National Aeronautics and Space Administration

TUNING MONOTONIC BASIN HOPPING:
IMPROVING THE EFFICIENCY OF STOCHASTIC SEARCH AS APPLIED TO LOW-THRUST TRAJECTORY OPTIMIZATION

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Overview

- Monotonic Basin Hopping (MBH) is a stochastic global search method which is very effective on constrained single-objective optimization problems with many locally optimal solutions
  - Hybrid of a stochastic search stage with a constrained nonlinear programming (NLP) local optimization stage
- MBH has been applied to a wide variety of optimization problems
  - Molecular structure (Leary 2000, Locatelli and Schoen 2003, Locatelli 2005)
  - Packing spheres in a box (Grosso *et al.* 2010)
  - Trajectory optimization (Vasile *et al.* 2008, Yam *et al.* 2010, Addis *et al.* 2011, Englander and Conway 2012, Ellison *et al.* 2013 and 2014, Englander *et al.* 2014)
- In this work we develop a new understanding of MBH for low-thrust mission design
- The results of this paper will also be applicable to other classes of problem

Introduction to Low-Thrust Mission Design

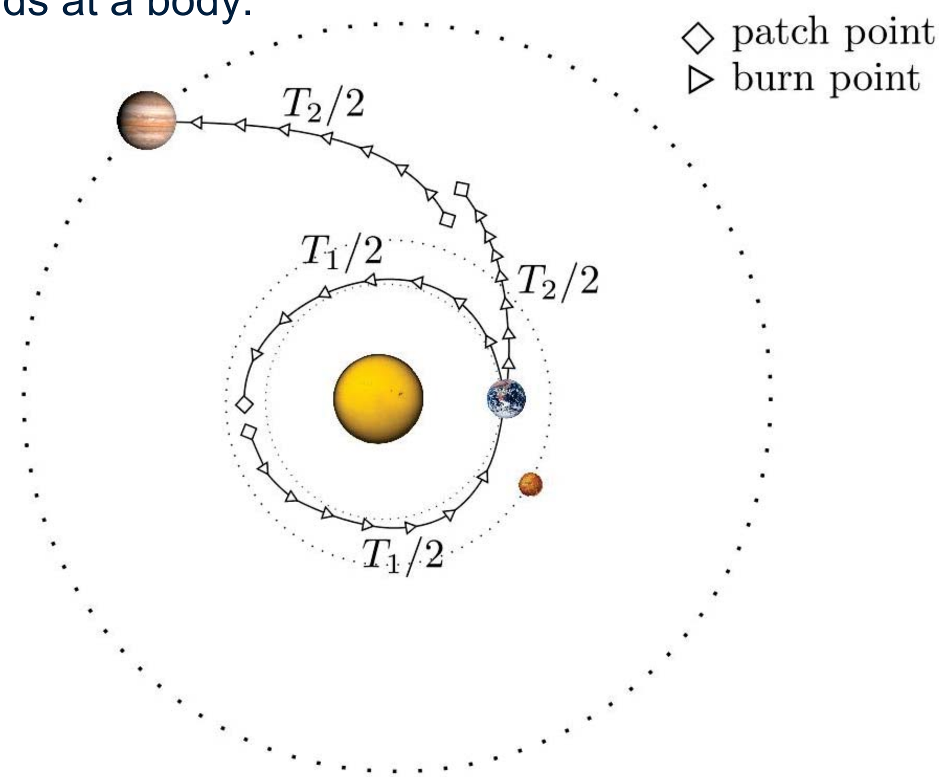
- Low-thrust electric propulsion enables access to difficult targets
  - Comets and asteroids
  - Mercury
  - Outer planets (with sufficient power supply)
- Low-thrust electric propulsion is characterized by high power requirements but also very high specific impulse (I_{sp}), leading to very good mass fractions
- Low-thrust trajectory design is a very different process from chemical trajectory design
  - Like chemical design, must find the optimal launch date, flight time, and dates of each flyby (if applicable)
  - Unlike chemical design, must find a time-history of thrust control for the entire mission
- It is desirable to automate the low-thrust design process as much as possible
- Computer time is CHEAP. Analyst time is EXPENSIVE

Stochastic Global Search in Low-Thrust Mission Design

- The design space of a low-thrust mission is highly complex
  - Hundreds of variables
  - Tens of constraints
  - Many locally optimal solutions
  - Space is too large to be evaluated in a grid search
- Best solution can be non-intuitive
  - Sometimes a reduced fidelity initial guess can be used, sometimes not
  - Changes in the problem assumptions (propulsion, flybys, etc) can significantly alter the problem
- An autonomous stochastic method, Monotonic Basin Hopping hybridized with a Nonlinear Programming-based local search, is very effective in exploring the problem space
- Since the method is autonomous, a single human designer can explore many variations of a mission simultaneously
- This method is implemented in Goddard's automated interplanetary low-thrust mission design tool, the Evolutionary Mission Trajectory Generator (EMTG)

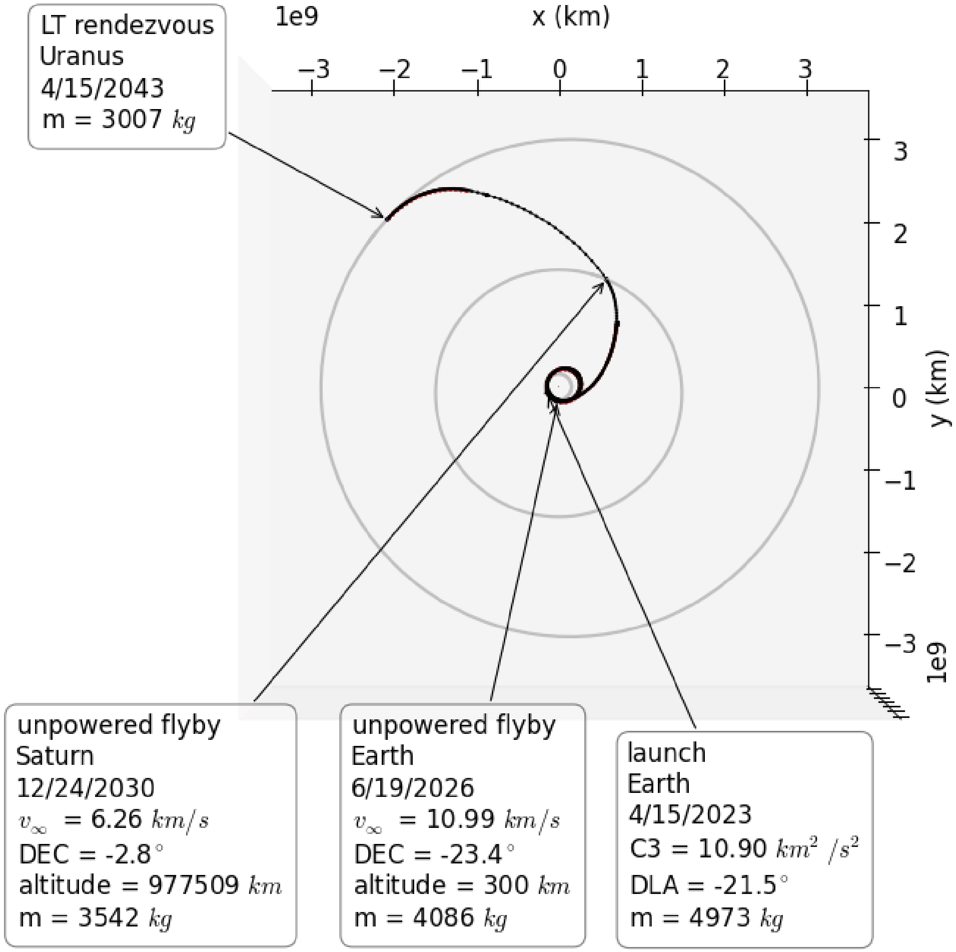
Multiple Gravity Assist with Low-Thrust (MGALT) via the Sims-Flanagan Transcription

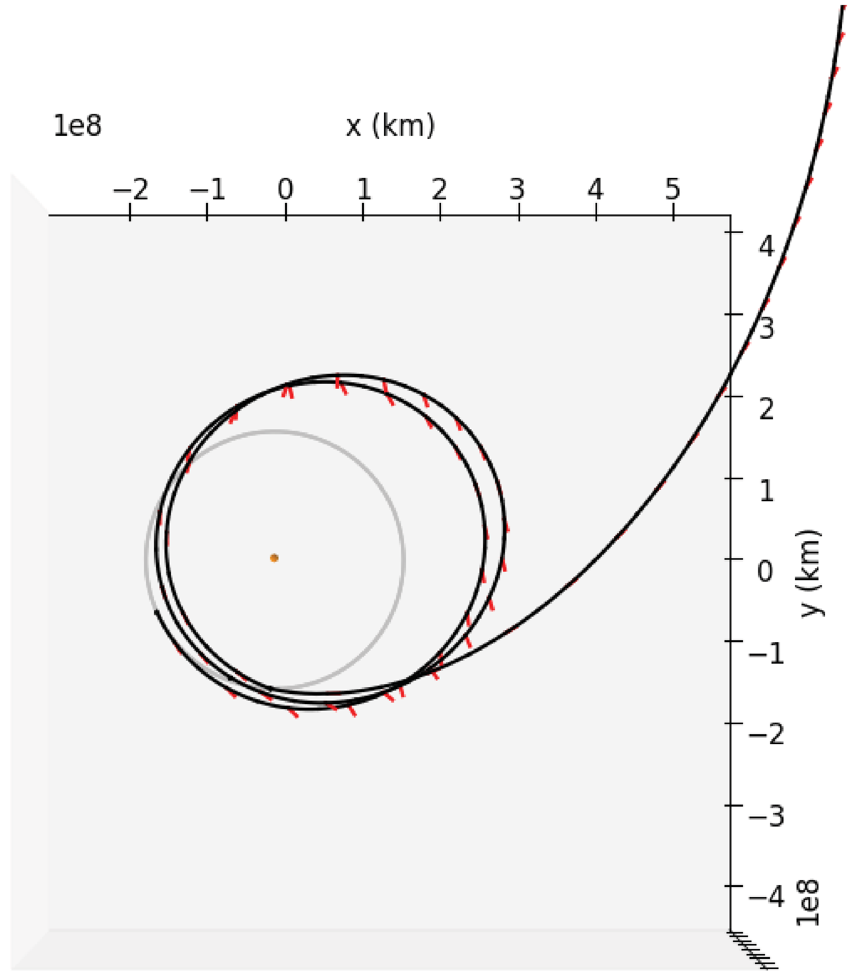
- Break mission into phases. Each phase starts and ends at a body.
- Sims-Flanagan Transcription
  - Break phases into time steps
  - Insert a small impulse in the center of each time step, with bounded magnitude
  - Optimizer Chooses:
    - Launch date
    - For each phase:
      - Initial velocity vector
      - Flight time
      - Thrust-impulse vector at each time step
      - Mass at the end of the phase
      - Terminal velocity vector



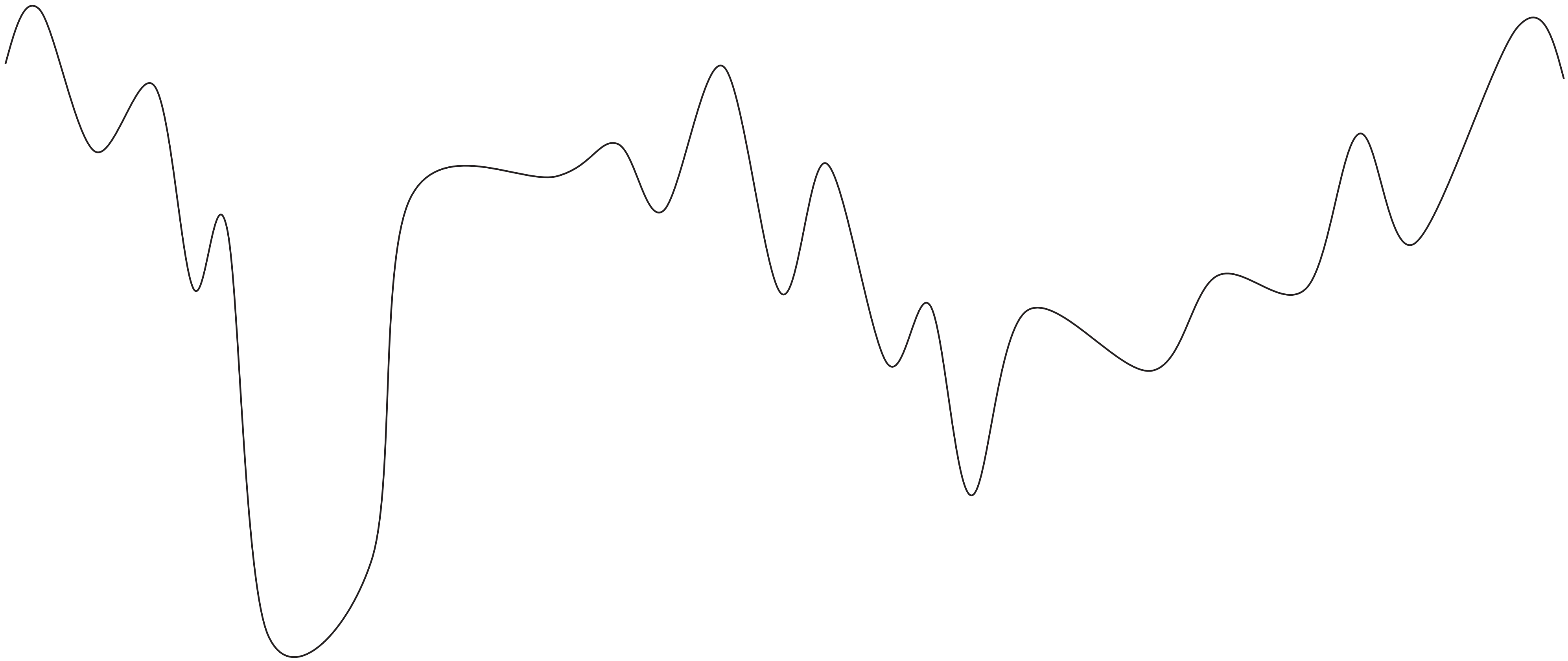
- Assume two-body force model; propagate by solving Kepler's problem
- Propagate forward and backward from phase endpoints to a "match point"
- Enforce nonlinear state continuity constraints at match point
- Enforce nonlinear velocity magnitude and altitude constraints at flyby

Test Problem: VSI Mission to Uranus





Monotonic Basin Hopping + SNOPT Trajectory Optimization



Tuning Monotonic Basin Hopping (MBH)

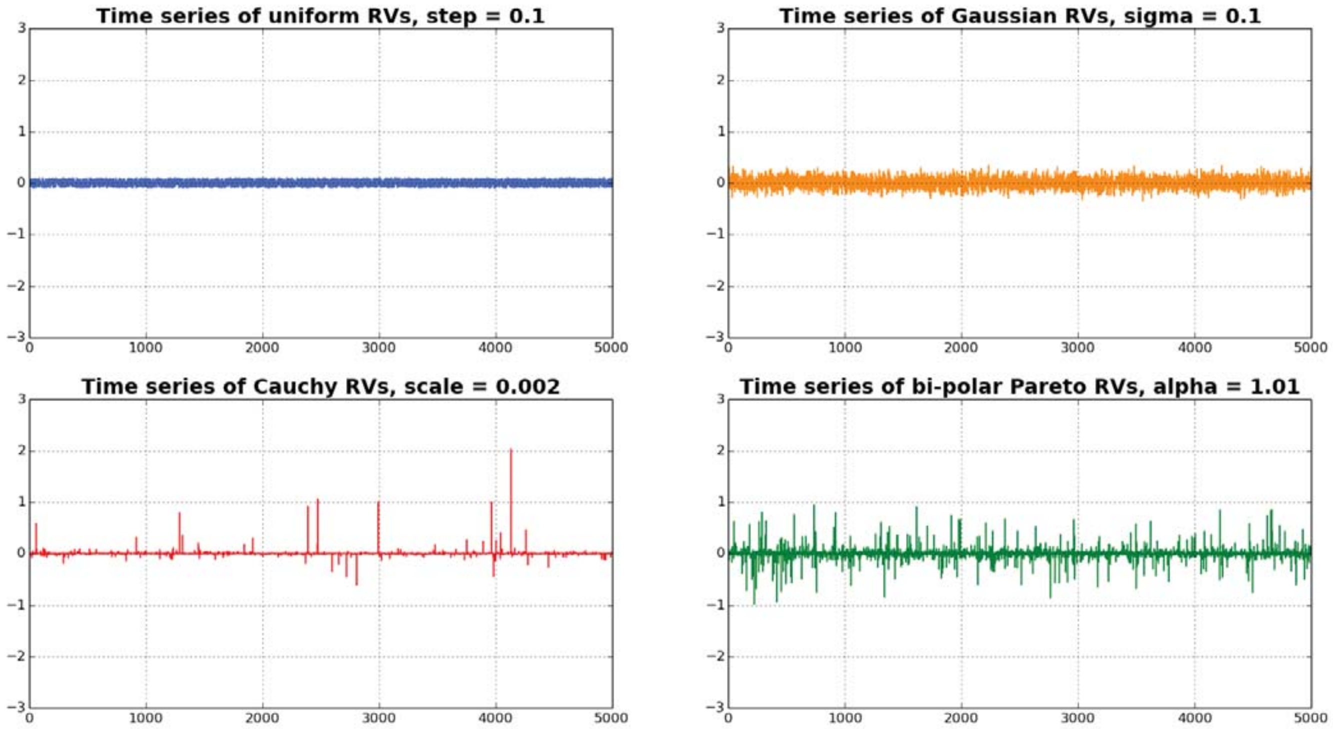
- We examined two components of classical MBH:
  - In classical MBH, random hops driven by a uniform probability distribution; hops can occur in a ball of some user-defined radius around current best point
  - There is a concept of “impatience” – a certain number of iterations where the solution does not improve, after which the algorithm resets
- In this work we consider:
  - Alternative probability distributions, especially Cauchy and Pareto, which hop with “long-tailed” probabilities, sometimes jumping “wildly”, getting “unstuck”
  - Given the above, that the concept of “impatience” may not be necessary when using alternative probability distributions
- Our objective was to find an alternative to classical MBH that would be:
  - Efficient (find better solutions in less time)
  - Robust (work well on highly constrained problems and not be sensitive to tuning parameters)

Probability Distributions and Their Tuning Parameters

Distribution	RV Generator	Excursion Parameter
Uniform	$2\rho(r - 0.5)$	ρ: ball size, impatience
Gaussian	$\frac{s}{\sigma\sqrt{2\pi}} e^{-\frac{r^2}{2\sigma^2}}$	σ: standard deviation
Cauchy	$\rho \tan\big(\pi(r - 0.5)\big)$	ρ: scale
Bi-polar Pareto	$\frac{s}{\epsilon} \frac{(\alpha - 1.0)}{\left(\frac{\epsilon}{\epsilon + r}\right)^{-\alpha}}$	α: "parameter"

$r = uniform(0.0,1.0)$, s is a fair coin flip, $\epsilon = 1.0 * 10^{-13}$

RV Generators

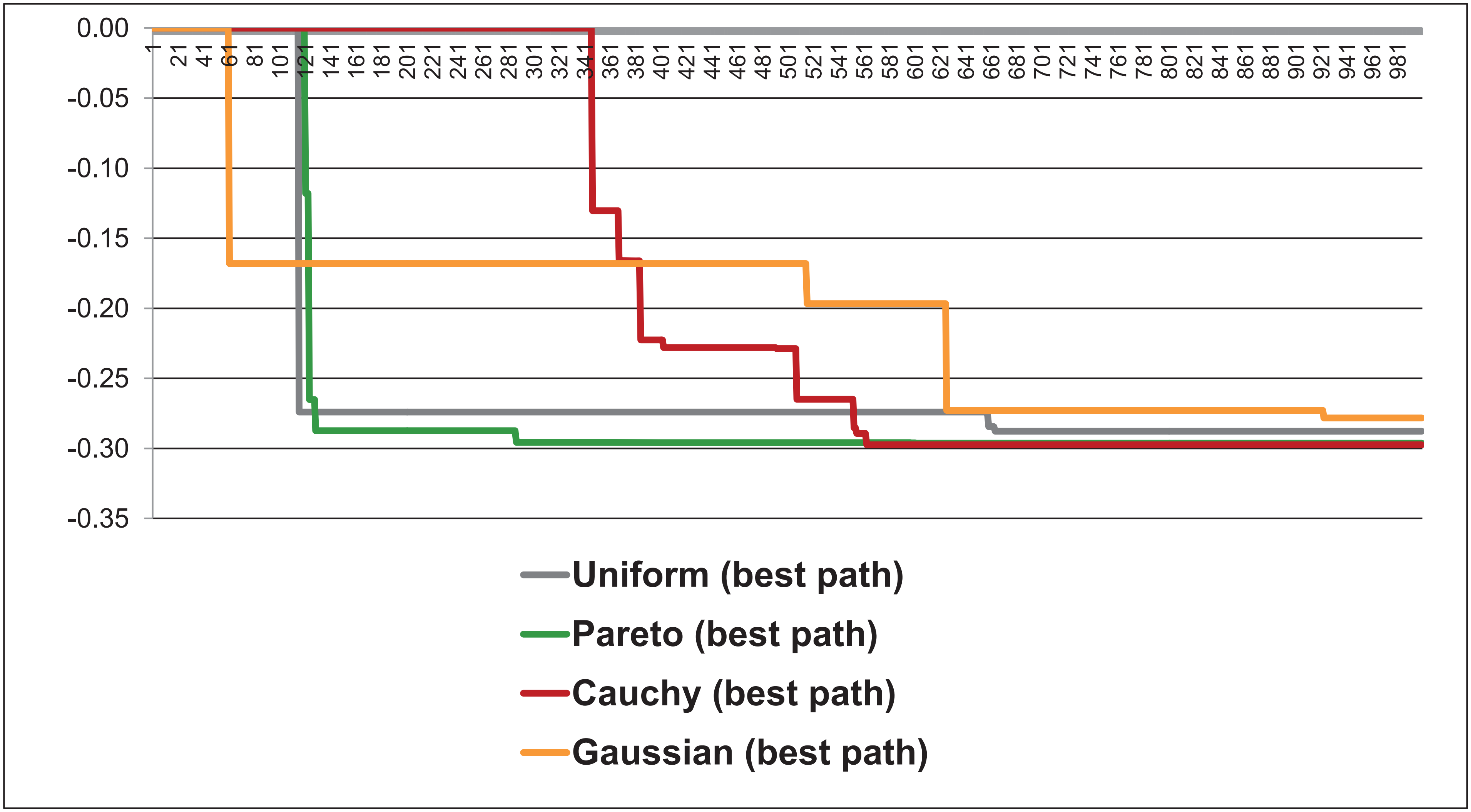


We want a distribution that not only takes lots of small steps to "exploit" the local region, but also takes frequent large steps to "explore" the rest of the space.

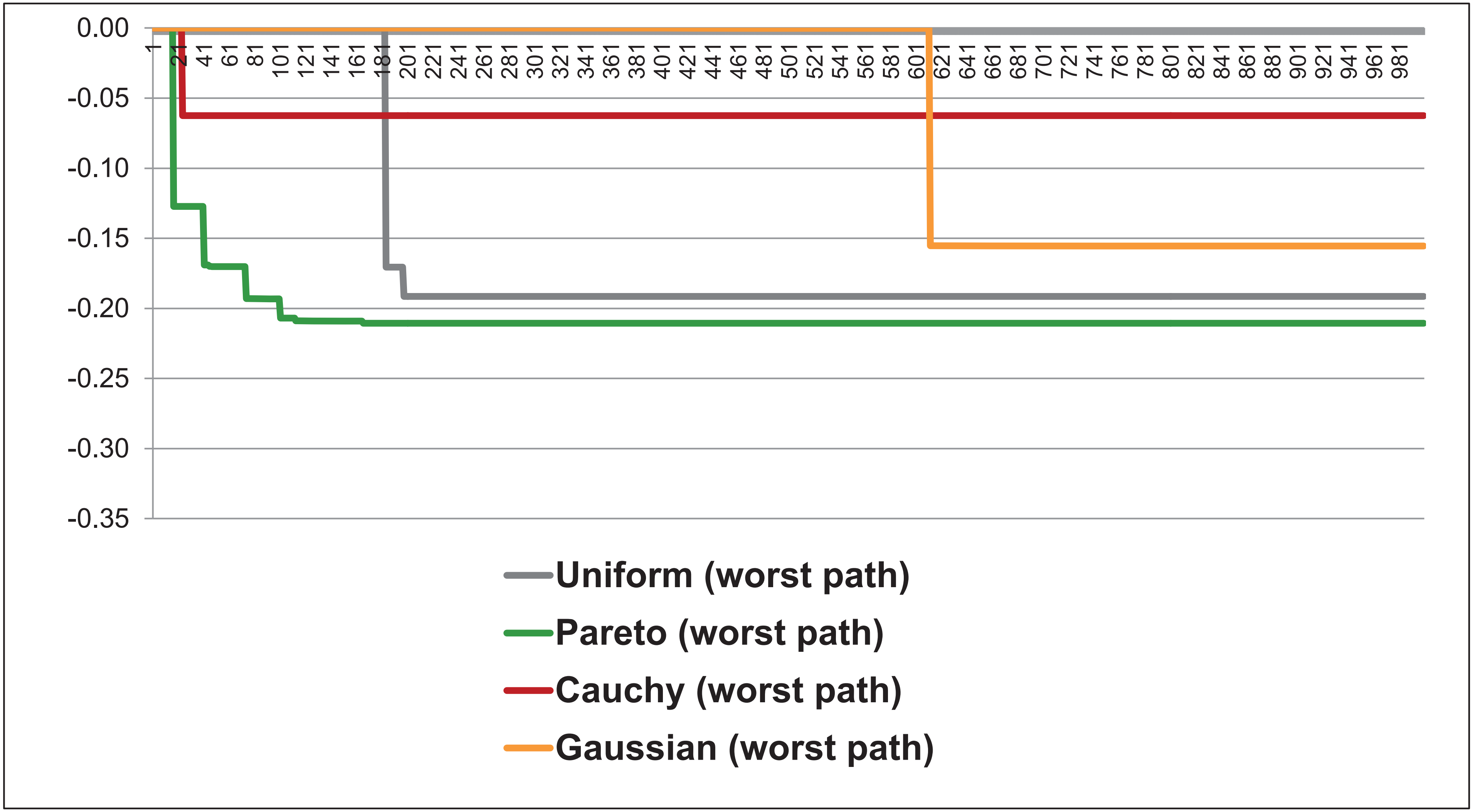
The Experiment

- 16 four-day (10000 step) runs of EMTG were conducted for each distribution
- Each of the 16 runs had a different value of the excursion parameter
- Impatience was turned off, i.e. MBH was never allowed to reset during the experiment
  - This was necessary to see how effectively each distribution could random-walk around the decision space
  - Our preliminary results suggest that, for non-classical MBH driven by long-tailed distributions such as Cauchy or bi-polar Pareto, resets and the concept of “impatience” are no longer needed

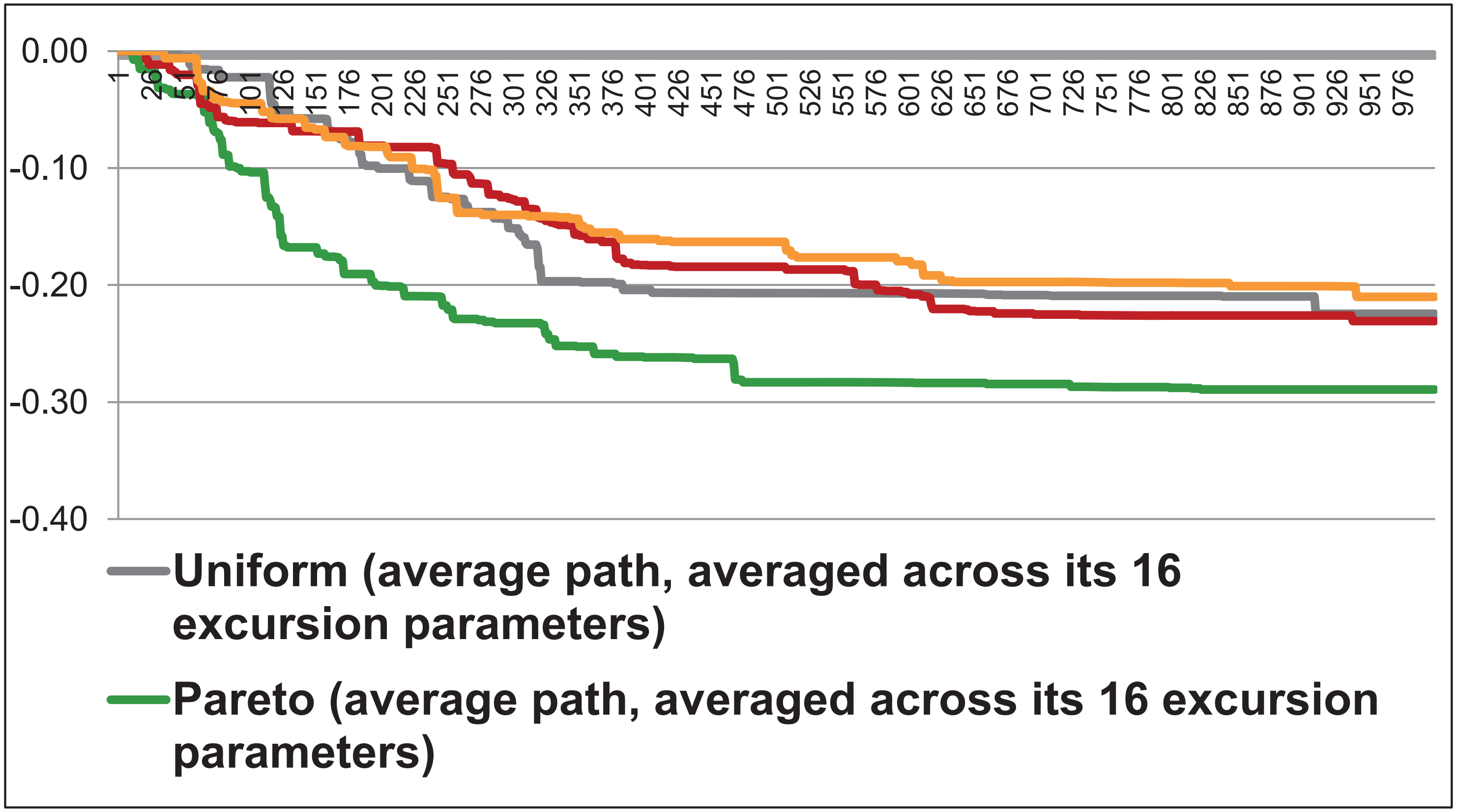
Results – Best Path



Results – Worst Path



Results – Average Performance



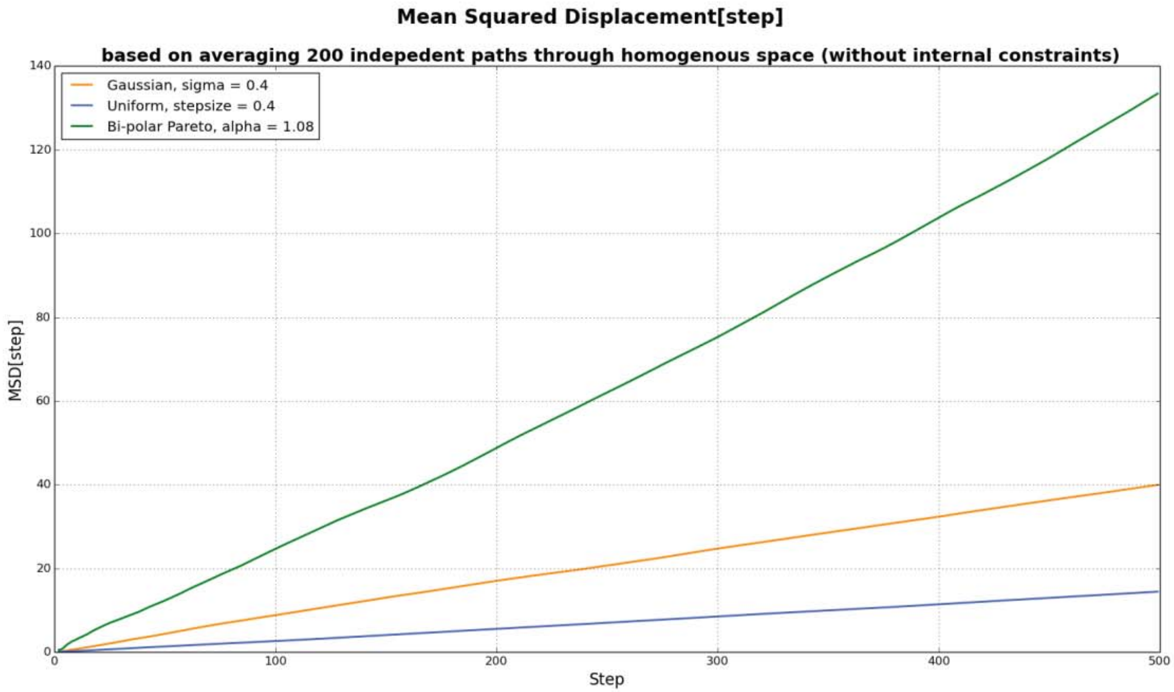
Bi-polar Pareto-driven MBH is most efficient (better solution in less time) and most robust (insensitive to tuning parameters)

Why?

- Random walks (RWs) can be compared in terms of their mean squared displacement (MSD)
  - A higher MSD means that a RW travels the problem space faster and more thoroughly than a RW that has a lower MSD
- MSD can be used to describe RWs as diffusions through media
- In diffusion through homogeneous media (i.e. unconstrained problem spaces), RWs driven by independent identically distributed (i.i.d.) distributions with finite variance are considered “normally diffusive”
  - MSD proportional to the number of steps
- RWs driven by i.i.d. distributions with infinite variance (or, in practice, with very long tails) are “super-diffusive”
  - MSD proportional to the number of steps raised to some power

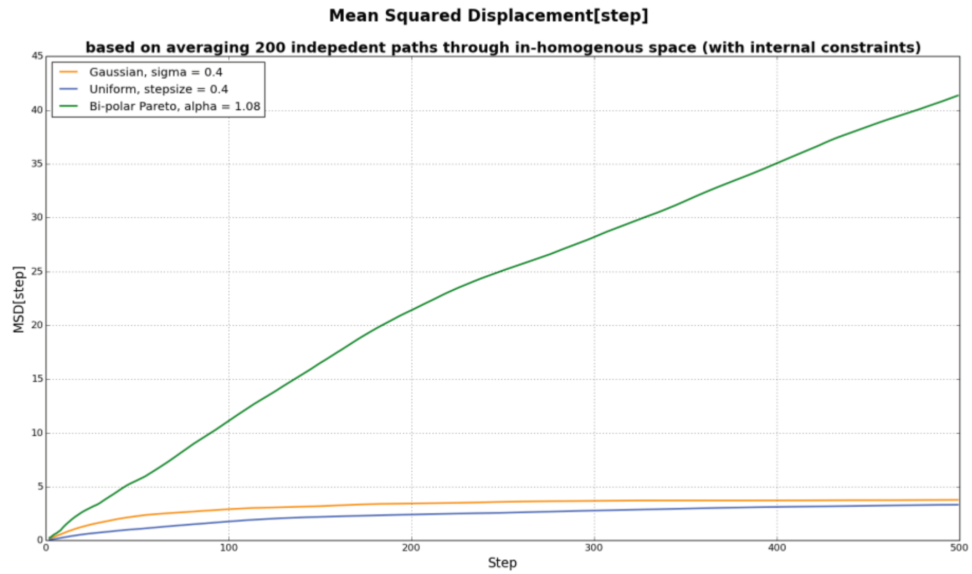
Why? Continued…

- In a simplified test problem the bi-polar Pareto RW is super-diffusive while the uniform and Gaussian RWs are normally diffusive
- It is difficult to plot MSD of the Cauchy RW on the same graph because Cauchy distributions do not have a mean



What about constraints?

- Constraints introduce serial negative auto-correlations
  - A constraint effectively restricts the RW from moving in a certain direction, i.e. into the constraint
- Stochastic global search in constrained problem spaces can be described as diffusions through in-homogenous media
- When constraints are added to the simplified test problem, the uniform and Gaussian RWs become sub-diffusive but the bi-polar Pareto distribution is still super-diffusive



Conclusions

- The purpose of this work was to find an alternative to classical MBH that is:
  - Efficient (better solutions in less time)
  - Robust (works well in constrained problems and is insensitive to change in the excursion parameter)
- We found that the bi-polar Pareto distribution meets our criteria as “efficient and robust” on a very challenging low-thrust trajectory optimization problem
- In order to explain this result, we compare stochastic global search in a constrained space to diffusion through in-homogenous media
- The method developed in this work has already proved useful in solving many low-thrust trajectory optimization problems
- We expect our results to be generalizable to other problems, both inside and outside the field of Astrodynamics
- The results of this investigation have been implemented in EMTG

Thank You

EMTG is available open-source at

https://sourceforge.net/projects/emtg/