Analytic couple modeling introducing device design factor, fin factor, thermal diffusivity factor, and inductance factor

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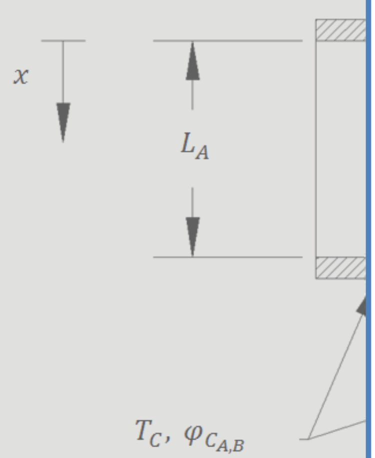
Classic Model



Thermal-

Electrical-

System-

Classic Parameters

Objectives

- Investigate couple configurations analytically:
  - Rectangular
  - Cylindrical
- Investigate additional physics from classic case:
  - Thermal resistance of shoe material
  - Lateral heat transfer
  - Variable material properties
  - Transient operation
- Establish a set of simple design guidelines, for lab couple demonstration purposes
  - Applicable to automotive, power, electronic, and other industries

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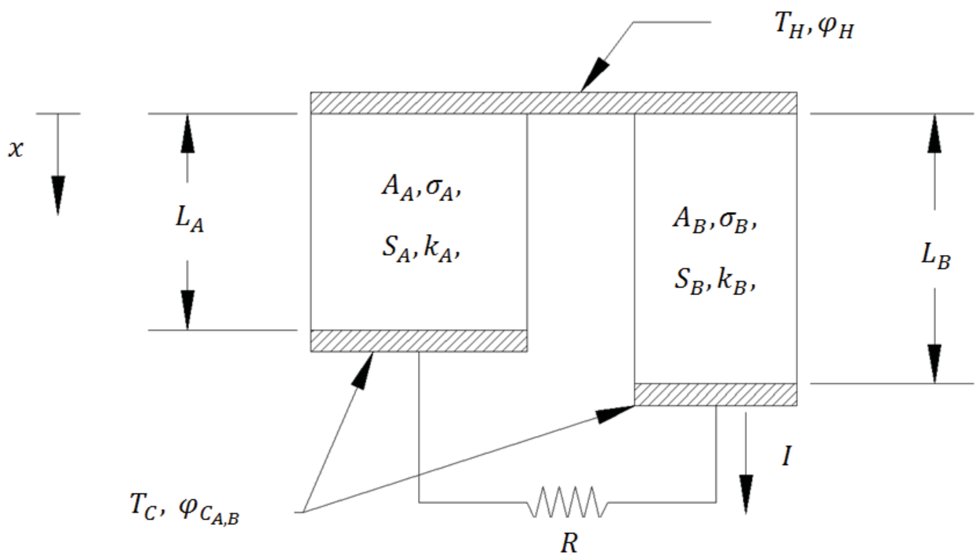

Thermal-

$$\frac{d}{dx}\left[-k_{A,B}\frac{dT_{A,B}}{dx}\right] + \frac{I_{A,B}\tau_{A,B}}{A_{A,B}}\frac{dT_{A,B}}{dx} - \frac{{I_{A,B}}^2}{{A_{A,B}}^2\sigma_{A,B}} = 0$$

Electrical-

$$\frac{d\varphi_{A,B}}{dx} = -S_{A,B}\frac{dT_{A,B}}{dx} - \frac{I_{A,B}}{A_{A,B}\sigma_{A,B}}$$

System-

$$\varphi_B(L_B) - \varphi_A(L_A) = IR$$

Classic Parameters

Geometric-

$$X = \frac{A_B L_A}{A_A L_B}$$

Load-

$$Y = \frac{R}{\frac{L_B}{\sigma_B A_B} + \frac{L_A}{\sigma_A A_A}}$$

Materials-

$$Z(X) = \frac{(S_B - S_A)^2}{\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X}\right)(k_A + k_B X)}$$

Classic Solution

$$\eta_{opt} = \frac{\eta_c Y_{opt}}{\frac{(1+Y_{opt})^2}{T_h Z(X_{opt})} + (1+Y_{opt}) - \frac{1}{2}\eta_c}$$

$$\mathrm{X_{opt}} = \sqrt{\frac{\mathrm{k_A}\sigma_\mathrm{A}}{\mathrm{k_B}\sigma_\mathrm{B}}}$$

$$\mathrm{Y_{opt}} = \sqrt{1 + \mathrm{Z(X_{opt})T_{avg}}}$$

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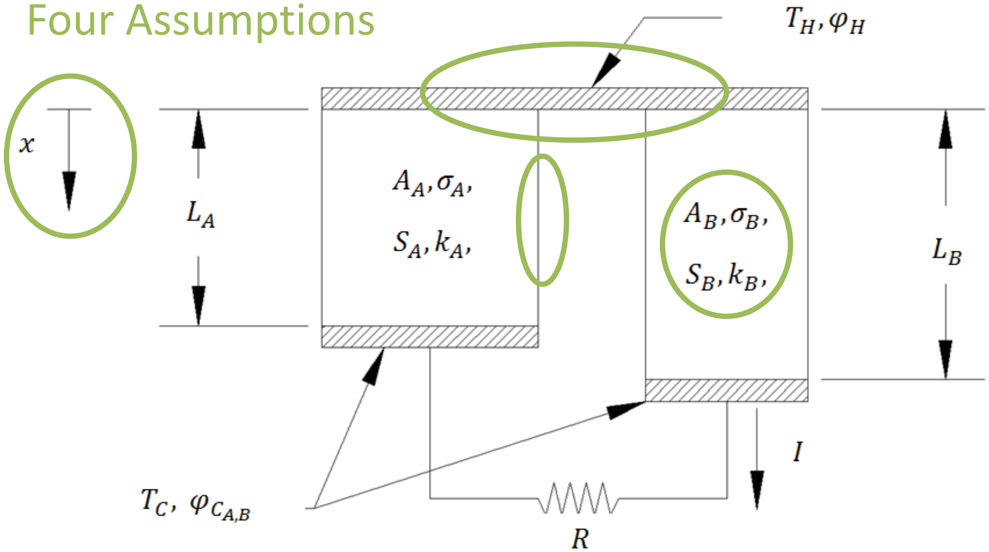
Classic Model

Four Assumptions



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$$\frac{d\varphi_{A,B}}{dx}=-S_{A,B}\frac{dT_{A,B}}{dx}-\frac{I_{A,B}}{A_{A,B}\sigma_{A,B}}$$

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$$\varphi_B(L_B)-\varphi_A(L_A)=IR$$

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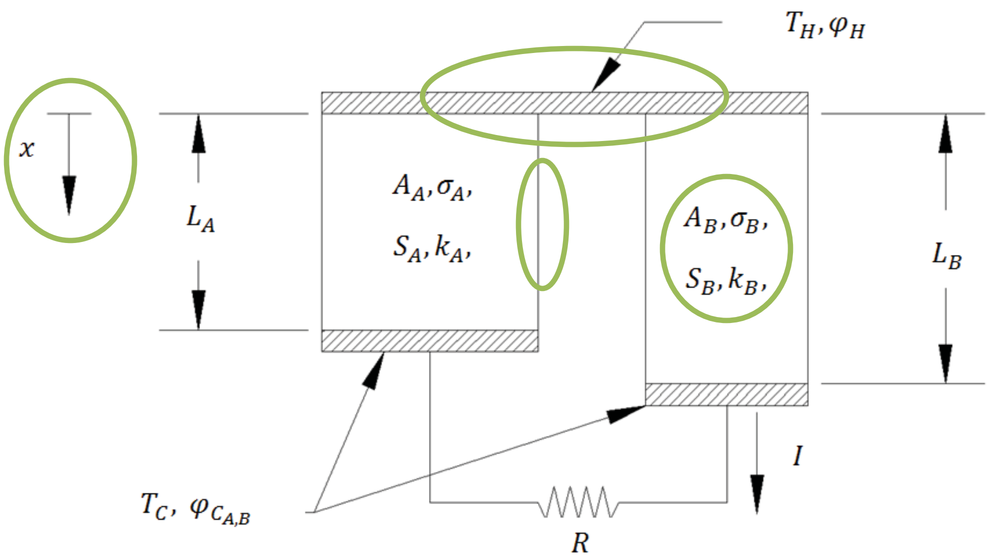

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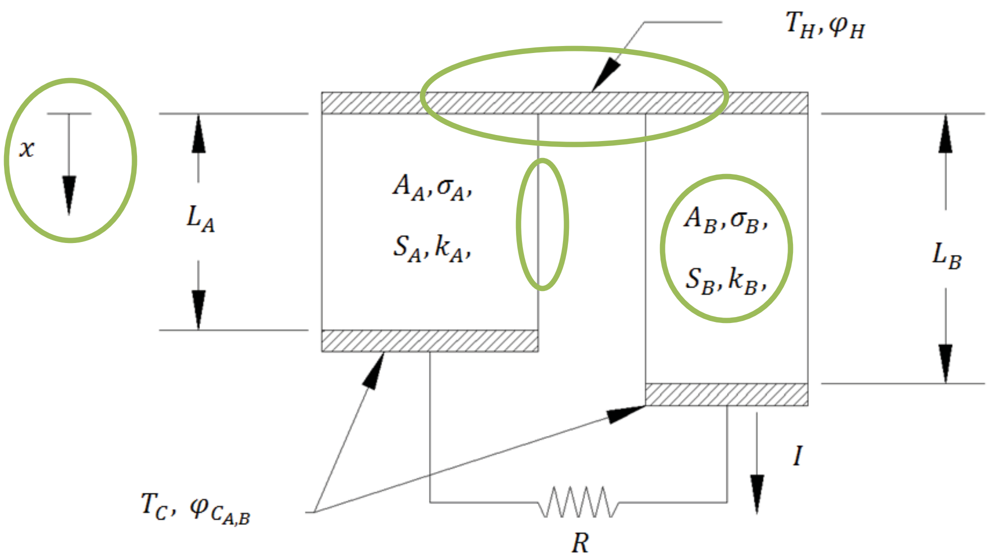

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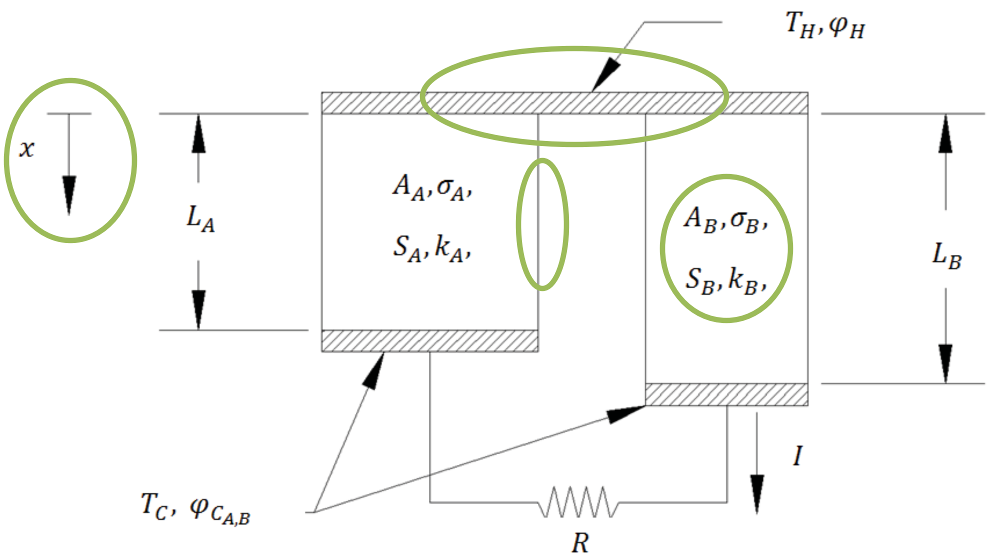

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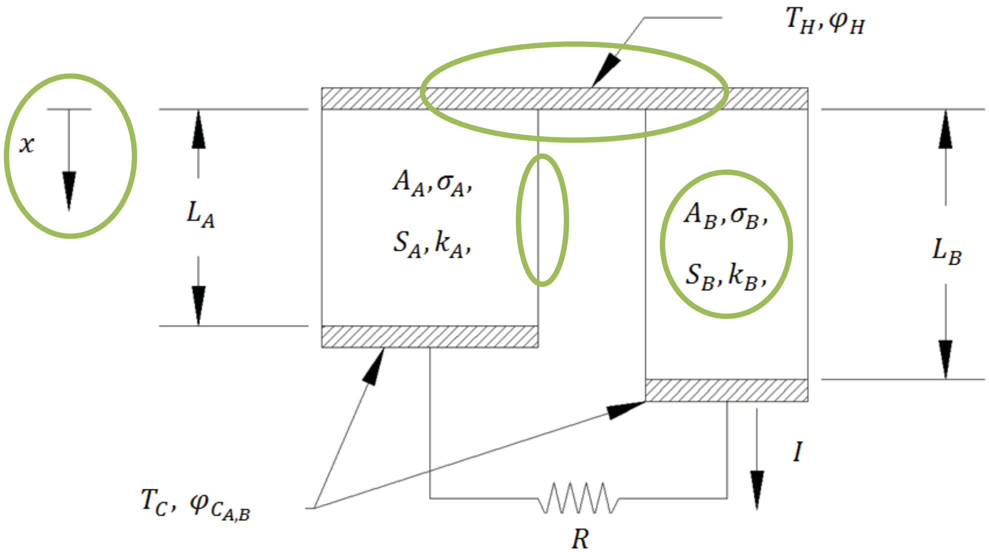

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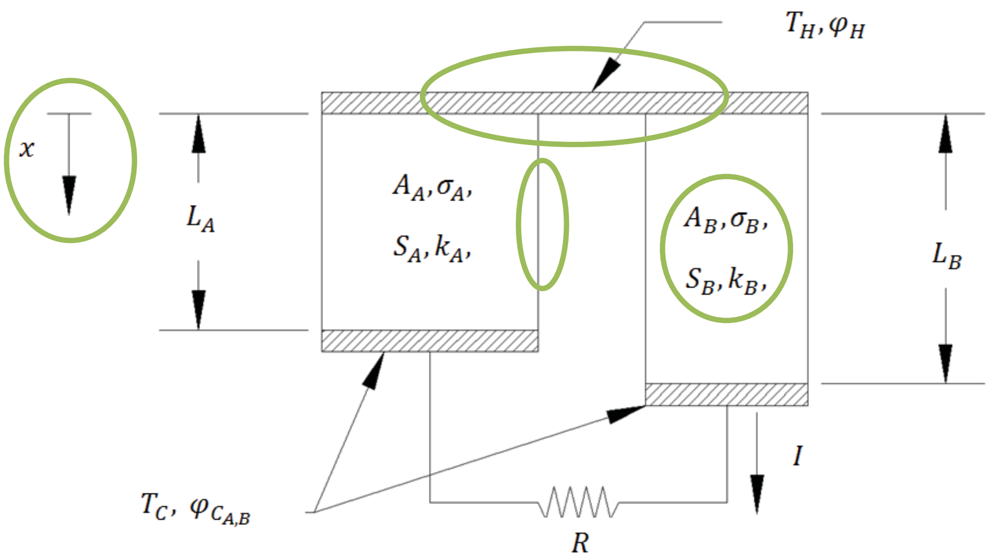

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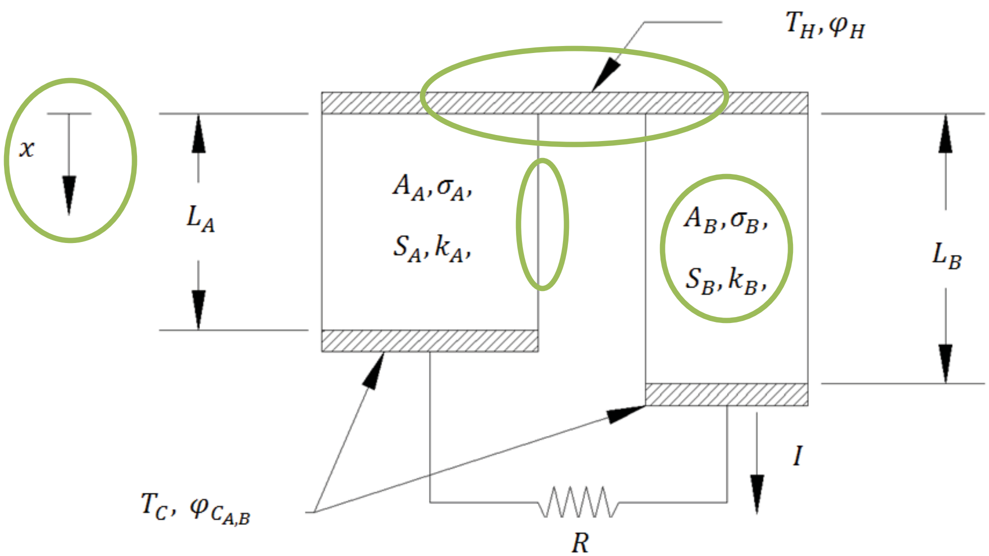

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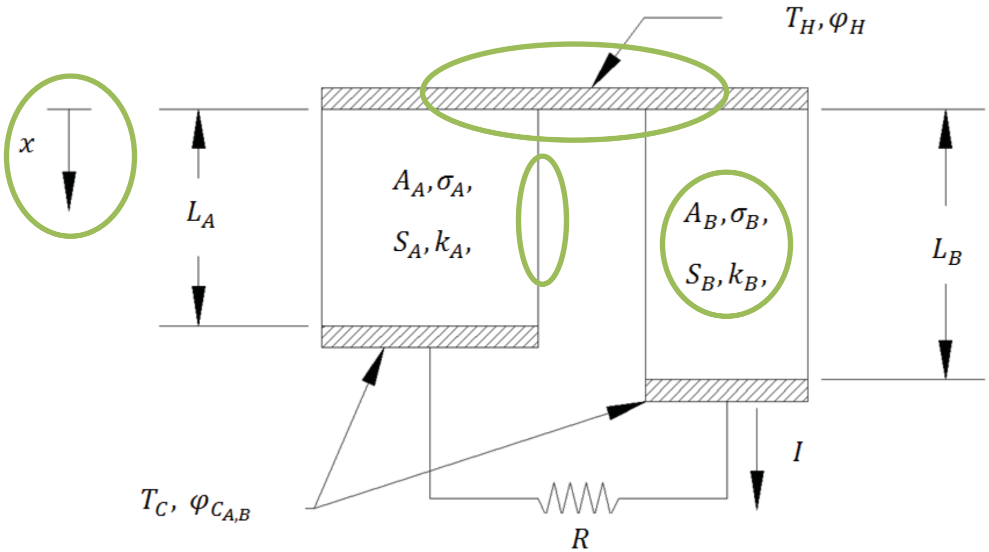

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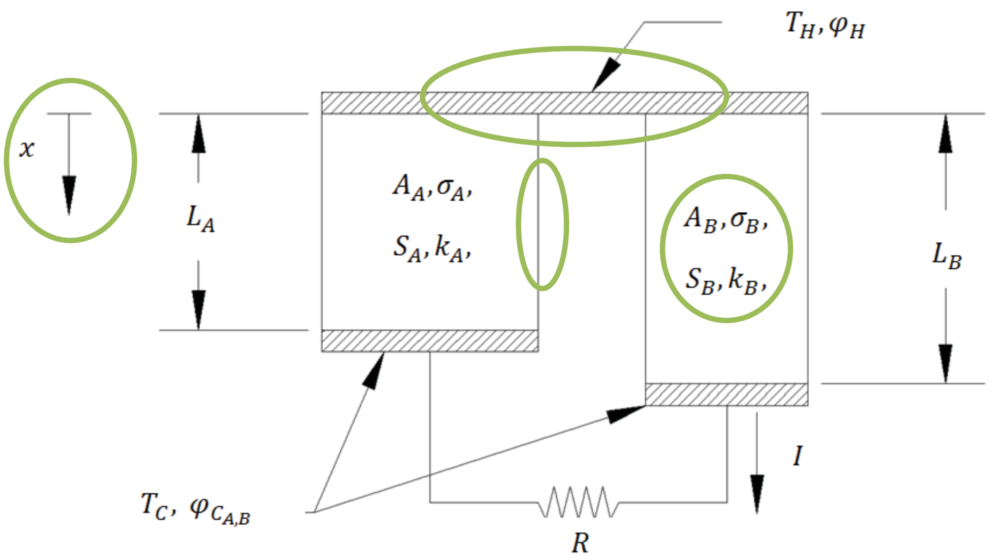

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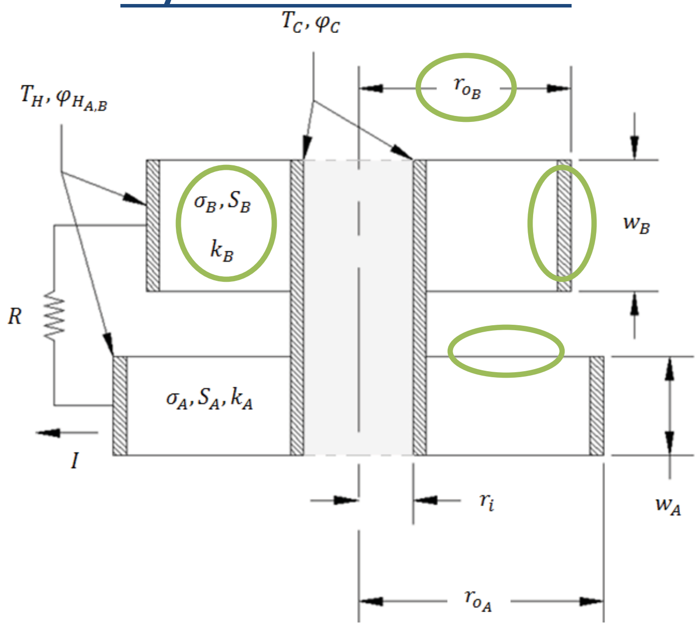
Cylindrical Model



Thermal- $$\frac{d}{dr}\left[-k_{A,B}\, r\, \frac{dT_{A,B}}{dr}\right] + \frac{I_{A,B}\tau_{A,B}}{2\pi w_{A,B}}\frac{dT_{A,B}}{dr} - \frac{{I_{A,B}}^2}{4\pi^2\, w_{A,B}^2 r\, \sigma_{A,B}} = 0$$

Electrical- $$\frac{d\varphi_{A,B}}{dr} = -S_{A,B}\frac{dT_{A,B}}{dr} - \frac{I_{A,B}}{2\pi w_{A,B} r \sigma_{A,B}}$$

System- $$\varphi_B(L_B) - \varphi_A(L_A) = IR$$

Cylindrical Parameters

Geometric- $$X = \frac{w_B \ln\left({r_{o,A}}/{r_i}\right)}{w_A \ln\left({r_{o,B}}/{r_i}\right)}$$

Load- $$Y = \frac{R}{\frac{\ln\left({r_{o,B}}/{r_i}\right)}{2\pi\sigma_B w_B} + \frac{\ln\left({r_{o,A}}/{r_i}\right)}{2\pi\sigma_A w_A}}$$

Materials- $$Z(X) = \frac{(S_B - S_A)^2}{\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X}\right)(k_A + k_B X)}$$

Classic Solution

$$\eta_{opt} = \frac{\eta_c Y_{opt}}{\frac{\left(1 + Y_{opt}\right)^2}{T_h Z(X_{opt})} + \left(1 + Y_{opt}\right) - \frac{1}{2}\eta_c}$$

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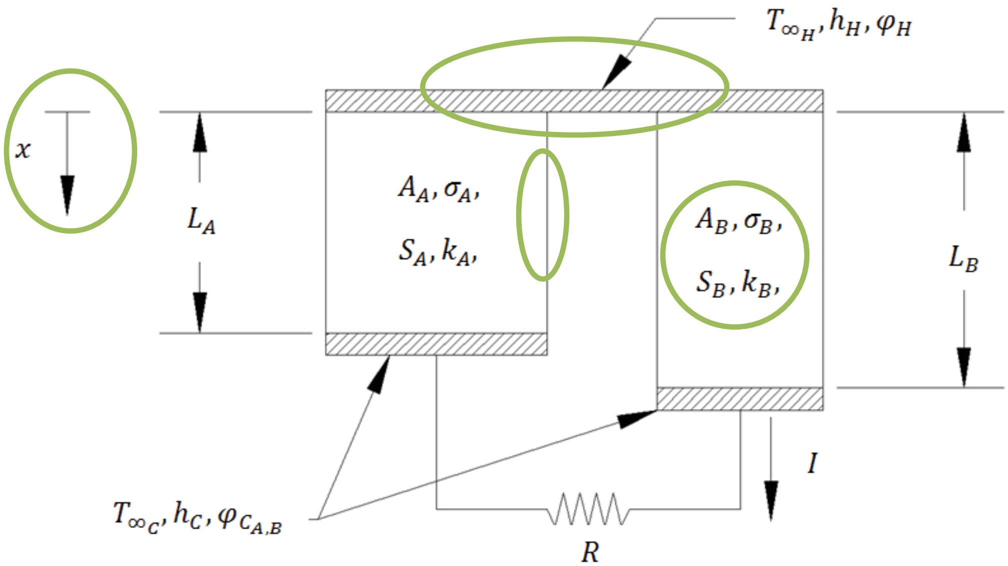
B.C. Model



Boundary Conditions (B.C.)-

$$-k_{A,B}\frac{dT_{A,B}(0)}{dx}+\frac{I_{A,B}S_{A,B}}{A_{A,B}}T_{A,B}(0)=h_h\left(T_{\infty h}-T_{a,b}(0)\right)$$

$$-k_{A,B}\frac{dT_{A,B}(L_{A,B})}{dx}+\frac{I_{A,B}S_{A,B}}{A_{A,B}}T_{A,B}(L_{A,B})=h_c\left(T_{A,B}(L_{A,B})-T_{\infty c}\right)$$

$$h_{h/c}^{-1}=h^{-1}+\sum_j\frac{L_j}{k_j}+\frac{1}{\varepsilon\sigma(T_s+T_\infty)(T_s^2+T_\infty^2)}$$

B.C. Parameters

Device Design- $$D_{A,B}=\frac{1}{1+\frac{k_{A,B}(h_h+h_c)}{L_{A,B}h_h h_c}}$$

Geometric- $$X=\frac{A_B L_A}{A_A L_B}$$

Load- $$Y=\frac{R}{\frac{L_B}{\sigma_B A_B}+\frac{L_A}{\sigma_A A_A}}$$

Materials- $$Z(X)=\frac{(D_B S_B-D_A S_A)^2}{\left(\frac{1}{\sigma_A}+\frac{1}{\sigma_B X}\right)(D_A k_A+D_B k_B X)}$$

B.C. Solution

$$X_{opt}=\sqrt{\frac{k_A\sigma_A D_A}{k_B\sigma_B D_B}}$$

$$Y_{opt}=\sqrt{1+Z(X_{opt})\left[T_{\infty_H}\frac{S_B-S_A}{D_B S_B-D_A S_A}(1-D_{avg})-\frac{\Delta T_\infty}{2}\right]}$$

$$Z(X_{opt},D_A,D_B)=\frac{(D_B S_B-D_A S_A)^2}{\left(\sqrt{\frac{k_A D_A}{\sigma_A}}+\sqrt{\frac{k_B D_B}{\sigma_B}}\right)^2}$$

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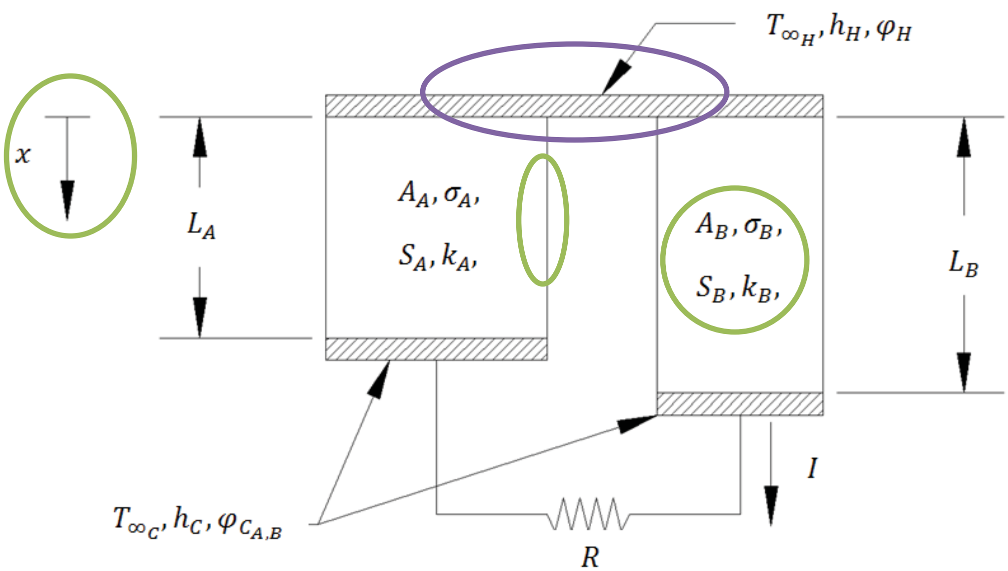

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$$h_{h/c}^{-1}=h^{-1}+\sum_j\frac{L_j}{k_j}+\frac{1}{\varepsilon\sigma(T_s+T_\infty)(T_s^2+T_\infty^2)}$$

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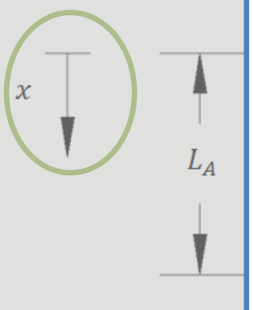
B.C. Solution

$$X_{opt}=\sqrt{\frac{k_A\sigma_A D_A}{k_B\sigma_B D_B}}$$

$$Y_{opt}=\sqrt{1+Z(X_{opt})\left[T_{\infty_H}\frac{S_B-S_A}{D_B S_B-D_A S_A}(1-D_{avg})-\frac{\Delta T_\infty}{2}\right]}$$

$$Z(X_{opt},D_A,D_B)=\frac{(D_B S_B-D_A S_A)^2}{\left(\sqrt{\frac{k_A D_A}{\sigma_A}}+\sqrt{\frac{k_B D_B}{\sigma_B}}\right)^2}$$

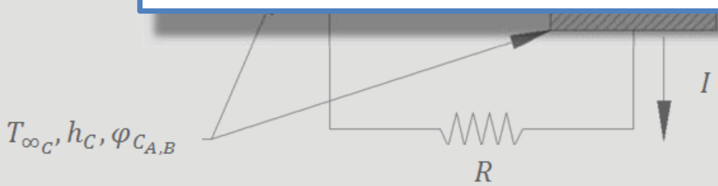


Classic Parameters

Geometric- $$X = \frac{A_B L_A}{A_A L_B}$$

Load- $$Y = \frac{R}{\frac{L_B}{\sigma_B A_B} + \frac{L_A}{\sigma_A A_A}}$$

Materials- $$Z(X) = \frac{(S_B - S_A)^2}{\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X}\right)(k_A + k_B X)}$$



B.C. Parameters

Device Design- $$D_{A,B} = \frac{1}{1 + \frac{k_{A,B}(h_h + h_c)}{L_{A,B} h_h h_c}}$$

Geometric- $$X = \frac{A_B L_A}{A_A L_B}$$

Load- $$Y = \frac{R}{\frac{L_B}{\sigma_B A_B} + \frac{L_A}{\sigma_A A_A}}$$

Materials- $$Z(X) = \frac{(D_B S_B - D_A S_A)^2}{\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X}\right)(D_A k_A + D_B k_B X)}$$

Boundary Conditions (B.C.)-

$$-k_{A,B}\frac{dT_{A,B}(0)}{dx} + \frac{I_{A,B}S_{A,B}}{A_{A,B}}T_{A,B}(0) = h_h\left(T_{\infty h} - T_{a,b}(0)\right)$$

$$-k_{A,B}\frac{dT_{A,B}(L_{A,B})}{dx} + \frac{I_{A,B}S_{A,B}}{A_{A,B}}T_{A,B}(L_{A,B}) = h_c\left(T_{A,B}(L_{A,B}) - T_{\infty c}\right)$$

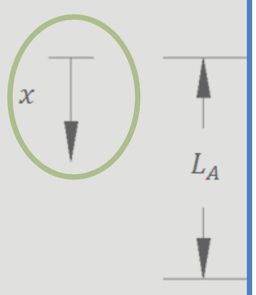
$$h_{h/c}^{-1} = h^{-1} + \sum_j \frac{L_j}{k_j} + \frac{1}{\varepsilon\sigma(T_s + T_\infty)(T_s^2 + T_\infty^2)}$$

B.C. Solution

$$X_{opt} = \sqrt{\frac{k_A \sigma_A D_A}{k_B \sigma_B D_B}}$$

$$Y_{opt} = \sqrt{1 + Z(X_{opt})\left[T_{\infty H}\frac{S_B - S_A}{D_B S_B - D_A S_A}(1 - D_{avg}) - \frac{\Delta T_\infty}{2}\right]}$$

$$Z(X_{opt}, D_A, D_B) = \frac{(D_B S_B - D_A S_A)^2}{\left(\sqrt{\frac{k_A D_A}{\sigma_A}} + \sqrt{\frac{k_B D_B}{\sigma_B}}\right)^2}$$



$T_{\infty c}, h_C, \varphi$

Classic Parameters

Geometric- $$X = \frac{A_B L_A}{A_A L_B}$$

Load- $$Y = \frac{R}{\frac{L_B}{\sigma_B A_B} + \frac{L_A}{\sigma_A A_A}}$$

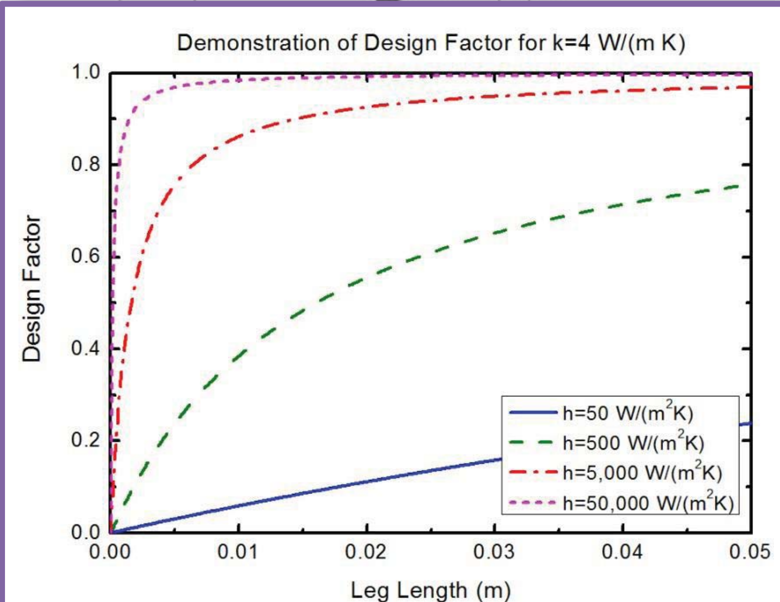
Materials- $$Z(X) = \frac{(S_B - S_A)^2}{\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X}\right)(k_A + k_B X)}$$

Boundary

$-k_{A,B}$

$-k_{A,B} \frac{dT_A}{}$

h



B.C. Parameters

Device Design- $$D_{A,B} = \frac{1}{1 + \frac{k_{A,B}(h_h + h_c)}{L_{A,B} h_h h_c}}$$

Geometric- $$X = \frac{A_B L_A}{A_A L_B}$$

Load- $$Y = \frac{R}{\frac{L_B}{\sigma_B A_B} + \frac{L_A}{\sigma_A A_A}}$$

Materials- $$Z(X) = \frac{(D_B S_B - D_A S_A)^2}{\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X}\right)(D_A k_A + D_B k_B X)}$$

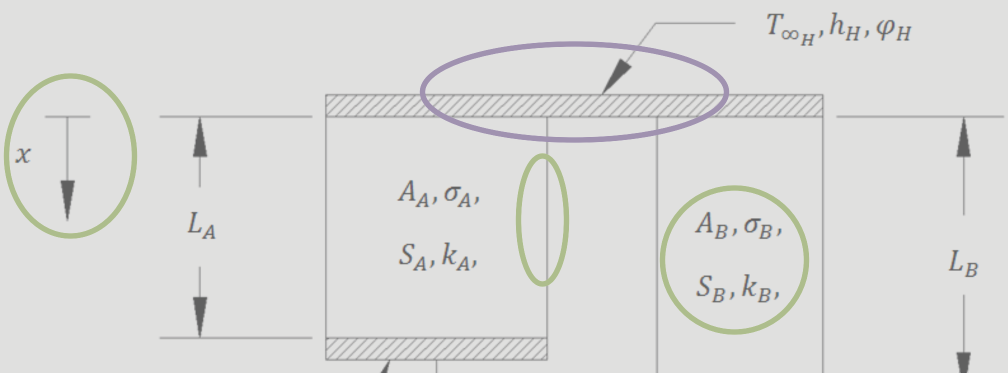
B.C. Solution

$$X_{opt} = \sqrt{\frac{k_A \sigma_A D_A}{k_B \sigma_B D_B}}$$

$$Y_{opt} = \sqrt{1 + Z(X_{opt})\left[T_{\infty H} \frac{S_B - S_A}{D_B S_B - D_A S_A}(1 - D_{avg}) - \frac{\Delta T_\infty}{2}\right]}$$

$$Z(X_{opt}, D_A, D_B) = \frac{(D_B S_B - D_A S_A)^2}{\left(\sqrt{\frac{k_A D_A}{\sigma_A}} + \sqrt{\frac{k_B D_B}{\sigma_B}}\right)^2}$$

B.C. Model



B.C. Parameters

Device Design- $D_{A,B} = \dfrac{1}{1 + \dfrac{k_{A,B}(h_h + h_c)}{L_{A,B} h_h h_c}}$

Geometric- $X = \dfrac{A_B L_A}{A_A L_B}$

Load- $Y = \dfrac{R}{\dfrac{L_B}{\sigma_B A_B} + \dfrac{L_A}{\sigma_A A_A}}$

Classic Solution

$$\eta_{opt} = \frac{\eta_c Y_{opt}}{\frac{(1+Y_{opt})^2}{T_h Z(X_{opt})} + (1+Y_{opt}) - \frac{1}{2}\eta_c}$$

$$X_{opt} = \sqrt{\frac{k_A \sigma_A}{k_B \sigma_B}}$$

$$Y_{opt} = \sqrt{1 + Z(X_{opt}) T_{avg}}$$

$$Z(X_{opt}) = \frac{(S_B - S_A)^2}{\left(\sqrt{\frac{k_A}{\sigma_A}} + \sqrt{\frac{k_B}{\sigma_B}}\right)^2}$$

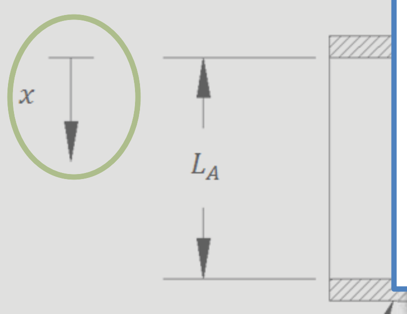
B.C. Solution

$$\eta = \frac{\eta_{c\infty} Y_{opt}}{\frac{(1+Y_{opt})^2}{T_{\infty H} Z(X_{opt}, D_B, D_A)} + \frac{(1+Y_{opt})(S_B - S_A)}{(D_B S_B - D_A S_A)}\left[1 - \frac{\eta_{c\infty}}{2}\{1 - D_{avg}\}\right] - \frac{1}{2}\eta_{c\infty}}$$

$$X_{opt} = \sqrt{\frac{k_A \sigma_A D_A}{k_B \sigma_B D_B}}$$

$$Y_{opt} = \sqrt{1 + Z(X_{opt})\left[T_{\infty H}\frac{S_B - S_A}{D_B S_B - D_A S_A}(1 - D_{avg}) - \frac{\Delta T_\infty}{2}\right]}$$

$$Z(X_{opt}, D_A, D_B) = \frac{(D_B S_B - D_A S_A)^2}{\left(\sqrt{\frac{k_A D_A}{\sigma_A}} + \sqrt{\frac{k_B D_B}{\sigma_B}}\right)^2}$$



meters

$$\frac{1}{1+\frac{k_{A,B}(h_h+h_c)}{L_{A,B}h_h h_c}}$$

$$X=\frac{A_B L_A}{A_A L_B}$$

$$=\frac{R}{\frac{L_B}{\sigma_B A_B}+\frac{L_A}{\sigma_A A_A}}$$

Example Calculation

Convection (W/m-K)	Design Factor	Max Efficiency (%)	Max Power Density (W/m²)
∞	1.00	6.15	17,733
50,000	0.98	6.05	17,118
500	0.38	2.28	2,300

Classic Solution

$$\eta_{opt}=\frac{\eta_c Y_{opt}}{\frac{(1+Y_{opt})^2}{T_h Z(X_{opt})}+(1+Y_{opt})-\frac{1}{2}\eta_c}$$

$$\mathrm{X_{opt}}=\sqrt{\frac{\mathrm{k_A\sigma_A}}{\mathrm{k_B\sigma_B}}}$$

$$\mathrm{Y_{opt}}=\sqrt{1+\mathrm{Z(X_{opt})T_{avg}}}$$

$$\mathrm{Z(X_{opt})}=\frac{(\mathrm{S_B}-\mathrm{S_A})^2}{\left(\sqrt{\frac{\mathrm{k_A}}{\sigma_A}}+\sqrt{\frac{\mathrm{k_B}}{\sigma_B}}\right)^2}$$

B.C. Solution

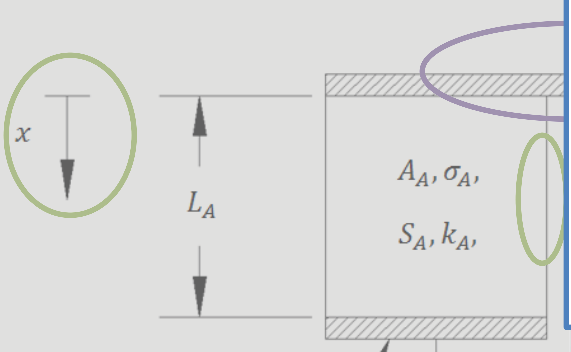
$$\eta=\frac{\eta_{c\infty}Y_{opt}}{\frac{(1+Y_{opt})^2}{T_{\infty H}Z(X_{opt},D_B,D_A)}+\frac{(1+Y_{opt})(S_B-S_A)}{(D_B S_B-D_A S_A)}\left[1-\frac{\eta_{c\infty}}{2}\{1-D_{avg}\}\right]-\frac{1}{2}\eta_{c\,\infty}}$$

$$\mathrm{X_{opt}}=\sqrt{\frac{\mathrm{k_A\sigma_A D_A}}{\mathrm{k_B\sigma_B D_B}}}$$

$$\mathrm{Y_{opt}}=\sqrt{1+\mathrm{Z(X_{opt})}\left[\mathrm{T_{\infty H}}\frac{\mathrm{S_B}-\mathrm{S_A}}{\mathrm{D_B S_B}-\mathrm{D_A S_A}}(1-\mathrm{D_{avg}})-\frac{\Delta \mathrm{T_\infty}}{2}\right]}$$

$$\mathrm{Z(X_{opt},D_A,D_B)}=\frac{(\mathrm{D_B S_B}-\mathrm{D_A S_A})^2}{\left(\sqrt{\frac{\mathrm{k_A D_A}}{\sigma_A}}+\sqrt{\frac{\mathrm{k_B D_B}}{\sigma_B}}\right)^2}$$

B.C. Model



B.C. Parameters

e Design-

$$D_{A,B} = \frac{1}{1 + \frac{k_{A,B}(h_h + h_c)}{L_{A,B} h_h h_c}}$$

etric-

$$X = \frac{A_B L_A}{A_A L_B}$$

$$Y = \frac{R}{\frac{L_B}{\sigma_B A_B} + \frac{L_A}{\sigma_A A_A}}$$

Design Guideline

$$L_D \geq \frac{D(h_H + h_C)k}{(1-D)h_H h_C}$$

$$L_{99\%} = \frac{99(h_H + h_C)k}{h_H h_C}$$

Classic Solution

$$\eta_{opt} = \frac{\eta_c Y_{opt}}{\frac{(1+Y_{opt})^2}{T_h Z(X_{opt})} + (1+Y_{opt}) - \frac{1}{2}\eta_c}$$

$$X_{opt} = \sqrt{\frac{k_A \sigma_A}{k_B \sigma_B}}$$

$$Y_{opt} = \sqrt{1 + Z(X_{opt}) T_{avg}}$$

$$Z(X_{opt}) = \frac{(S_B - S_A)^2}{\left(\sqrt{\frac{k_A}{\sigma_A}} + \sqrt{\frac{k_B}{\sigma_B}}\right)^2}$$

B.C. Solution

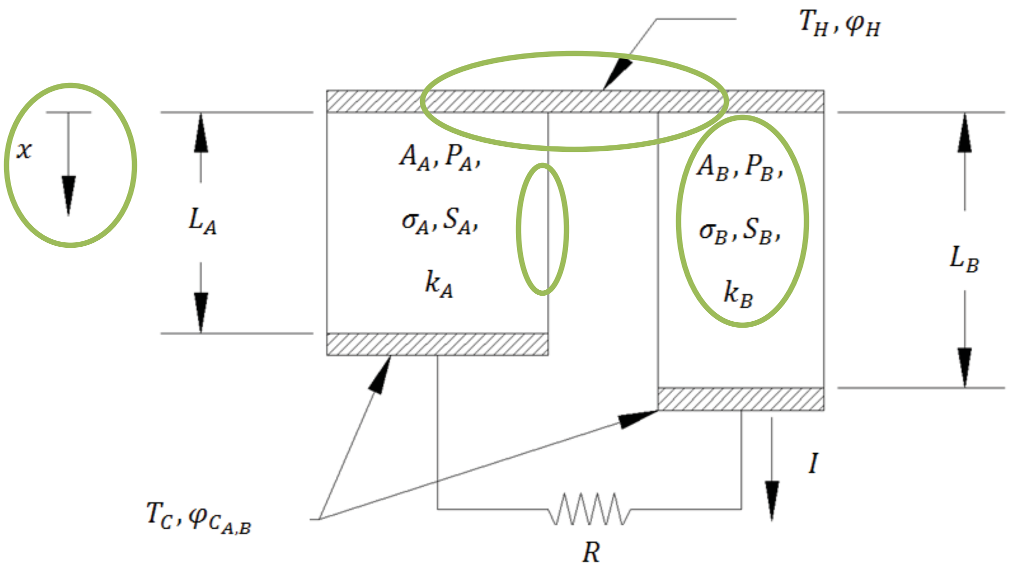
$$\eta = \frac{\eta_{c\infty} Y_{opt}}{\frac{(1+Y_{opt})^2}{T_{\infty H} Z(X_{opt}, D_B, D_A)} + \frac{(1+Y_{opt})(S_B - S_A)}{(D_B S_B - D_A S_A)}\left[1 - \frac{\eta_{c\infty}}{2}\{1 - D_{avg}\}\right] - \frac{1}{2}\eta_{c\,\infty}}$$

$$X_{opt} = \sqrt{\frac{k_A \sigma_A D_A}{k_B \sigma_B D_B}}$$

$$Y_{opt} = \sqrt{1 + Z(X_{opt})\left[T_{\infty H} \frac{S_B - S_A}{D_B S_B - D_A S_A}(1 - D_{avg}) - \frac{\Delta T_\infty}{2}\right]}$$

$$Z(X_{opt}, D_A, D_B) = \frac{(D_B S_B - D_A S_A)^2}{\left(\sqrt{\frac{k_A D_A}{\sigma_A}} + \sqrt{\frac{k_B D_B}{\sigma_B}}\right)^2}$$

Fin Model



Thermal Governing Equation-

$$\frac{d}{dx}\left[-k_{A,B}\frac{d\theta_{A,B}}{dx}\right]+\frac{I_{A,B}\tau_{A,B}}{A_{A,B}}\frac{d\theta_{A,B}}{dx}+\frac{P_{A,B}h_{A,B}}{A_{A,B}}\theta_{A,B}-\frac{{I_{A,B}}^2}{{A_{A,B}}^2\sigma_{A,B}}=0$$

$$\theta_{A,B}=T_{A,B}-T_\infty$$

Fin Parameters

Fin Factor- $$F_{A,B}=L_{A,B}\sqrt{\frac{P_{A,B}h_{A,B}}{k_{A,B}A_{A,B}}}$$

Geometric Fin- $$G=\sqrt{\frac{P_BA_Bh_Bk_A}{P_AA_Ah_Ak_B}\frac{\tanh(F_A)}{\tanh(F_B)}}$$

Load- $$Y=\frac{R}{\frac{L_B}{\sigma_BA_B}+\frac{L_A}{\sigma_AA_A}}$$

Materials- $$Z(X,G)=\frac{(S_B-S_A)^2}{\left(\frac{1}{\sigma_A}+\frac{1}{\sigma_BX}\right)(k_A+k_BG)}$$

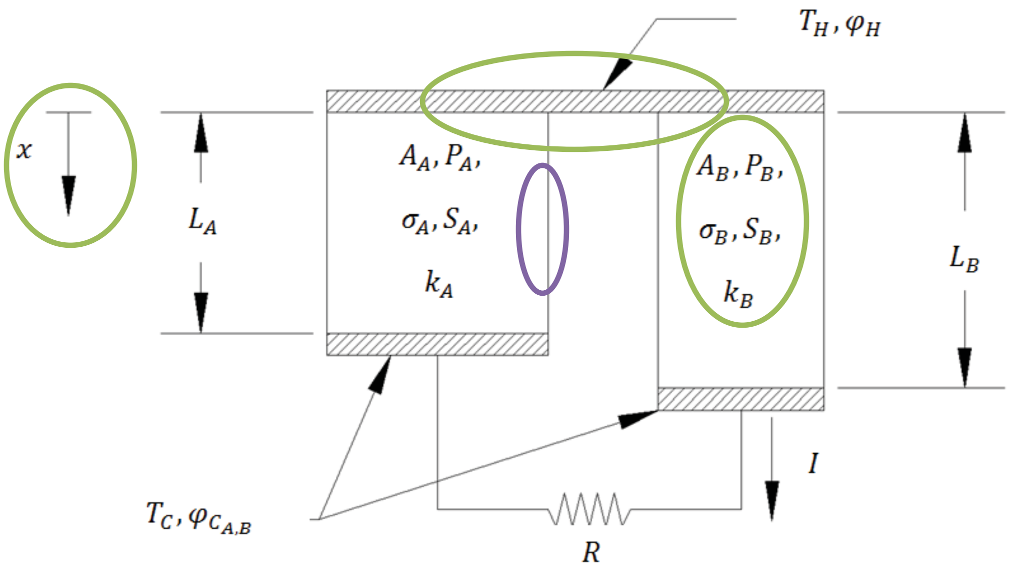
Fin Solution

$$G_{opt}=\sqrt{\frac{\mathrm{k_A\sigma_A}}{\mathrm{k_B\sigma_B}}}$$

$$\mathrm{Y_{opt}}=\sqrt{1+\mathrm{Z(X_{opt},G)T_{avg}}}$$

$$\mathrm{Z(X_{opt},G)}=\frac{(S_B-S_A)^2}{\left(\frac{1}{\sigma_A}+\frac{1}{\sigma_BX_{opt}}\right)(k_A+k_BG)}$$

Fin Model



Thermal Governing Equation-

$$\frac{d}{dx}\left[-k_{A,B}\frac{d\theta_{A,B}}{dx}\right] + \frac{I_{A,B}\tau_{A,B}}{A_{A,B}}\frac{d\theta_{A,B}}{dx} + \frac{P_{A,B}h_{A,B}}{A_{A,B}}\theta_{A,B} - \frac{{I_{A,B}}^2}{{A_{A,B}}^2\sigma_{A,B}} = 0$$

$$\theta_{A,B} = T_{A,B} - T_\infty$$

Fin Parameters

Fin Factor- $F_{A,B} = L_{A,B}\sqrt{\frac{P_{A,B}h_{A,B}}{k_{A,B}A_{A,B}}}$

Geometric Fin- $G = \sqrt{\frac{P_B A_B h_B k_A}{P_A A_A h_A k_B}\frac{\tanh(F_A)}{\tanh(F_B)}}$

Load- $Y = \frac{R}{\frac{L_B}{\sigma_B A_B} + \frac{L_A}{\sigma_A A_A}}$

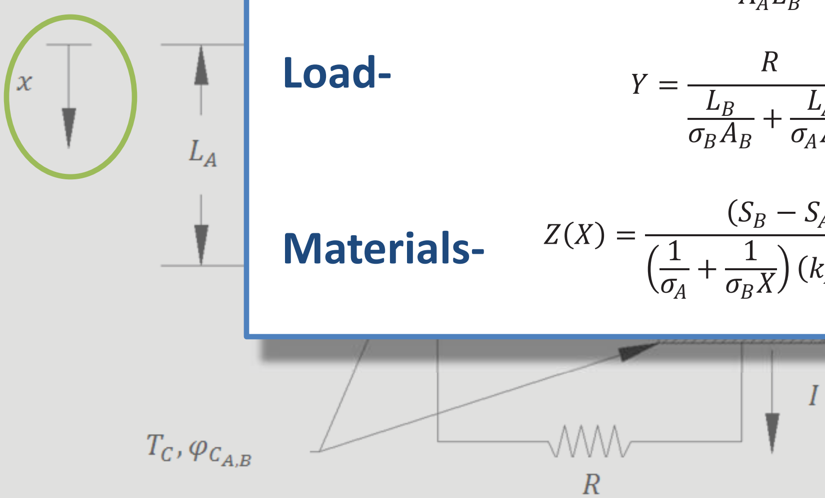
Materials- $Z(X,G) = \frac{(S_B - S_A)^2}{\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X}\right)(k_A + k_B G)}$

Fin Solution

$$G_{opt} = \sqrt{\frac{k_A \sigma_A}{k_B \sigma_B}}$$

$$Y_{opt} = \sqrt{1 + Z(X_{opt}, G)T_{avg}}$$

$$Z(X_{opt}, G) = \frac{(S_B - S_A)^2}{\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X_{opt}}\right)(k_A + k_B G)}$$



Classic Parameters

Geometric- $$X = \frac{A_B L_A}{A_A L_B}$$

Load- $$Y = \frac{R}{\frac{L_B}{\sigma_B A_B} + \frac{L_A}{\sigma_A A_A}}$$

Materials- $$Z(X) = \frac{(S_B - S_A)^2}{\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X}\right)(k_A + k_B X)}$$

Fin Parameters

Fin Factor- $$F_{A,B} = L_{A,B}\sqrt{\frac{P_{A,B} h_{A,B}}{k_{A,B} A_{A,B}}}$$

Geometric Fin- $$G = \sqrt{\frac{P_B A_B h_B k_A}{P_A A_A h_A k_B}\frac{\tanh(F_A)}{\tanh(F_B)}}$$

Load- $$Y = \frac{R}{\frac{L_B}{\sigma_B A_B} + \frac{L_A}{\sigma_A A_A}}$$

Materials- $$Z(X, G) = \frac{(S_B - S_A)^2}{\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X}\right)(k_A + k_B G)}$$

Thermal Governing Equation-

$$\frac{d}{dx}\left[-k_{A,B}\frac{d\theta_{A,B}}{dx}\right] + \frac{I_{A,B}\tau_{A,B}}{A_{A,B}}\frac{d\theta_{A,B}}{dx} + \frac{P_{A,B} h_{A,B}}{A_{A,B}}\theta_{A,B} - \frac{{I_{A,B}}^2}{{A_{A,B}}^2\sigma_{A,B}} = 0$$

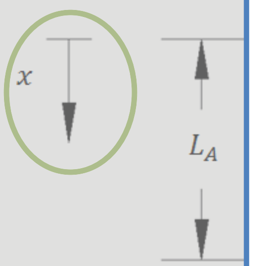
$$\theta_{A,B} = T_{A,B} - T_\infty$$

Fin Solution

$$G_{opt} = \sqrt{\frac{k_A \sigma_A}{k_B \sigma_B}}$$

$$Y_{opt} = \sqrt{1 + Z(X_{opt}, G) T_{avg}}$$

$$Z(X_{opt}, G) = \frac{(S_B - S_A)^2}{\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X_{opt}}\right)(k_A + k_B G)}$$



T_C, φ_{C_A}

Thermal

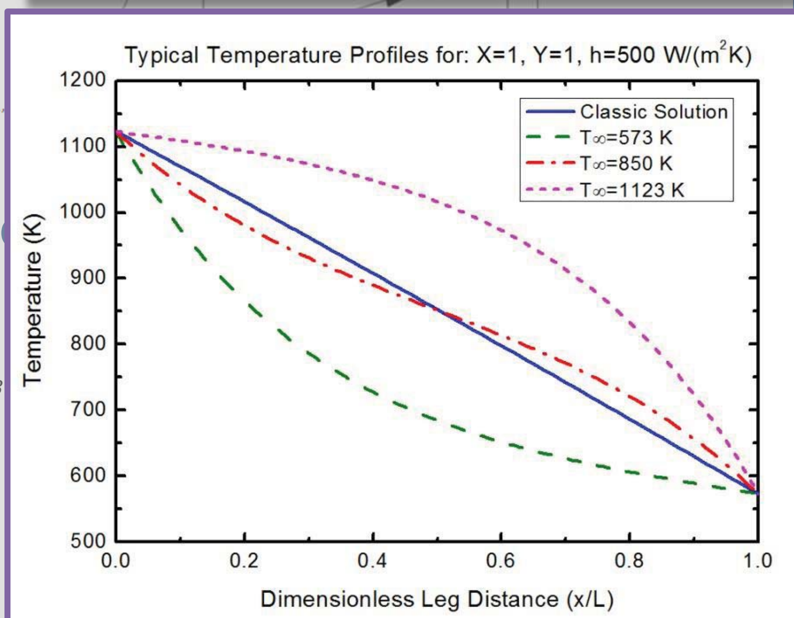
$\frac{d}{dx}\left[-k_{A,B}\right.$

Classic Parameters

Geometric- $$X = \frac{A_B L_A}{A_A L_B}$$

Load- $$Y = \frac{R}{\frac{L_B}{\sigma_B A_B} + \frac{L_A}{\sigma_A A_A}}$$

Materials- $$Z(X) = \frac{(S_B - S_A)^2}{\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X}\right)(k_A + k_B X)}$$



Fin Parameters

Fin Factor- $$F_{A,B} = L_{A,B}\sqrt{\frac{P_{A,B} h_{A,B}}{k_{A,B} A_{A,B}}}$$

Geometric Fin- $$G = \sqrt{\frac{P_B A_B h_B k_A}{P_A A_A h_A k_B} \frac{\tanh(F_A)}{\tanh(F_B)}}$$

Load- $$Y = \frac{R}{\frac{L_B}{\sigma_B A_B} + \frac{L_A}{\sigma_A A_A}}$$

Materials- $$Z(X, G) = \frac{(S_B - S_A)^2}{\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X}\right)(k_A + k_B G)}$$

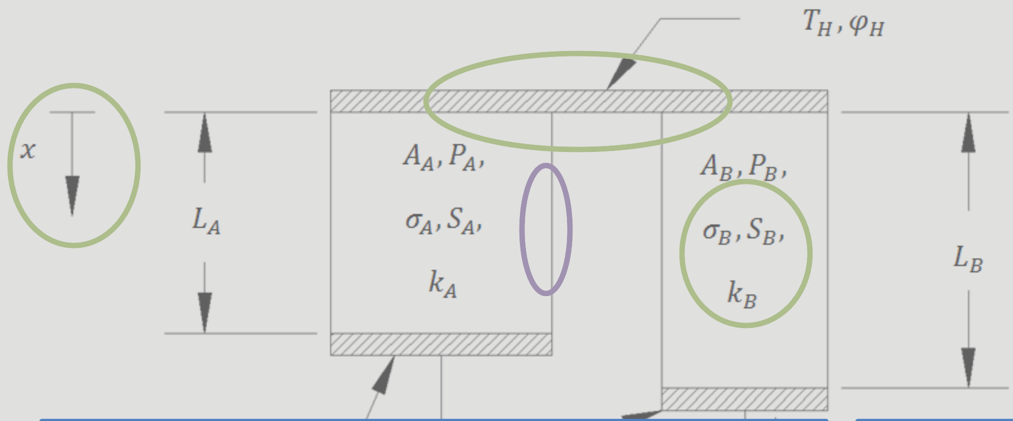
Fin Solution

$$G_{opt} = \sqrt{\frac{k_A \sigma_A}{k_B \sigma_B}}$$

$$Y_{opt} = \sqrt{1 + Z(X_{opt}, G) T_{avg}}$$

$$Z(X_{opt}, G) = \frac{(S_B - S_A)^2}{\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X_{opt}}\right)(k_A + k_B G)}$$

Fin Model



Fin Parameters

Fin Factor- $F_{A,B} = L_{A,B}\sqrt{\frac{P_{A,B}h_{A,B}}{k_{A,B}A_{A,B}}}$

Geometric Fin- $G = \sqrt{\frac{P_B A_B h_B k_A}{P_A A_A h_A k_B}\frac{\tanh(F_A)}{\tanh(F_B)}}$

Load- $Y = \frac{R}{\frac{L_B}{\sigma_B A_B} + \frac{L_A}{\sigma_A A_A}}$

Classic Solution

$$\eta_{opt} = \frac{\eta_c Y_{opt}}{\frac{\left(1+Y_{opt}\right)^2}{T_h Z\left(X_{opt}\right)} + \left(1+Y_{opt}\right) - \frac{1}{2}\eta_c}$$

$$\mathrm{X_{opt}} = \sqrt{\frac{\mathrm{k_A \sigma_A}}{\mathrm{k_B \sigma_B}}}$$

$$\mathrm{Y_{opt}} = \sqrt{1 + \mathrm{Z(X_{opt})T_{avg}}}$$

$$\mathrm{Z(X_{opt})} = \frac{\mathrm{(S_B - S_A)^2}}{\left(\sqrt{\frac{\mathrm{k_A}}{\mathrm{\sigma_A}}} + \sqrt{\frac{\mathrm{k_B}}{\mathrm{\sigma_B}}}\right)^2}$$

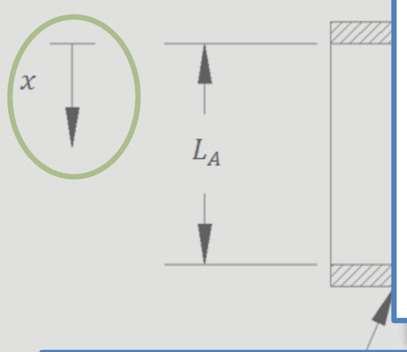
Fin Solution

$$\eta_{opt} = \frac{\eta_c Y_{opt}}{\frac{F_A\left(1+Y_{opt}\right)^2}{\tanh(F_A)\, T_H Z\left(X_{opt}, G\right)} + \left(1+Y_{opt}\right) - \eta_c \frac{\tanh\left(\frac{F_A}{2}\right)\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B G(F/2)}\right)}{F_A\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X}\right)}}$$

$$G_{opt} = \sqrt{\frac{\mathrm{k_A \sigma_A}}{\mathrm{k_B \sigma_B}}}$$

$$\mathrm{Y_{opt}} = \sqrt{1 + \mathrm{Z(X_{opt}, G)T_{avg}}}$$

$$\mathrm{Z(X_{opt}, G)} = \frac{(S_B - S_A)^2}{\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X_{opt}}\right)(k_A + k_B G)}$$



(Para)meters

$$= L_{A,B}\sqrt{\frac{P_{A,B}h_{A,B}}{k_{A,B}A_{A,B}}}$$

$$\sqrt{\frac{P_B A_B h_B k_A}{P_A A_A h_A k_B}}\frac{\tanh(F_A)}{\tanh(F_B)}$$

$$= \frac{R}{\frac{L_B}{\sigma_B A_B}+\frac{L_A}{\sigma_A A_A}}$$

Example Calculation

Convection (W/m-K)	Fin Factor	Max Efficiency (%)	Max Power Density (W/m²)
0	0.00	6.15	17,733
5	0.32	6.05	17,733
500	0.38	2.70	17,733

Classic Solution

$$\eta_{opt} = \frac{\eta_c Y_{opt}}{\frac{(1+Y_{opt})^2}{T_h Z(X_{opt})} + (1+Y_{opt}) - \frac{1}{2}\eta_c}$$

$$X_{opt} = \sqrt{\frac{k_A \sigma_A}{k_B \sigma_B}}$$

$$Y_{opt} = \sqrt{1 + Z(X_{opt})T_{avg}}$$

$$Z(X_{opt}) = \frac{(S_B - S_A)^2}{\left(\sqrt{\frac{k_A}{\sigma_A}} + \sqrt{\frac{k_B}{\sigma_B}}\right)^2}$$

Fin Solution

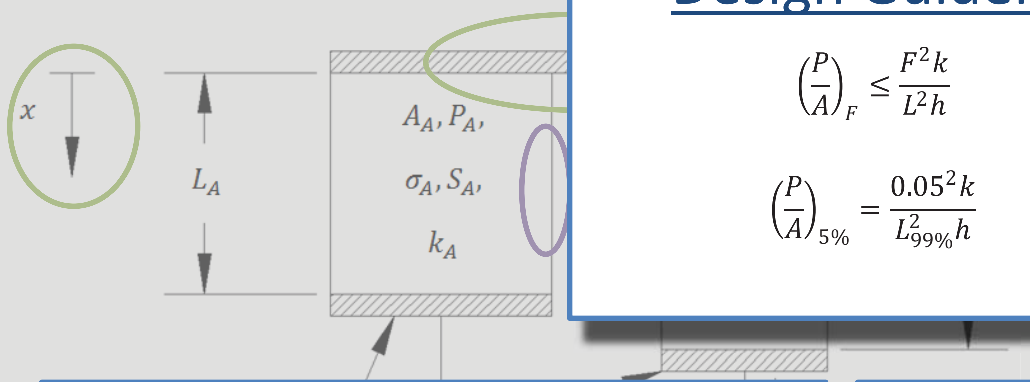
$$\eta_{opt} = \frac{\eta_c Y_{opt}}{\frac{F_A(1+Y_{opt})^2}{\tanh(F_A)\, T_H Z(X_{opt}, G)} + (1+Y_{opt}) - \eta_c \frac{\tanh\left(\frac{F_A}{2}\right)\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B G(F/2)}\right)}{F_A\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X}\right)}}$$

$$G_{opt} = \sqrt{\frac{k_A \sigma_A}{k_B \sigma_B}}$$

$$Y_{opt} = \sqrt{1 + Z(X_{opt}, G)T_{avg}}$$

$$Z(X_{opt}, G) = \frac{(S_B - S_A)^2}{\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X_{opt}}\right)(k_A + k_B G)}$$

Fin Model



Fin Parameters

...ctor-

$$F_{A,B} = L_{A,B}\sqrt{\frac{P_{A,B}h_{A,B}}{k_{A,B}A_{A,B}}}$$

...etric Fin-

$$G = \sqrt{\frac{P_B A_B h_B k_A}{P_A A_A h_A k_B}\frac{\tanh(F_A)}{\tanh(F_B)}}$$

$$Y = \frac{R}{\frac{L_B}{\sigma_B A_B} + \frac{L_A}{\sigma_A A_A}}$$

Design Guideline

$$\left(\frac{P}{A}\right)_F \leq \frac{F^2 k}{L^2 h}$$

$$\left(\frac{P}{A}\right)_{5\%} = \frac{0.05^2 k}{L_{99\%}^2 h}$$

Classic Solution

$$\eta_{opt} = \frac{\eta_c Y_{opt}}{\frac{(1+Y_{opt})^2}{T_h Z(X_{opt})} + (1+Y_{opt}) - \frac{1}{2}\eta_c}$$

$$\mathrm{X_{opt}} = \sqrt{\frac{\mathrm{k_A \sigma_A}}{\mathrm{k_B \sigma_B}}}$$

$$\mathrm{Y_{opt}} = \sqrt{1 + \mathrm{Z(X_{opt})T_{avg}}}$$

$$\mathrm{Z(X_{opt})} = \frac{(\mathrm{S_B} - \mathrm{S_A})^2}{\left(\sqrt{\frac{\mathrm{k_A}}{\mathrm{\sigma_A}}} + \sqrt{\frac{\mathrm{k_B}}{\mathrm{\sigma_B}}}\right)^2}$$

Fin Solution

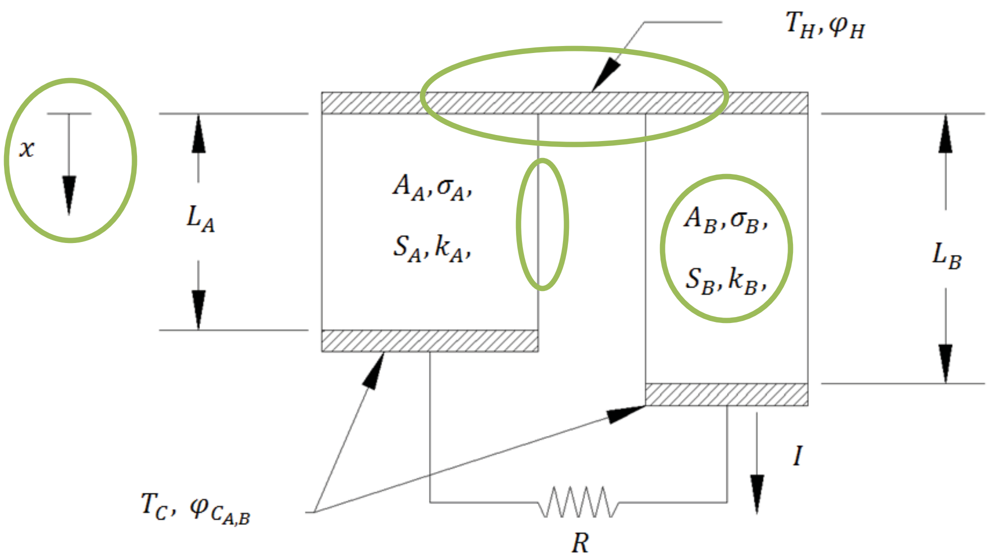
$$\eta_{opt} = \frac{\eta_c Y_{opt}}{\frac{F_A(1+Y_{opt})^2}{\tanh(F_A)\, T_H Z(X_{opt}, G)} + (1+Y_{opt}) - \eta_c \frac{\tanh\left(\frac{F_A}{2}\right)\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B G(F/2)}\right)}{F_A\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X}\right)}}$$

$$G_{opt} = \sqrt{\frac{\mathrm{k_A \sigma_A}}{\mathrm{k_B \sigma_B}}}$$

$$\mathrm{Y_{opt}} = \sqrt{1 + \mathrm{Z(X_{opt}, G)T_{avg}}}$$

$$\mathrm{Z(X_{opt}, G)} = \frac{(S_B - S_A)^2}{\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X_{opt}}\right)(k_A + k_B G)}$$

Variable Model



Material Properties by Asymptotic Expansion-

$$\sigma(T) = \tilde{\sigma}\frac{\sigma(T)}{\tilde{\sigma}} = \tilde{\sigma}(\sigma_0 + \epsilon\sigma_1 T)$$

$$S(T) = \tilde{S}\frac{S(T)}{\tilde{S}} = \tilde{S}(S_0 + \epsilon S_1 T)$$

$$k(T) = \tilde{k}\frac{k(T)}{\tilde{k}} = \tilde{k}(k_0 + \epsilon k_1 T)$$

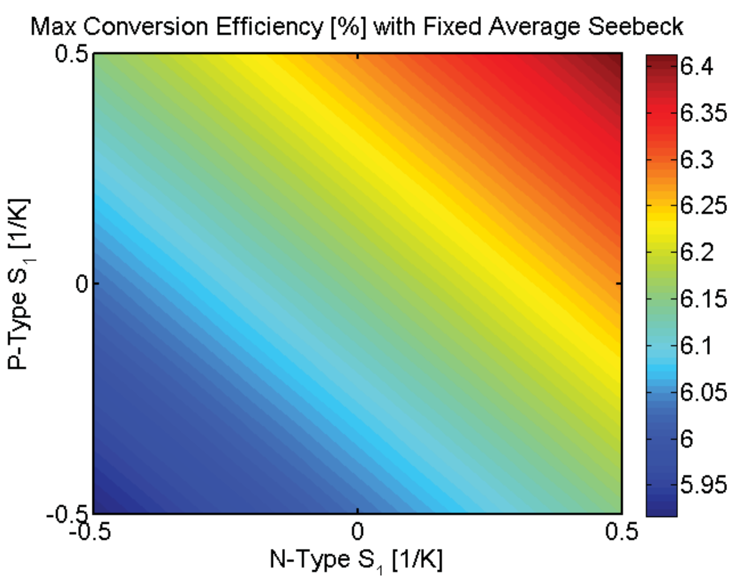
Asymptotic Expansion

$$\hat{T} = \frac{T}{\Delta T} \qquad \hat{\varphi} = \frac{\varphi}{\Delta S \Delta T} \qquad \hat{I} = \frac{IR}{\Delta S \Delta T}$$

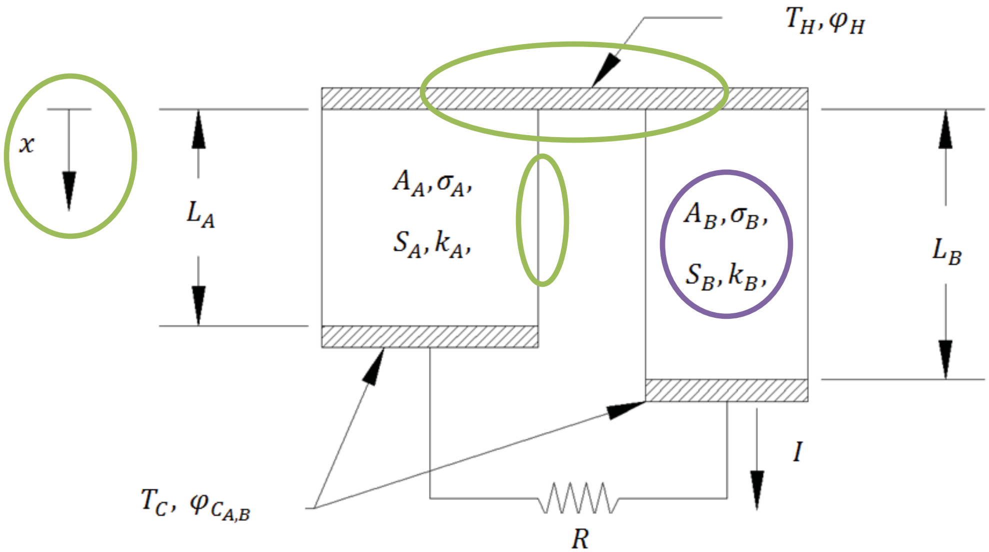
$$\hat{T} = T_0 + \epsilon T_1$$

$$\hat{\varphi} = \varphi_0 + \epsilon\varphi_1$$

Variable Solution



Variable Model



Material Properties by Asymptotic Expansion-

$$\sigma(T) = \tilde{\sigma}\frac{\sigma(T)}{\tilde{\sigma}} = \tilde{\sigma}(\sigma_0 + \epsilon\sigma_1 T)$$

$$S(T) = \tilde{S}\frac{S(T)}{\tilde{S}} = \tilde{S}(S_0 + \epsilon S_1 T)$$

$$k(T) = \tilde{k}\frac{k(T)}{\tilde{k}} = \tilde{k}(k_0 + \epsilon k_1 T)$$

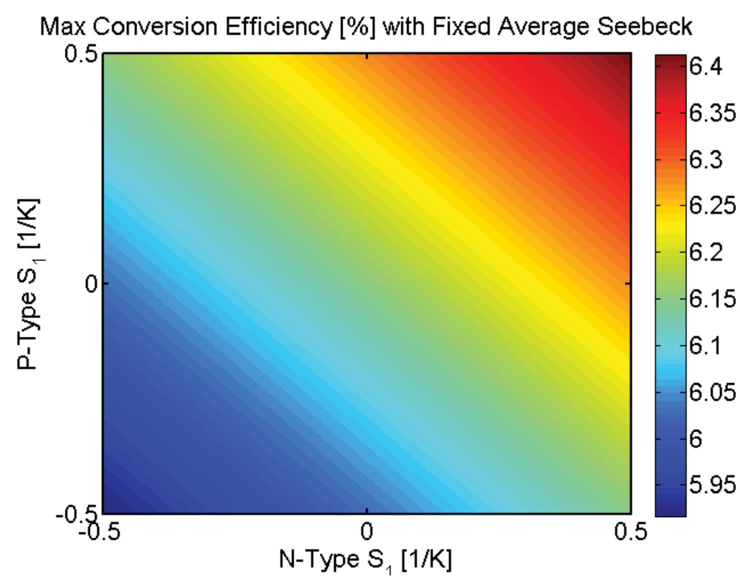
Asymptotic Expansion

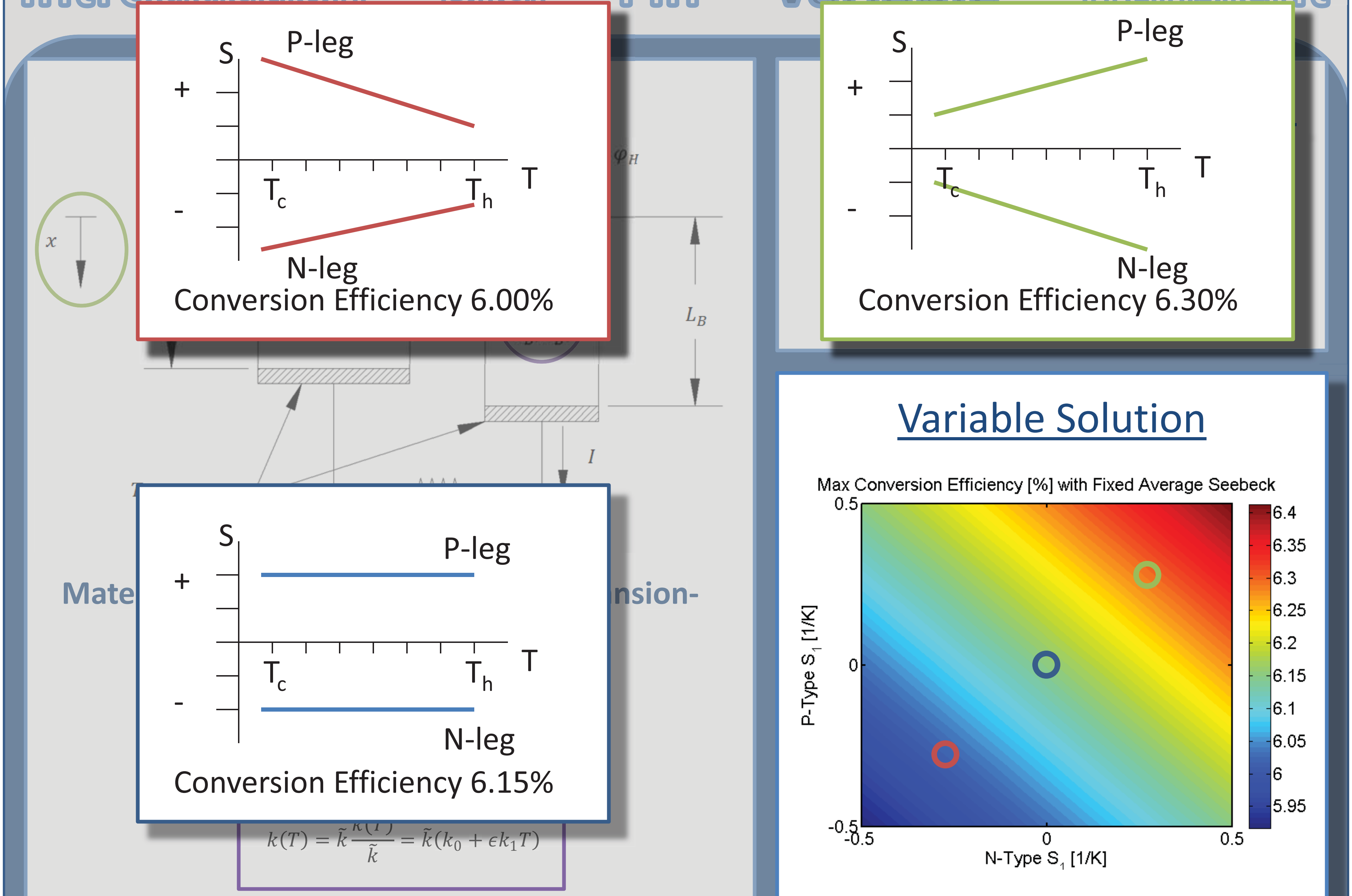
$$\hat{T} = \frac{T}{\Delta T} \qquad \hat{\varphi} = \frac{\varphi}{\Delta S \Delta T} \qquad \hat{I} = \frac{IR}{\Delta S \Delta T}$$

$$\hat{T} = T_0 + \epsilon T_1$$

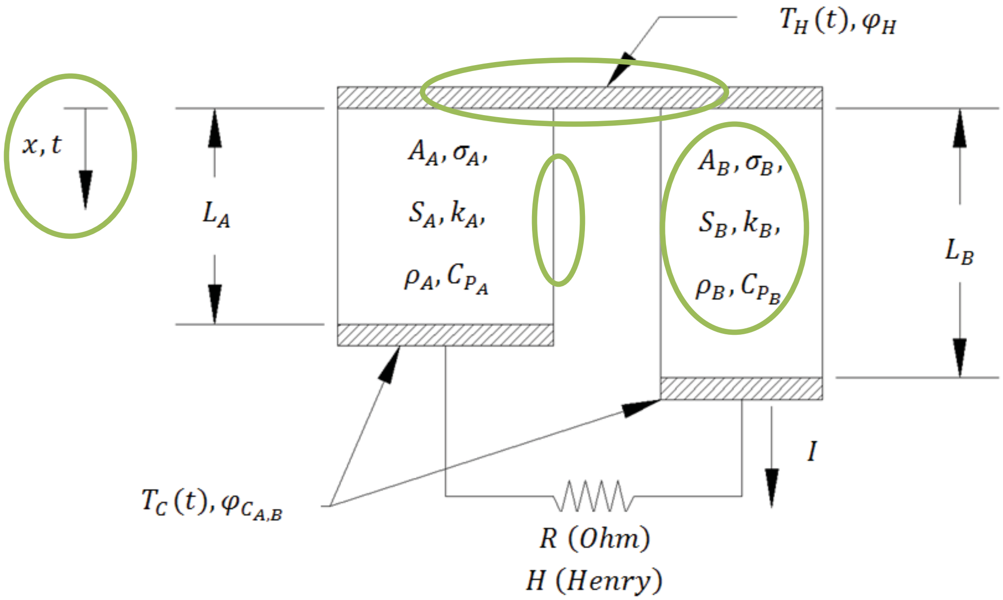
$$\hat{\varphi} = \varphi_0 + \epsilon\varphi_1$$

Variable Solution




S
P-leg
+
-
T_c
T_h
T
N-leg
Conversion Efficiency 6.00%
S
P-leg
+
-
T_c
T_h
T
N-leg
Conversion Efficiency 6.30%
S
P-leg
+
-
T_c
T_h
T
N-leg
Conversion Efficiency 6.15%
Variable Solution
Max Conversion Efficiency [%] with Fixed Average Seebeck
P-Type S_1 [1/K]
N-Type S_1 [1/K]
0.5
0
-0.5
6.4
6.35
6.3
6.25
6.2
6.15
6.1
6.05
6
5.95

Transient Model

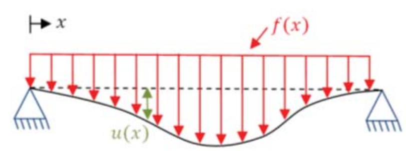


Thermal- $$\frac{\partial}{\partial x}\left[-k_{A,B}\frac{\partial T_{A,B}}{\partial x}\right] + \frac{I_{A,B}\tau_{A,B}}{A_{A,B}}\frac{\partial T_{A,B}}{\partial x} - \frac{{I_{A,B}}^2}{{A_{A,B}}^2\sigma_{A,B}} = \rho_{A,B}c_{p_{A,B}}\frac{\partial T_{A,B}}{\partial t}$$

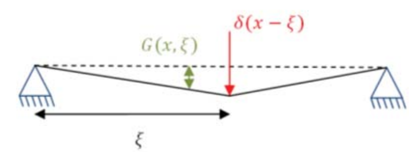
Electrical- $$\frac{\partial \varphi_{A,B}}{\partial x} = -S_{A,B}\frac{\partial T_{A,B}}{\partial x} - \frac{I_{A,B}}{A_{A,B}\sigma_{A,B}}$$

System- $$\varphi_B(L_B) - \varphi_A(L_A) = IR + H\frac{dI}{dt}$$

Green's Function Solution



$$\mathcal{L}u(x) = f(x)$$



$$\mathcal{L}^*G(x,\xi) = \delta(x-\xi)$$

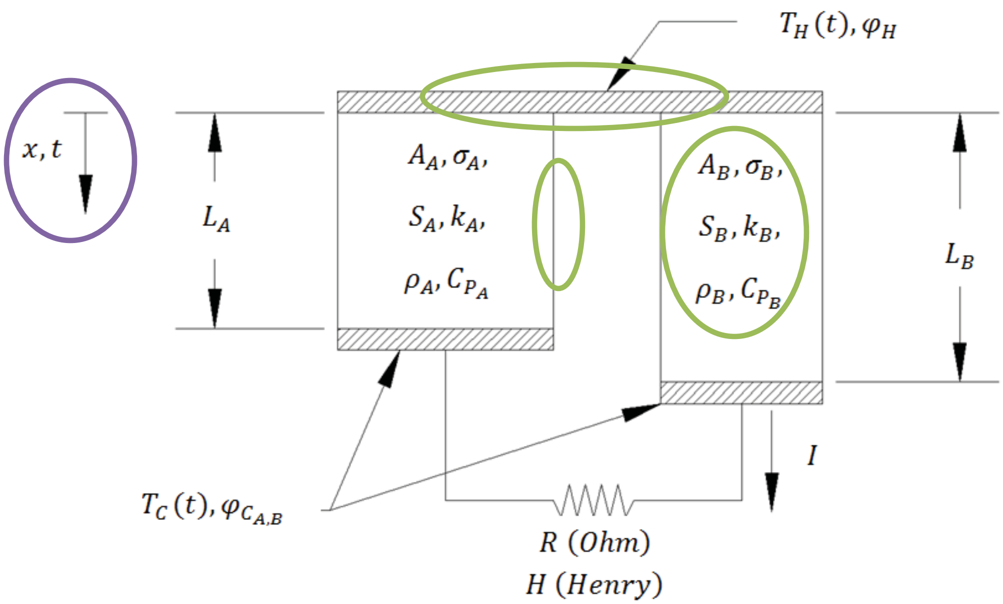
$$u(x) = \int G(x,\xi)f(\xi)d\xi$$

Transient Parameters

Thermal diffusivity factor- $$\Gamma_{A,B} = \frac{\alpha_{avg}L_{A,B}^2}{\alpha_{A,B}L_{avg}^2}$$

Inductance factor- $$\beta = \frac{H\alpha_{avg}}{RL_{avg}^2}$$

Transient Model

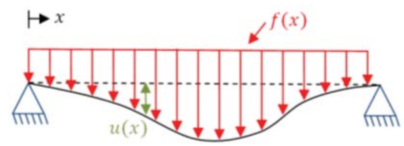


Thermal- $\frac{\partial}{\partial x}\left[-k_{A,B}\frac{\partial T_{A,B}}{\partial x}\right] + \frac{I_{A,B}\tau_{A,B}}{A_{A,B}}\frac{\partial T_{A,B}}{\partial x} - \frac{{I_{A,B}}^2}{{A_{A,B}}^2\sigma_{A,B}} = \rho_{A,B}c_{p_{A,B}}\frac{\partial T_{A,B}}{\partial t}$

Electrical- $\frac{\partial \varphi_{A,B}}{\partial x} = -S_{A,B}\frac{\partial T_{A,B}}{\partial x} - \frac{I_{A,B}}{A_{A,B}\sigma_{A,B}}$

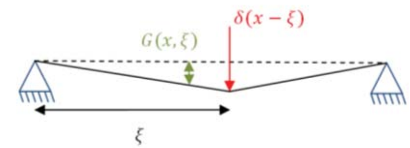
System- $\varphi_B(L_B) - \varphi_A(L_A) = IR + H\frac{dI}{dt}$

Green's Function Solution



$\mathcal{L}u(x) = f(x)$

$\mathcal{L}^*G(x,\xi) = \delta(x-\xi)$



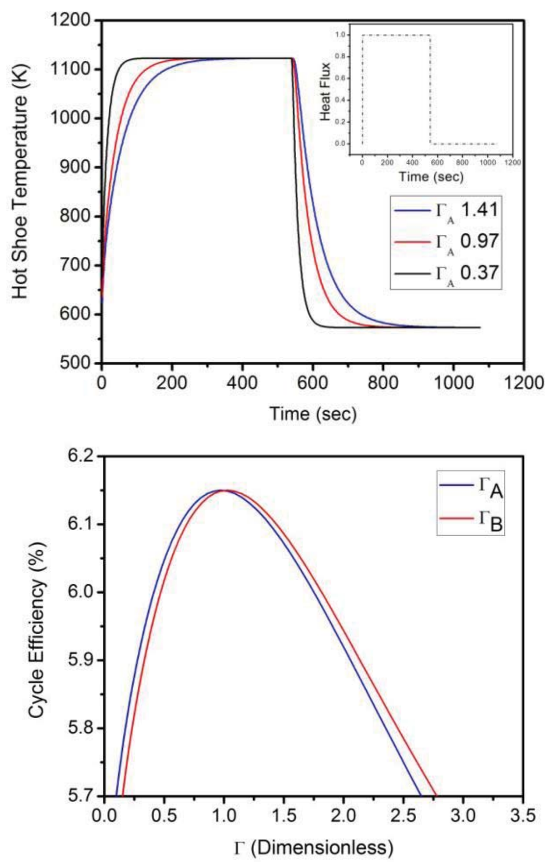
$u(x) = \int G(x,\xi)f(\xi)d\xi$

Transient Parameters

Thermal diffusivity factor- $\Gamma_{A,B} = \frac{\alpha_{avg}L_{A,B}^2}{\alpha_{A,B}L_{avg}^2}$

Inductance factor- $\beta = \frac{H\alpha_{avg}}{RL_{avg}^2}$

Periodic On/Off Operation



Design Guideline

$$\frac{L_A}{L_B} = \frac{\sqrt{2a}+1}{2a-1}$$

$$a = 1 + \frac{\alpha_B}{\alpha_A}$$

$$\mathcal{L}u(x) = f(x)$$

$$u(x) = \int G(x,\xi) f(\xi) d\xi$$

Transient Parameters

Thermal diffusivity factor- $\Gamma_{A,B} = \frac{\alpha_{avg} L_{A,B}^2}{\alpha_{A,B} L_{avg}^2}$

Inductance factor- $\beta = \frac{H\alpha_{avg}}{RL_{avg}^2}$

Conclusion

- Several new design factors can have a large influence on couple behavior
  - Device Design Factor
  - Fin Factor
  - Thermal Diffusivity Factor
  - Inductance Factor
- The introduced design guidelines must be considered in couple design

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JPL

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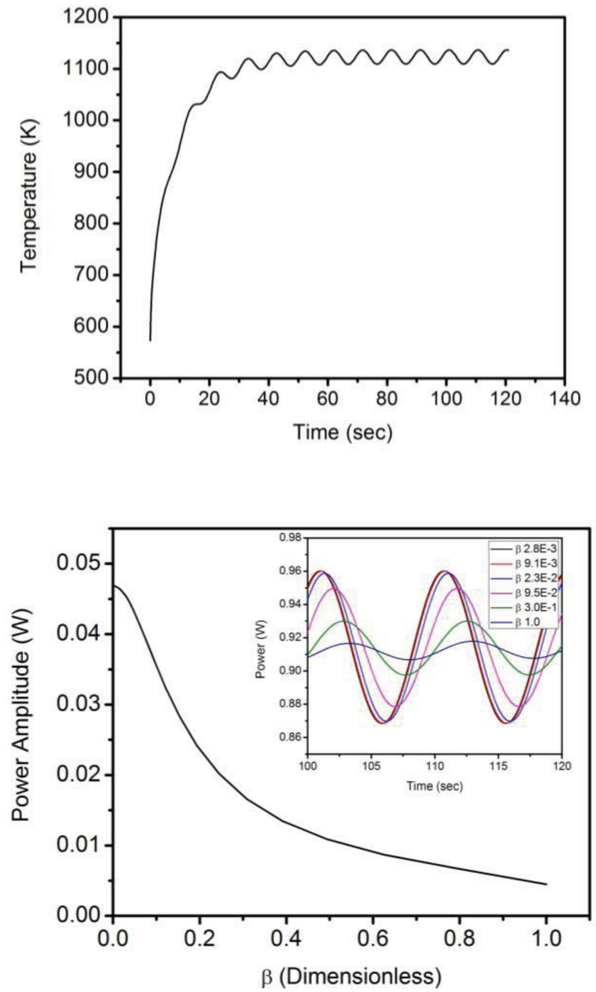
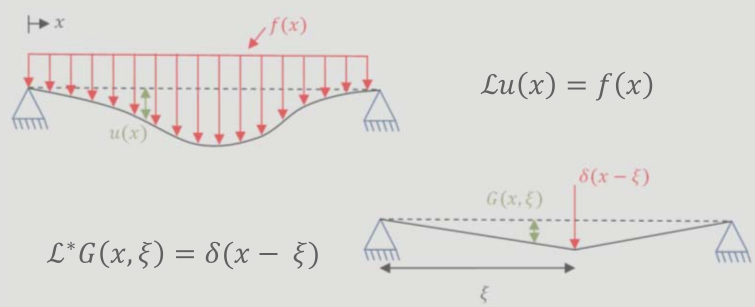
Appendix

Sinusoidal Operation



Green's Function Solution



$$\mathcal{L}u(x) = f(x)$$

$$\mathcal{L}^*G(x,\xi) = \delta(x-\xi)$$

$$u(x) = \int G(x,\xi) f(\xi) d\xi$$

Transient Parameters

Thermal diffusivity factor- $\Gamma_{A,B} = \frac{\alpha_{avg} L_{A,B}^2}{\alpha_{A,B} L_{avg}^2}$

Inductance factor- $\beta = \frac{H\alpha_{avg}}{RL_{avg}^2}$