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## Abstract

Unmanned Aerial Vehicles (UAV) are playing an increasing role in aviation. Various methods exist for the computation of UAV attitude based on low cost microelectromechanical systems (MEMS) and Global Positioning System (GPS) receivers. There has been a recent increase in UAV autonomy as sensors are becoming more compact and onboard processing power has increased significantly. Correct UAV attitude estimation will play a critical role in navigation and separation assurance as UAVs share airspace with civil air traffic. This paper describes attitude estimation derived by post-processing data from a small low cost Inertial Navigation System (INS) recorded during the flight of a subscale commercial off the shelf (COTS) UAV. Two discrete time attitude estimation schemes are presented here in detail. The first is an adaptation of the Kalman Filter to accommodate nonlinear systems, the Extended Kalman Filter (EKF). The EKF returns quaternion estimates of the UAV attitude based on MEMS gyro, magnetometer, accelerometer, and pitot tube inputs. The second scheme is the complementary filter which is a simpler algorithm that splits the sensor frequency spectrum based on noise characteristics. The necessity to correct both filters for gravity measurement errors during turning maneuvers is demonstrated. It is shown that the proposed algorithms may be used to estimate UAV attitude. The effects of vibration on sensor measurements are discussed. Heuristic tuning comments pertaining to sensor filtering and gain selection to achieve acceptable performance during flight are given. Comparisons of attitude estimation performance are made between the EKF and the complementary filter. Additionally, GPS heading estimates are compared with EKF and complementary filter heading estimates.

## Introduction

There is an extensive body of literature regarding attitude estimation using various sensor inputs [1-11]. Methods vary from computationally expensive particle filters to fast direct methods using complementary filtering. Estimator choice is based on available hardware and cost constraints. The application covered here is based on the use of existing attitude estimation schemes. These schemes are used to post-process flight data from a subscale

Unmanned Aerial Vehicle (UAV). While particle filtering in principle can produce superior state estimates for highly nonlinear systems that are perturbed by unknown distributions, they suffer from a large computational burden. The Unscented Kalman Filter (UKF) reduces the computational burden by picking the “particles”, (that is the sigma points) deterministically in a manner that preserves the mean and variance through nonlinear transformation [12]. The Extended Kalman Filter (EKF) uses a linearized representation of the nonlinear system to estimate the state [13]. While the EKF is computationally faster than the UKF, it does not perform as well as the UKF in systems that are highly nonlinear [14]. An even faster nonlinear state estimation developed for attitude estimation is the complementary filter [15-19]. This filter may be tuned in the frequency domain to select areas of sensor spectrum containing the most state information while rejecting the spectral content most corrupted by noise. Because the application studied here is for UAVs, which are mildly nonlinear, the EKF and the complementary filter are chosen to post-process the flight data. The resulting attitude estimations are compared to those of the GPS-aided ArduPilot, (a popular hobbyist grade autopilot utilizing a complementary filter for attitude estimation). It is shown that reasonable attitude estimation cannot be obtained without the use of air data or GPS to correct acceleration measurements during turns. Reasonable attitude estimations may be obtained by modifying the EKF and complementary filter to account for centrifugal accelerations resulting from turns. The utility of the modified EKF and complementary filter is demonstrated and shown to be ideal methods for attitude estimation using low cost sensors for UAV application without the need for GPS.

## Mathematical Development

Reference frame transformations and algorithms for the EKF and complementary filter are described in this section. The algorithms are presented for discrete time implementation and references given describing development and additional experimental outcomes.

### *Reference Frames*

Several reference frames may be used to describe the orientation of an aircraft in three-

dimensional space [6, 20]. The body frame is the reference frame with the origin at the aircraft's center of gravity, positive z pointing out of the bottom of the aircraft, positive x out of the nose, and positive y in the direction of the right wing. For the inertial frame, positive x points to magnetic north, positive z points toward the earth's center of gravity, and positive y is normal to the x-z plane, approximately east; this forms the North, East, Down, (NED) frame.

Before the accelerometer readings can be used to estimate aircraft attitude, they must be compensated to reflect only the effects of gravity and not accelerations due to turning [17, 19]. The airspeed  $V_{air}$ , angle of attack measurements  $\alpha$ , and the 3-axis angular rate gyros  $\omega_{x,y,z}$  may be used to estimate the component of the measured acceleration  $\vec{a}_m$  that is due to gravity. This true gravity reading  $\vec{a}_g$  is needed to level the inertial platform. The discrete time algorithm below performs this task at every time step  $k$  and returns the normalized acceleration estimate due to gravity only. In the algorithm below,  $\times$  is the vector cross product,  $u(k)$  and  $w(k)$  are the aircraft's linear velocity components in the  $x$  and  $y$  direction, and  $|\vec{a}_g(k)|$  is the magnitude of the true gravity vector.

$$u(k) = V_{air}(k) \cdot \cos \alpha(k)$$

$$w(k) = V_{air}(k) \cdot \sin \alpha(k)$$

$$\vec{\Omega}(k) = [\omega_x(k) \quad \omega_y(k) \quad \omega_z(k)]^T$$

$$\vec{a}_c(k) = \vec{\Omega}(k) \times [u(k) \quad 0 \quad w(k)]^T$$

$$\vec{a}_m(k) = [a_x(k) \quad a_y(k) \quad a_z(k)]^T$$

$$\vec{a}_g(k) = \vec{a}_m(k) - \vec{a}_c(k)$$

$$\vec{a}_g(k) = \vec{a}_g(k) / |\vec{a}_g(k)|$$

Sensor measurements in one frame may be represented in another through a series of matrix transformations [20, 21]. Consider transforming the measurement vector from the body frame to the inertial frame. This transformation may be accomplished through an ordered series of three rotation matrices. First, the compensated accelerometer signals  $\vec{a}_g$  are used to roll about the x-axis until the acceleration in the y-axis,  $y_1$  is zero. The vector  $(x_1, y_1, z_1)$  is an intermediate vector used in the next rotation. Also,  $atan2$  is the inverse tangent that places the angle in the correct quadrant.

$$\varphi = atan2\left(\frac{-a_y}{a_z}\right)$$

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi & \sin \varphi \\ 0 & -\sin \varphi & \cos \varphi \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

The intermediate values of  $x_1, y_1, z_1$  are next used to pitch about the y-axis until  $a_x$  is zero.

$$\theta = atan2\left(\frac{x_1}{z_1}\right) \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$$

At this point the x-y plane is parallel to the surface of the earth. The next step is to rotate about the z-axis until the x-axis points to magnetic north. This is accomplished by using the transformed magnetometer vector. When pointing to magnetic north, the leveled magnetometer will read zero in the y-axis. A final rotation accomplishes this as shown below where  $(m_x, m_y, m_z)$  is the magnetic reference vector.

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix} \cdot$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi & \sin \varphi \\ 0 & -\sin \varphi & \cos \varphi \end{bmatrix} \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}$$

$$\psi = atan2\left(\frac{y_2}{y_1}\right)$$

$$\mathbf{NED} = \begin{bmatrix} \cos \Psi & \sin \Psi & 0 \\ -\sin \Psi & \cos \Psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi & \sin \varphi \\ 0 & -\sin \varphi & \cos \varphi \end{bmatrix}$$

The matrix  $\mathbf{NED}$  will transform the body reference vector to the inertial, or that is magnetic north, east down (NED) reference frame. Also, the transpose of  $\mathbf{NED}$  will convert the inertial frame to the body frame. The relation is shown below where the accelerometer and magnetometer vectors have been normalized.

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \mathbf{NED} \cdot \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \quad \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \mathbf{NED}^T \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ 0 \\ M_z \end{bmatrix} = \mathbf{NED} \cdot \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} \quad \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = \mathbf{NED}^T \cdot \begin{bmatrix} M_x \\ 0 \\ M_z \end{bmatrix}$$

In the above equations  $M_x, 0, M_z$  are the measured components of the normalized magnetic vector when the body frame has been aligned with the inertial frame. These measures are relative to geographic location.

The transpose of the matrix  $NED$  is the Directional Cosine Matrix ( $DCM$ ). The  $DCM$  may be used to compute the Euler angles as shown below.

$$\begin{aligned}\varphi &= \text{atan2}\left(\frac{DCM_{2,3}}{DCM_{3,3}}\right) && \text{roll} \\ \theta &= -\text{asin}(DCM_{1,3}) && \text{pitch} \\ \psi &= \text{atan2}\left(\frac{DCM_{1,2}}{DCM_{1,1}}\right) && \text{yaw}\end{aligned}$$

The subscripts refer to the row, column  $DCM$  entries. The above attitude estimation is based on accelerometer and magnetometer readings and returns an approximation to the true attitude. The gyro measures may be used to improve the accuracy of the attitude estimation. This is done by both the EKF and the complementary filter.

### Extended Kalman Filter

An Extended Kalman Filter is used to fuse together the sensor measurements for attitude estimation [7, 8]. Aircraft attitude may be modeled using a quaternion based on angular velocity measures from a three-axis gyroscope. The continuous time varying model is as follows:

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

The quaternion model is shown below in compact form where  $\vec{q}$  is the quaternion vector and  $\mathbf{\Omega}$  is the matrix of angular velocity measurements given by the gyro.

$$\dot{\vec{q}} = \frac{1}{2} \cdot \mathbf{\Omega} \cdot \vec{q}$$

This continuous time equation must be transformed to the discrete time domain for computations. An approximation suitable for real time implementations is to use the first few terms of the Taylor series representation [22].

$$\dot{\vec{q}}(k) = \frac{1}{2} \cdot \mathbf{A} \cdot \vec{q}(k-1) + \vec{w}(k)$$

$$\text{where } \mathbf{A} = e^{\mathbf{\Omega}T_s} \cong \mathbf{I} + \mathbf{A} \cdot T_s + \frac{1}{2!} \cdot (\mathbf{A} \cdot T_s)^2 + \frac{1}{3!} \cdot (\mathbf{A} \cdot T_s)^3 + \dots$$

Here  $\mathbf{A}$  is the discrete time representation of  $\mathbf{\Omega}$ ,  $T_s$  is the sampling period in seconds, and  $\vec{w}$  is zero mean Gaussian noise. The EKF requires a system output equation to update the state error. A combination of three-axis accelerometer and three-axis magnetometer is used to update the quaternion state estimations as shown below [23].

$$\begin{bmatrix} a_x \\ a_y \\ a_z \\ \psi \end{bmatrix} = \begin{bmatrix} 2(q_1q_3 - q_0q_2) \\ 2(q_2q_3 - q_0q_1) \\ q_0^2 - q_1^2 - q_2^2 + q_3^2 \\ \text{atan2}\left(\frac{2(q_1q_2 + q_3q_0)}{q_0^2 + q_1^2 - q_2^2 - q_3^2}\right) \end{bmatrix} + \vec{v}$$

This equation may be written in compact form as shown.

$$\vec{y}(k) = g\{\vec{q}(k)\} + \vec{v}(k)$$

Here  $a_x$ ,  $a_y$ , and  $a_z$  are the compensated accelerations  $\vec{a}_g$  with positive  $a_z$  down and with yaw  $\psi$  in radians. The yaw is taken from the  $DCM$  yaw estimation. The term  $\vec{v}$  is additive zero mean Gaussian measurement noise.

The equations for  $\dot{\vec{q}}(k)$  and  $\vec{y}(k)$  form the discrete time system model used to estimate the aircraft attitude in terms of the quaternion. This representation is both time varying and nonlinear. The state may be estimated using an EKF as shown below.

$$\dot{\vec{q}}(k) = \frac{1}{2} \cdot \mathbf{A} \cdot \vec{q}(k-1)$$

$$\mathbf{P}(k) = \mathbf{A}(k) \cdot \mathbf{P}(k-1) \cdot \mathbf{A}^T(k) + \mathbf{Q}$$

$$\vec{K}(k) = \mathbf{P}(k) \cdot \mathbf{G}^T(k) \cdot (\mathbf{G}(k) \cdot \mathbf{P}(k) \cdot \mathbf{G}^T(k) + \mathbf{R})^{-1}$$

$$\dot{\vec{q}}(k) = \dot{\vec{q}}(k) + \vec{K}(k) \cdot (\vec{y}(k) - g\{\vec{q}(k)\})$$

$$\mathbf{P}(k) = (\mathbf{I} - \vec{K}(k) \cdot \mathbf{G}(k)) \cdot \mathbf{P}(k)$$

In the EKF algorithm  $\mathbf{G}(k)$  is the Jacobian matrix of  $g\{\vec{q}(k)\}$  and may be found by taking the partial derivative of each row of  $g\{\vec{q}(k)\}$  with respect to each element of  $\vec{q}(k)$ .

The EKF returns estimates of the aircraft attitude in terms of quaternions. The Euler angles may be determined from the quaternions as shown below [23].

$$\begin{aligned}\phi &= \text{atan2}\left(\frac{2(q_0q_1 + q_3q_2)}{q_0^2 - q_1^2 - q_2^2 + q_3^2}\right) \\ \theta &= \text{asin}(2(q_0q_2 - q_1q_3)) \\ \psi &= \text{atan2}\left(\frac{2(q_1q_2 + q_3q_0)}{q_0^2 + q_1^2 - q_2^2 - q_3^2}\right)\end{aligned}$$

### Complementary Filter

The complementary filter may be used to compute attitude based on sensor measurements [15, 16, 18]. This technique requires fewer computations, which makes it more suitable for less powerful processors with slower clock speeds. The algorithm is presented below.

$$\begin{aligned}\tilde{\mathbf{R}}(k) &= \hat{\mathbf{R}}^T(k) \cdot \mathbf{DCM}(k) \\ \boldsymbol{\pi}_a(k) &= \frac{1}{2} \cdot (\tilde{\mathbf{R}}(k) - \tilde{\mathbf{R}}^T(k)) \\ \boldsymbol{\Omega}(k) &= \begin{bmatrix} 0 & -\omega_z(k) & \omega_y(k) \\ \omega_z(k) & 0 & -\omega_x(k) \\ -\omega_y(k) & \omega_x(k) & 0 \end{bmatrix} \\ \boldsymbol{\Omega}(k) &= \boldsymbol{\Omega}(k) - \mathbf{bias}(k) \\ \mathbf{A}(k) &= \mathbf{I} - \frac{\boldsymbol{\Omega}(k) \cdot \sin(|\boldsymbol{\Omega}(k)| \cdot T_s)}{|\boldsymbol{\Omega}(k)|} \\ &\quad + \frac{\boldsymbol{\Omega}(k)^2 \cdot (1 - \cos(|\boldsymbol{\Omega}(k)| \cdot T_s))}{|\boldsymbol{\Omega}(k)|^2} \\ \hat{\mathbf{R}}(k+1) &= \hat{\mathbf{R}}(k) \cdot (\mathbf{A}(k) + K_{est} \cdot \boldsymbol{\pi}_a(k)) \\ \mathbf{bias}(k+1) &= \mathbf{bias}(k) + T_s \cdot k_b \cdot \boldsymbol{\pi}_a(k)\end{aligned}$$

In the above algorithm  $\mathbf{DCM}$  is the directional cosine matrix derived from the sensor measurements. The matrix  $\hat{\mathbf{R}}$  may be initialized to identity;  $k$  is the time step and  $T_s$  is the sample rate in seconds. The matrix elements  $\omega_x(k)$ ,  $\omega_y(k)$ , and  $\omega_z(k)$  are the gyro readings in radians/second and  $\mathbf{bias}(k)$  is the gyro bias at time step  $k$ . The gains  $K_{est}$  and  $k_b$  may be tuned for the desired application. The attitude estimate may be obtained from the matrix  $\hat{\mathbf{R}}$ , which is an improved estimate of the  $\mathbf{DCM}$  since it includes gyro measurements.

The Complementary filter may also be implemented directly without need to compute the  $\mathbf{DCM}$  [17, 19]. The quaternion based algorithm is shown below for discrete time implementation.

$$u(k) = V_{air} \cdot \cos \alpha$$

$$\begin{aligned}w(k) &= V_{air} \cdot \sin \alpha \\ \bar{\boldsymbol{\Omega}}(k) &= [\omega_x(k) \quad \omega_y(k) \quad \omega_z(k)]^T \\ \bar{\mathbf{a}}_c(k) &= (\bar{\boldsymbol{\Omega}}(k) - \bar{\mathbf{b}}(k)) \times [u(k) \quad 0 \quad w(k)]^T \\ \bar{\mathbf{a}}_m(k) &= [a_x(k) \quad a_y(k) \quad a_z(k)]^T \\ \bar{\mathbf{a}}_g(k) &= \bar{\mathbf{a}}_m(k) - \bar{\mathbf{a}}_c(k) \\ \bar{\mathbf{a}}_g(k) &= \bar{\mathbf{a}}_g(k) / |\bar{\mathbf{a}}_g(k)| \\ \hat{\boldsymbol{\Omega}}(k) &= \bar{\boldsymbol{\Omega}}(k) + k_p \cdot \bar{\mathbf{e}}(k-1) + k_I \cdot \bar{\mathbf{b}}(k-1) \\ \mathbf{A}(k) &= \mathbf{e}^{\frac{1}{2}T_s \cdot \hat{\boldsymbol{\Omega}}_x} \\ \bar{\mathbf{q}}(k) &= \mathbf{A}(k) \cdot \bar{\mathbf{q}}(k-1) \\ \bar{\mathbf{q}}(k) &= \bar{\mathbf{q}}(k) / |\bar{\mathbf{q}}(k)| \\ \hat{\mathbf{a}}(k) &= \begin{bmatrix} 2(q_1q_3 - q_0q_2) \\ 2(q_2q_3 + q_0q_1) \\ q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix} \\ \bar{\mathbf{e}}(k) &= \bar{\mathbf{a}}_g(k) \times \hat{\mathbf{a}}(k) \\ \bar{\mathbf{b}}(k) &= \bar{\mathbf{b}}(k-1) + k_I \cdot T_s \cdot \bar{\mathbf{e}}(k)\end{aligned}$$

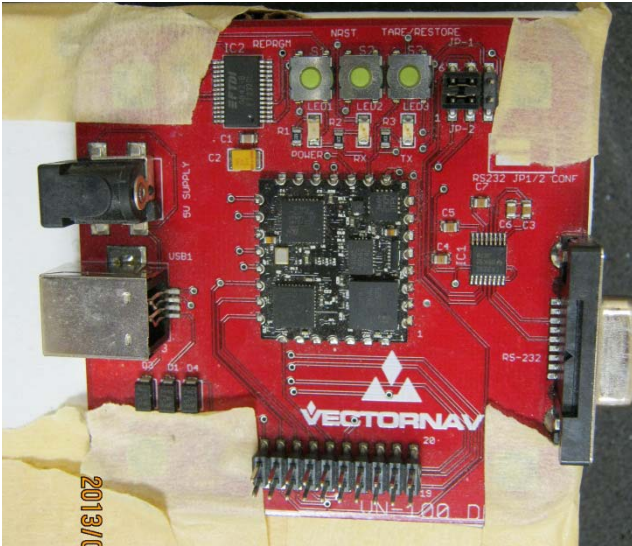
In the above algorithm  $k_p$  and  $k_I$  are gains to be set by the user and the Euler angles may be obtained from the quaternion estimation. Also, the acceleration has been corrected for turning maneuvers. The vector  $\bar{\mathbf{a}}_m(k)$  is the measured acceleration at time step  $k$ ,  $\bar{\mathbf{a}}_c(k)$  is the acceleration due to centrifugal force, and the vector  $\bar{\mathbf{a}}_g(k)$  is due to gravity. It is interesting to note that this algorithm does not use the magnetometer measurements. The quaternion solution returned is valid for pitch and roll, but yaw must be computed by integrating the gyro signal or using the magnetometer. The equation shown below uses the magnetometer measurements to compute the yaw and is the method used here.

$$\begin{aligned}\psi &= \\ &= -\text{atan2}\left(\frac{m_y \cos \phi - m_z \sin \phi}{m_x \cos \theta + m_y \sin \phi \sin \theta + m_z \cos \phi \sin \theta}\right)\end{aligned}$$

### Experimental Setup

Two test platforms were used to obtain data for attitude estimation. The first is the VECTORNAV VN100 development board, manufactured by VectorNav Technologies LLC., shown in Figure 1 [24, 25].





**Figure 1. VN100 Development Board**

The VN100 data was gathered using a serial interface to a computer using MATLAB to record sensor data (3-axis accelerometer, gyroscope, and magnetometer) and attitude estimations returned by the VN100. The data was generated by randomly moving the VN100 by hand for two minutes. This data was post-processed by the EKF and complementary filter algorithms to estimate the attitude and compare the estimated attitude with that returned by the VN100. The purpose of this experiment was to test for correct algorithm implementation in a controlled environment and make observations in regards to any deviations in comparing the resulting attitude estimations.

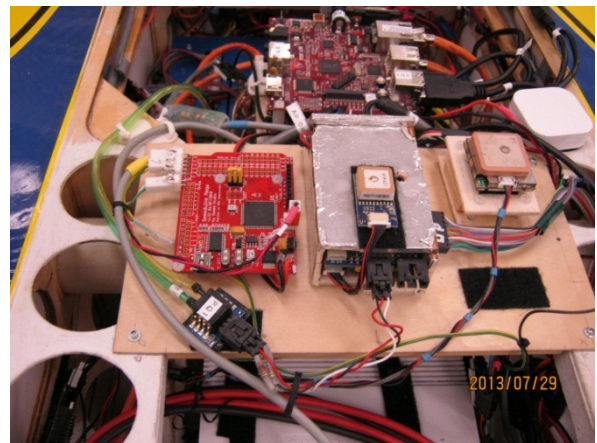
The second experimental platform was the UAV shown in Figure 2 mounted nose down in a trifilar.



**Figure 2. Edge 540T UAV**

This UAV is a 33% scaled Edge 540T, manufactured by SIG Manufacturing Company, Inc., with electric propulsion powered by lithium polymer batteries. The Edge is 98 inches long with a 100 inch wing span, weighs 47.4 lbs., and has 1881 in<sup>2</sup> of wing area with an average wing load of 0.025 psi. The power system consists of two brushless DC electric motors mounted in tandem to drive a 26 inch propeller. The motor assembly turns the propeller up to 6000 RPM to develop about 37 pounds of thrust. Its airspeed ranges from a stall of 12 m/s to a dash of about 40 m/s (23-77 knots).

There are two INS systems onboard the Edge. The first is the VN100 and the second the ArduPilot, manufactured by 3DRobotics Inc., shown in Figure 3.



**Figure 3. ArduPilot Mounted in Edge**

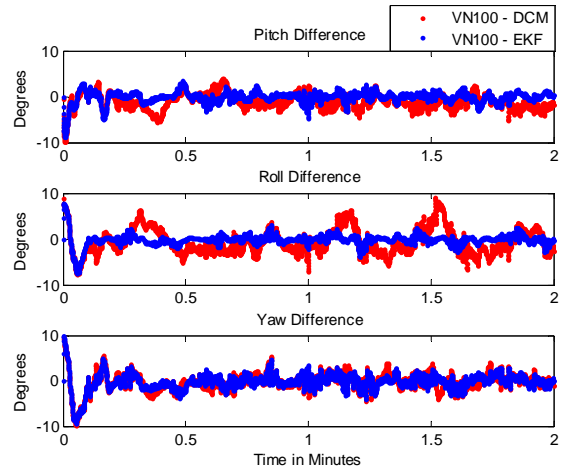
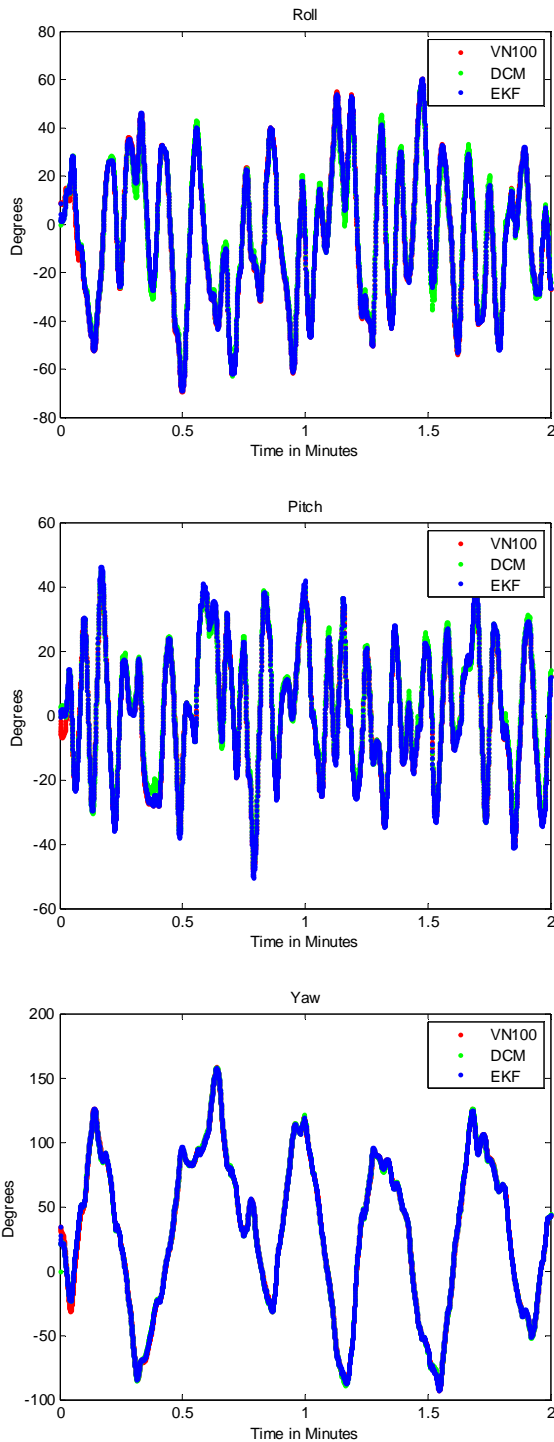
The ArduPilot is located within the center silver box. In the rear of Figure 3 is the PC104 used for data recording. Data from the ArduPilot sensors and the ArduPilot's attitude estimation were recorded during flight along with the VN100 sensor data and attitude estimation. This data was then post-processed to determine attitude estimations using both the EKF and the complementary filter. Post-processed attitude estimations were compared to estimations from the VN100, ArduPilot, and heading estimates from GPS.

## Laboratory Results

To demonstrate the attitude estimation capabilities of both the EKF and the complementary filter, data was gathered using the VN100 development board. Comparisons were made with the unfiltered DCM solution, EKF and complementary filter solutions, and the attitude solution returned by the VN100.

### Extended Kalman filter results

Shown in Figure 4 is the ability of the EKF to estimate the Euler angles of the VN100 development board during random motion by hand.

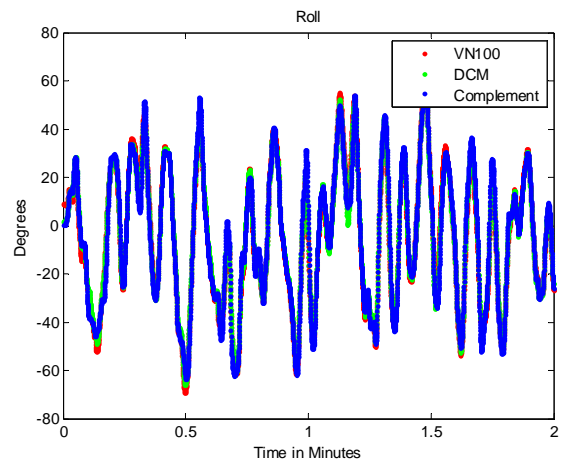


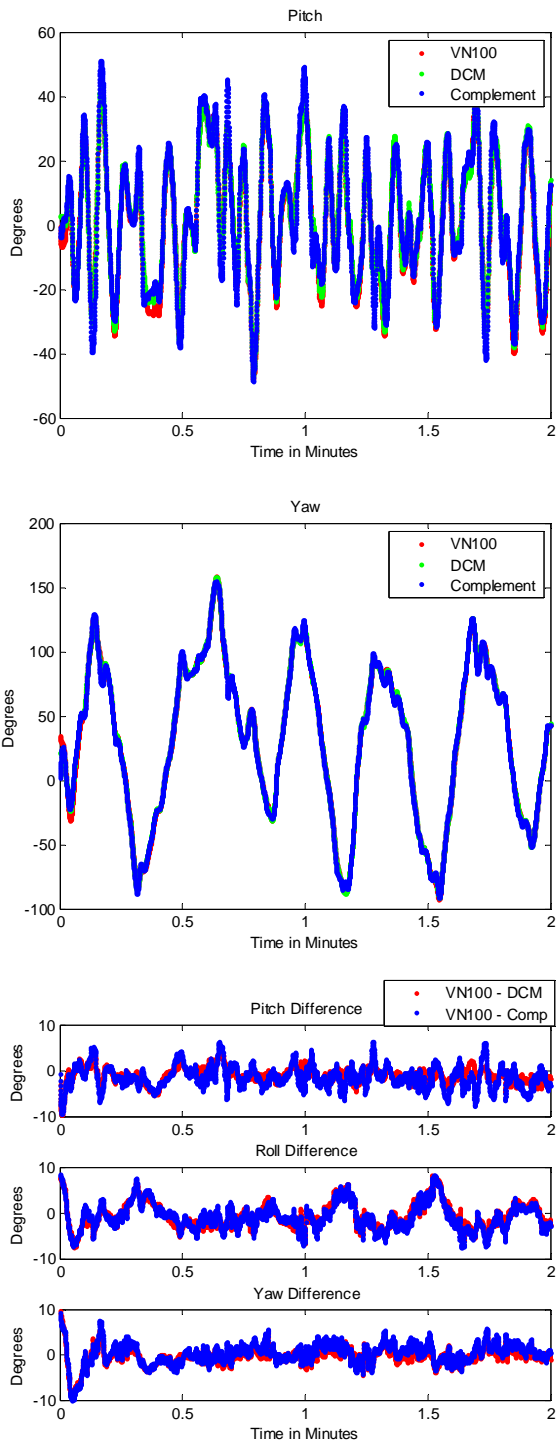
**Figure 4. EKF Euler Angles and Differences**

As seen in the attitude plots of Figure 4, the EKF returned the same estimates, (blue), as the VN100, (red). Also shown are the DCM attitude estimations, (green), which were derived from the accelerometer and magnetometer without use of the gyrometers. The DCM solution is not as immune to noise since no filtering is taking place. Overall, these experimental results demonstrate the ability of the EKF implementation to very closely match that of the VN100.

### Complementary filter results

Figure 5 demonstrates the ability of the complementary filter to estimate the attitude returned by the VN100 during random motion by hand. As can be seen from the attitude plots, the complementary filter performed similar to the VN100 solution.





**Figure 5. Complementary Filter Estimates and Differences**

The attitude plots show reasonable agreement between the filtering techniques. However, a small difference can be seen when comparing the VN100 and complementary filter solutions. This small

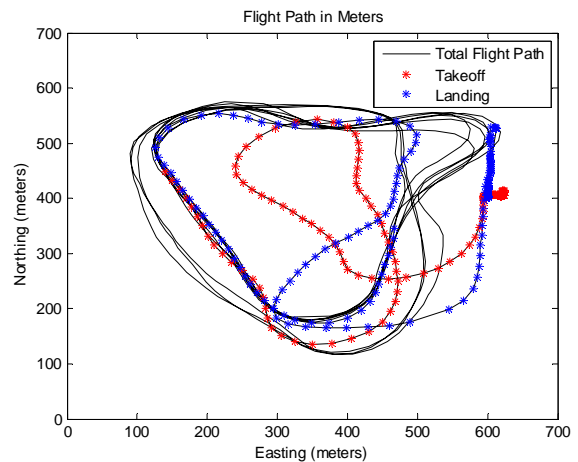
difference may be attributed to the fact that they are different estimation schemes and does not necessarily reflect a more accurate solution of one technique in comparison with the other. Overall the solutions are very similar and indicate the characteristic of the complementary filter to return a reasonable solution while requiring fewer computations.

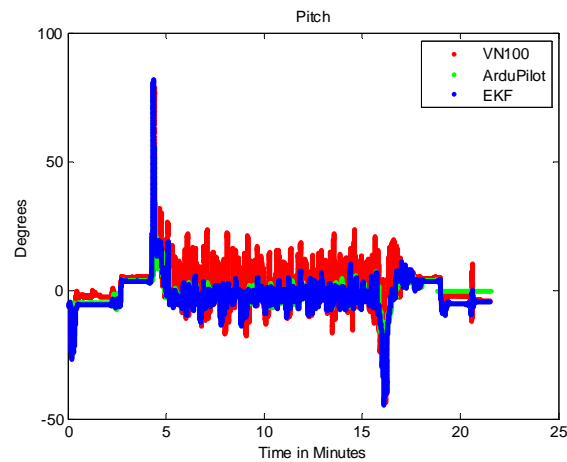
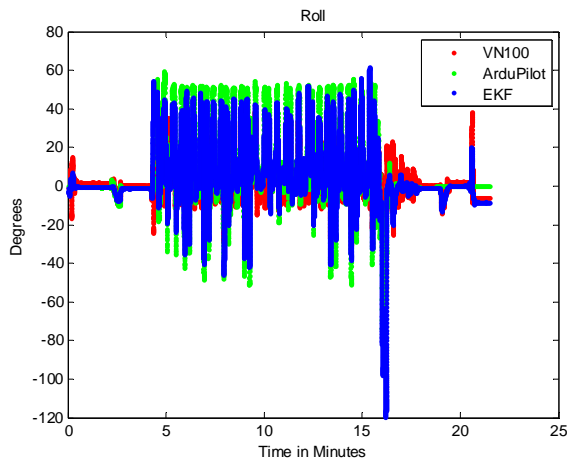
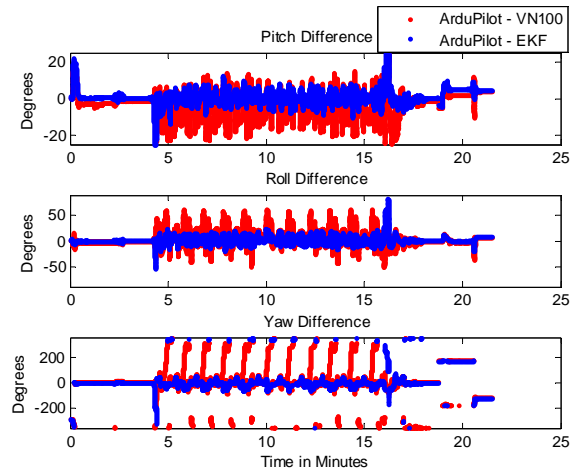
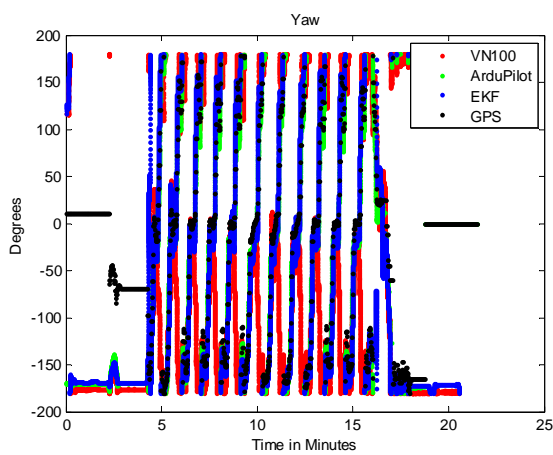
## Flight Results

Both the EKF and complementary filter may be used to estimate the attitude of the Edge aircraft by post-processing the sensor data recorded during flight. It was found that both filters returned reasonable results for attitude estimates when the acceleration measurements were corrected during turn maneuvers.

### *EKF application to flight data*

In this section the application of the EKF to flight data is presented. The sensor data from the accelerometer, gyrometer, and magnetometer is post-processed using the EKF algorithm to compute estimates of the quaternion attitude representation. Figure 6 shows the flight path and the resulting attitude estimates given by the VN100, ArduPilot, and EKF. The flight path is relative to an arbitrary zero point and northing, easting are the relative distances north and east. The yaw estimate plot also shows the GPS heading [26].





**Figure 6. EKF Euler Angles and Differences**

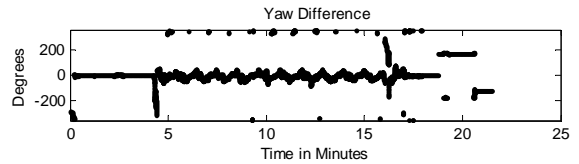
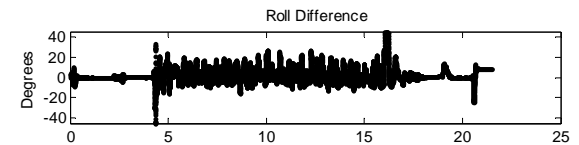
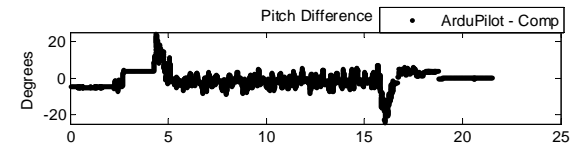
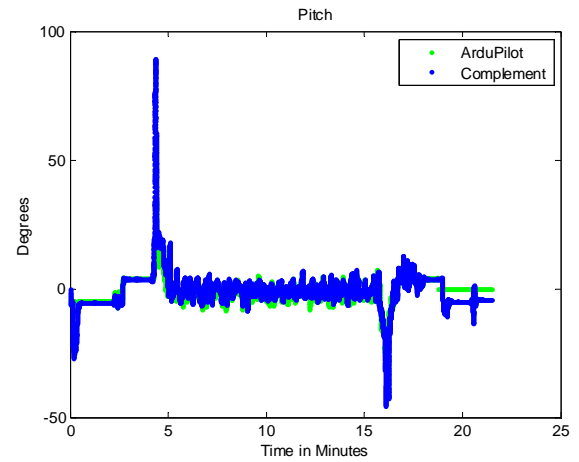
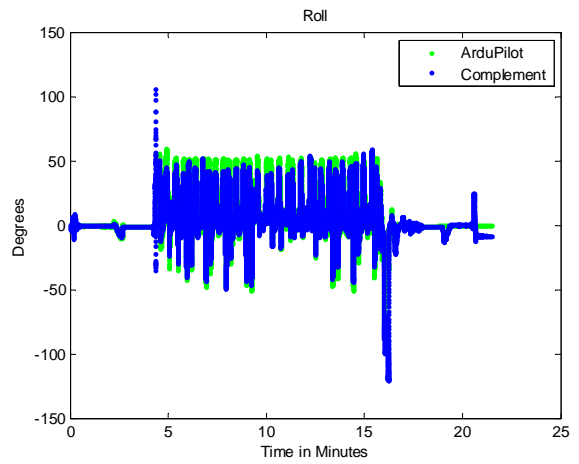
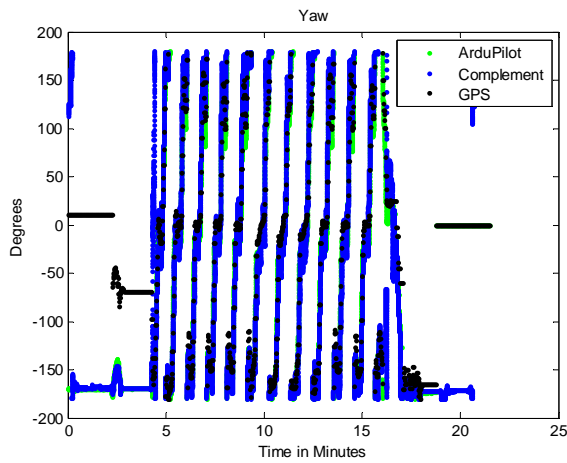
The ArduPilot attitude solution and that returned by the EKF are approximately the same given the high noise level and low cost instrumentation used in the flight. However, the attitude estimation given by the VN100, (red), is unacceptable as can be clearly seen by examining the yaw solution and difference plots of Figure 6. This is due to centrifugal force effects on the VN100 accelerometer during turns. In the first experiment when the VN100 development board was randomly moved by hand, there were no appreciable centrifugal forces. However, during aircraft turn maneuvers the accelerometers are measuring the acceleration due to turning as well as gravity. Since the aircraft attitude calculations are dependent on the gravity vector pointing straight down for accurate attitude estimations, the body frame of the aircraft cannot be properly aligned with the inertial frame for meaningful estimates. This requires that the acceleration measurements be compensated using velocity and/or GPS measurements to correct the attitude estimates during turning. The VN100 does not have this capability. However, the EKF implementation presented here uses the pitot tube to determine the aircraft velocity combined with the gyros to produce correct attitude estimates. GPS is used here only to show heading agreement.

*Complementary filter application to flight data*

The complementary filter may also be applied to the flight data to compute the correct attitude. Figure 7 shows the resulting attitude for the same flight used by the EKF. As can be seen, the ArduPilot and the complementary filter return similar attitude



estimations. The observed differences may be attributed to different tuning gains, noise levels, and instrumentation differences. The MEMs used for the ArduPilot solution are slightly less accurate than those of the VN100. The VN100 sensor suite and the pitot tube were used as the data sources for the complementary filter. The GPS heading is shown only for comparison and was not used in the complementary filter calculations. It is interesting that the complementary filter returns an attitude estimate that appears to be just as accurate as that of the EKF but with far fewer computations.



**Figure 7. Complementary Filter Estimates and Differences**

## Conclusions

Low cost Inertial Navigation Systems provide reasonable attitude estimation performance when centrifugal forces can be accounted for during turns. The applications presented here demonstrate the use of the Extended Kalman Filter and complementary filter for both a low noise lab experiment and an unmanned aerial vehicle flight. Results demonstrate reasonable attitude tracking performance of both filters when the filters are provided with velocity measurements derived from a pitot tube to correct gravity measurements during turns. The direct implementation of the complementary filter is of special interest due to low computational burden. From visual inspection of the attitude plots it was found that the complementary filter is an ideal candidate for low cost attitude estimation where processor memory and speed are restricted. This work also highlights the need to provide the low cost microelectromechanical systems with additional information, such as velocity measurements, to correct attitude estimates during turns.

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**REPORT DOCUMENTATION PAGE**

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<b>14. ABSTRACT</b>  Unmanned Aerial Vehicles (UAV) are playing an increasing role in aviation. Various methods exist for the computation of UAV attitude based on low cost microelectromechanical systems (MEMS) and Global Positioning System (GPS) receivers. There has been a recent increase in UAV autonomy as sensors are becoming more compact and onboard processing power has increased significantly. Correct UAV attitude estimation will play a critical role in navigation and separation assurance as UAVs share airspace with civil air traffic. This paper describes attitude estimation derived by post-processing data from a small low cost Inertial Navigation System (INS) recorded during the flight of a subscale commercial off the shelf (COTS) UAV. Two discrete time attitude estimation schemes are presented here in detail. The first is an adaptation of the Kalman Filter to accommodate nonlinear systems, the Extended Kalman Filter (EKF). The EKF returns quaternion estimates of the UAV attitude based on MEMS gyro, magnetometer, accelerometer, and pitot tube inputs. The second scheme is the complementary filter which is a simpler algorithm that splits the sensor frequency spectrum based on noise characteristics. The necessity to correct both filters for gravity measurement errors during turning maneuvers is demonstrated. It is shown that the proposed algorithms may be used to estimate UAV attitude. The effects of vibration on sensor measurements are discussed. Heuristic tuning comments pertaining to sensor filtering and gain selection to achieve acceptable performance during flight are given. Comparisons of attitude estimation performance are made between the EKF and the complementary filter.					
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