Elastic Model Transitions: A Hybrid Approach Utilizing Quadratic Inequality Constrained Least Squares (LSQI) and Direct Shape Mapping (DSM)

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A method for transitioning linear time invariant (LTI) models in time varying simulation is proposed that utilizes a hybrid approach for determining physical displacements by augmenting the original quadratically constrained least squares (LSQI) algorithm with Direct Shape Mapping (DSM) and modifying the energy constraints. The approach presented is applicable to simulation of the elastic behavior of launch vehicles and other structures that utilize discrete LTI finite element model (FEM) derived mode sets (eigenvalues and eigenvectors) that are propagated throughout time. The time invariant nature of the elastic data presents a problem of how to properly transition elastic states from the prior to the new model while preserving motion across the transition and ensuring there is no truncation or excitation of the system.

A previous approach utilizes a LSQI algorithm with an energy constraint to effect smooth transitions between eigenvector sets with no requirement that the models be of similar dimension or have any correlation. This approach assumes energy is conserved across the transition, which results in significant non-physical transients due to changing quasi-steady state energy between mode sets, a phenomenon seen when utilizing a truncated mode set.

The computational burden of simulating a full mode set is significant so a subset of modes is often selected to reduce run time. As a result of this truncation, energy between mode sets may not be constant and solutions across transitions could produce non-physical transients. In an effort to abate these transients an improved methodology was developed based on the aforementioned approach, but this new approach can handle significant changes in energy across mode set transitions. It is proposed that physical velocities due to elastic behavior be solved for using the LSQI algorithm, but solve for displacements using a two-step process that independently addresses the quasi-steady-state and non-steady-state contributions to the elastic displacement.

For structures subject to large external forces, such as thrust or atmospheric drag, it is imperative to capture these forces when solving for elastic displacement. To simplify the mathematical formulation, assumptions are made regarding mass matrix normalization, constant external forcing, and constant viscous damping. These simplifications allow for direct solutions to the quasi-steady-state displacements

through a process titled Direct Shape Mapping. DSM solves for the displacements using the eigenvalues of the elastic modes and the external forcing and returns a set of elastic displacements dictated by the eigenvectors of the post-transition mode set. For the non-steady-state contributions to displacement we formulate a LSQI problem that is constrained by energy of the non-steady state terms. The contributions from the quasi-steady-state and non-steady state solutions are then combined to obtain the physical displacements associated with the new set of eigenvectors.

Results for the LSQI-DSM approach show significant reduction/complete removal of transients across mode set transitions while maintaining elastic motion from the prior state. For time propagation applications employing discrete elastic models that need to be transitioned in time and where running with full a full mode set is not feasible, the method developed offers a practical solution to simulating vehicle elasticity.