Smooth Pursuit of Flicker-defined Motion

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Scott B. Stevenson
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A (familiar?) Model of Eye Movement Control

The topic of today's talk

The delayed feedback paradigm

Effect of Changing Feedback Delay on Spontaneous Oscillations in Smooth Pursuit Eye Movements of Monkeys

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Boxology 201

Figure from Goldreich, Krauzlis & Lisberger (1992)
Try it yourself!

- Launch demo
Period vs. delay - summary
A little math - definitions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e(t)$</td>
<td>eye position (EP)</td>
</tr>
<tr>
<td>$p(t)$</td>
<td>target position</td>
</tr>
<tr>
<td>$\dot{e}(t)$</td>
<td>eye velocity (EV)</td>
</tr>
<tr>
<td>$\dot{p}(t)$</td>
<td>target velocity</td>
</tr>
<tr>
<td>$\ddot{e}(t)$</td>
<td>eye acceleration (EA)</td>
</tr>
<tr>
<td>$\ddot{p}(t)$</td>
<td>target acceleration</td>
</tr>
</tbody>
</table>
A little math - definitions

\begin{align*}
    p(t) - e(t) & \quad \text{retinal position error (RPE)} \\
    \dot{p}(t) - \dot{e}(t) & \quad \text{retinal velocity error (RVE)} \\
    \ddot{p}(t) - \ddot{e}(t) & \quad \text{retinal acceleration error (RAE)}
\end{align*}
A little math - control laws

\[ \ddot{e}(t) = k_1 \left[ p(t - \delta_1) - e(t - \delta_1) \right] \quad \text{RPE drives EA} \]

\[ \ddot{e}(t) = k_2 \left[ \dot{p}(t - \delta_1) - \dot{e}(t - \delta_1) \right] \quad \text{RVE drives EA} \]

\[ \ddot{e}(t) = k_3 \left[ \ddot{p}(t - \delta_1) - \ddot{e}(t - \delta_1) \right] \quad \text{RAE drives EA} \]
A little math - control laws

\[
\ddot{e}(t) = k_2 \left[ p(t - \delta_1) - e(t - \delta_1) \right] \quad \text{RPE drives EV}
\]

\[
\ddot{e}(t) = k_2 \left[ \dot{p}(t - \delta_1) - \dot{e}(t - \delta_1) \right] \quad \text{RVE drives EA}
\]
A little math - possible stimuli

\[ p(t) = k \] stationary target

\[ p(t) = e(t) \] ideal stabilization

\[ p(t) = e(t - \epsilon) \] lab stabilization

\[ p(t) = e(t) + d(t) \] open-loop

\[ p(t) = e(t) - e(t - \delta_2) \] transient stabilization
A little math - delayed feedback & model

\[ p(t) = e(t) - e(t - \delta_2) \quad \text{transient stabilization} \]

\[ \ddot{e}(t) = k_1 \left[ p(t - \delta_1) - e(t - \delta_1) \right] \quad \text{RPE drives EA} \]

\[ \ddot{e}(t) = k_1 \left[ e(t - \delta_1) - e(t - \delta_1 - \delta_2) - e(t - \delta_1) \right] \]

\[ \ddot{e}(t) = -k_1 e(t - \delta_1 - \delta_2) \]

\[ \ddot{e}(t) = -k_1 e(t - \delta) \]
A little math - sinusoidal solution

\[ \ddot{e}(t) = -k_1 e(t - \delta) \]

\[ e(t) = e^{i\omega t} \quad \text{trial solution} \]

satisfied if \( k_1 = \omega^2 \) and \( \lambda = \delta \),

where \( \lambda \equiv \frac{2\pi}{\omega} \).
Period vs. delay - summary

\[ \lambda = \delta \quad \text{RPE drives EA} \]

\[ \lambda = 4\delta \quad \text{RVE drives EA} \]

\[ \lambda = 2\delta \quad \text{RAE drives EA} \]

\[ \lambda = \frac{4\delta}{4n + 2 - N_d} \]

\[ \lambda = \frac{4}{3} \delta \quad \text{RPE drives eye jerk} \]
Period vs. delay - summary

![Graph showing period vs. total delay with lines for RVE, RAE, and RPE]
Period vs. delay - summary

![Graph showing the relationship between period and feedback delay with RVE, RAE, and RPE as markers.]
Figure 4. Relationship between the period of spontaneous oscillation and the total feedback delay. Each graph plots the duration of the first half period of the oscillations as a function of the sum of the natural latency for pursuit and the artificial delay added by the computer. A-C show results for 3 monkeys. Open symbols represent data obtained with a big, bright target and short natural delays, and filled symbols represent data obtained with a smaller, dim target and longer natural delays. The lines describe the relationship: period of oscillation equals 2 or 4 times total delay. As described in the appendix, these lines represent the theoretical limits of the relationship between the period of oscillation and the total delay. The speeds of target motion were 30°/s in monkeys F and N and 15°/s in monkey J.

Figure from Goldreich, Krauzlis & Lisberger (1992)
Period vs. delay - summary

**Figure 4.** Relationship between the period of spontaneous oscillation and the total feedback delay. Each graph plots the duration of the first half period of the oscillations as a function of the sum of the natural latency for pursuit and the artificial delay added by the computer. A-C: results for 3 monkeys. Open symbols represent data obtained with a big, bright target and short natural delays, and filled symbols represent data obtained with a small, dim target and longer natural delays. The lines describe the relationship period of oscillation equals 2 or 4 times total delay. As described in the captions, these lines represent the theoretical limits of the relationship between the period of oscillation and the total delay. The speeds of target motion were 30°/s in monkey J and 8 and 13°/s in monkey N.

Figure from Goldreich, Krauzlis & Lisberger (1992)
The lines describe the relationships: period of oscillation equals 2 or 4 times total delay. As described in the APPENDIX, these lines represent the theoretical limits of the relationship between the period of oscillation and the total delay.
Figure from Goldreich, Krauzlis & Lisberger (1992)
Old results - laser spot on spiral background

Slopes: 1.3 - 1.6
The topic of today's talk

The stimulus - flicker defined motion

- Black/white pattern minimizes effects of gamma nonlinearity
- Carrier pattern: dense (Julesz) random dot pattern
- Stimulus pattern: probability of dot polarity reversal
- Real-time generation via nVidia CUDA (120 Hz to CRT)
- Demos
Sample trial - black spot
First trial - twinkle spot
Later trial - twinkle spot
Later trial - twinkle spot
Period-vs-Delay - black spot

- JBM, black, smooth
  - slope = -1.649, x_int = -0.203
- SBS, black, smooth
  - slope = 1.739, x_int = 0.126
Period-vs-Delay - rigid spot

- JBM, rigid, smooth
  - slope = 1.783, x_int = -0.215
- SBS, rigid, smooth
  - slope = 1.886, x_int = -0.112
Period-vs-Delay - twinkle spot

- JHM, twink, smooth
  - slope = 1.804, x_int = -0.189
- SBS, twink, smooth
  - slope = 1.735, x_int = -0.19
Period-vs-Delay - static spot

```
   period (seconds)  
  1.5  
  1.25  
  1.0  
  0.75  
  0.5  
  0.25  
  0.0  

   delay (seconds)  
  0.0  
  0.25  
  0.5  
  0.75  
  1.0  
  1.25  
  1.5  

JBM, static, smooth
slope = 1.695, x_int = -0.36

SBS, static, smooth
slope = 1.556, x_int = -0.37
```
Summary

- Flicker-defined motion produces weak motion sensation
- But response to flicker-defined spot is similar to that of standard target
- Largest difference seen for cue-conflict "static" spot
- BUT only small change in period-versus-delay slope!? 
- Modeling required to understand the role of position inputs to pursuit

- THANK YOU!