LUNAR RECONNAISSANCE ORBITER ORBIT DETERMINATION ACCURACY ANALYSIS

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Abstract: *Results from operational OD produced by the NASA Goddard Flight Dynamics Facility for the LRO nominal and extended mission are presented. During the LRO nominal mission, when LRO flew in a low circular orbit, orbit determination requirements were met nearly 100% of the time. When the extended mission began, LRO returned to a more elliptical frozen orbit where gravity and other modeling errors caused numerous violations of mission accuracy requirements. Prediction accuracy is particularly challenged during periods when LRO is in full-Sun. A series of improvements to LRO orbit determination are presented, including implementation of new lunar gravity models, improved spacecraft solar radiation pressure modeling using a dynamic multi-plate area model, a shorter orbit determination arc length, and a constrained plane method for estimation. The analysis presented in this paper shows that updated lunar gravity models improved accuracy in the frozen orbit, and a multi-plate dynamic area model improves prediction accuracy during full-Sun orbit periods. Implementation of a 36-hour tracking data arc and plane constraints during edge-on orbit geometry also provide benefits. A comparison of the operational solutions to precision orbit determination solutions shows agreement on a 100- to 250-meter level in definitive accuracy.*

Keywords: *LRO, Orbit determination, Gravitation, Moon.*

1. Introduction

The Goddard Space Flight Center (GSFC) Flight Dynamics Facility (FDF) under the direction of the GSFC Navigation and Mission Design Branch (NMDB) has responsibility for daily orbit determination and related product generation in support of Lunar Reconnaissance Orbiter (LRO) mission operations and science teams. NMDB engineers are responsible for LRO maneuver planning and calibration.

The requirements on the FDF for LRO orbit determination (OD) and prediction accuracy are:

- Definitive ephemeris accuracy of 500 meters total position root mean squared (RMS) and 18 meters radial RMS,
- Predicted orbit accuracy less than 800 meters root sum squared (RSS) over an 84-hour prediction span.

The LRO Lunar Orbiter Laser Altimeter (LOLA) science team also receives and processes LRO tracking data for the purpose of high-precision orbit reconstruction and gravity field estimation in support of the LRO laser altimeter. The LOLA science team has previously published work describing their results performing high-precision OD for LRO using GEODYN [1, 2].

This report reviews the FDF orbit determination results obtained during both the LRO nominal and extended missions, describes the challenges to orbit determination accuracy, and assesses implemented improvements, including effects of new gravity models, modified tracking arc length, investigation of spacecraft modeling during full-Sun orbit periods, and application of constrained plane methods to lunar orbit determination.

2. Mission Description

The ~~Lunar Reconnaissance Orbiter~~LRO launched on 18 June 2009 from Kennedy Space Center aboard an Atlas V. LRO utilized a direct transfer orbit to the Moon, without phasing loops, and entered lunar orbit on 23 June 2009. LRO initially orbited in an approximately 40 km by 180 km altitude commissioning orbit from 27 June 2009 until 15 September 2009, when it entered a circular polar orbit at a mean altitude of 50 km for its nominal mission phase, from 15 September 2009 until 11 December 2011. On 11 December 2011, LRO ended its nominal mission and was returned to the 40 x 180 km altitude commissioning frozen orbit for its extended mission phase, where it currently operates. The higher orbit will permit a much extended mission life without monthly station-keeping maneuvers to maintain the orbit.

During its nominal mission phase, LRO executed a station-keeping (SK) maneuver every 27 days and a momentum dump maneuver (DH) typically every 14 days, although the frequency of DH events varies with Beta angle and spacecraft activity. Momentum dumps were executed in conjunction with SK maneuvers when they occurred in the same week. In the current extended mission orbit SK maneuvers are only required yearly, and momentum dump maneuvers are still required every two to four weeks.

LRO is tracked primarily by a NASA tracking station at White Sands, New Mexico designated WS1, and by Universal Space Network (USN) stations in Perth, Australia; Hawaii, USA; Weilheim, Germany; and Kiruna, Sweden. LRO gets a total of about 13 S-band tracking passes each day, with the White Sands passes typically 60 minutes in duration and the USN passes typically 30 minutes in duration. LRO also receives occasional tracking support from the NASA Deep Space Network (DSN), typically only during SK or DH maneuvers, to close the link when the High Gain Antenna (HGA) is off-pointing from the Earth due to the maneuver. Figure 1 show the typical LRO tracking station distribution as a fraction of all tracking. As ~~Fig.~~Figure 1 illustrates, LRO tracking is primarily (50% to 60%) from the White Sands station.

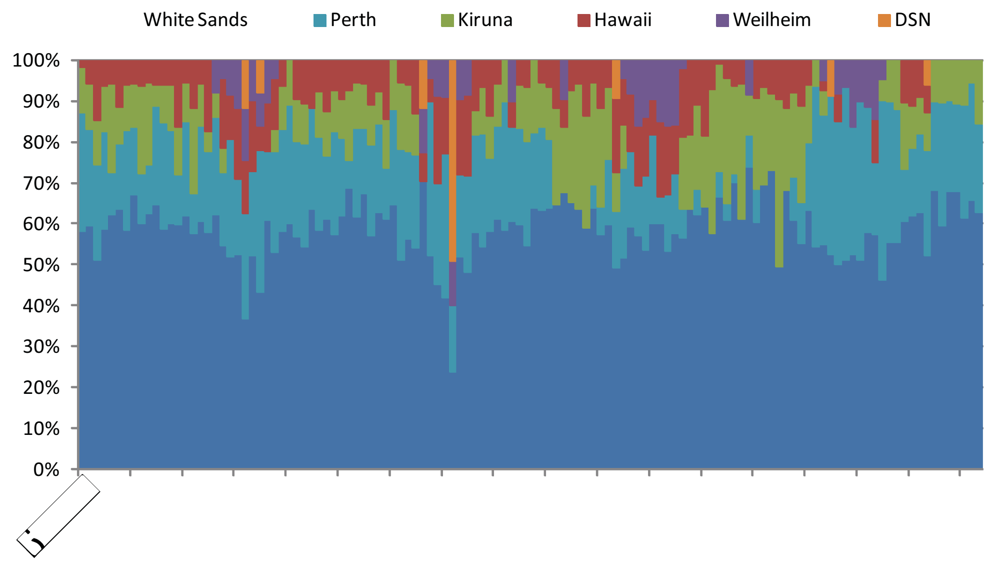

Figure 1. LRO tracking by station, as a fraction of total tracking

The White Sands and USN stations provide range and range-rate tracking observations. The White Sands station also provides angle observations, but angle data is not used in the FDF OD. The FDF assumes observation noise of 10 meters on all range tracking, noise of 3 mm/sec on USN and White Sands range-rate, and 1 mm/sec range-rate noise on DSN tracking. As noted by Mazarico, et. al. [1], the actual tracking data performance is better than these assumed values.

LRO tracking data from the USN and White Sands is affected by systematic observation measurement and timing biases [1, 3]. USN range-rate tracking exhibits an observation measurement bias of approximately -1.0 cm/sec on all USN tracking stations. USN range tracking exhibits an approximately -2 millisecond observation time-tag bias. White Sands range-rate tracking does not exhibit a systematic measurement bias, but White Sands ranging exhibits an approximate +6 to +7 millisecond observation time-tag bias. Range-rate tracking does not exhibit a significant time-tag bias on any station. Other tracking data anomalies encountered and resolved during the LRO early mission are documented in Nicholson, et. al. [3]. DSN tracking of LRO does not exhibit systematic biases of any kind.

LRO also receives one-way laser ranging from International Laser Ranging Service (ILRS) stations. [4] LRO laser ranging data is produced from post-processed telemetry received in the LOLA Mission Operations Center (MOC) and is not used in FDF OD processing.

3. LRO Modeling for Orbit Determination and Prediction

The FDF employs the Goddard Trajectory Determination System (GTDS) for LRO OD and prediction. GTDS is a batch least-squares estimator. A number of enhancements to GTDS were implemented in support of LRO OD accuracy requirements, including gravity modeling up to 360x360, solid lunar tide modeling, lunar albedo and lunar thermal emissivity force modeling, and multi-plate spacecraft area modeling for solar and lunar radiation pressure [3]. As will be described in subsequent sections, the FDF LRO force modeling has been updated in recent years as more accurate gravity models became available. However, the modeling remained the same during the entire 50 km altitude circular nominal mission phase. GTDS modeling parameters for LRO during the nominal mission phase and the current extended mission phase are summarized in ~~Tab.~~Table 1. Details of these updates are described in subsequent sections below.

Table 1. GTDS modeling parameters

Parameter	Nominal Mission	Update in the Extended Mission
Lunar gravity	LP150Q, 150x150	GSFC-GRAIL-270, 200x200
Planetary ephemeris	DE421	
Integration step size	5 sec	
Non-central bodies	Earth, Sun, Jupiter	
Solar radiation pressure (SRP)	Applied, Cr = 1.0	Applied, Cr = 1.67
Spacecraft model for SRP	Spherical, 14 m^2 area	
Lunar radiation pressure (LRP) and lunar albedo	Not used	
Lunar tide modeling	K2 = 0.0248, K3 = 0.0, Phase Shift = 0 degrees	
Solar irradiance	1358 W/m^2 at 1 AU	
Tracking data arc	60 hours	36 hours
Observation data types	Range, range-rate	

In addition to the LRO orbit state, FDF OD for LRO estimates the approximate -1.0 cm/sec range-rate measurement bias on USN data and also estimates range-rate biases on the White Sands station, although the White Sands range-rate measurement bias is typically close to zero. FDF OD actually estimates multiple range-rate biases for each station across each tracking data arc by segmenting the data arc by station and time, and estimating separate range-rate biases for each segment. Segmented arc bias estimation was originally implemented during the early mission when poorer gravity models made it unclear whether the range-rate biases were constant or time-dependent. With the new high-accuracy gravity models, the WS1 and USN range-rate biases are not observed to be time dependent to first order or over short spans, although there is some evidence for long-term second-order variations. Range biases are not routinely estimated, but mean range biases on the order of 10 meters, determined by FDF metric tracking data analysis, are applied for each station.

The timing biases noted on the range data for both White Sands and USN range observations are not estimated or applied in the FDF OD. While pre-processing techniques to accommodate estimation of these biases was considered, it was determined that it such estimation was unneeded to meet the FDF OD and prediction accuracy requirements.

LRO OD accuracy is impacted by two effects dependent on Earth-LRO orbit geometry. The first is an observed degradation in definitive radial accuracy that depends on the relative orientation of the orbit plane as seen from the Earth. In “face-on” orientation, when the LRO orbit normal vector is aligned with the Earth-Moon line and the entire orbit is visible from Earth, tracking provides poor observability of the radial component of the LRO orbit. This effect is easily seen in trending plots of the LRO radial definitive position accuracy.

The cross-track orbit uncertainty also exhibits a periodicity with orbit geometry, 90 degrees out of phase with the radial component. Cross-track OD accuracy is best in face-on geometry, and poorest in edge-on geometry, when the LRO orbit normal is perpendicular to the Earth-Moon line and the LRO orbit is viewed “edge-on” from Earth. These effects are discussed in further detail in combination with results presented below.

Finally, twice a year, LRO enters a full-Sun exposure period, where it does not experience any umbra or penumbra due to lunar eclipse. These periods, in November to January and May to July, each last about 36 days and have a significant effect on FDF LRO prediction accuracy. In FDF operations for LRO, orbit prediction, including delta-v modeling for upcoming SK and DH maneuvers, is accomplished by propagating the OD state from GTDS in Analytical Graphics’ (AGI) Systems Tool Kit (STK) using the Astrogator module. In both GTDS and STK, a simple cannonball spacecraft model is used without any attitude or spacecraft plate modeling. A more sophisticated box-and-wing spacecraft model for lunar orbiters was added to GTDS for LRO support, and investigation of the use of this model is described below.

4. Method of Assessing Orbit Determination Accuracy

From each new LRO orbit determination solution, the FDF generates an ephemeris file. An ephemeris file of any format or origin can be thought of as consisting of two spans; the *definitive span* (sometimes also called a *fit span*), and the *predictive span*. In an ephemeris derived from batch least-squares OD, the definitive span is that portion of the ephemeris which covers the tracking data arc used to generate the solution vector. The predictive span is the portion of ephemeris data after the end of the tracking data arc, which represents the future prediction of the spacecraft orbit based on the tracking data.

In the FDF, definitive accuracy for batch least-squares OD is assessed by comparison of sequential ephemeris files over a common overlapping definitive span. This is commonly called a *definitive overlap* compare. Predictive accuracy is assessed by comparison of the predictive portion of one ephemeris file with the definitive portion of a subsequent file (called the *baseline* ephemeris). This comparison assesses the accuracy of the prediction in the prior ephemeris, to within the uncertainty of the definitive portion of the baseline ephemeris.

It is understood that these methods actually only assess ephemeris consistency, and that a systematic error such as a bias in any component of the orbit is not observable by this technique. A preferable technique is comparison to an independent and high-accuracy solution from precision OD. Such solutions are rarely available to operations facilities, but in the case of LRO, the LOLA science team performs precision orbit determination for geodetic mapping and gravity model estimation. Section 8 below presents details of a comparison between the FDF solutions and a particular set of LOLA precision ephemeris files, allowing an assessment of the true accuracy of the FDF solutions and evaluation of the validity of using ephemeris consistency as a proxy for ephemeris accuracy.

5. FDF LRO Orbit Determination Results from the Nominal Mission

The LRO nominal mission began on 15 September 2009 when LRO entered its 50 km mean altitude orbit, and ended on 11 December 2011 when it returned to the 40 x 180 km mean altitude commissioning frozen orbit. Figure 2, ~~Fig.~~Figure 3, and ~~Fig.~~Figure 4 show the timeline of results obtained for RMS definitive radial accuracy, RMS total position accuracy, and 84-hour total predicted position accuracy (the maximum prediction error in 84 hours). Each point on the plot represents a daily solution. Results from the early commissioning orbit are also included for context, and to illustrate the effect that orbit regime has on accuracy. The numerical results for the nominal mission only, approximately 730 solutions, are summarized in ~~Tab.~~Table 2. All the solutions included in these results were computed using the LP150Q gravity model at 150x150. Predictions that cross an SK or DH event have been removed from the figures, as well as from the data reported in ~~Tab.~~Table 2.

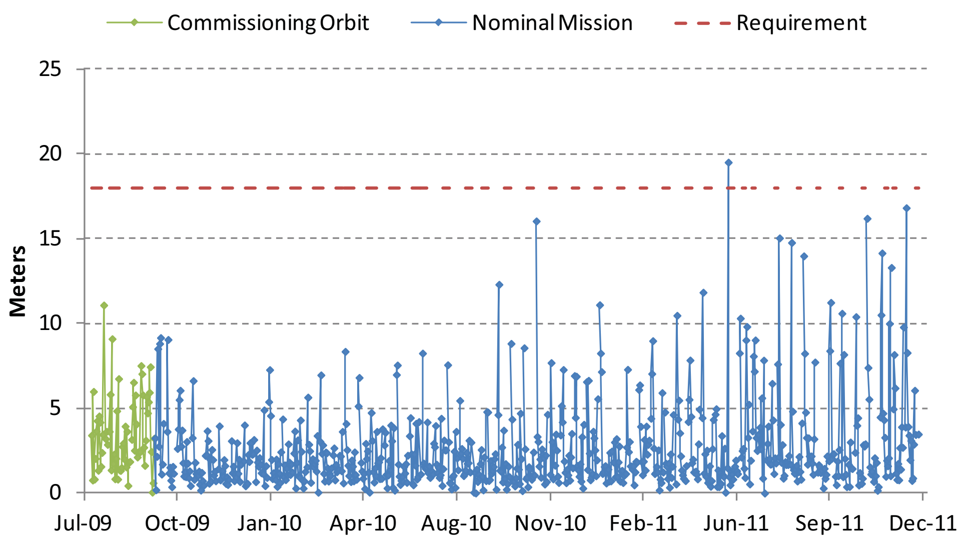

Figure 2. RMS radial position definitive accuracy, early commissioning orbit and complete nominal mission orbit

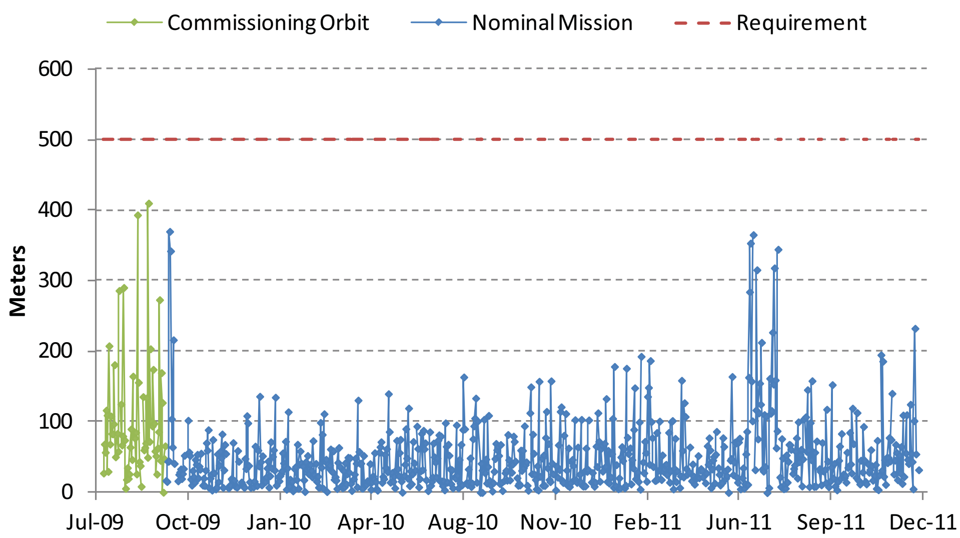

Figure 3. RMS total position definitive accuracy, early commissioning orbit and complete nominal mission orbit

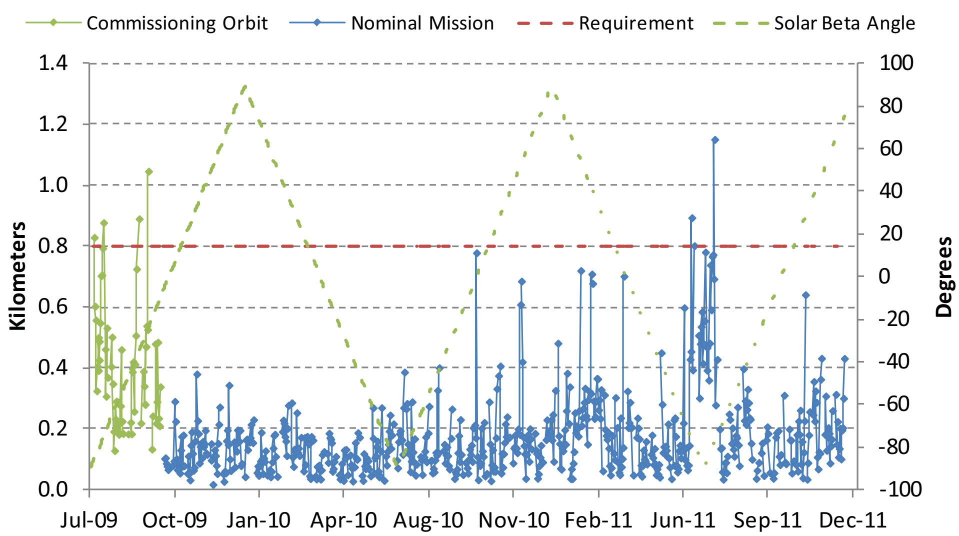

Figure 4. Maximum 84-hour predicted position error, and solar Beta angle, early commissioning orbit and complete nominal mission orbit

Table 2. FDF LRO OD results during the nominal mission

	Mean (meters)	Standard Deviation (meters)
RMS radial definitive position	3	3
RMS in-track definitive position	32	37
RMS cross-track definitive position	29	34
RMS total definitive position	48	47
84-hour total position prediction	171	138

As the above figures illustrate, there was one failure of the RMS radial definitive position requirement, no failures of the RMS total definitive position requirement, and three failures of the 84-hour predicted requirement during the nominal mission. All of the predicted accuracy failures occurred during the June-July 2011 full-Sun period when prediction accuracy is challenged by the coarse solar radiation pressure (SRP) and spacecraft modeling employed for routine OD operations.

Figure 2 and ~~Fig.~~Figure 3 illustrate the orbit geometry effect on definitive accuracy. The periodic spikes above the baseline noise level correlate to the two-week cycle of face-on/edge-on observation of the LRO orbit plane. This is shown in more detail in ~~Fig.~~Figure 5 below. The largest spikes in radial error occur when the angle between the Moon-Earth line and the LRO orbit normal vector is near 0 or 180 degrees, that is, when the plane of the LRO orbit is viewed face-on from the Earth.

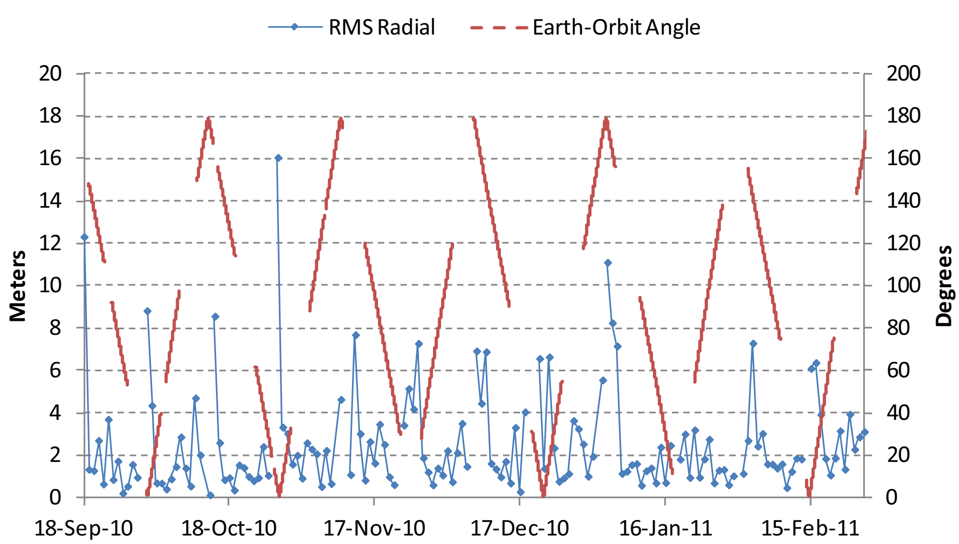

Figure 5. Detail plot of RMS radial position accuracy, and the angle between the Moon-Earth line and the LRO orbit normal vector

Figure 3 and ~~Fig.~~Figure 4 additionally illustrate the considerable disparity in orbit accuracy between the commissioning orbit and the mission orbit when using the LP150Q gravity model. Average total definitive accuracy is worse in the commissioning orbit than in the nominal orbit. The 84-hour prediction accuracy in the early commissioning orbit is at least partially degraded due to a full-Sun orbit condition. The LP150Q model is derived from Lunar Orbiter, Apollo sub-satellites, Clementine, and Lunar Prospector mission data and is likely better tuned for 30 to 100 km circular orbits than the elliptical 40 x 180 km LRO commissioning orbit. [5, 6, 7]

6. Orbit Determination Accuracy Improvements in the Extended Mission

The LRO nominal mission ended on 11 December 2011, when the spacecraft returned to a 40 x 180 km altitude elliptical frozen orbit similar to the early mission commissioning orbit. At the beginning of the extended mission, the FDF was still using the LP150Q gravity model for OD, and the poor performance of this model observed during the early commissioning orbit quickly returned. The FDF began to experience a significant increase in the frequency of requirement violations, and an effort was launched to study potential improvements to LRO OD.

6.1. Improved Lunar Gravity Models

In support of their instrument's science, the LOLA team has conducted LRO precision orbit determination since launch using GEODYN. As a part of their processing they have produced new lunar gravity models, some particularly tuned for the LRO orbit. [1] Since it was already known that the LP150Q lunar gravity model performs adequately for the 50-km circular nominal mission orbit, but not as well for the commissioning orbit, in July 2012 the FDF contacted the LOLA team and obtained a version of their latest gravity model at the time, designated LRO Lunar Gravity Model 2 (LLGM-2). [1, 2] LLGM-2 is a 150x150 model, derived from data for Lunar Orbiter, Apollo sub-satellites, Lunar Prospector, Clementine, and 2.5 years of LRO tracking. Therefore, the LLGM-2 model incorporated LRO mission data from both the nominal mission and early extended mission. The LLGM-2 model was evaluated against the LP150Q model in a test series covering June 2012 and was found to be a significant improvement.

At the same time that the LLGM-2 model was provided, the first models from the GRAIL mission were being released. Shortly after receiving LLGM-2, the FDF obtained a GRAIL-derived model to degree and order 270, designated GSFC-GRAIL-270. The LLGM-2 and GRAIL models were reevaluated, this time over a longer time span of about two months of data, covering May to July 2012. In this series, LLGM-2 at degree and order 150 performed comparably to the GSFC-GRAIL-270 model at 270x270. It is assumed that this is because the LLGM-2 model incorporates LRO tracking in its formulation, and in particular some data from the early extended mission, so is better tuned for LRO estimation and prediction in that orbit regime. The GSFC-GRAIL-270 model does not incorporate LRO tracking.

Running gravity models of degree and order 270 or higher is currently impractical for FDF routine OD operations, given the constraints of daily delivery deadlines. Performing an OD with a 10-day prediction on current FDF hardware takes about 25 minutes when using a 150x150 model, about 40 minutes when using a 200x200 model, and about 60 minutes when using a 270x270 model. Furthermore, once the OD is completed in GTDS, the orbit must be repropagated again using STK for momentum unload and SK modeling, which could add up to an additional 70 minutes for a 270x270 model. Since the LLGM-2 gravity model at 150x150 was showing comparable accuracy to the GSFC-GRAIL-270 model at 270x270 in June 2012, the FDF began using the LLGM-2 gravity model at 150x150 for operational LRO OD starting on 3 September 2012.

However, while LLGM-2 performance initially looked comparable to GRAIL-GSFC-270, perhaps due to its incorporation of recent early extended mission LRO tracking, it began to show stress by February 2013, when the FDF started again to experience recurring violations of the radial position definitive accuracy requirement in face-on orbit geometry, prompting a reevaluation of the GRAIL-GSFC-270 model. This evaluation, performed over the time span of 11 March 2013 to 11 July 2013, showed significant improvements when using both a 200x200 truncation and the full GSFC-GRAIL-270 model. See below in ~~Tab.~~Table 3. For comparison, later evaluation runs using 200x200 and 360x360 truncations of the GRAIL GRGM660PRIM model [8] are included.

Table 3. Gravity model evaluation runs, 11 March 2013 to 11 July 2013, with number of failures of the associated accuracy requirement

	RMS Radial Definitive Position Error (m) Average / Std. Dev.	**RMS Total Definitive Position Error (m) Average / Std. Dev.**	**Maximum Total 84-Hour Predicted Position Error (m) Average / Std. Dev.**
LLGM-2	6 / 6 7 failures	158 / 139 3 failures	539 / 332 22 failures
GSFC-GRAIL-270 (200x200)	2 / 2 0 failures	83 / 87 0 failures	461 / 317 14 failures
GSFC-GRAIL-270 (270x270)	2 / 1 0 failures	79 / 85 0 failures	447 / 314 14 failures
GRGM660PRIM (200x200)	2 / 2 0 failures	83 / 87 0 failures	456 / 317 14 failures
GRGM660PRIM (360x360)	2 /1 0 failures	78 / 85 0 failures	445 / 316 13 failures

The performance degradation observed with the LLGM-2 model is consistent with what was reported by Mazarico, et. al. [2], and is ascribed there to a combination of drift in LRO orbit inclination and tuning of the LLGM-2 field model to the data span used in its estimation. As a result of this analysis, the FDF switched from the LLGM-2 model to GSFC-GRAIL-270 on 8 September 2013. Since 200x200 models such as GSFC-GRAIL-270 provide more than adequate accuracy for current orbit requirements and the full model provides only marginal improvement for much greater processing time, the FDF currently employs a 200x200 truncation of the GSFC-GRAIL-270 model for operations.

In these results, the new gravity models don't produce as dramatic an improvement in predictive accuracy as definitive accuracy. This is because the statistics reported here include a full-Sun period, where the predictive accuracy is degraded due to coarse SRP and spacecraft area modeling. This issue is discussed in further detail below.

6.2. Orbit Prediction Accuracy during Full-Sun Periods

As previously noted, LRO has twice-yearly full-Sun exposure periods, where it does not experience any umbra or penumbra due to lunar eclipse. These periods, occurring between November to January and May to July, each last about 36 days on average and have a significant effect on FDF LRO prediction accuracy, causing frequent violations of the 800 meter 84-hour predicted accuracy requirement. The prediction errors are larger and more frequent in the extended mission orbit than were observed in the nominal mission orbit. Furthermore, the significantly improved gravity models do little to improve this issue. This clearly points to ~~solar radiation pressure~~SRP modeling as the likely source of the prediction errors.

Throughout the entire LRO mission, the FDF has employed a simple "cannonball" area model for the LRO spacecraft and modeled ~~solar radiation pressure (~~SRP~~)~~ using an umbra/penumbra shadow model, and a constant spacecraft area and coefficient of solar radiation pressure (C_R). Attempts to estimate a well-determined C_R ~~coefficient of solar radiation pressure~~ were unsuccessful using both the LP150Q and LLGM-2 gravity models. Estimated values of C_R using those models were very inconsistent and sometimes unphysical. However, estimation of C_R using a cannonball area model is possible for non-full-Sun orbit periods with the GSFC-GRAIL-270 model. An analysis series estimating C_R for such a period yielded a new value of 1.67, which was implemented as an applied C_R in FDF operations on 19 November 2013. Consistent estimation of C_R, even with the GRAIL gravity model, is still not possible during full-Sun periods when using the cannonball area model, so FDF LRO operations currently only applies the value of 1.67 and does not attempt to estimate C_R in operational OD.

Applying the updated value of C_R improves prediction accuracy during regular eclipse periods, but does not solve the prediction issues during full-Sun periods. Prior to LRO launch, an enhanced multi-plate spacecraft area model, implementing both fixed and moving surfaces, and individual coefficients of specular and diffuse reflectivity for each surface, was added to GTDS for LRO support. This accommodates modeling LRO as a box, with a moving solar array and moving ~~high-gain antenna (~~HGA~~)~~. The spacecraft and solar array attitudes may be provided via external attitude files, or computed analytically, using nadir and Sun-pointing constraints, but the HGA motion currently must be provided via an external attitude file.

Test runs were executed over the 11 March to 11 July 2013 span, using this multi-plate model with both analytically computed attitude modeling and definitive attitude files. Since modeling of the HGA cannot currently be computed analytically, this surface was left out of the analytic runs. C_R was not estimated for these runs. For all these runs a C_R value of 1.67, determined using the cannonball model, was applied. Results are shown in ~~Tab.~~Table 4.

Table 4. OD estimation and prediction results using cannonball and multi-plate area models, 11 March 2013 to 11 July 2013. All runs employ the GSFC-GRAIL-270 (200x200) model.

Spacecraft Area Model	Maximum Total 84-Hour Predicted Position Error (m) Average / Std. Dev.
Cannonball	325 / 325 8 failures
Multi-plate with analytic attitude (no HGA modeling)	364 / 358 12 failures
Multi-plate with definitive attitude	128 / 84 0 failures

From these results, it is clear that both the cannonball model and the multi-plate model with analytic spacecraft attitude are inadequate for full-Sun orbit prediction. The multi-plate model with definitive spacecraft, solar array, and HGA attitudes yields significant improvement.

The timeline of predictive accuracy results from these runs is shown in ~~Fig.~~Figure 6 for the cannonball area model and the multi-plate area model with definitive attitude files. In this analysis series, LRO entered full-Sun on 22 May and exited full-Sun on 29 June. In ~~Fig~~Figure. 6, the multi-plate model shows some degradation in prediction accuracy during the nominal eclipse period, but this may be due to the fact that the multi-plate model run applied a value of C_R (1.67) that was determined using the cannonball area model, and it may not be an optimal value for use with the multi-plate model.

The analysis runs presented in ~~Tab.~~Table 4 and ~~Fig.~~Figure 6 also employed a constrained plane technique ~~which~~ that further improves OD and prediction accuracy for LRO. This technique is described in Section 6.4 below.

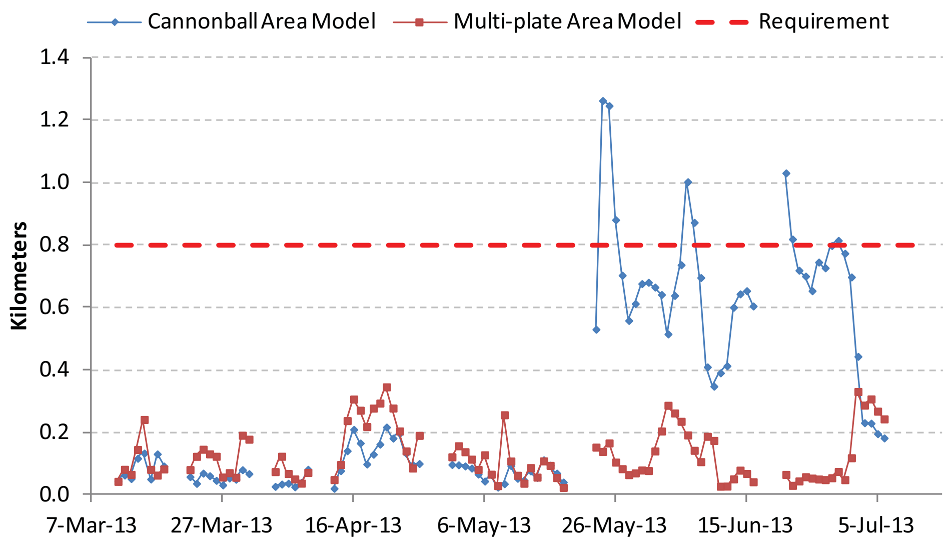

Figure 6. 84-hour maximum predicted position error for cannonball area model and multi-plate model with definitive attitude data. Predictions crossing a momentum unload have been removed.

The FDF receives definitive spacecraft, solar array, and HGA attitude data from the LRO ~~mission operations center (~~MOC~~)~~, but this data is not available in time for use in daily operations, and predicted solar array and HGA pointing data is not currently available at all. Obtaining accurate predicted attitude data clearly has the potential to improve orbit prediction accuracy, but this has not yet been pursued. As a result, the FDF currently still employs the cannonball area model for LRO OD and prediction.

The multi-plate area model for lunar orbiters implemented in GTDS also accommodates modeling of lunar albedo and lunar thermal radiation pressure. These forces were not applied in any of these runs because they are assumed to be quite small (perhaps smaller even than spacecraft thermal radiation pressure, which is also not modeled) [9], and the application of the lunar albedo force modeling dramatically increases computation time.

6.3. Orbit Determination Arc Length

OD and prediction accuracy are sensitive to tracking data arc length. Based on previous history with the Lunar Prospector mission [10], a 60-hour arc was initially adopted for LRO OD. This span was re-evaluated and an arc length of 36 hours was considered. Results of evaluation runs using 60-hour and 36-hour arcs for orbit determination are shown in ~~Tab.~~Table 5.

Table 5. OD estimation and prediction results using 60- and 36-hour OD arcs, 11 March 2013 to 11 July 2013. All runs employ the GSFC-GRAIL-270 (200x200) model and use a cannonball spacecraft model, applying a CR of 1.67.

	RMS Radial Definitive Position Error (m) Average / Std. Dev.	RMS Total Definitive Position Error (m) Average / Std. Dev.	Maximum Total 84-Hour Predicted Position Error (m) Average / Std. Dev.
60-hour arc	4 / 2 0 failures	108 / 169 5 failures	516 / 583 26 failures
36-hour arc	3 / 2 0 failures	74 / 87 0 failures	382 / 373 14 failures

The 36-hour tracking arc shows improvements to both definitive and predictive accuracy. It is natural that, up to a point, a shorter arc length yields more consistent definitive overlaps. Fewer observations means fewer orbit constraints to fit, and dynamical modeling errors, particularly time-dependent ones like solar radiation pressure variations due to variable spacecraft area, do not accumulate as much as for longer arcs. The shorter arc is also clearly beneficial to prediction accuracy, presumably because the estimated orbit state is more biased toward recent data in a 36-hour arc than is the case in a 60-hour arc. Based on these results, a 36-hour arc for orbit determination was adopted for FDF LRO operations on 3 September 2012.

6.4. Constrained Plane Method

As noted previously, both radial and cross-track accuracy are strongly affected by the observation geometry of the LRO orbit plane, driven by the two-week face-on/edge-on cycle. For different phases of the mission, orbit geometry has driven the radial error anywhere from 10 to 70 meters above baseline noise. The cross-track effect is even worse, peaking from 100 meters to as high as 800 meters for past mission phases. See ~~Fig.~~Figure 7.

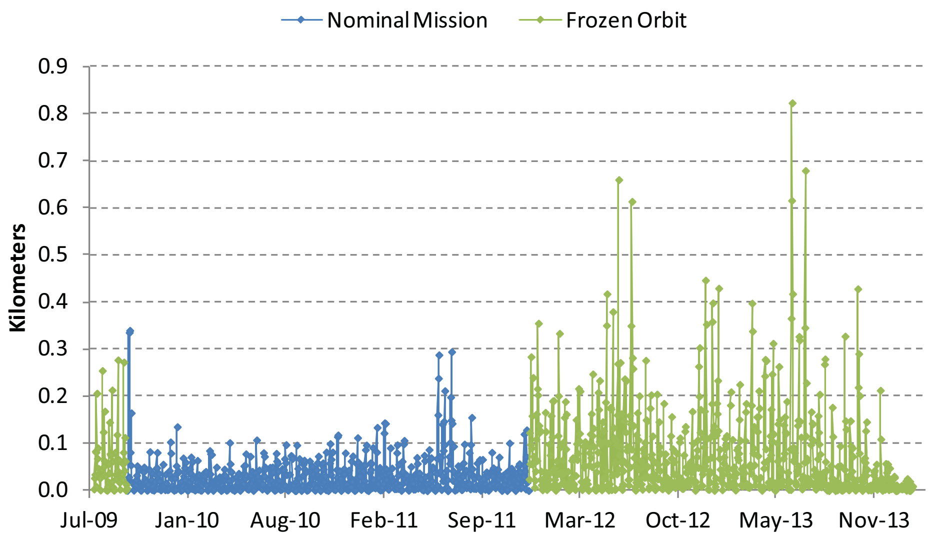

Figure 7. RMS cross-track definitive position error for the nominal mission and early and late mission commissioning (frozen) orbit

The FDF has for many years used constrained plane methods to improve cross-track accuracy of short-arc orbit determination for geosynchronous satellites, where observation of the plane is challenged by abbreviated tracking data arcs. A constrained plane is implemented by starting the batch least-squares estimation from an initial state with a well-estimated plane, and applying tight a priori variances to the inclination and right ascension of the ascending node of the initial state. This forces the estimation, which may suffer from a shortened arc or poor observability of the plane, to stay close to the well-estimated prior plane, which can considerably improve cross-track estimation accuracy in these situations.

For LRO, cross-track estimation accuracy is best when the orbit plane is viewed face-on and poorest when viewed edge-on. As a consequence of this, a scheme was tested whereby the a priori variances were constrained when the LRO orbit plane was within 45 degrees of the Earth-Moon line, in other words, when the orbit plane is viewed near edge-on to the Earth. At other times, when the orbit is viewed more face-on to the Earth, the plane is left unconstrained. By this method, an accurate estimation of the plane is obtained during near face-on geometry, and this estimation is retained and propagated forward through a poor edge-on period. Once the orbit exits the edge-on condition, the constraint is released, allowing the plane to reset and the OD to remove any errors that may have accumulated during the constrained period.

This analysis was run over the 11 March 2013 to 11 July 2013 evaluation span and the results are shown in ~~Fig.~~Figure 8 and ~~Tab.~~Table 6.

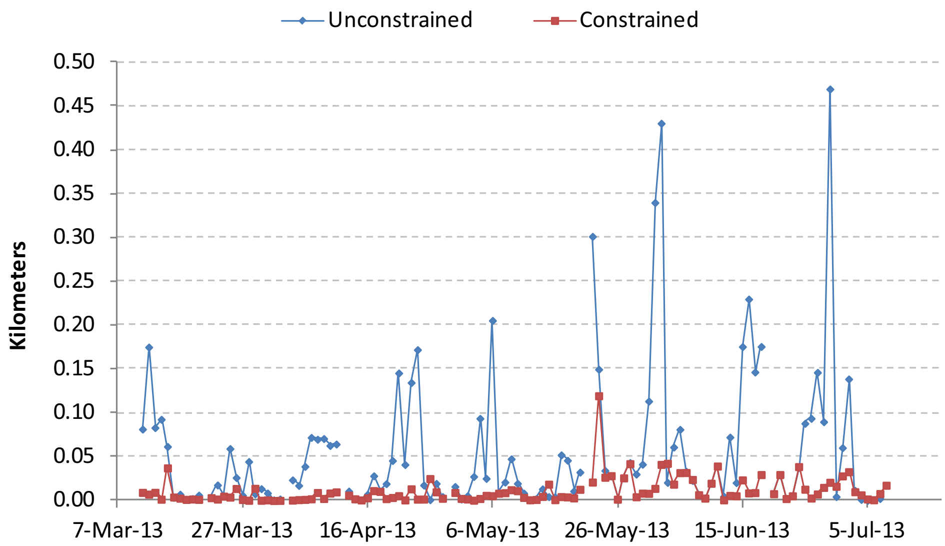

Figure 8. RMS cross-track definitive position error for unconstrained and constrained solutions, 11 March 2013 – 11 July 2013

Table 6. RMS and total definitive position error for unconstrained and constrained solutions, 11 March 2013 to 11 July 2013

	RMS Cross-track Definitive Position Error (m) Average / Std. Dev.	**RMS Total Definitive Position Error (m) Average / Std. Dev.**
GSFC-GRAIL-270 (200x200) Unconstrained	50 / 80	74 / 87
GSFC-GRAIL-270 (200x200) Constrained	11 / 15	32 / 36

As is evident from ~~Fig.~~Figure 8, application of constraints largely removes the periodic errors due to orbit geometry. The well-determined plane estimates during the face-on "troughs" are carried forward across the edge-on "peaks" by the constraint. For the constrained runs, a root variance constraint of 1×10^{-6} degrees was applied to the a priori inclination and right ascension of the ascending node (RAAN) when the orbit plane was within 45 degrees of the Earth-Moon line. Since there was a clear improvement in cross-track accuracy, this method of constraining and un-constraining the LRO orbit plane was implemented in FDF operations for LRO starting on 5 November 2013. The value of 1×10^{-6} degrees chosen as the operational constraint is a typical value historically used in the FDF for constraints in other orbit regimes. There is reasonable possibility that this constraint is too strict, and it merits closer examination in the future with further analysis. In addition, the FDF currently does not modify the constraint

Formatted: Not Superscript/ Subscript methodology when LRO executes out-of-plane (cross-track) delta-H momentum dumps, but to date this has not caused any large errors.

7. FDF LRO Orbit Determination Results from the Extended Mission

Figures 9-11 show the timeline of results obtained for RMS definitive radial accuracy, RMS total position accuracy, and 84-hour total predicted position accuracy in the LRO extended mission respectively. Results from the late nominal mission orbit are also included for context. In these figures, the triangle sigils denote the time of implementation of the LLGM-2 gravity model and a 36-hour data arc for OD, adoption of the GSFC-GRAIL-270 gravity model, and introduction of the constrained plane method for OD, respectively from left to right. In ~~Fig.~~Figure 11, the solar Beta angle is also shown, illustrating the correlation between Beta angle (and full-Sun periods) and prediction accuracy. As previously described, LRO experiences no lunar shadow at all near Beta angles of ±90 degrees, and prediction accuracy degrades due to poor spacecraft area modeling.

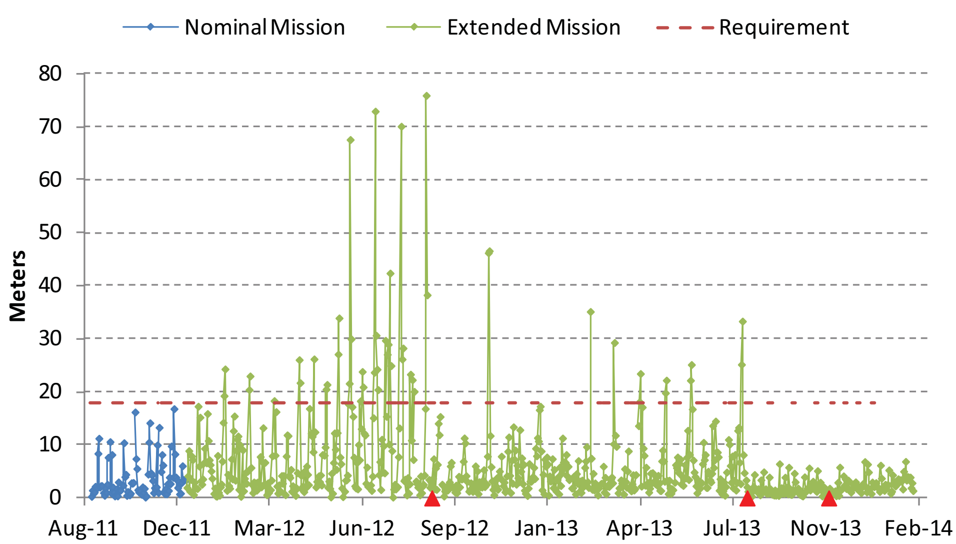

Figure 9. RMS radial position definitive accuracy, late nominal mission orbit and extended mission orbit

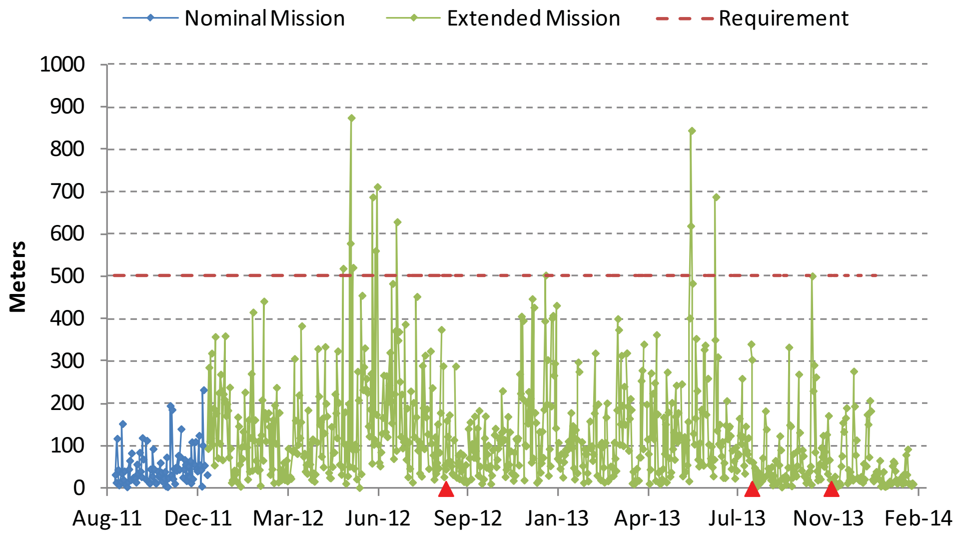

Figure 10. RMS total position definitive accuracy, late nominal mission orbit and extended mission orbit

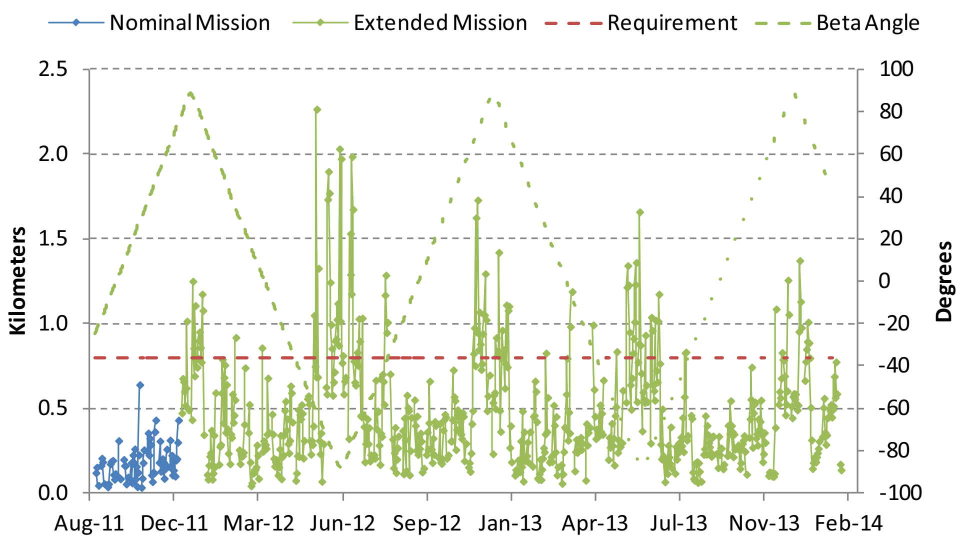

Figure 11. Maximum 84-hour predicted position error, and solar Beta angle, late nominal mission orbit and extended mission orbit

Table 7 summarizes the results from the extended mission, for each of the improvements as they were implemented. Since the 84-hour prediction accuracy is degraded during full-Sun periods, ~~Tab.~~Table 7 shows results for 84-hour prediction accuracy both including and excluding the full-

Sun periods. There was no full-Sun orbit period between 9 Aug 2013 and 5 November 2013 (the GSFC-GRAIL-270 unconstrained series). The values tabulated here are the mean and standard deviation of the daily FDF ephemeris definitive overlap and predictive RMS differences.

Table 7. FDF LRO OD results during the extended mission

Modeling		Mean (meters)	Standard Deviation (meters)
LP150Q (11-Dec-11 to 3-Sep-12)	RMS radial definitive position	9	11
	RMS in-track definitive position	111	115
	RMS cross-track definitive position	86	96
	RMS total definitive position	154	136
	84-hour total position prediction (all)	540	406
	84-hour total position prediction (ex. full-Sun)	385	284
LLGM-2 (3-Sep-12 to 9-Aug-13)	RMS radial definitive position	6	6
	RMS in-track definitive position	83	71
	RMS cross-track definitive position	85	107
	RMS total definitive position	132	115
	84-hour total position prediction (all)	472	319
	84-hour total position prediction (ex. full-Sun)	339	191
GSFC-GRAIL-270 Unconstrained Plane (9-Aug-13 to 5-Nov-13)	RMS radial definitive position	2	1
	RMS in-track definitive position	41	44
	RMS cross-track definitive position	52	80
	RMS total definitive position	73	86
	84-hour total position prediction (all)	291	131
	84-hour total position prediction (ex. full-Sun)	291	131
GSFC-GRAIL-270 Constrained Plane (5-Nov-13 to 31-Jan-14)	RMS radial definitive position	3	1
	RMS in-track definitive position	41	51
	RMS cross-track definitive position	17	28
	RMS total definitive position	47	56
	84-hour total position prediction (all)	523	299
	84-hour total position prediction (ex. full-Sun)	329	163

The improvement in definitive accuracy achieved as a result of the new gravity models is evident. Radial consistency has been reduced from 10 meters to 3 meters, and total definitive position accuracy has improved from 154 meters to 47 meters. Improvement in predicted accuracy is not as dramatic, mainly due to the prediction errors from coarse SRP modeling, particularly during full-Sun periods.

8. Comparison of FDF Definitive OD to High-Precision LOLA Ephemeris Files

The ~~LRO laser altimeter (~~LOLA~~)~~ team produces high-accuracy orbit solutions for LRO using GEODYN [1, 2, 11]. These solutions use the same S-band tracking data used in the FDF solutions, but employ much higher fidelity force modeling and media corrections, and are documented as having total position accuracy of about 10 meters [2]. Having an independent high-accuracy OD source facilitates an assessment of the true definitive accuracy of the FDF solutions, and an evaluation of the definitive overlap consistency method as a measure of definitive accuracy.

Figure 12 displays the distribution of definitive total (RSS) position differences between LOLA definitive ephemeris files derived using a GRAIL gravity model [11] and the FDF operational solutions for the nominal and extended mission orbits (using the LP150Q gravity model for both series), as well as a test series in the extended mission using the GSFC-GRAIL-270 gravity model. The distributions represent the total position difference between the LOLA and FDF solutions measured point-by-point every ten minutes over particular spans for each mission phase. Radial differences between the FDF and LOLA ephemerides are mostly 10 meters or less, so the largest components of the total difference are the along-track and cross-track components. Figure 12 illustrates how the accuracy of the LP150Q model degrades in the extended mission orbit. For the nominal mission using LP150Q, 95% of the differences are less than 115 meters. For the extended mission using LP150Q, the 95th percentile is much larger, at 225 meters. It is also evident that GRAIL models represent a significant improvement for the extended mission orbit. For the extended mission using GSFC-GRAIL-270, the 95th percentile is at 100 meters. These values are consistent with comparisons between FDF and LOLA solutions assessed using altimeter geolocation methods reported in Mazarico, et. al. [2]

The comparison span for the nominal mission data is 11 March 2011 to 9 June 2011. The extended mission comparison span is 11 March 2013 to 22 May 2013. These spans exclude any solutions during full-Sun periods. As described earlier, the FDF full-Sun solutions are degraded for reasons unrelated to gravity modeling. Inclusion of data during full-Sun spans reduces the overall performance of the FDF solutions compared to the LOLA baseline, especially in the extended mission, moving the 95th percentile for the GSFC-GRAIL-270 series out to 250 meters.

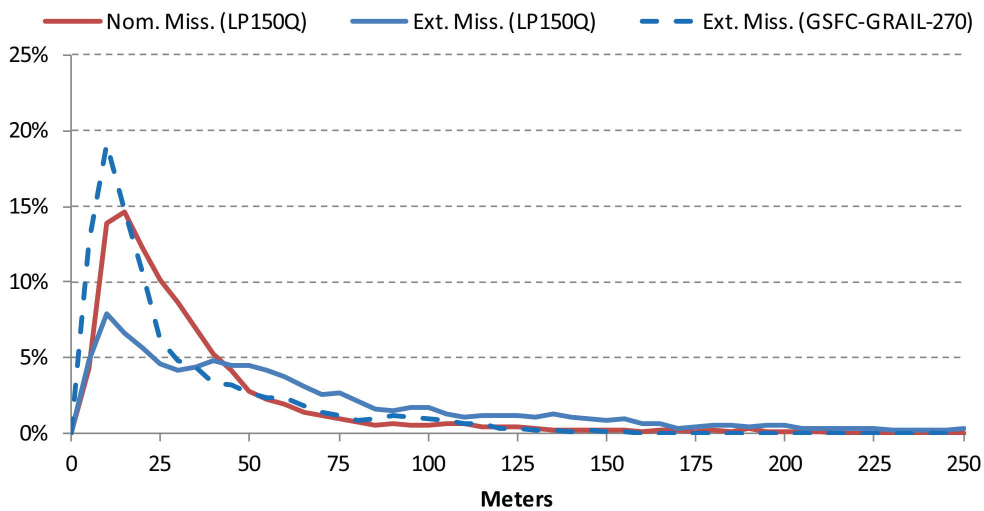

Figure 12. Distribution of total position difference between LOLA and FDF solutions using LP150Q and GSFC-GRAIL-270, for the nominal and extended missions

Finally, we can also use the LOLA high-accuracy ephemeris to evaluate the definitive overlap consistency method of determining definitive accuracy. Figure 13 presents the distribution of definitive total (RSS) position difference, measured in the same fashion as described above, between the LOLA precision ephemeris and the FDF GSFC-GRAIL-270 series in the extended mission period (the same GSFC-GRAIL-270 series reported in ~~Fig.~~Figure 12), along with the definitive overlap differences of consecutive FDF solutions, applying the operational FDF method of measuring definitive accuracy. It is evident that the definitive overlap method used operationally by the FDF to assess definitive accuracy does not suffer from major biases, as both distributions are qualitatively similar. As reported above, the 95th percentile for the FDF-LOLA comparisons is 100 meters, while the 95th percentile for the FDF overlap compares illustrated in ~~Fig.~~Figure 13 is 115 meters.

In general, the FDF definitive overlap method seems to overestimate definitive error. For the nominal mission, the 2-sigma definitive error inferred from the FDF-LOLA ephemeris differences is 115 meters. From ~~Tab.~~Table 2, the mean plus 2-sigma definitive accuracy from FDF-FDF overlap comparisons is 142 meters. For the extended mission using the GSFC-GRAIL-270 model, the 2-sigma definitive error from FDF-LOLA comparisons is 100 meters (excluding full-Sun periods), but from ~~Tab.~~Table 7, the inferred mean plus 2-sigma accuracy for FDF-FDF overlap comparisons is 245 meters.

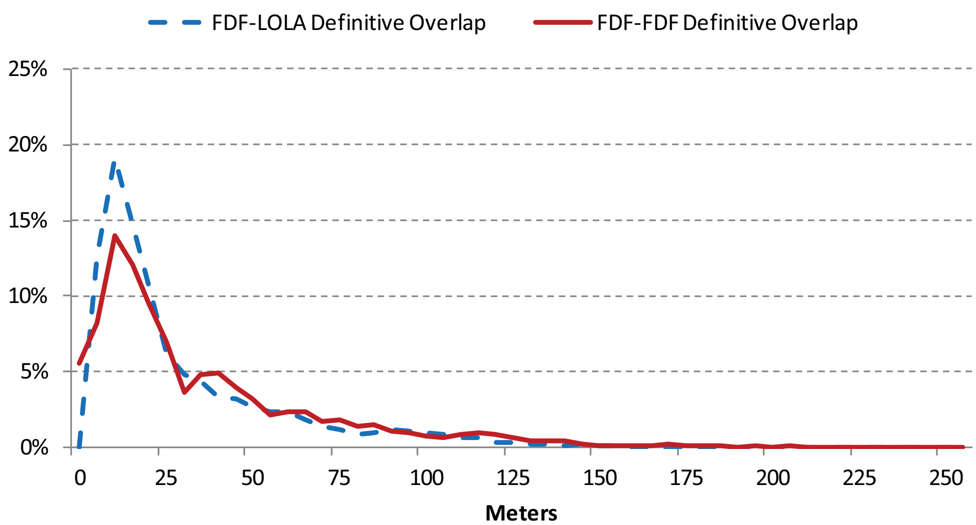

Figure 13. Distribution of total position difference between LOLA and FDF solutions using GSFC-GRAIL-270, and the FDF definitive overlap differences

9. Conclusion

GSFC FDF orbit determination results from the LRO nominal and extended mission have been presented. During the nominal mission (a 50 km altitude circular orbit), using the LP150Q lunar gravity model, there was one violation of the RMS radial definitive position requirement, no violations of the RMS total definitive position requirement, and three violations of the 84-hour predicted requirement. All of the predicted accuracy failures occurred during the June-July 2011 full-Sun period when prediction accuracy is challenged by the coarse SRP and spacecraft modeling employed for routine OD operations.

OD requirement violations became much more frequent beginning in December 2011, after returning to the frozen commissioning orbit for the LRO extended mission. The extended mission frozen orbit exposed modeling deficiencies and shortcomings of the then-current lunar gravity models, prompting the implementation of newly-derived gravity models, a modified data arc length for OD, and constrained plane methods for estimation. Coarse spacecraft area modeling was identified as the cause of poor predicted accuracy during full-Sun orbit periods. Implementing a multi-plate spacecraft model using definitive attitude data for the spacecraft and appendages significantly improves prediction accuracy during full-Sun periods, but this is not currently practical for daily operations, since definitive attitude data is not available in time for daily OD. The FDF continues to investigate improvements to spacecraft modeling, including improving the analytical multi-plate spacecraft model, or using predicted nominal attitude data as input to the multi-plate spacecraft model.

Comparison of FDF solutions to precision OD performed by the LRO LOLA team shows that accuracy of the FDF solutions is about 100 meters, both during the nominal mission using LP150Q and the extended mission using GSFC-GRAIL-270. Full-Sun periods in the extended mission degrade the definitive accuracy considerably, however. Despite this fact, the new GRAIL models are a significant improvement over LP150Q for the extended mission frozen orbit.

As a result of the GRAIL mission, lunar geodesy is proceeding at a rapid pace, and new high-accuracy lunar gravity models are emerging. The current best lunar gravity models are 660x660, and even larger models are being considered. However, the constraints of daily operational deadlines, current hardware, and operational flight dynamics software like GTDS and STK don't permit practical use of models higher than about 200x200 for routine flight dynamics operations. The results of this report show that excellent accuracy can be achieved with 200x200 truncations of modern lunar gravity models. The benefits of lunar gravity models beyond 200x200 truncations are measurable but marginal given LRO accuracy requirements, so there is no current motivation for the FDF to pursue improvements to processing capability. Nevertheless, the operations community might benefit from a future high-accuracy lunar model estimated to order 200x200. Such a model might have the benefit of forcing some of the high-order potential into lower terms, possibly recouping some accuracy that is currently lost in truncation.

10. Acknowledgments

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