, our model ϕ_m , if adjusted with a suitable multiplier $1/b$, is relevant for an entire slow evolution if that evolution contains a single state to which ϕ_m applies exactly. In particular, the qualitative potential shape, within present approximations, is correctly represented by the model. This also follows from the fact that the parameters $\bar{B}(0)$, β_2 of Figure 2 and \bar{B}_1 , β_2 of Figure 4 are conserved under Eq. (34). In the following, we will refer to our model as the “relaxed model” if adjustment of the electric potential by a constant multiplier is admitted.

VI. APPLICATION TO TWO EARLIER SIMULATIONS

To gain further insight into the usefulness of our model, we applied it to two simulations carried out earlier.²⁶ Surprisingly, we found that in both cases Eq. (39) holds approximately for suitably adjusted values of the parameter b so that the relaxed model proves applicable.

The results are shown in Figures 5 and 6 for the two simulations. Plotted are the simulation magnetic field, the simulation potential ϕ , the model potential ϕ_m , and the adjusted model potential ϕ_a .

Surprisingly, in both cases the relaxed model curves (small-scale-broken lines) show fair agreement with the simulation (solid line), while the model ϕ_m (dashed curves) deviates grossly from the simulation potential. The parameter b was found by a best fit procedure, which gave $b = 3.6$ for the first simulation (Fig. 5) and $b = 6.0$ for the second (Fig. 6).

It is instructive to confirm that the potential shapes of the simulations can be identified with the corresponding model shapes in Figures 2 and 4, respectively. The magnetic step of Figure 5 has the (simulation) parameters $\bar{B}_1 = 0.06$, $\beta_2 = 0.10$, which is close to the red curve in Fig. 4, on which ϕ_m is exactly U shaped, consistent with the approximate U shape of the simulation potential. The approximate U shape of the simulation potential in Figure 6 has the parameters $\bar{B}(0) = 0.67$, $\beta_2 = 0.05$, which in Figure 2 lie in the U shape

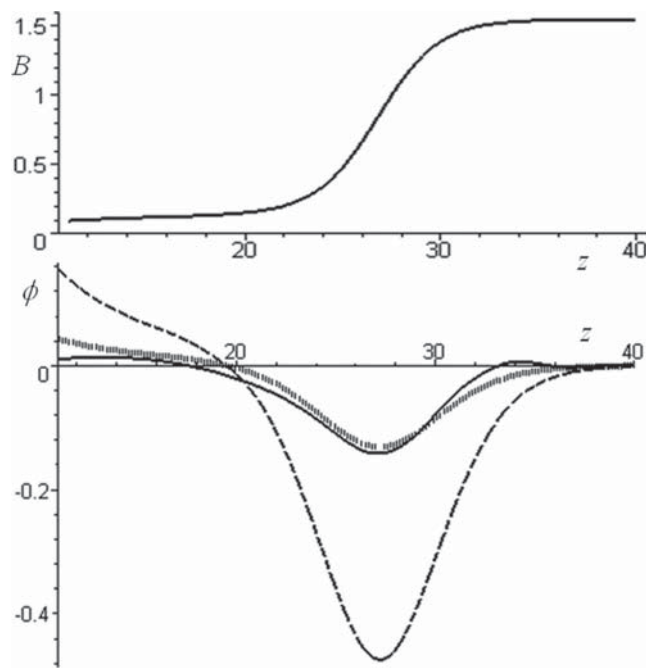


FIG. 5. The figure provides an application of the present model to an earlier particle simulation (Section IV of Ref. 26, interval $10 \leq z \leq 40$). The upper graph shows the magnetic field profile. The lower graph gives the electric potential of the simulation (solid curve), the model curve ϕ_m based on simulation data (dashed line) and the adjusted model potential ϕ_a (fine-scale broken line). The parameter $b = 3.6$ was determined by a minimum variance fit.

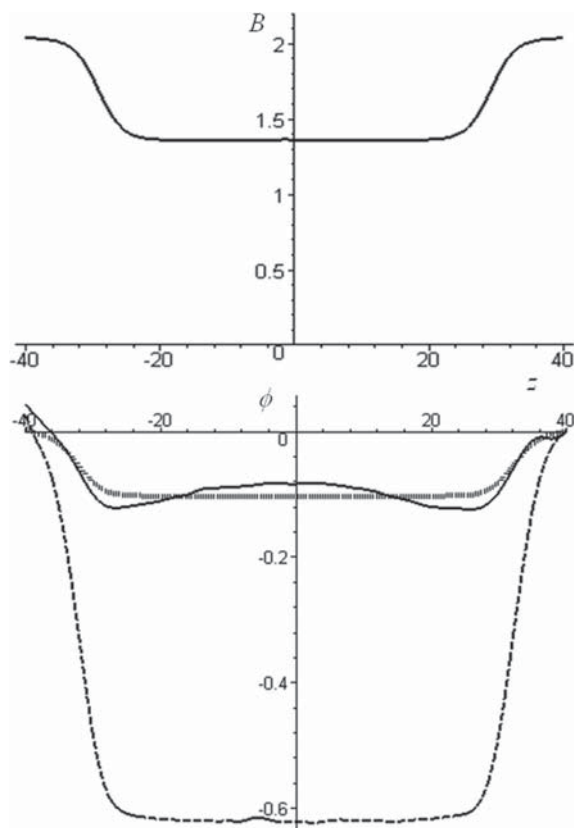


FIG. 6. This figure corresponds to Figure 5 for the second simulation (Section V of Ref. 26). The parameter $b = 6.0$ was determined by a minimum variance fit.

regime between the blue and green curves. Alternatively, we may consider the right flank of the U shaped ϕ profile in Figure 6 as a step with parameters $\bar{B}_1 = 0.67$, $\beta_2 = 0.05$, which, as expected, yields a step in Figure 4.

VII. SUMMARY AND DISCUSSION

This paper addresses properties of electric potentials that exist in localized steady plasma structures, specifically thin current sheets embedded in uniform or weakly varying magnetic fields. A two-dimensional model is presented that is based on finite ion gyro-radius effects and expresses the potential in terms of ion temperature, density, and magnetic flux tube volume (Eq. (27)). The model is valid for a particular class of distribution functions given by Eq. (4). Further, it uses an expansion for small deviation from gyrotropy and besides quasineutrality it assumes that electrons and ions have the same number of particles with their generalized gyrocenter on any given magnetic field line, the P-condition. There is no magnetic field component in the invariant direction. The one-dimensional specialization gives the electric potential in terms of density and magnetic field magnitude (Eq. (28)) and if (via pressure balance) the density is eliminated, the potential is expressed in terms of the magnetic field alone. For homogeneous asymptotic states at large $|z|$, the model assumes a particularly simple form (Eq. (30) or Eq. (33)).

One might wonder why the expressions for the electric potential, which is the result of an expansion into ion non-gyrotropy, do not involve the ion mass. The reason is that the zeroth order of the P-condition is satisfied identically, so that the next higher nonvanishing order determines the zeroth order of G_i . This property is carried over to the potential ϕ , so that it is determined by zero-order quantities alone. The electron temperature drops out, because in the present limit of $m_e/m_i \rightarrow 0$, the electrons do not contribute to finite-gyroradius effects.

The 1D model is employed to study the electric potential forms of magnetic dips, bumps, and steps (Figures 1–4). Each of these elementary magnetic structures can be associated with a variety of different potential profiles, their qualitative shape depending only on two parameters. A magnetic dip can be associated either with a potential dip (“U” shape) or a potential bump, transitional forms can also occur (Figure 2). A magnetic step can have an electric potential that is either “S” shaped or “U” shaped or of a transitional shape (Fig. 4).

Electric potential shapes play a central role in observed quasi-steady auroral arcs. U-shaped potentials above the acceleration region corresponding to converging perpendicular electric fields are frequently observed to be associated with downward acceleration of electrons and upward electrical currents.^{34,39–41} S-shapes and transitional shapes have been observed also. For instance, Marklund *et al.*³⁴ presented spacecraft data indicating a transition from a U-shaped to an S-shaped potential.

If our model applies to an initial state of a slow evolution driven by external action, it remains to be applicable during the evolution, however, only in its relaxed form

allowing the potential to be adjusted by a constant multiplier. This does not affect the qualitative potential shapes, which are conserved in the entire evolution sequence.

Density and magnetic field strength have to be bounded away from zero to avoid breakdown of the expansion. The expansion can readily be extended to higher orders, should this be required for particular applications. Our model does not cover plasmas for which the mass ratio m_e/m_i is not small (e.g., electron-positron plasma), but the method can easily be generalized to include such cases.

When, tentatively, we applied the model to two earlier simulations (Figs. 5 and 6), it came as a pleasant surprise that both cases were found to be covered by the relaxed model. How is this possible, given the fact that our model is subject to stringent or arbitrary assumptions such as the weak gyrotropy condition and the P-condition? Weak gyrotropy means that ε_i , the ratio of the ion gyroradius over the length scale of the magnetic field, must be small compared to 1, but this condition is alleviated by the fact that the error is of order ε_i^4 . In fact, the weak gyrotropy condition is reasonably satisfied in the cases in question; the maximum error amounts to 6% for the first simulation (Fig. 5) and is near 10^{-6} for the second (Fig. 6). Regarding the other assumptions, the situation is less clear. It might be significant that the agreement between model and simulations applies only to the relaxed version of the model which involves a fitting parameter that widens the range of applicability of the relaxed model compared with the non-relaxed model substantially. For example, the agreement could be explained if the simulation state can be understood as a member of a set of slowly evolving equilibria with one member being covered by the non-relaxed model (see Section V). It seems also possible that the actual range of applicability is wider than it might seem from our derivation. Investigations beyond the present scope seem required to provide a firm answer.

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