

Introduction: The initial accretion of primitive bodies (asteroids and TNOs) from freely-floating nebula particles remains problematic. Here we focus on the asteroids where constituent particle (read “chondrule”) sizes are observationally known; similar arguments will hold for TNOs, but the constituent particles in those regions will be smaller, or will be fluffy aggregates, and are unobserved. Traditional growth-by-sticking models encounter a formidable “meter-size barrier” [1] (or even a mm-cm-size barrier [2]) in *turbulent* nebulae, while *nonturbulent* nebulae form large asteroids too quickly to explain long spreads in formation times, or the dearth of melted asteroids [3]. Even if growth by sticking could somehow breach the meter size barrier, other obstacles are encountered through the 1-10km size range [4].

Another clue regarding planetesimal formation is an apparent 100km diameter peak in the pre-depletion, pre-erosion mass distribution of asteroids [5]; scenarios leading directly from independent nebula particulates to this size, which avoid the problematic m-km size range, could be called “leapfrog” scenarios [6-8]. The leapfrog scenario we have studied in detail involves formation of dense clumps of aerodynamically selected, typically *mm-size particles* in turbulence, which can under certain conditions shrink inexorably on 100-1000 orbit timescales and form 10-100km diameter sandpile planetesimals. The typical sizes of planetesimals and the rate of their formation [7,8] are determined by a statistical model with properties inferred from large numerical simulations of turbulence [9]. Nebula turbulence can be described by its Reynolds number $Re = (L/\eta)^{4/3}$, where $L = H\alpha^{1/2}$ is the largest eddy scale, H is the nebula gas vertical scale height, and α the nebula turbulent viscosity parameter, and η is the Kolmogorov or smallest scale in turbulence (typically about 1km), with eddy turnover time t_η . In the nebula, Re is far larger than any numerical simulation can handle, so some physical model is needed to extend the results of numerical simulations to nebula conditions.

Cascade model and multiplier distributions: The spatial distribution of particle concentration can be captured statistically by a *cascade model* [7,9] which predicts the probability distribution functions (PDFs) for dense particle clumps; these PDFs are essentially volume fractions in the nebula having the necessary properties (solids mass and local vorticity) for planetesimal formation. A cascade model presumes that, as energy flows from large eddies to smaller ones, particles and fluid properties are partitioned unequally at each “level” of the cascade from “parent” into “daughter” eddies. The probability distribution functions (pdfs) of the so-called *multipliers* by which particle and fluid properties are partitioned have widths that are given by $1/\beta$; smaller β means larger width, or a higher probability of strongly asymmetrical partitioning which, repeated over a number of levels, leads to clumpier or more *intermittent* local values of particle and fluid properties.

The wide range of scales between L and η is loosely called the inertial range; in this range the equations of motion are scale-free, and the multiplier pdfs for dissipation of turbulent kinetic energy ϵ , which, like our preferred particles,

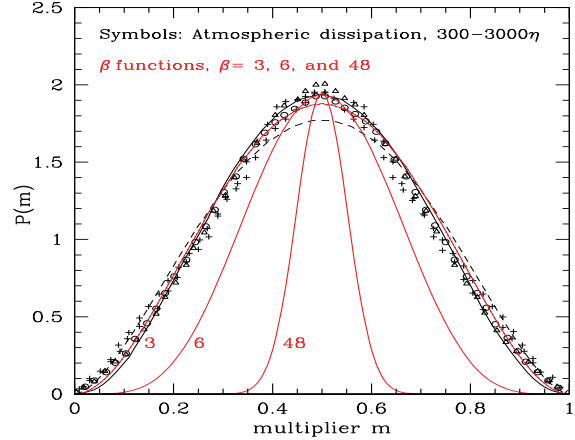


Figure 1: The symbols indicate the multiplier pdfs for turbulent dissipation in high- Re atmospheric turbulence [10], showing scale invariance over $300-3000\eta$. The red β -functions ($\beta=3, 6, \text{ and } 48$) provide context for β values in figure 2. In [7] we adopted level-independent multiplier pdfs with $\beta \sim 3$ based on our own simulations.

is characterized by Kolmogorov eddy timescales, are known to be scale-independent in atmospheric flows across hundreds to thousands of Kolmogorov lengthscales [10; figure 1]. This fact motivated us to assume in [7] that the multiplier pdfs for *particle concentration*, which we denote by Φ , were also independent of nebula turbulent eddy lengthscale. While this assumption was checked and seemingly validated (see figure 4 of [9]), we have always been concerned about it [7], and discrepancies between our model predictions and recent results of [11] have led us to determine more precisely the range of scales over which our assumption *is or is not* valid.

Here we report a scale dependence of the widths of the multiplier pdfs for *both* ϵ and Φ , over a range of the *largest* scales, covering a decade or more between the energy-containing or integral scale L and the top of a more rigorously scale-free inertial range. This scale dependence probably explains the differences between [7] and [11], and will affect the formation rate and masses of primary planetesimals in the scenario of [7,8].

Method and results: We obtained a complete dataset of particle trajectories in direct simulations covering 6 large eddy turnover times (T_L) in high Re isotropic turbulence, for $2 - 6 \times 10^5$ “inertial” particles at each of a range of gas drag stopping times ($t_s > 0$), and for fluid tracers ($t_s = 0$) [12,13]. The original simulation was on a cube of 2048η on a side, and had an integral scale of about 1024η . The data are thus similar to [9,11] but at a considerably higher Re , and showed a clear inertial range [12] extending over tens to nearly 1000η based on the second order velocity structure function. The particle locations were spatially binned into snapshots in time, initially

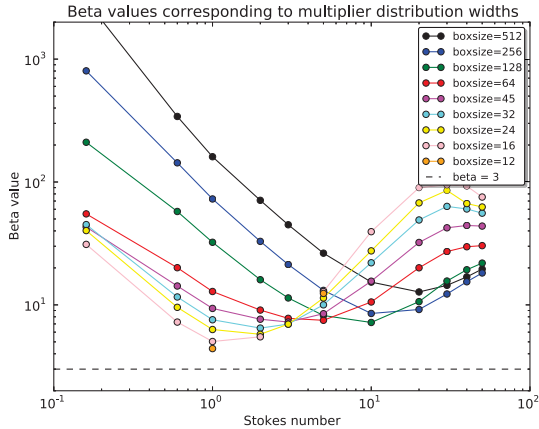


Figure 2: The width of multiplier pdfs for particle concentration is the inverse of the β value, plotted here as a function of particle Stokes number $St = t_s/t_\eta$. Each curve represents the situation at a different lengthscale (in units of η). Note that particles with $St > 1$ but still *on the order of unity* approach $\beta \sim 5 - 10$ by $64\eta \sim L/16$ and generally stay there with the minimum β shifting slowly to smaller St at smaller scales and deepening towards the asymptotic value $\beta \sim 3$ (dashed line).

separated by $T_L/2$ to ensure their independence. We then determined the multiplier pdfs in the manner described by [9], but now over a range of spatial scales between roughly $12-512\eta$ and a range in $St \equiv t_s/t_\eta = 0.6 - 70$, roughly corresponding to 1mm - 10cm particle diameter under nebula conditions in the asteroid belt region. The dissipation ϵ was known at the locations of the tracers, so its multiplier pdfs could also be determined as a function of spatial binning scale. The spatial distribution of the $St = 0$ tracers agreed well with a Poisson distribution, as they do not undergo preferential concentration.

Because the number of inertial and tracer particles in this study was smaller than used in [9,11], the average number of particles per 32η -scale grid cell (for example) was only about 2.5. This led to several small-number-statistics sampling biases that needed to be allowed for. These biases led to artificial intermittency; that is, the widths of the *apparent* multiplier pdfs were *larger* than their true values. This effect becomes *worse* at smaller scales where we typically find fewer particles per bin, even if there are more bins. We ran a number of Monte Carlo simulations, and found that the bias was predictable and iteratively correctable, depending only on the number of particles in a bin and the true width of the multiplier pdf. For instance, a very broad pdf (small β) was not biased as much by this effect as was a very narrow one. We also realized that smaller bins could be sampled on shorter timescales than $T_L/2$, because of the shorter eddy times and faster refresh rates associated with smaller lengthscales; this provided improved statistics for the smaller bins.

Figure 2 implies that, at least for the top decade or so of eddy lengthscales, multiplier pdfs are slightly narrower (larger β) than we had assumed *and* particles of $St \sim 2 - 10$ are more concentrated (*ie*, have broader multiplier pdfs than those

for $St = 1$). By contrast, particles of dm -size and larger have narrow multiplier pdfs at all but the very largest scales. Our preliminary results for ϵ are similar to the particle results, in that the multiplier pdf which is so level-independent over a wide range of scales in atmospheric turbulence (figure 1) also starts out very narrow at the largest scales and only approaches $\beta \sim 3$ at 64η or so, roughly 1.5 decades below L . We have not yet completed the correction of the ϵ results for bias as of this writing.

Thus, the multiplier pdfs for both ϵ and Φ paint a consistent picture. In the top decade of lengthscales, turbulent stretching and vortex tube formation (see [8] and references therein) have not yet reached their fully developed state, so the statistics of ϵ and Φ of particles with Kolmogorov-eddy stopping times do not either. However, both ϵ and Φ multipliers approach the asymptotic value of $\beta \sim 3$ at slightly different but comparable scales of $\sim L/30$, and we know from atmospheric turbulence that this asymptotic value then continues to hold *over the entire inertial range*, at least for ϵ . The important implication is that multiplier statistics are not characterized by their scale as a *multiple of η* but as a *fraction of L* . Observed level independence of ϵ to 3000η in atmospheric flows [10] is consistent with their much larger L , such that $3000\eta < L/16$.

The cascade model for planetesimal formation by turbulent concentration [10] can now be modified easily to allow for the observed level-dependence in the multipliers (figure 2), and since scales are resolved in the calculations reported here to almost the scale where level-independent pdfs are attained, future uncertainties in calculated IMFs for planetesimal formation will be greatly reduced. Moreover, particles that become most densely concentrated (are most intermittent, or have broader multiplier pdfs) in the range of eddy scales near the inertial scale have stopping times perhaps 2-10 times larger than we previously assumed, consistent with some previous work [11,14]. At smaller scales, $St = 1$ particles will increasingly dominate (figure 2). Because nebula properties such as turbulent intensity and gas density are not known to factors of order unity, these findings do not qualitatively change the outlook for the scenario (note the basic timescale revision found by [8]), but will lead to quantitative changes in predictions of planetesimal IMF and chondrule size distributions.

References: [1] Brauer, F. et al 2008 A&A 480, 859; [2] Zsom A. et al 2010, A&A 513, id.A57; [3] Cuzzi J.N. and Weidenschilling S.J. 2006 in "Meteorites and the Early Solar System II, Univ. Arizona Press; [4] Gressel et al 2012 MNRAS 422, 1140, and references therein; [5] Bottke W. et al 2005, Icarus 175, 111; [6] Johansen, A. et al 2007 Nature, 448, 1022; [7] Cuzzi J.N., R.C. Hogan, and W.F. Bottke 2010 Icarus, 208, 518 [8] Cuzzi J.N. and R.C. Hogan 2012 43rd LPSC; id. 2536 [9] Hogan, R.C. and J.N. Cuzzi 2007 Phys. Rev. E 75, id. 056305; [10] Sreenivasan K.R. and G. Stolovitsky 1995, J. Stat. Phys. 78, 311; [11] Pan, L. et al 2011, ApJ 740, article id. 6; [12] Bec J. et al. 2010, J. Fluid Mech. 646, 527; [13] Calzavarini E. et al 2008; J. Fluid Mech. 607, 13; [14] Bec, J. et al 2007, Phys. Rev. Lett. 98, 084502-1084502-4. We thank E. Calzavarini and F. Toschi for their help in providing the numerical data, and NASA's Origins of Solar Systems program for supporting this work.