Statistical Issues in Galaxy Cluster Cosmology

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Short Version

The number and growth of massive galaxy clusters are sensitive probes of cosmological structure formation. Surveys at various wavelengths can detect clusters to high redshift, but the fact that cluster mass is not directly observable complicates matters, requiring us to simultaneously constrain scaling relations of observable signals with mass. The problem can be cast as one of regression, in which the data set is truncated, the (cosmology-dependent) underlying population must be modeled, and strong, complex correlations between measurements often exist. [1,2]

Long Version

Simulations of cosmological structure formation provide a robust prediction for the number of clusters in the Universe as a function of mass and redshift (the mass function), but they cannot reliably predict the observables used to detect clusters in sky surveys (e.g. X-ray luminosity). Consequently, observers must constrain observable–mass scaling relations using additional data, and use the scaling relation model in conjunction with the mass function to predict the number of clusters as a function of redshift and luminosity.

Selection Bias: However, our census of clusters in the Universe is incomplete, and so the data available to constrain scaling relations necessarily represent a biased sample of the population being modeled (Malmquist bias). The effect of this selection bias is illustrated in Fig. 1, below.

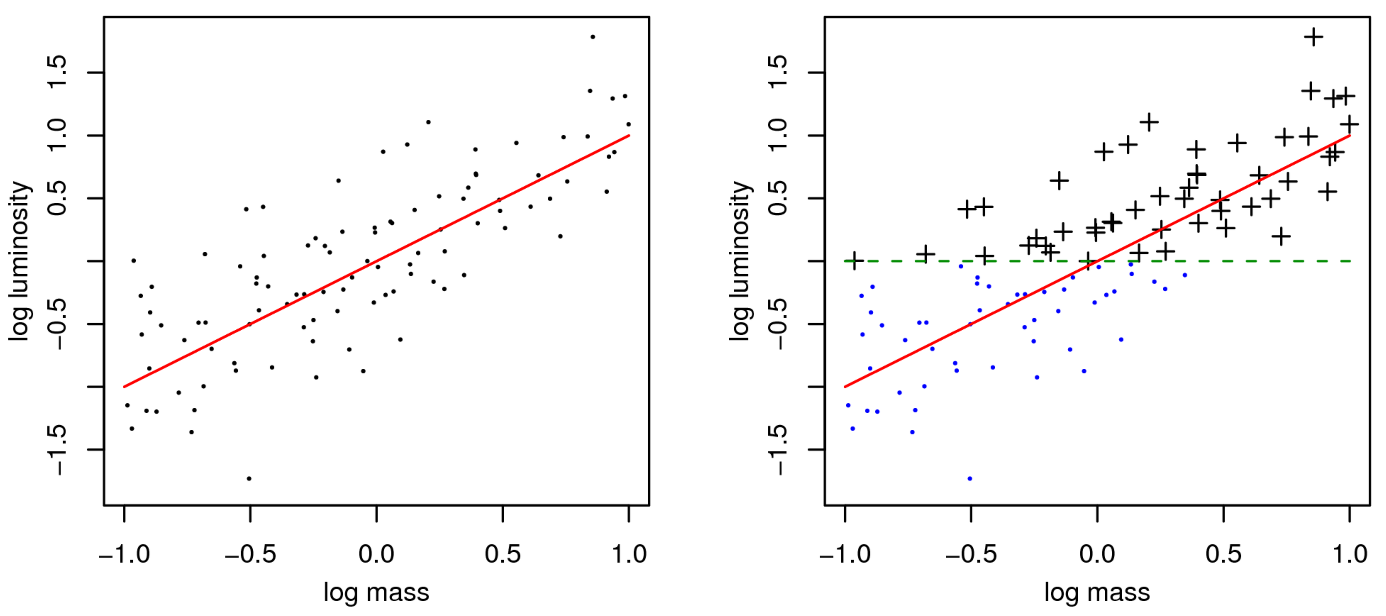

Fig. 1: Cartoon illustrating the effect of selection (Malmquist) bias. Left: represents the true values of luminosity and mass for all clusters in a fictional universe, which follow a power-law relation (red line) with a log-normal intrinsic scatter. Right: a very simple survey, requiring a constant threshold luminosity for detection (green, dashed line) results in the selection of clusters shown as black crosses, while undetected clusters appear as blue points. Even if perfect mass measurements can be made for all the detections, a naive fit to the data will not reproduce the true, underlying scaling relation.

Degeneracy with Cosmology: The distribution of cluster masses shown in Fig. 1 is unrealistic. In reality, the mass function decreases rapidly with mass in the cluster regime ($\propto M^{-3}$). The parameters governing this function are exactly the cosmological parameters that ultimately we would like to constrain. Yet, the properties of the mass function influence the available scaling relation data (Fig. 2), and so they must be accounted for even if cosmological constraints are not the goal of the study. In other words, the cosmological and scaling relation problems must be solved simultaneously.

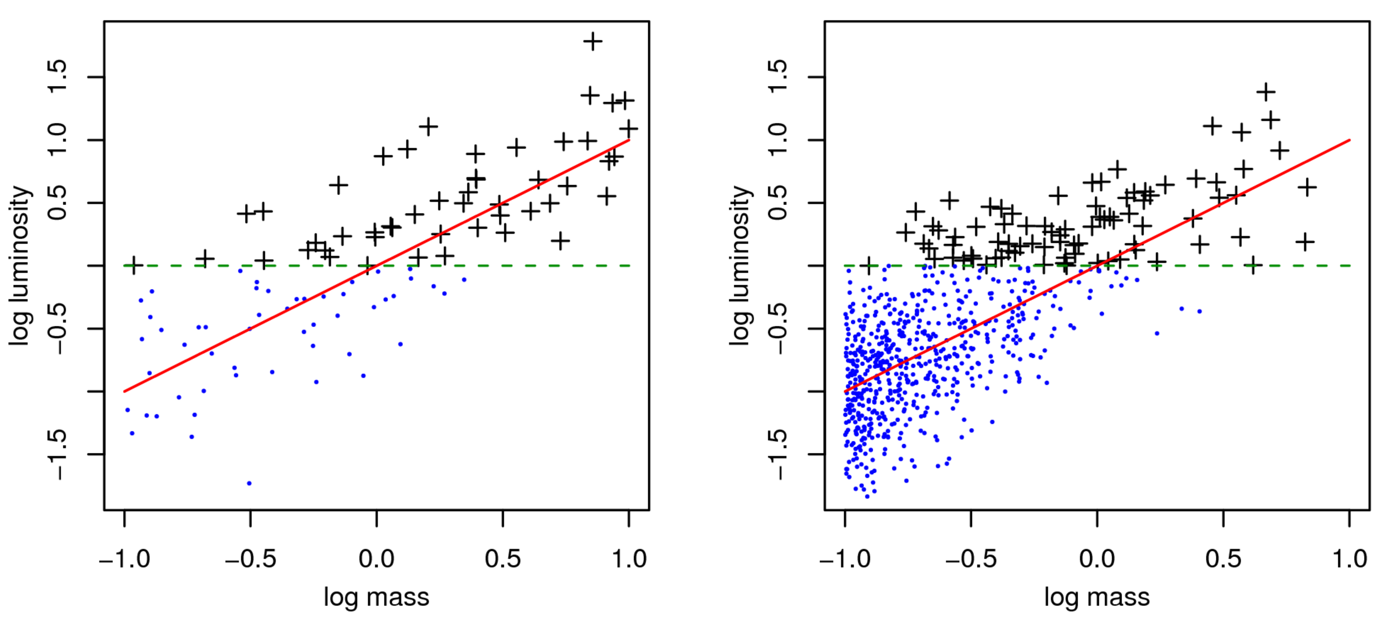

Fig 2: Illustration of the influence of cosmology (through the mass function) on scaling relation constraints. Comparing the left (unrealistic mass function) and right (more realistic, steeper mass function) panels, the number of heavily biased points at the low mass end is clearly sensitive to the underlying cosmology. (In the context of counting sources, this is called Eddington bias.) Evaluating the likelihood of these points requires information about the mass function, meaning that scaling relations cannot be constrained in isolation – the problem must be solved simultaneously with the cosmological analysis. For example, for the true scaling relation (shown) the lowest-mass detection is a several-sigma outlier from the mean. To correctly assign a (reasonable) likelihood to the scaling model given this data point, one needs to know the probability of such an outlier being realized (i.e. the number of clusters in the universe at that mass). In particular, simply conditioning the sampling distributions of luminosity on detection (truncating at the threshold), without considering the mass function, is insufficient.

Intrinsic Covariance: The issues above are clear when the scaling relation of interest involves the observable used to detect the clusters. However, even if the scaling relation involves a different observable, selection effects can be important. The size of the bias depends on the intrinsic covariance of the two observables at fixed mass. Fig. 3 illustrates this for the case of a temperature–mass relation using clusters selected on luminosity.

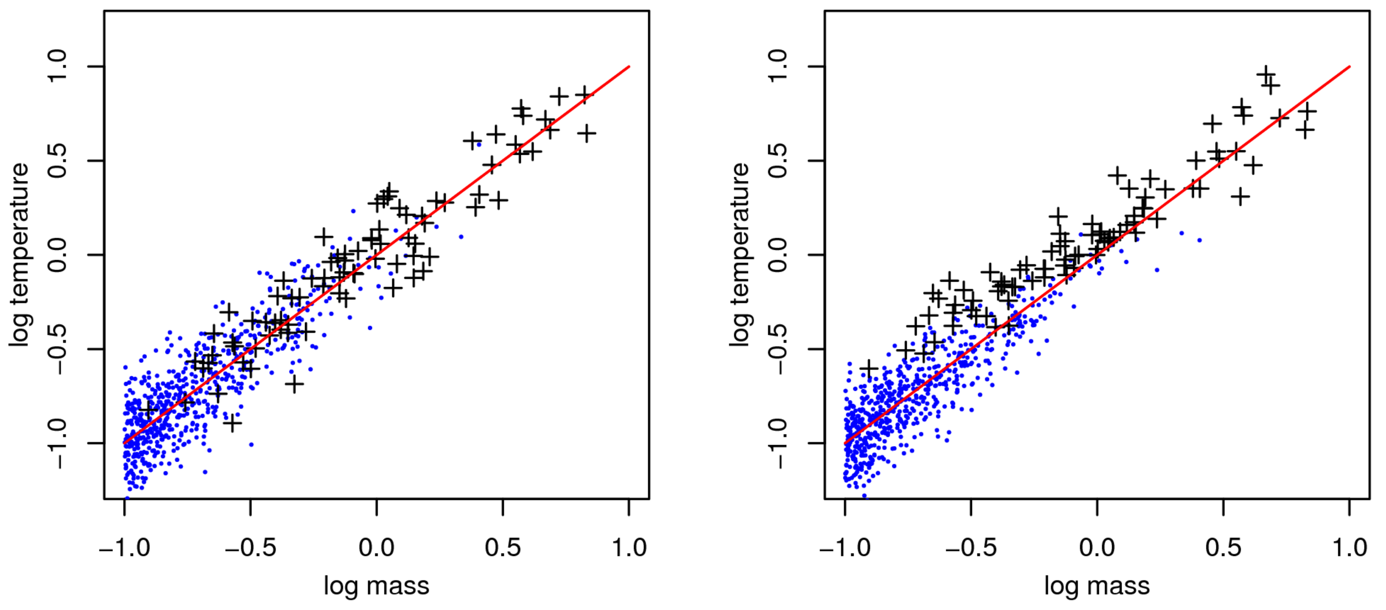

Fig 3: Cartoon temperature–mass data for the cartoon universe of Fig. 2b. Black crosses and blue points correspond to the detected and undetected clusters in that figure. The marginal intrinsic scatter for this relation is assumed to be smaller than for luminosity–mass, consistent with observations. Left: the correlation of the multivariate, log-normal intrinsic scatter of luminosity and temperature at fixed mass is set to 0.1. Here, the detected clusters represent the true scaling relation (red line) reasonably well. Right: the correlation is set to 0.9. For strong intrinsic correlation, the effect of selection on luminosity is evident in biasing the apparent slope of the observed data.

Measurement Covariance is generically present in scaling quantities measured from clusters. This is clear in the case where, for example, both luminosity and mass are measured from the same X-ray observation, since Poisson noise affects both quantities. More broadly, it occurs due to the conventional definition of a characteristic radius within which to measure the scaling quantities:

$$M_\Delta = \frac{4\pi\Delta}{3}\rho_{\rm crit} r_\Delta^3.$$

A solution for the cluster mass profile provides both M_Δ and r_Δ. However, quantities that are measured within r_Δ, even from completely independent observations, necessarily covary with M_Δ as a result. The correlation, and in general the detailed shape of the multivariate sampling distribution, must be taken into account (see also work by Brandon Kelly, this conference).

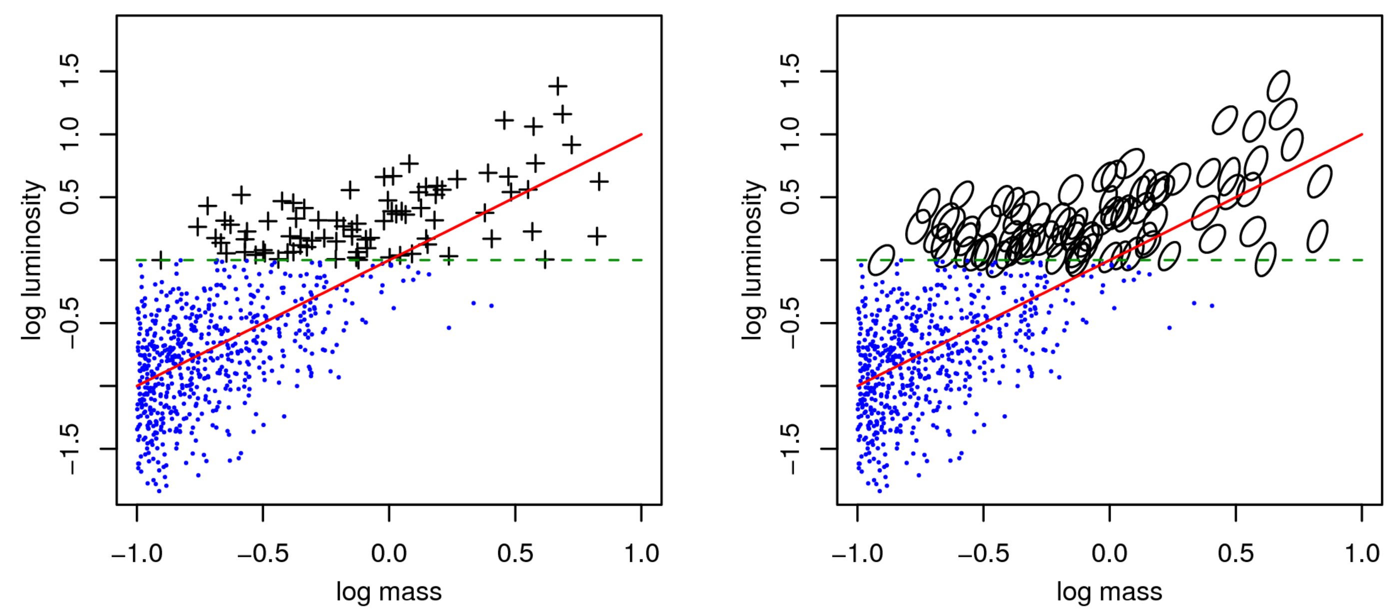

Fig. 4: Very often, measurement errors are not independent (left) but correlated (right), which can bias results if not accounted for.

In Practice: The procedure sketched above was applied to an X-ray flux limited cluster sample from the ROSAT All-Sky Survey, consisting of 238 detections at redshifts $0 < z < 0.5$. For 94 clusters, temperatures and mass estimates were incorporated, using archival X-ray data from pointed Chandra and ROSAT observations (Fig. 5). Our main results constrain cosmological models simultaneously with a multivariate (X-ray luminosity and temperature given mass), power-law scaling relation with log-normal scatter [1,3]. Confidence regions for the parameters of interest appear in Fig. 6.

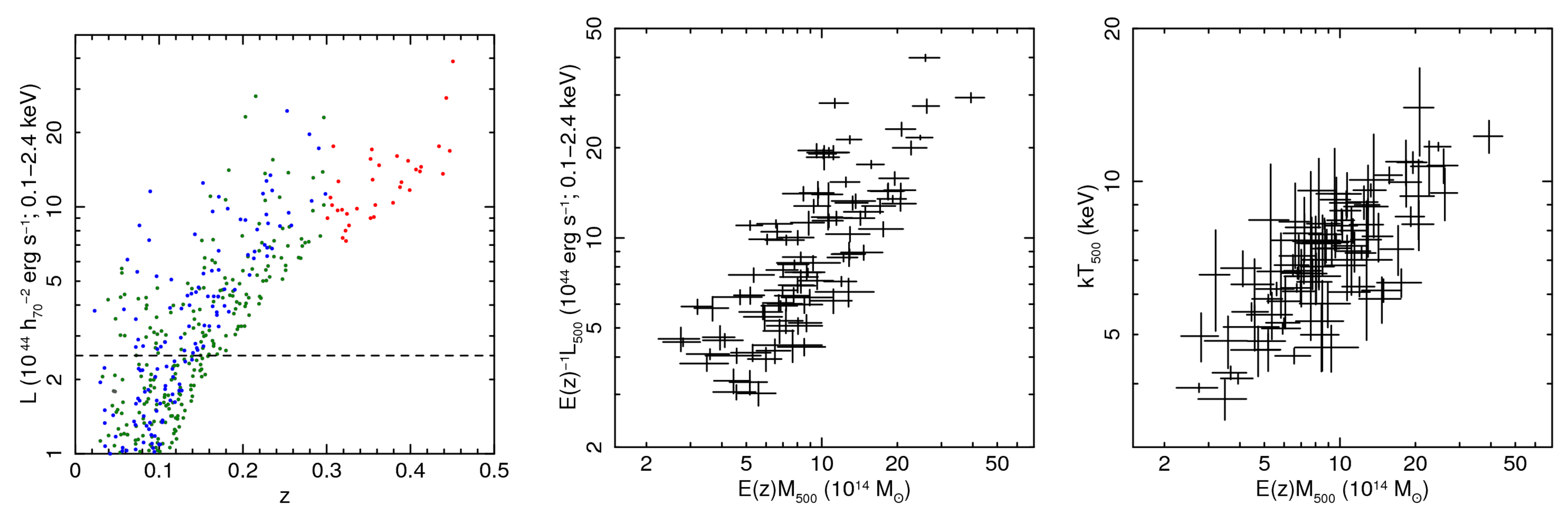

Fig 5: Cluster survey and follow-up data. Left: survey detections in luminosity–redshift space. Colors indicate different samples used to construct the data set. Selection required the threshold luminosity (dashed line) and flux limit for each sample to be met. Center: luminosities and masses from follow-up observations. The redshift dependent luminosity limit makes selection effects less obvious than in the cartoons above, though it is evident at the bottom-left of the panel. Right: temperature vs. mass from follow-up observations.

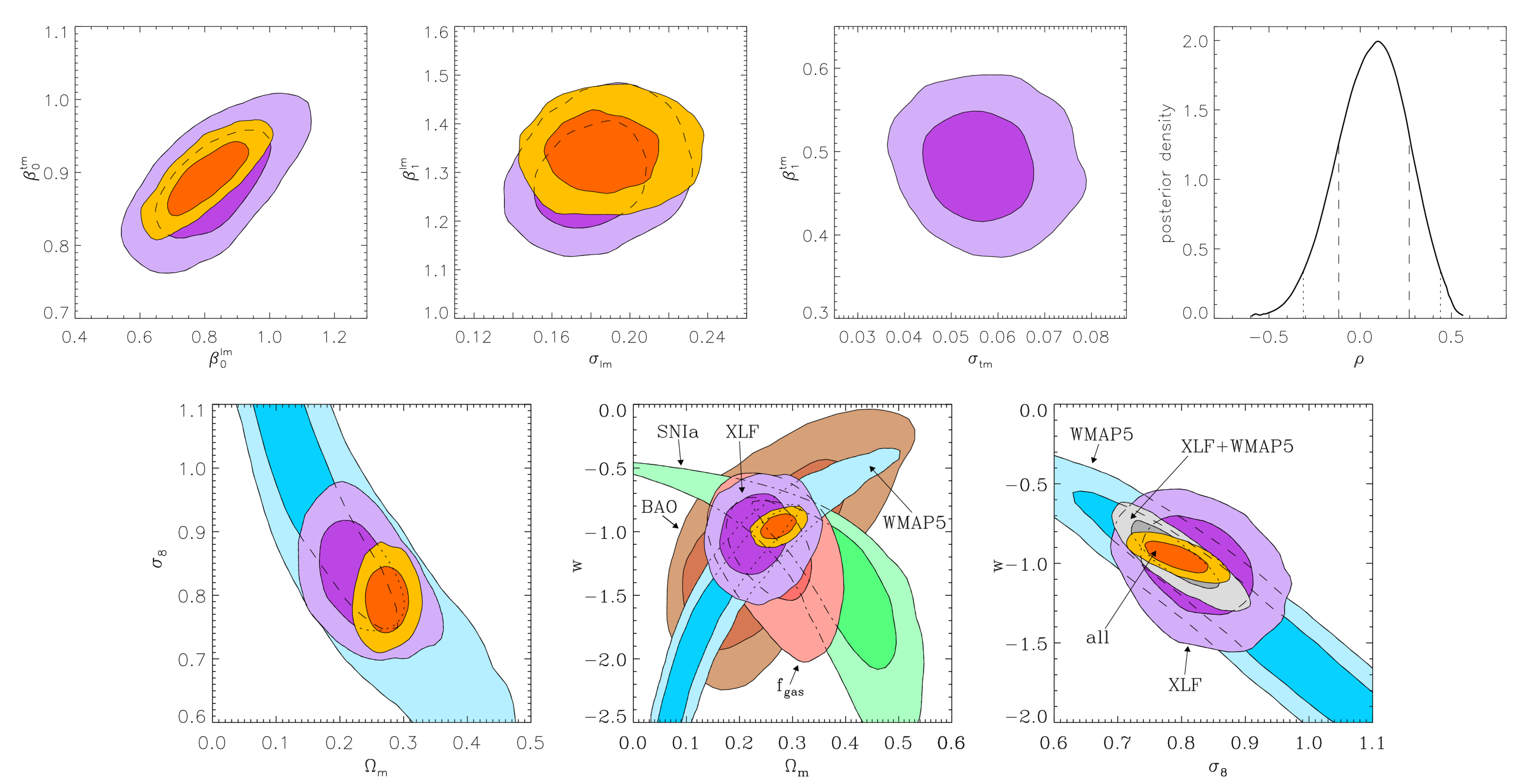

Fig 6: Confidence regions from the cluster growth analysis. Top: from left to right, constraints on the normalizations of the luminosity–mass and temperature–mass relations; the slope and marginal L–M intrinsic scatter; the slope and marginal T–M intrinsic scatter; and the intrinsic L–T scatter correlation. Bottom: cosmological constraints on $\Omega_{\rm m}$ (mean matter density), σ_8 (power spectrum amplitude) and w (dark energy equation of state). Results of the analysis described here are in purple and are labeled XLF for no good reason. Independent cosmological results are also show, and the combination of all these data appear in gold. The fact that adding external cosmological data has some effect on the L–M scaling relation (upper panels) reflects the degeneracy described above.

References: [1] A. Mantz, S. W. Allen, D. Rapetti, and H. Ebeling. MNRAS, 406, 1759, 2010 (arXiv:0909.3098).
[2] Allen S.W., Evrard A.E., Mantz A.B. 2011. *ARA&A*, in press (arXiv:1103.4829).
[3] A. Mantz, S. W. Allen, H. Ebeling, D. Rapetti, and A. Drlica-Wagner. MNRAS, 406, 1773, 2010 (arXiv:0909.3099).