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Computational Fluid Dynamics Uncertainty Analysis applied to Heat Transfer over a Flat Plate

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Problem

- Computational Fluid Dynamics (CFD) is being used without proper validation
- Experimental Data is expensive
  - Shrinking Budgets
- Pairing experimental data, uncertainty analysis, and analytical predictions provides a comprehensive approach to verification and is the current state of the art. (ASME V&V 20-2009)
- <u>A method is sought to conservatively envelop the exact solution using CFD only</u>
  - Example Heat Transfer over a flat plate is presented

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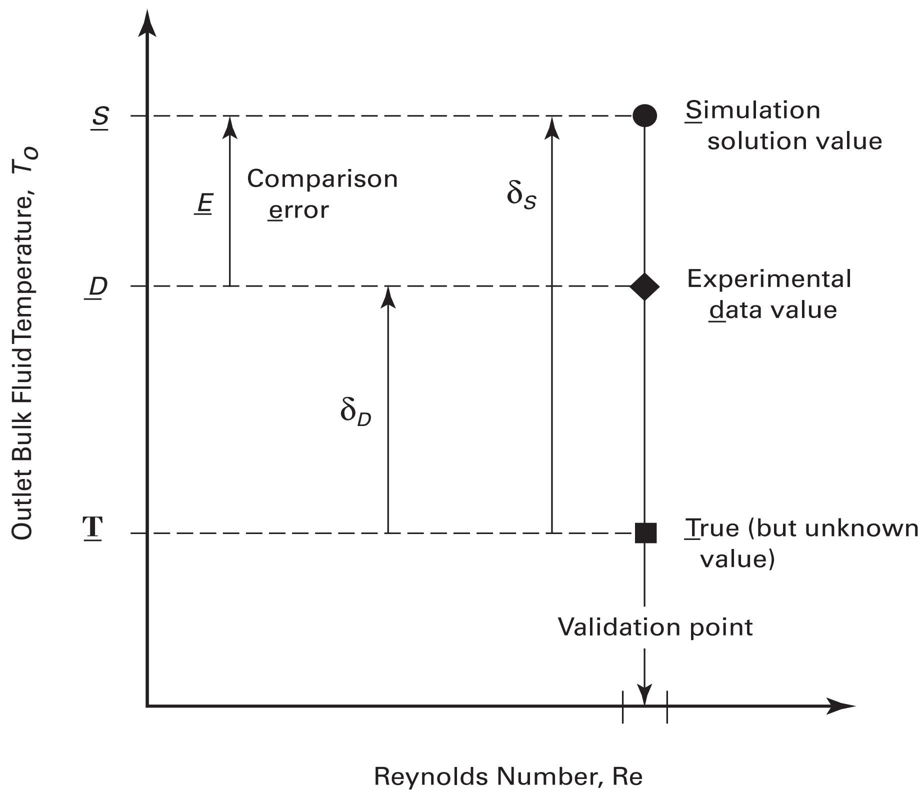
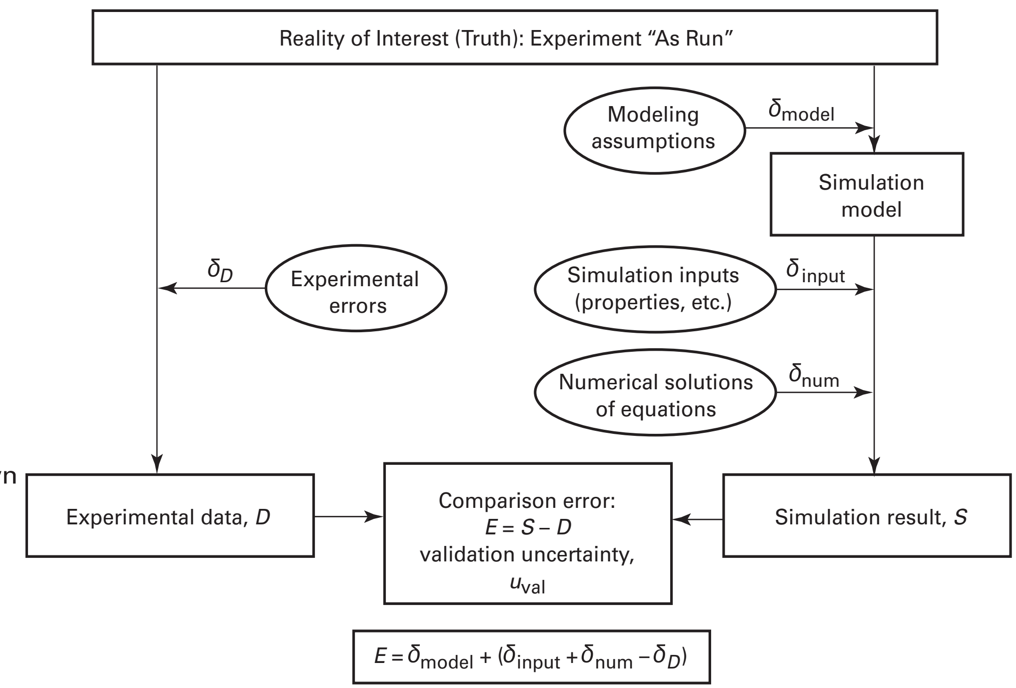
Summary ASME V&V 20-2009





$$\delta_{model}\ \varepsilon [E - u_{val}, E + u_{val}]$$

$$E = S - D$$

$$u_{val} = k\left(\sqrt{u_{num}{}^2 + u_{input}{}^2 + u_D{}^2}\right)$$

Without Experimental Data??

Proposed Methodology **conservative estimate to envelop true value

If there is no experimental data, D=0, δ_D=0, and u_D=0.

$$E = S - D = S$$

$$\delta s = S - T$$

$$E = S - D = T + \delta s - (T + \delta_D) = \delta_S - \delta_D = \delta_S$$

$$u_{val} = k\left(\sqrt{u_{num}{}^2 + u_{input}{}^2 + u_D{}^2}\right)$$

$$u_{val} = k\left(\sqrt{u_{num}{}^2 + u_{input}{}^2}\right)$$

Report the simulated result, S as

$$S \pm u_{val}$$

Without Experimental Data -continued

- Report $S \pm u_s$
- *k – value (Use Student-t Distribution)*
- Treat all input variables as ‘random’ and run separate CFD case
- Treat as an oscillatory convergence parameter

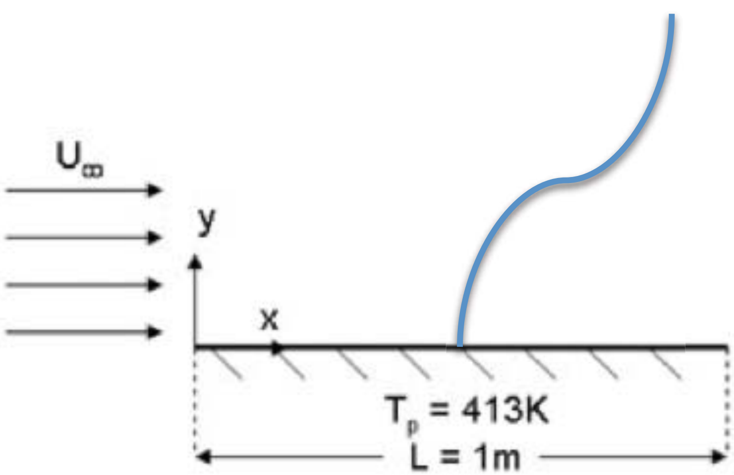
$$U_{Oscillatory} = \frac{1}{2}(S_U - S_L)$$

Number of Cases	Degrees of Freedom	Confidence 90%
2	1	6.314
3	2	2.92
4	3	2.353
5	4	2.132
6	5	2.015
7	6	1.943
8	7	1.895
9	8	1.86
10	9	1.833
11	10	1.812
12	11	1.796
13	12	1.782
14	13	1.771
15	14	1.761
16	15	1.753
17	16	1.746
18	17	1.74
19	18	1.734
20	19	1.729
21	20	1.725
22	21	1.721
23	22	1.717
24	23	1.714
25	24	1.711
26	25	1.708
27	26	1.706
28	27	1.703
29	28	1.701
30	29	1.699
31	30	1.697
41	40	1.684
51	50	1.676
61	60	1.671
81	80	1.664
101	100	1.66
121	120	1.658
infty	infty	1.645

Example Heat Transfer over Flat Plate

- Cornell University posted a Fluent Example Problem

$$h = c\left(\frac{\rho V L}{\mu}\right)^{4/5}\frac{k}{L}$$



U_∞ = 1 m/s

μ = 6.667e-7 kg/(m·s)

k = 9.4505e-4 W/(m·K)

C_p = 1006.43 J/(kg·K)

T_∞ = 353K

P_∞ = 101325 Pa

$Re_L = 1.5e6$ $\quad$ $Pr = 0.71$

<https://confluence.cornell.edu/display/SIMULATION/FLUENT+-+Forced+Convection+over+a+Flat+Plate>

Example Heat Transfer over Flat Plate

- Heat Transfer Correlations, Traditional
- The uncertainty analysis will follow the methodology laid out by Coleman and Steele (Experimentation and Uncertainty Analysis for Engineers, 2nd ed, J. Wiley and Sons, 1999). This methodology is in line with the ISO Guide to the Expression of Uncertainty in Measurement (1993).

$$U = \left(\underbrace{\sum_{i=1}^{J} \left\{ \left(\frac{\partial r}{\partial X_i} \right)^2 B_i^2 \right\}}_{\text{Bias}} + \underbrace{2 \sum_{i=1}^{J} \sum_{k=i+1}^{J} \left\{ \left(\frac{\partial r}{\partial X_i} \right) \left(\frac{\partial r}{\partial X_k} \right) [B_i B_k]_{correlated} \right\}}_{\text{Correlated}} + \underbrace{\sum_{i=1}^{J} \left\{ \left(\frac{\partial r}{\partial X_i} \right)^2 P_i^2 \right\}}_{\text{Random}} \right)^{1/2}$$

Bias Correlated Random

$$h = c \left(\frac{\rho V L}{\mu} \right)^{4/5} \frac{k}{L}$$

$$\frac{dh}{dV} = \frac{4ck\rho}{5\mu \left(\frac{LV\rho}{\mu} \right)^{\frac{1}{5}}} \qquad \frac{dh}{d\mu} = -\frac{4CVk\rho}{5\mu^2 \left(\frac{LV\rho}{\mu} \right)^{\frac{1}{5}}}$$

$$\frac{dh}{d\rho} = \frac{4ckV}{5\mu \left(\frac{LV\rho}{\mu} \right)^{\frac{1}{5}}} \qquad \frac{dh}{dL} = \frac{4CVk\rho}{5L\mu \left(\frac{LV\rho}{\mu} \right)^{\frac{1}{5}}} - \frac{Ck \left(\frac{LV\rho}{\mu} \right)^{4/5}}{L^2}$$

$$\frac{dh}{dk} = \frac{c}{L} \left(\frac{LV\rho}{\mu} \right)^{4/5} \qquad \frac{dh}{dC} = \frac{k}{L} \left(\frac{\rho V L}{\mu} \right)^{4/5}$$

Example Heat Transfer over Flat Plate

- Heat Transfer Correlation Uncertainty

$$U_h = \left(\left(\left(\frac{\partial h}{\partial V} \right)^2 B_V^2 \right) + \left(\left(\frac{\partial h}{\partial \rho} \right)^2 B_\rho^2 \right) + \left(\left(\frac{\partial h}{\partial k} \right)^2 B_k^2 \right) + \left(\left(\frac{\partial h}{\partial \mu} \right)^2 B_\mu^2 \right) + \left(\left(\frac{\partial h}{\partial L} \right)^2 B_L^2 \right) + \left(\left(\frac{\partial h}{\partial C} \right)^2 P_C^2 \right) + 2 \left(\frac{\partial h}{\partial \rho} \right) \left(\frac{\partial h}{\partial k} \right) B_\rho B_k + 2 \left(\frac{\partial h}{\partial \rho} \right) \left(\frac{\partial h}{\partial \mu} \right) B_\rho B_\mu + 2 \left(\frac{\partial h}{\partial k} \right) \left(\frac{\partial h}{\partial \mu} \right) B_k B_\mu \right)^{1/2}$$

Example Heat Transfer over Flat Plate

- Plug in Partial Derivatives

c	
Seban & Doughty	0.0236
Jakob	0.024
Sugawara	0.023
Fundamentals of Heat and Mass Transfer	0.0296
c middle	0.0263
c uncert (random)	0.0033

Variable	Bias
Velocity, V	3%
Density, rho	3%
Thermal Conductivity, k	3%
Viscosity, mu	3%

$$U_h = \Bigg(\left(\left(\frac{4ck\rho}{5\mu\left(\frac{LV\rho}{\mu}\right)^{\frac{1}{5}}} \right)^2 B_V^2 \right) + \left(\left(\frac{4ckV}{5\mu\left(\frac{LV\rho}{\mu}\right)^{\frac{1}{5}}} \right)^2 B_\rho^2 \right) + \left(\left(\frac{c}{L}\left(\frac{LV\rho}{\mu}\right)^{4/5} \right)^2 B_k^2 \right) +$$

$$\left(\left(\frac{\partial h}{\partial \mu} \right)^2 B_\mu^2 \right) + \left(\left(\frac{4CVk\rho}{5L\mu\left(\frac{LV\rho}{\mu}\right)^{\frac{1}{5}}} - \frac{Ck\left(\frac{LV\rho}{\mu}\right)^{4/5}}{L^2} \right)^2 B_L^2 \right) + \left(\left(\frac{k}{L}\left(\frac{\rho VL}{\mu}\right)^{4/5} \right)^2 P_C^2 \right) +$$

$$2\left(\frac{4ckV}{5\mu\left(\frac{LV\rho}{\mu}\right)^{\frac{1}{5}}} \right)\left(\frac{c}{L}\left(\frac{LV\rho}{\mu}\right)^{4/5} \right) B_\rho B_k + 2\left(\frac{4ckV}{5\mu\left(\frac{LV\rho}{\mu}\right)^{\frac{1}{5}}} \right)\left(-\frac{4CVk\rho}{5\mu^2\left(\frac{LV\rho}{\mu}\right)^{\frac{1}{5}}} \right) B_\rho B_\mu +$$

$$2\left(\frac{c}{L}\left(\frac{LV\rho}{\mu}\right)^{4/5} \right)\left(-\frac{4CVk\rho}{5\mu^2\left(\frac{LV\rho}{\mu}\right)^{\frac{1}{5}}} \right) B_k B_\mu \Bigg)^{1/2}$$

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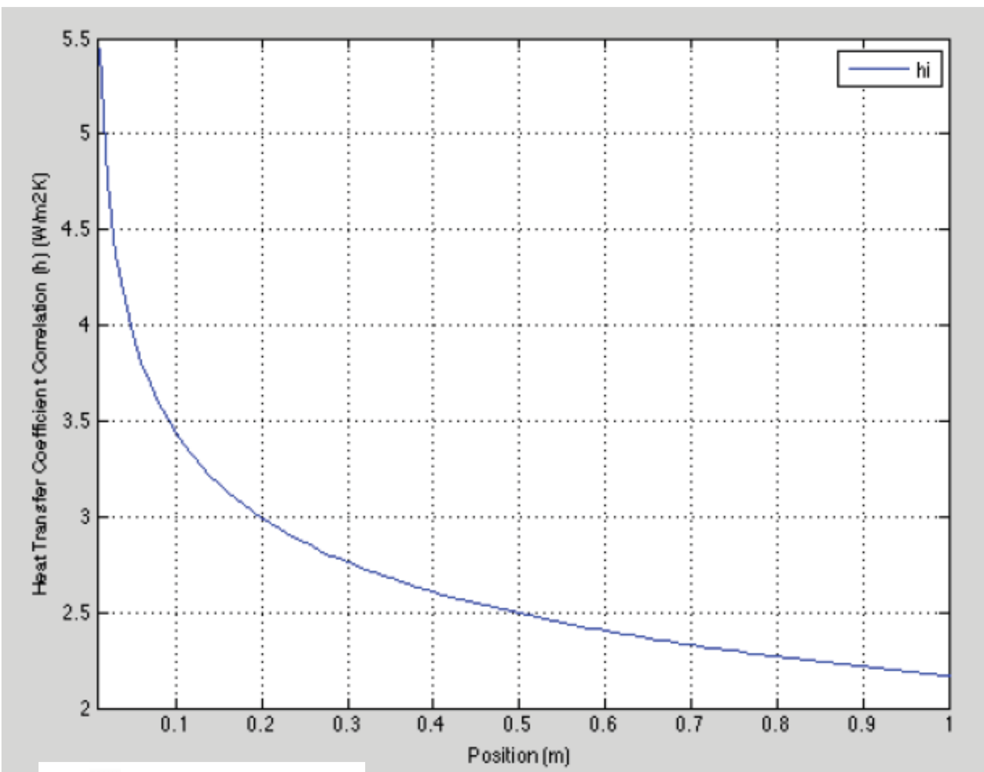
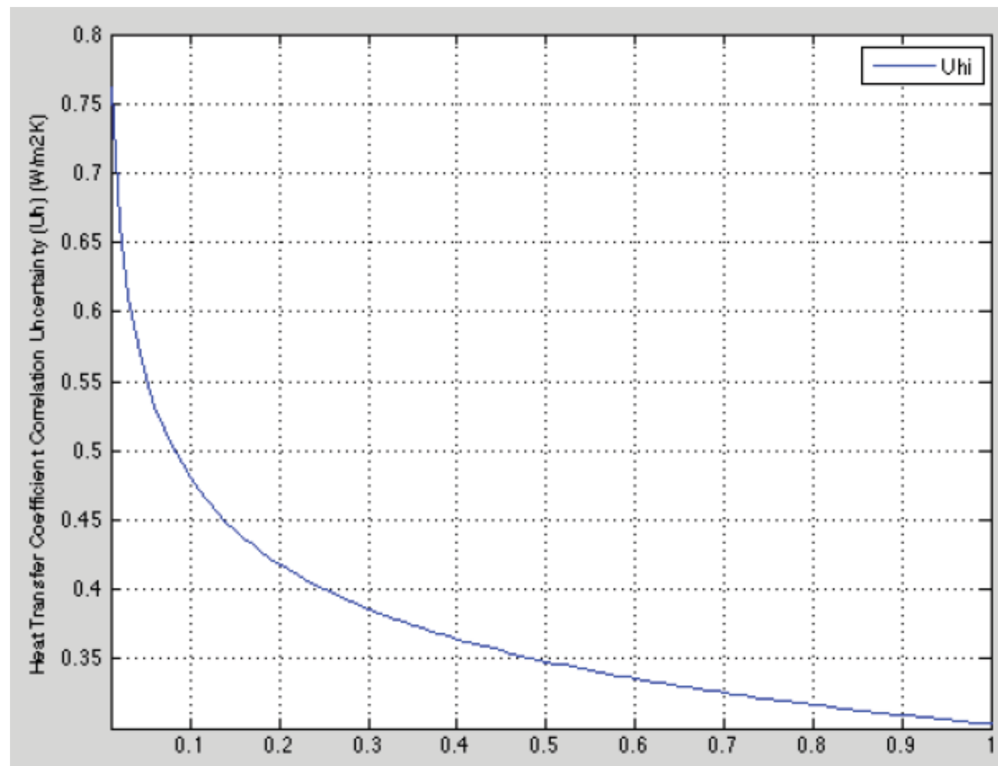
Example Heat Transfer over Flat Plate

- Numerically Evaluating (Traditional):

Heat Transfer Coefficient



Uncertainty in Heat Transfer Coefficient



Proposed Methodology using CFD Only

	CFD Uncertainty Cases
1	Coarse Grid
2	Medium Grid
3	Fine Grid
4	Velocity Low
5	Velocity High
6	Density Low
7	Density High
8	Thermal Conductivity High
9	Thermal Conductivity Low
10	Viscosity Low
11	Viscosity High
12	SA Turbulence Model
13	kwSST Turbulence Model

Number of Cases	Degrees of Freedom	Confidence 90%
2	1	6.314
3	2	2.92
4	3	2.353
5	4	2.132
6	5	2.015
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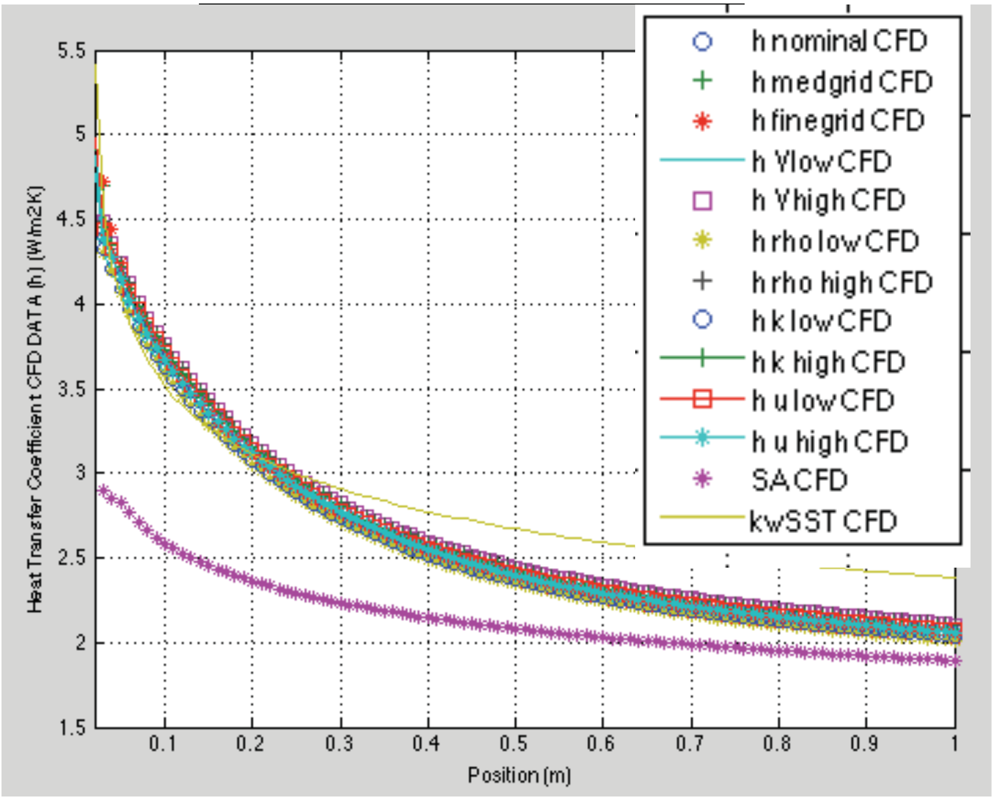
$$U_h = \left(\left(\left(\frac{\partial h}{\partial V}\right)^2 B_V^2 \right) + \left(\left(\frac{\partial h}{\partial \rho}\right)^2 B_\rho^2 \right) + \left(\left(\frac{\partial h}{\partial k}\right)^2 B_k^2 \right) + \left(\left(\frac{\partial h}{\partial \mu}\right)^2 B_\mu^2 \right) + \left(\left(\frac{\partial h}{\partial L}\right)^2 B_L^2 \right) + \left(\left(\frac{\partial h}{\partial C}\right)^2 P_C^2 \right) + 2\left(\frac{\partial h}{\partial \rho}\right)\left(\frac{\partial h}{\partial k}\right) B_\rho B_k + 2\left(\frac{\partial h}{\partial \rho}\right)\left(\frac{\partial h}{\partial \mu}\right) B_\rho B_\mu + 2\left(\frac{\partial h}{\partial k}\right)\left(\frac{\partial h}{\partial \mu}\right) B_k B_\mu \right)^{1/2}$$

$$u_{val} = 1.782 * \left|\frac{1}{2}(S_U - S_L)\right|$$

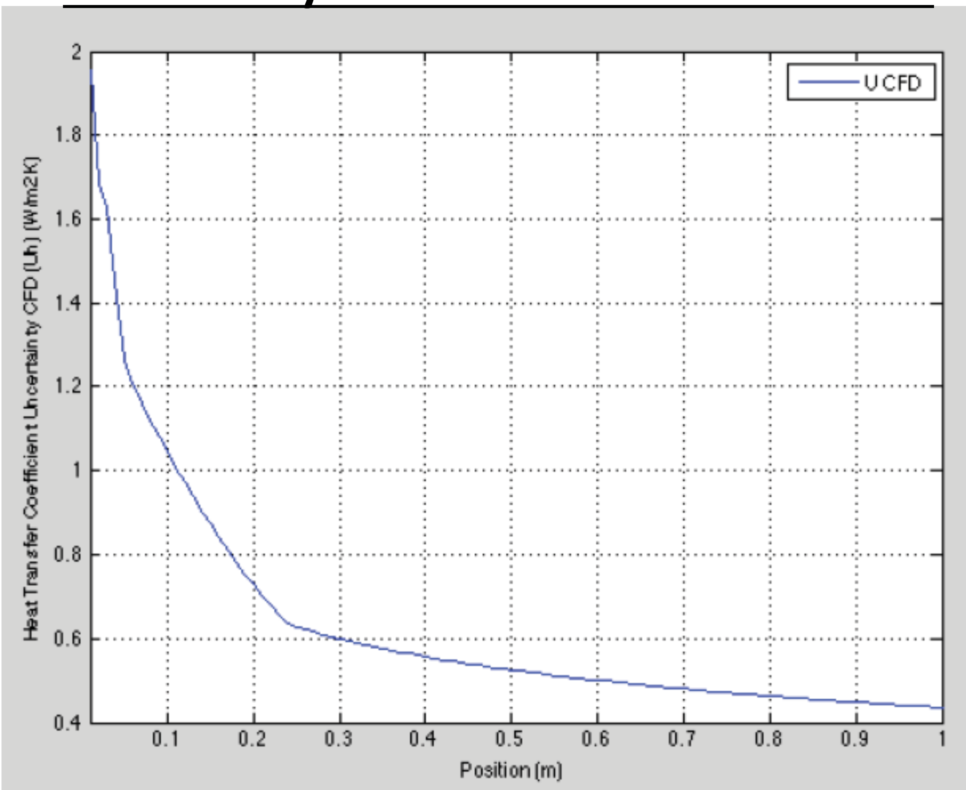
Proposed Methodology using CFD Only

- Results for proposed methodology

Heat Transfer Coefficient



Uncertainty in Heat Transfer Coefficient



	Average Difference in htc (W/m2K)	Ranking
Turbulence	0.693102673	1
Grid	0.130514851	2
Velocity	0.117431782	3
Density	0.117431683	4
k (thermal conductivity)	0.069466139	5
Viscosity	0.021837228	6

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Comparison of Methods

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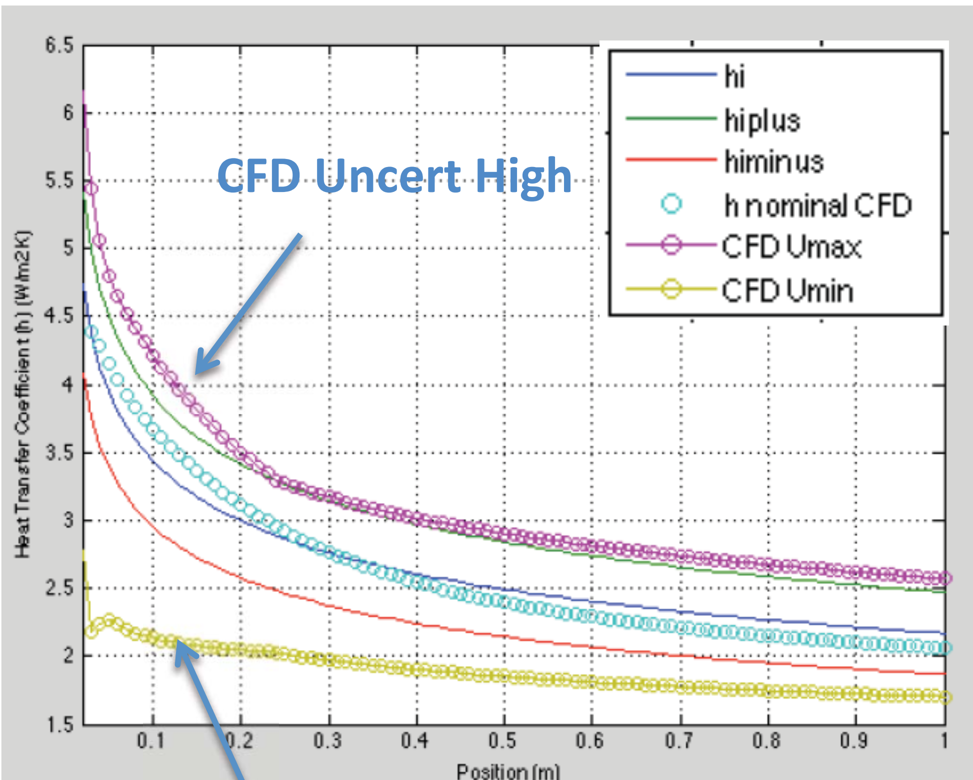
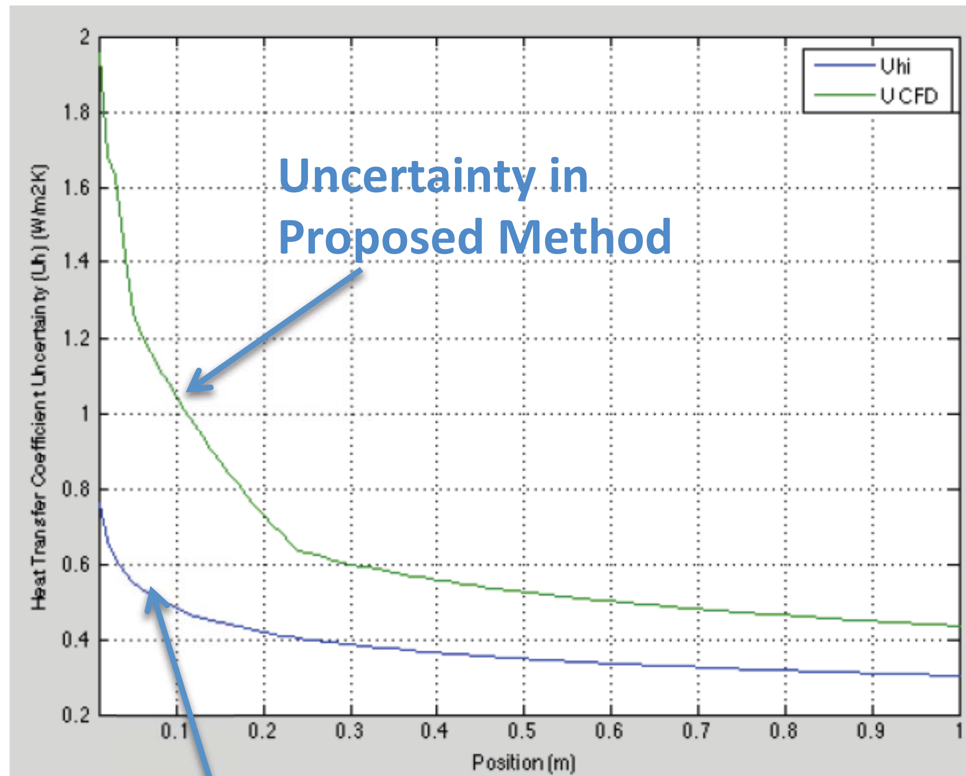
- Traditional vs. Proposed

Heat Transfer Coefficient



CFD Uncert Low

Uncertainty in Heat Transfer Coefficient



Uncertainty in
Traditional Method

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Comparison of Methods

- Traditional vs. Proposed Average Heat Transfer Coefficient over Flat Plate
  - Traditional

    h_{avg} = 2.66 +/- 0.74 [W/m2K]

  - Proposed CFD,

    h_{avg} = 2.73 +/- 1.39 [W/m2K]

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Conclusion

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- Proposed Method Envelops the True value and uses only CFD Data to Estimate the Uncertainty for Heat Transfer over a Flat Plate
  - No Testing

- Proposed methodology can be used to conservatively estimate the uncertainty in CFD models

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