Structural Dynamic Analysis in Rocket Engine Turbomachinery

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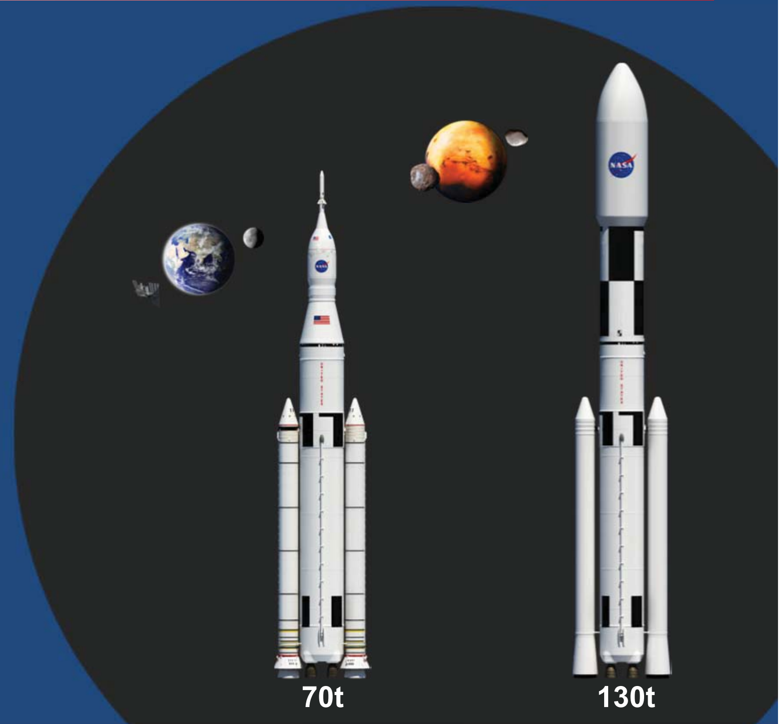
Travelling To and Through Space

Space Launch System (SLS) – America's Heavy-lift Rocket

- Provides initial lift capacity of 70 metric tons (t), evolving to 130 t
- Carries the Orion Multi-Purpose Crew Vehicle (MPCV) and significant science payloads
- Supports national and international missions beyond Earth's orbit, such as near-Earth asteroids and Mars




Solid Rocket Booster Test


Friction Stir Welding for Core Stage


Shell Buckling Structural Test


MPCV Stage Adapter Assembly


Selective Laser Melting Engine Parts

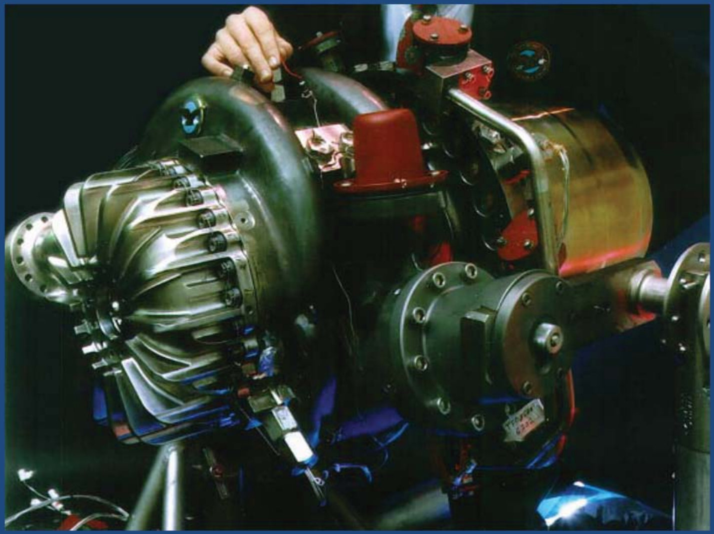

RS-25 (SSME) Core Stage Engines in Inventory

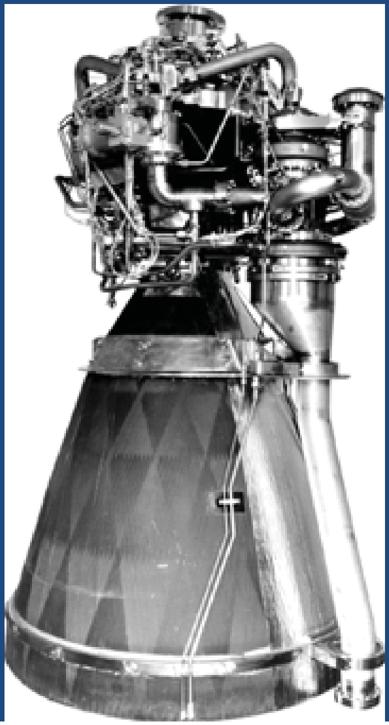
SLS is essential to US space exploration goals.

Agenda

- Motivation for Structural Dynamic Analysis of Turbomachinery
- How Turbomachinery is used in Rocket Engines
- Process for Analysis
  - Characterization of Excitation
  - Temporal Fourier series
  - Modeling and Modal Analysis
  - Forced (Frequency) Response
  - Campbell Diagrams (simple)
  - Cyclic symmetry
  - Modal orthogonality and Generalized Force
  - Impeller Triple crossover charts
  - Tyler Sofrin charts, more complex Campbell Diagrams
  - Spatial Fourier Analysis
- Conclusion

How turbomachinery is used in Rocket Engines





MSFC Fastrac engine

- Liquid Fuel (LH2, Kerosene) and Oxidizer (LO2) are stored in Fuel tanks at a few atmospheres.
- Turbines, driven by hot gas created by mini-combustors, tied with shaft to pump, which sucks in propellants and increases their pressures to several hundred atm.
- High pressure propellants sent to Combustion chamber, which ignites mixture with "injectors", very hot gas directed to converging diverging flow to increase to very high velocity for thrust.

Space Shuttle Main Engine (SLS RS25) Fun Facts!

- A little larger than a car engine, the SSME high-pressure fuel turbopump generates 150 Kw for each kilo of its weight, while car engine generates 0.75 Kw for each kilo of its weight.
- SSME weighs 1/7 as much as a train engine, its high-pressure fuel pump alone delivers as much power as 28 trains.
- The SSME high-pressure fuel turbopump main shaft rotates at 37,000 rpm compared to about 3,000 rpm for an car operating at 100 Kph.
- The discharge pressure of an SSME high-pressure fuel turbopump could send a column of liquid hydrogen 58 kilometers in the air.

SSME launch video

Motivation: Avoid High Cycle Fatigue Cracking in Turbomachinery

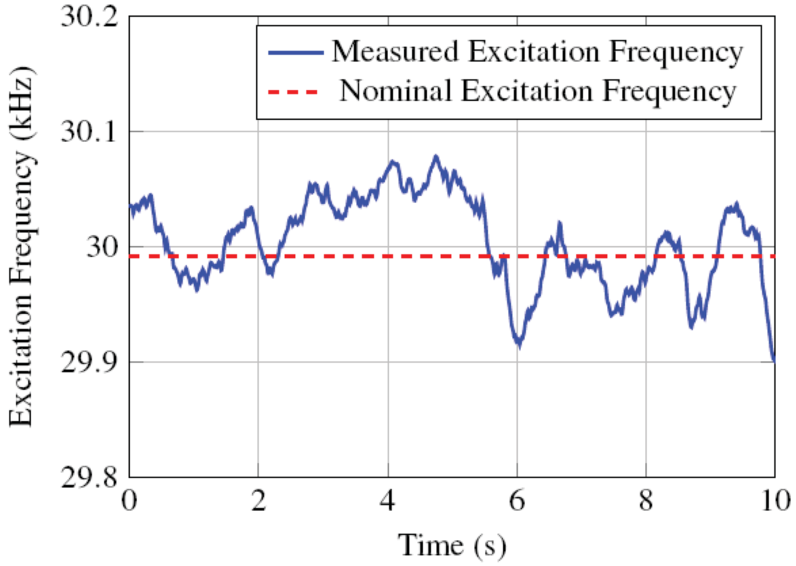
- Cracks found during ground-test program stop engine development
  - If cracks propagates, it could liberate a piece, which at very high rotational speeds could be catastrophic (i.e., engine will explode)

Characterization of Excitations – Speed Range

- First obtain speed range of operation from performance group.
  - For Rocket Engines, there are generally several "nominal" operating speeds dependent upon phase of mission (e.g., reduce thrust during "Max Q").
  - However, since flow is the controlling parameter, actual rotational speeds are uncertain (especially during design phase)
  - For new engine being built at MSFC, assuming possible variation +/-5% about each of two operating speeds.

Rated Power Level	70%	100%
Low Range	20759.4	26125
Nominal	21852	27500
High Range	22944.6	28875

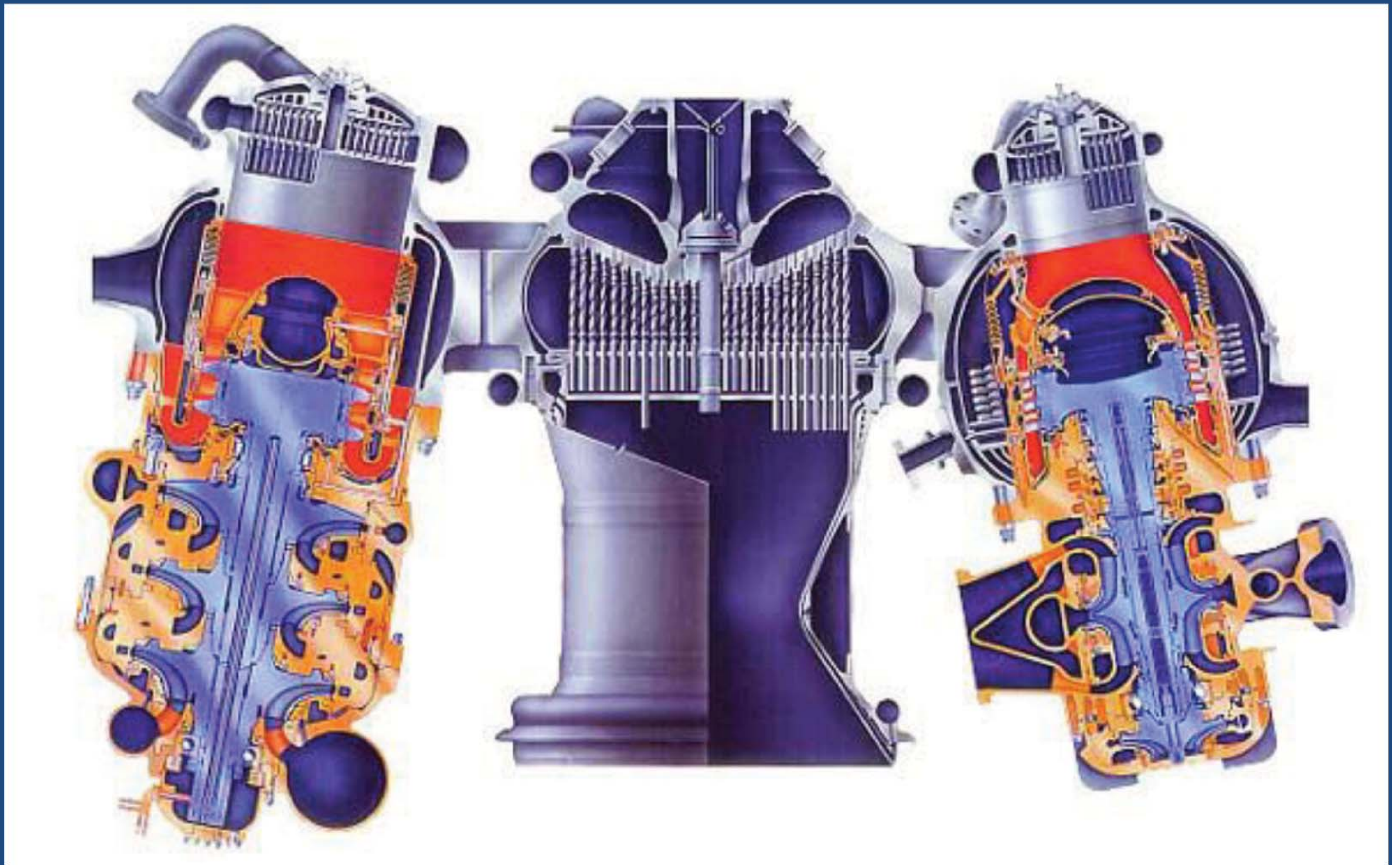
- In addition, speed generally isn't constant, but instead "dithers" – *come to seminar to learn more*.



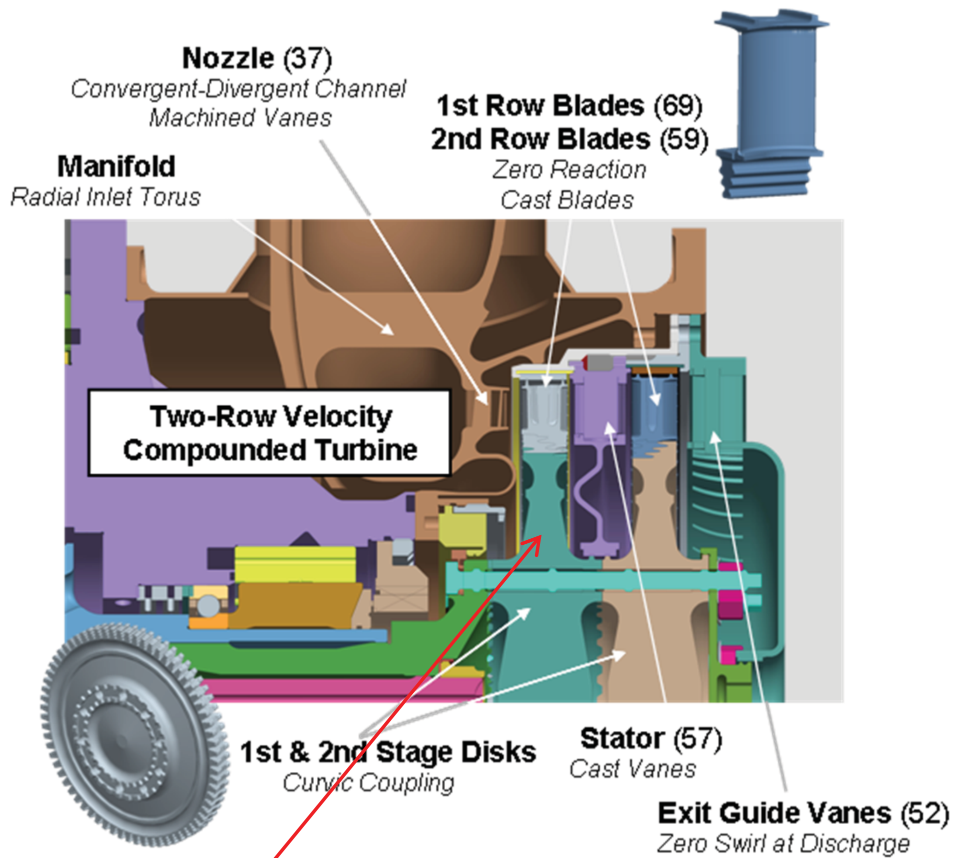
Characterization of Mechanical Excitation due to Unbalance

- Since all real machinery has some level of unbalance and bearing whirl, sinusoidal ("harmonic") loads at 1 and 2 times rotational speed ("1 and 2N") will be generated, along with up to their 3rd multiples (also called "harmonics", see Fourier slides) ➡ 3-6N throughout turbopump.

Space Shuttle Main Engine Powerhead Cross-Section

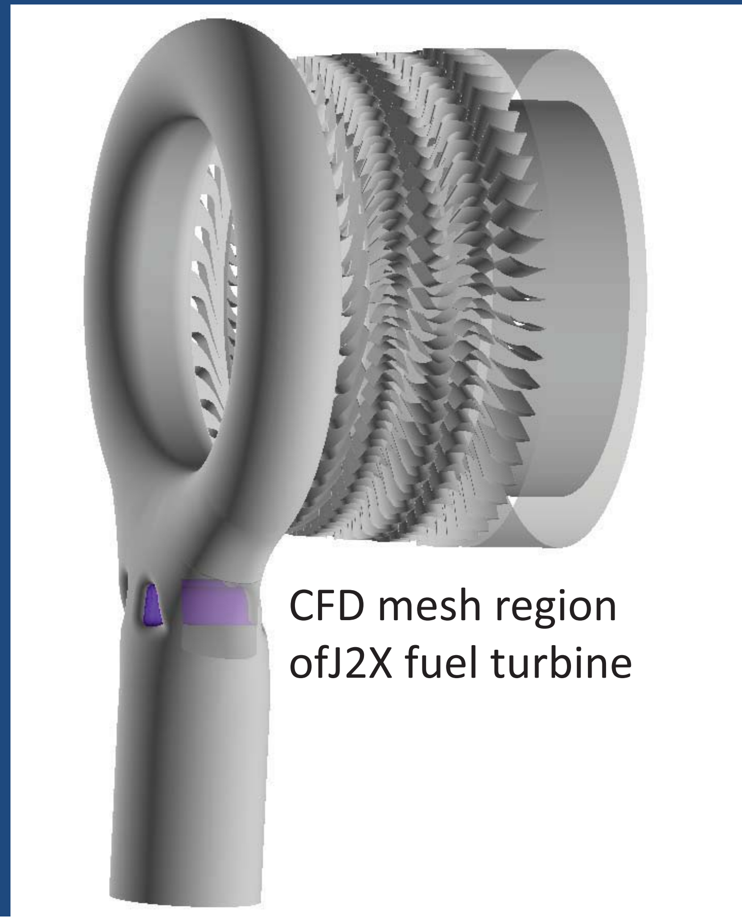


Characterization of Fluid Excitation



Bladed disk

- Harmonic excitation at engine order = Number of flow distortions and up to their 3rd multiples arising from adjacent upstream and downstream blade and vane counts.
  - Use CFD to generate Loading

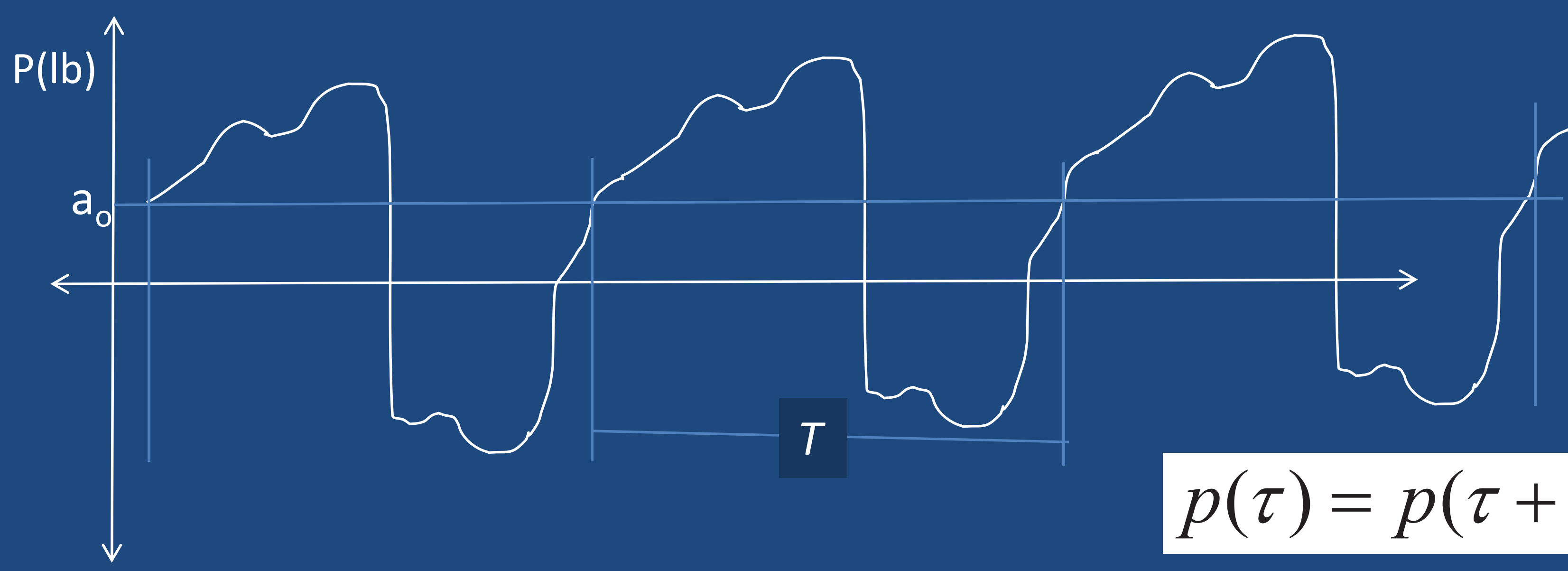


CFD mesh region ofJ2X fuel turbine

Quantify Engine Forces using Fourier Analysis

- Forces are not, in general, perfect sine waves (although sometimes they're close)
- We can deal with these in two ways:
  - Represent forces as sum of Sines (Spectral, Frequency Domain Approach), sum response to each Sine
  - Calculate response to actual temporal (time history) loading using "impulse response function"

- Spectral Approach: given a periodic but non-harmonic excitation



$$p(\tau) = p(\tau + T)$$

Fourier Analysis

- Jean Fourier realized we can write loading p(t) as sum of average, cosines, & sines:

$$p(t) = a_o + \sum_{n=1}^{\infty} [a_n \cos(n\Omega_1 t) + b_n \sin(n\Omega_1 t)]$$

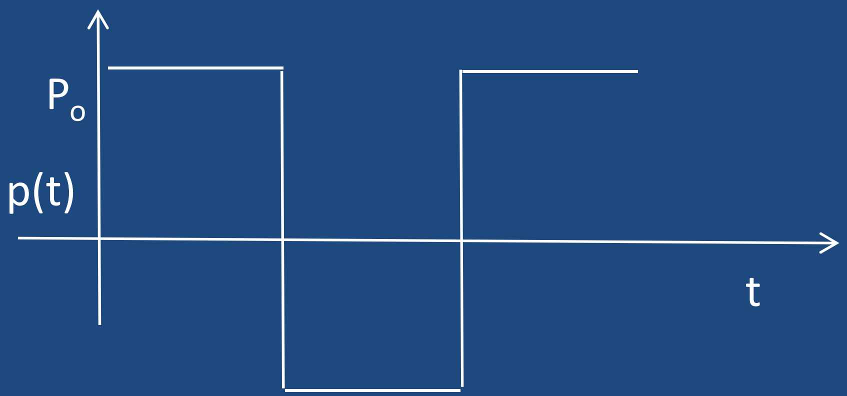
where

$$a_0 = \frac{1}{T_1} \int_{\tau}^{\tau+T} p(t)dt = \text{avg value of p(t)}$$

$$a_n = \frac{2}{T_1} \int p(t) \cos(n\Omega_1 t) dt$$

$$b_n = \frac{2}{T_1} \int p(t) \sin(n\Omega_1 t) dt$$

- Textbook Example: Using Fourier Series, represent square wave excitation as:



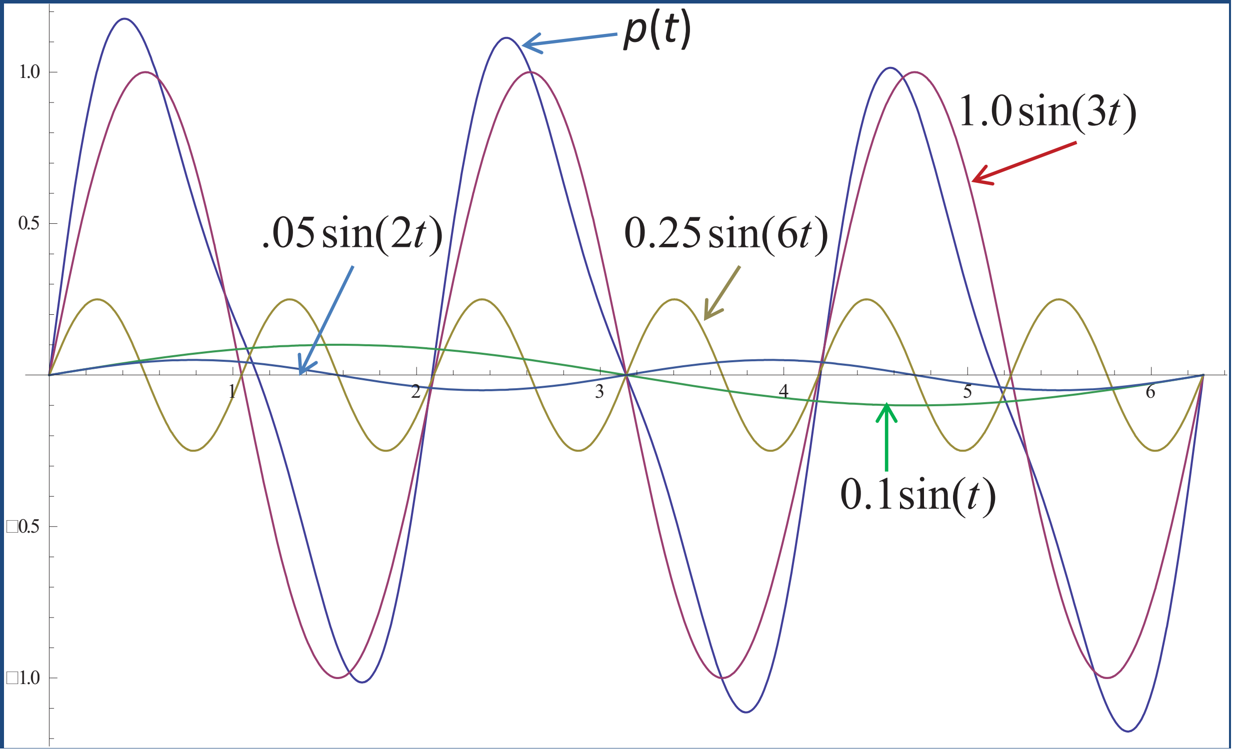
$$p(t) = \frac{4 p_0}{\pi} \sum_{n=1,3,5,\ldots} \left(\frac{1}{n}\right) \sin(n\Omega_1 t)$$

Engine Example of Application of Fourier Series

- Excitation wave based upon a pump with 3 primary distortions (e.g. diffusers), within slightly asymmetric overall field.
- Let primary excitation at 3N have an amplitude of 1, and asymmetric primary distortion have an amplitude of 0.1.
  - each of these will have a harmonic, since they aren't perfect sinusoidal distortions, such that the harmonic of the asymmetric is 0.05, and the amplitude of the harmonic of the primary distortion is 0.25.
  - So $p(t) = b_1 Sin(t) + b_2 Sin(2t) + b_3 Sin(3t) + b_6 Sin(6t)$

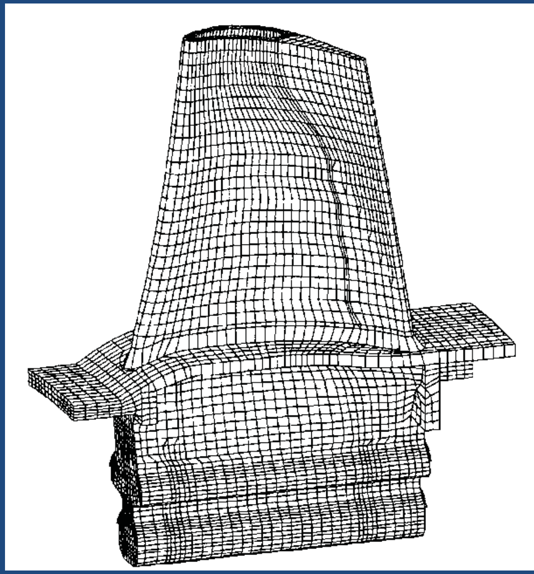
Where
b_1=0.1, b_2=0.05,
b_3=1.0, b_6=0.25

- Have to assess dynamics for each frequency component of excitation.

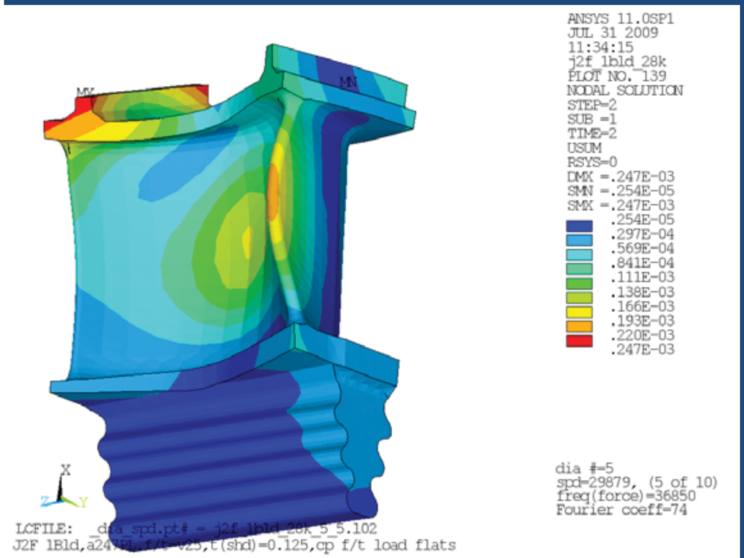


Now Structure: Create FEM of component, Modal Analysis

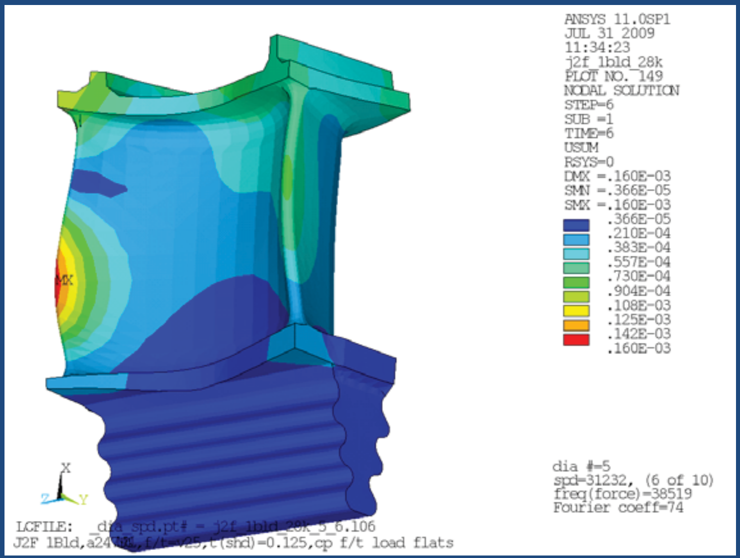
Example:
Turbine
Blades



Mode 12 at 36850 hz



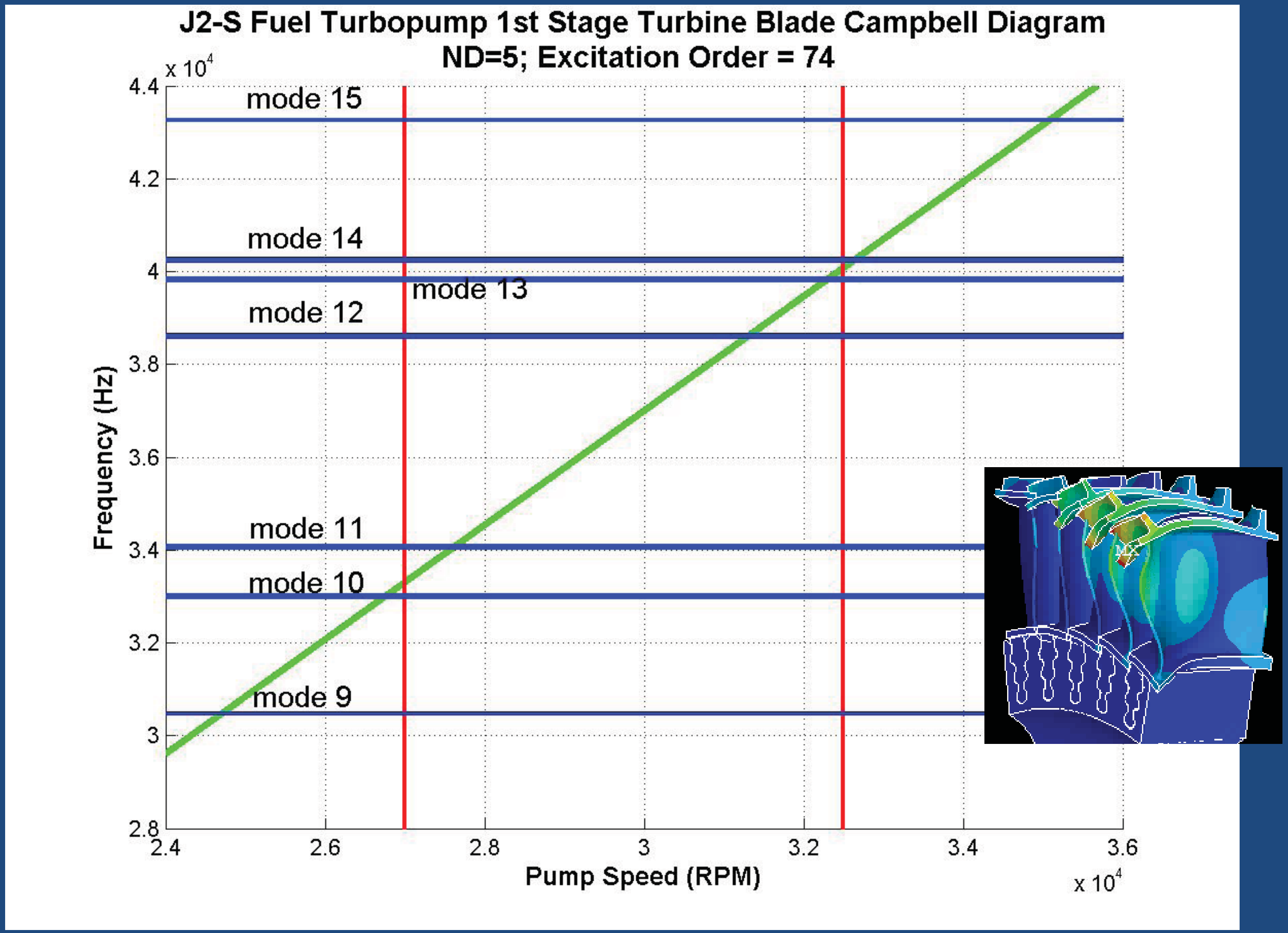
Mode 13 at 38519 hz



Modal Animations very Useful for Identifying problem modes, optimal damper locations

Create "Campbell Diagram"

- Simplest Version of Campbell Diagram is just a glorified Resonance Chart.

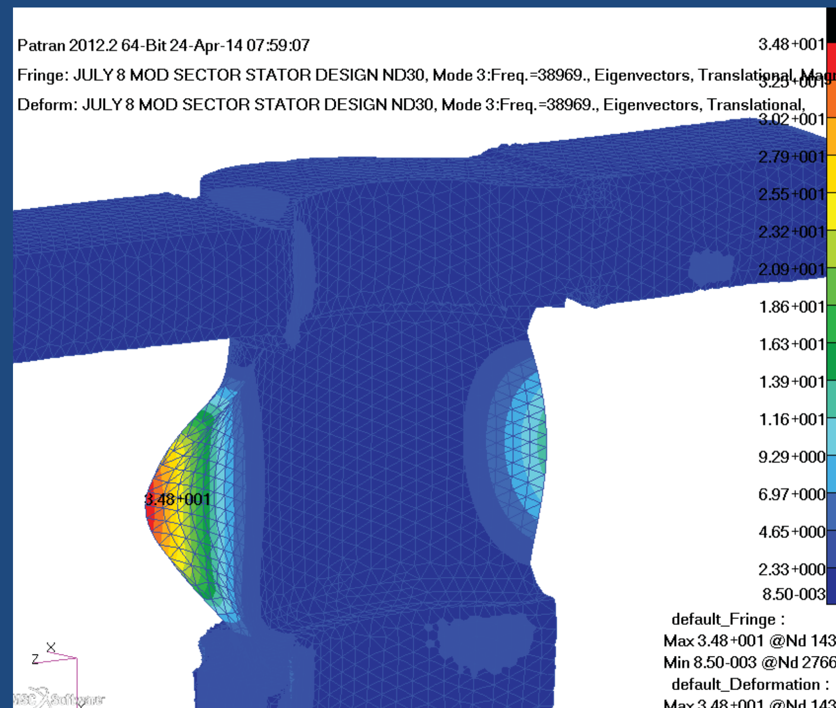


Modal Analysis has Multiple Uses

- Redesign Configuration to move excitations ranges away from natural frequencies
- Redesign component to move resonances out of operating range.
- Use as first step in "Forced Response Analysis" (applying forces and calculating structural response).

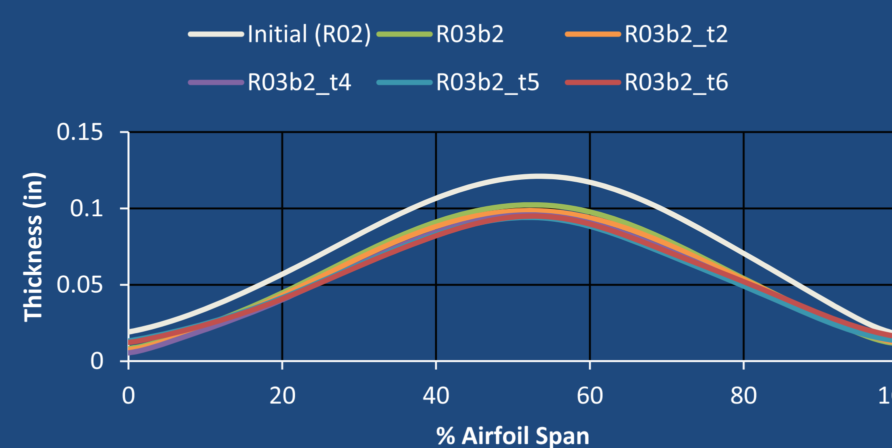
LPSP Turbine Stator Redesign to Avoid Resonance

- Modal analysis of original design indicated resonance with primary mode by primary forcing function.
  - Since excitation simultaneously from upstream and downstream blades, critical to change design to avoid resonance.
  - Extensive optimization effort performed to either move natural frequency out of range and/or change count of turbine blades to move excitation.



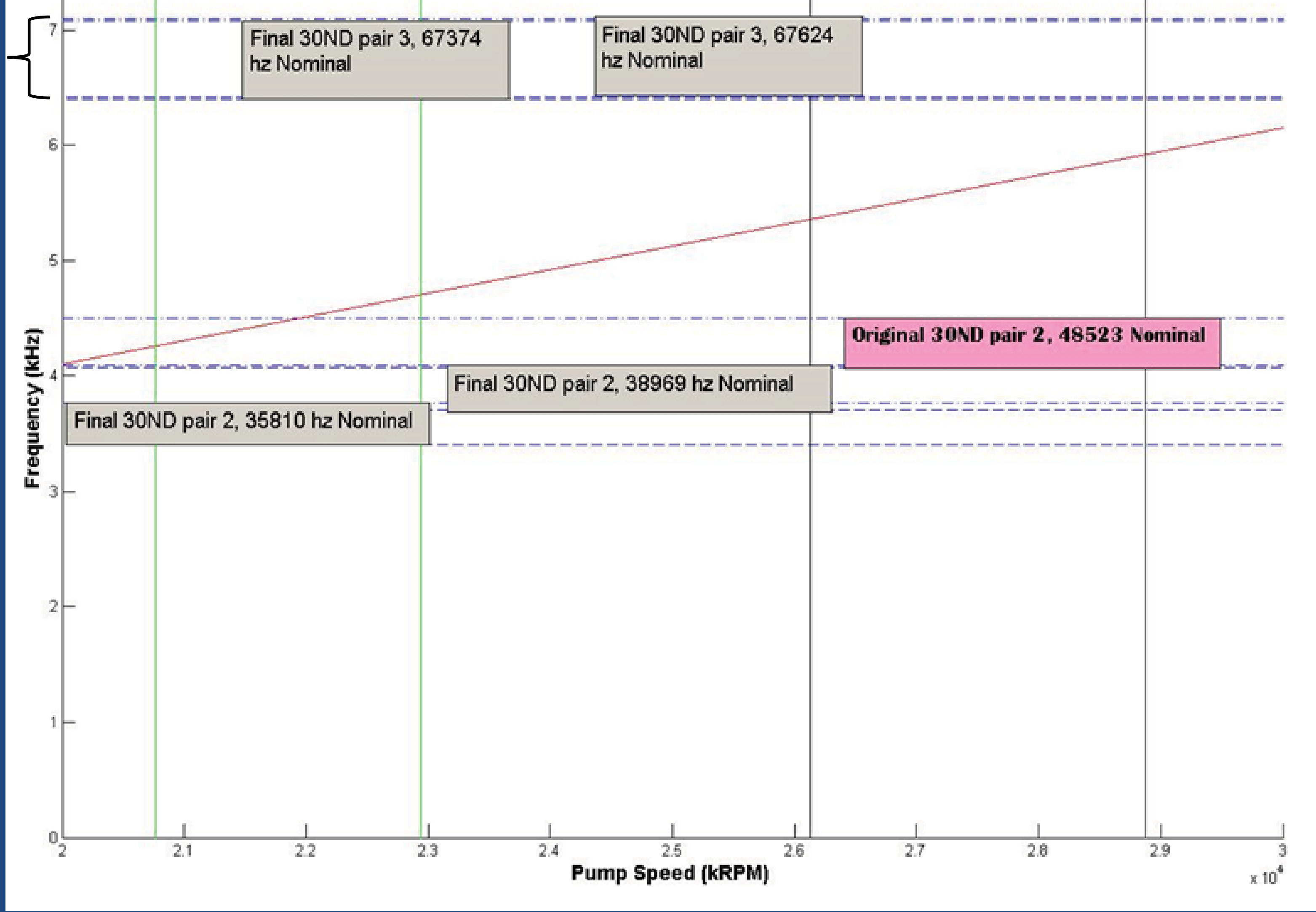
Courtesy D. O'N

Stator Airfoil Thickness Changes



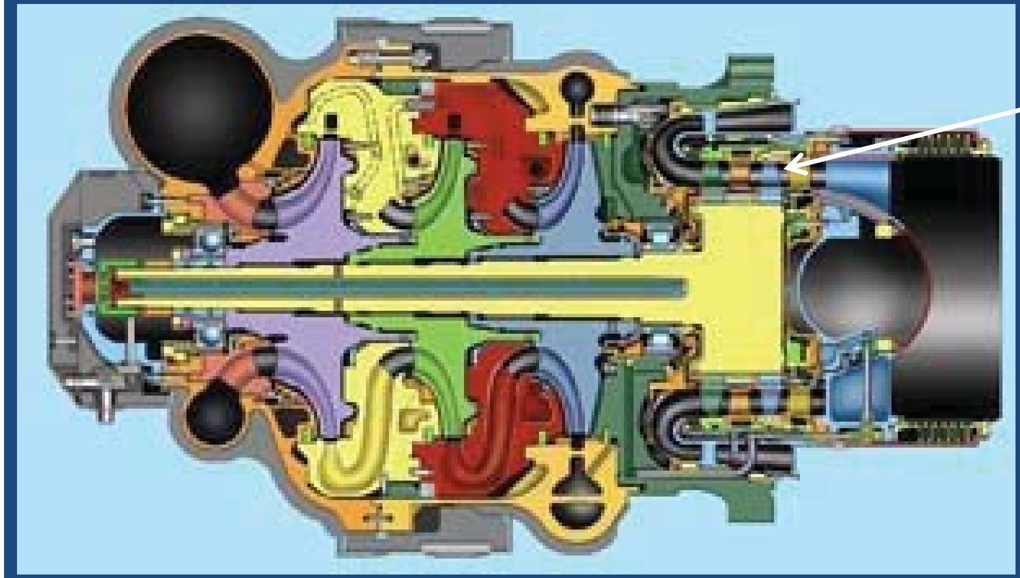
Final and Original Campbell of Modes for Stator Vane 30ND Family

Range of +/- 5% on natural frequencies to account for modeling uncertainty

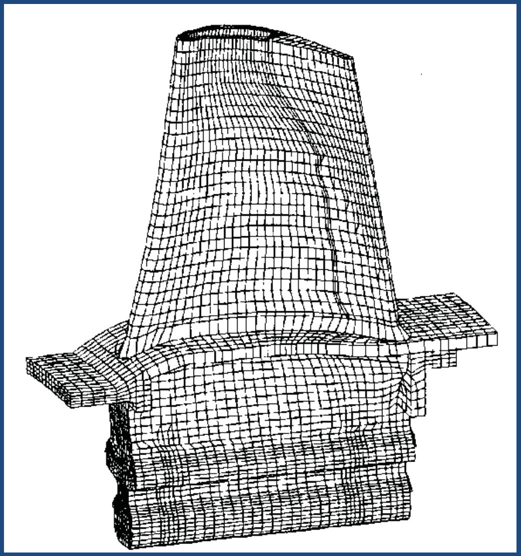


Can Also Use Modal Analysis in Failure Investigations

- Examination of *Modal Stress* Plots provides link to location of observed cracking.



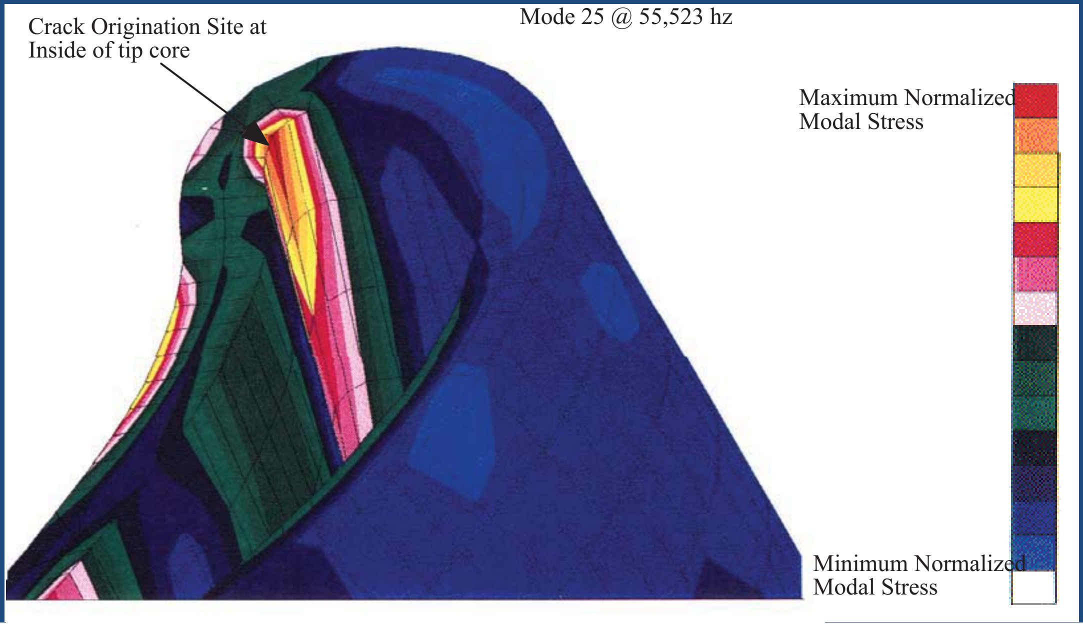
SSME
HPFTP
1st Stage
Turbine
Blade



Modal displacement

Modal stress

$$\phi^m = \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{Bmatrix}^m \rightarrow \phi_\sigma^m = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_N \end{Bmatrix}^m$$



Now, if resonance, forced response required, need to know about
Generalized Coordinates/Modal Superposition

- Frequency and Transient Response Analysis uses Concept of Modal Superposition using Generalized (or Principal Coordinates).

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{P(t)\}$$

- <u>Mode Superposition Method</u> – transforms to set of uncoupled, SDOF equations that we can solve using SDOF methods.
- First obtain $[\Phi]_{mass}$. Now, introduce coordinate transformation:

$$\{u\} = {}_{N}\overset{M}{[\Phi]}\{\eta\}_{M}$$

$$[M][\Phi]\{\ddot{\eta}\}+[C][\Phi]\{\dot{\eta}\}+[K][\Phi]\{\eta\}=\{P(t)\}$$

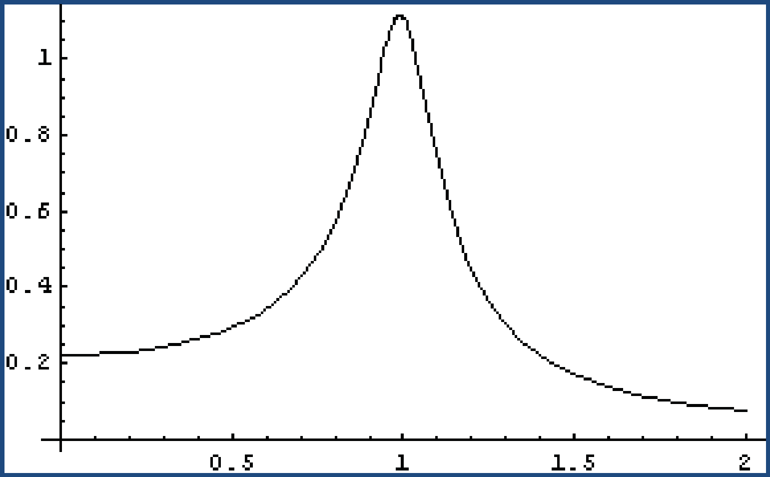
⇓

$$\left[{}^{\backslash}I_{\backslash}\right]\{\ddot{\eta}\}+\left[{}^{\backslash}\mathscr{C}_{\backslash}\right]\{\dot{\eta}\}+\left[{}^{\backslash}\kappa_{\backslash}\right]\{\eta\}=[\Phi]^{T}\{P(t)\}.$$

for the SDOF equation of motion,

$$m\ddot{u}+c\dot{u}+ku=F \rightarrow \ddot{u}+2\varsigma\omega\dot{u}+\omega^2 u=F$$

$$|U(\Omega)| = \frac{Fo}{k}\sqrt{\frac{1}{\left(1-\left(\frac{\Omega}{\omega}\right)^2\right)^2+\left(2\zeta\left(\frac{\Omega}{\omega}\right)\right)^2}}$$



$r = \frac{\Omega}{\omega}$

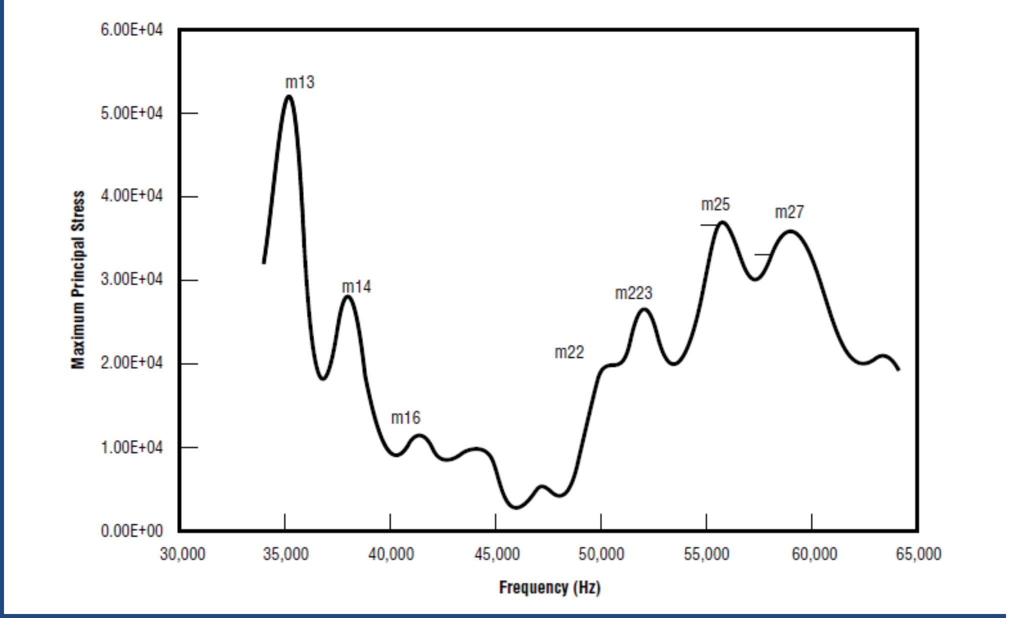
So we get the same equations in η:

$$\ddot{\eta}_m + 2\zeta_m\omega_m\dot{u}_m + \lambda_m\eta_m = \{\phi\}_m^T\{P(t)\}$$

$$|\eta_m(t)| = \frac{\{\phi\}_m^T\{F\}}{\lambda_m}\frac{1}{\sqrt{\left(1-\left(\frac{\Omega}{\omega_m}\right)^2\right)^2+\left(2\zeta_m\frac{\Omega}{\omega_m}\right)^2}}$$

- For "Frequency Response" Analysis, apply Fourier coefficients coming from CFD such that excitation frequencies match Campbell crossovers.

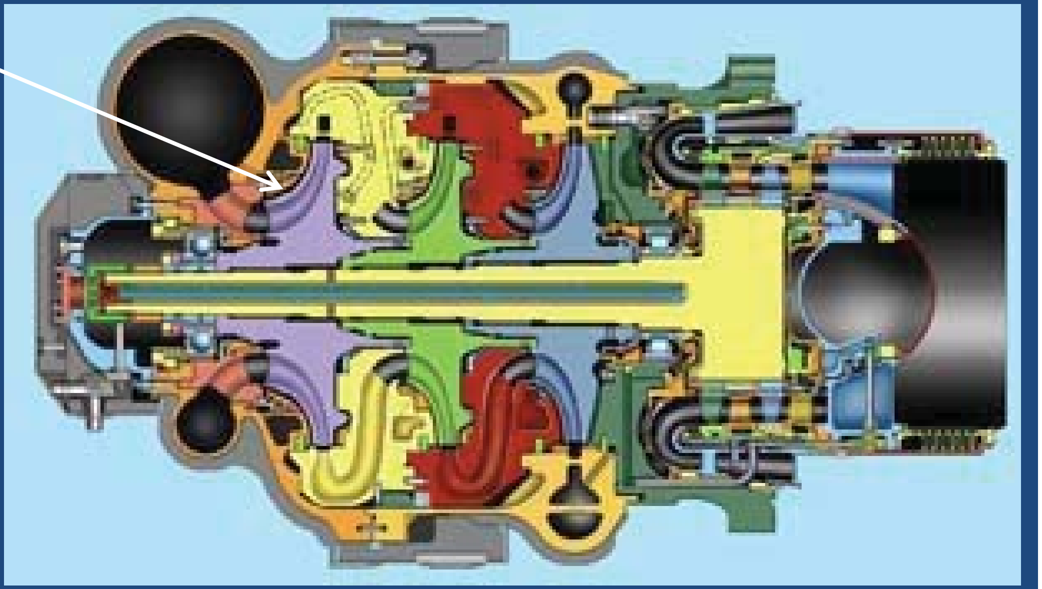
SSME HPFTP 1st Blade Frequency Response

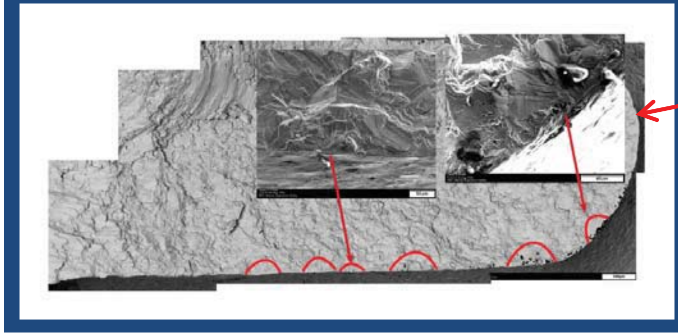


Modal Analysis in Failure Investigations (2)

- SSME HPFTP 1st Stage Impeller.

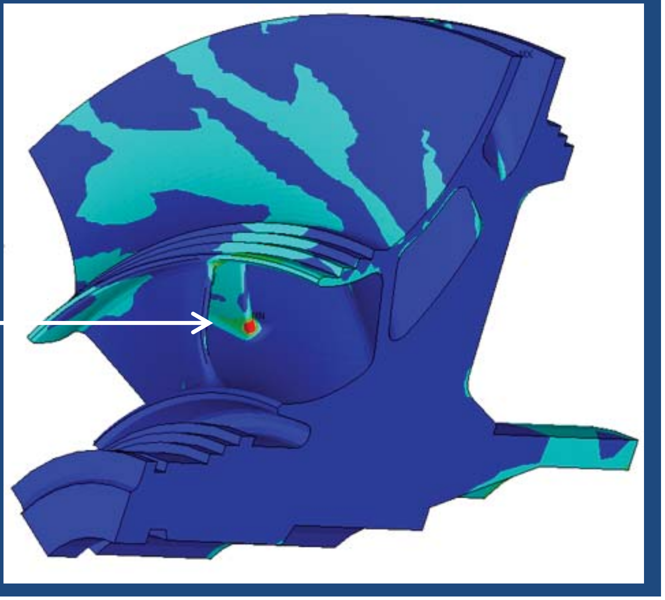
Mode shape





Crack location 1st splitter

Frequency Response Analysis



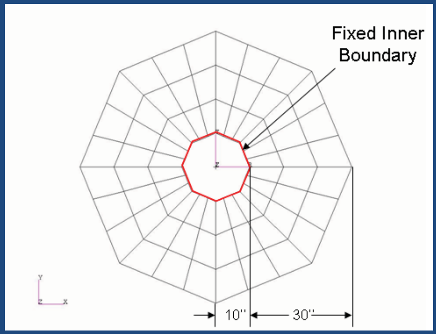
Cyclic Symmetry in Turbine Components

- Many structures of importance to engineering possess some kind of symmetry that can be used to simplify their analysis.
- A cyclically symmetric structure possesses rotational symmetry, i.e., the original configuration is obtained after the structure is rotated about the axis of symmetry by a given angle.
- Instead of modeling entire structure, only model one sector.
- For turbomachinery structures, structural analysis is generally only possible by taking advantage of huge reduction in model size by using cyclic symmetry.

Section courtesy of Dr. Eric Christensen, DCI Inc.

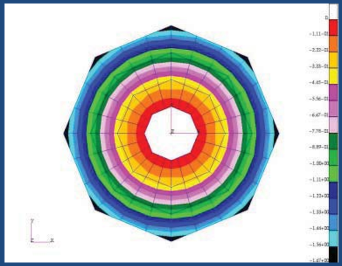
Characteristics of Cyclic Symmetric Modes

- Most Nodal Diameter modes exist in pairs, same shape but rotated by π/ND

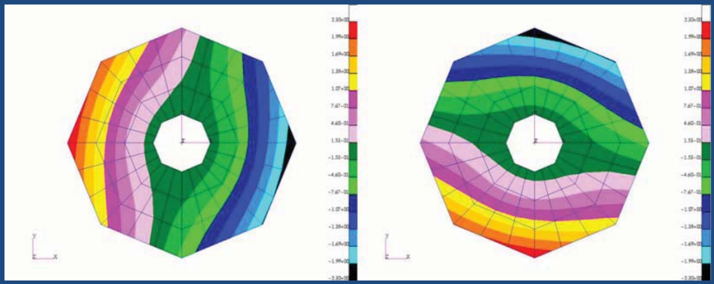


Flat Plate Example
N = 8 Segments

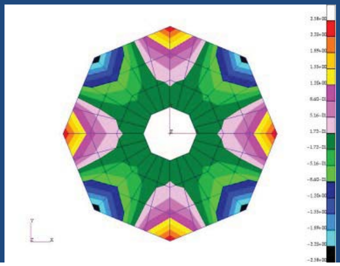
- First family of modes:
  - Has Unique eigenvalues
  - Has Unique eigenvectors
  - All segments have same mode shape



- The next family of modes:
  - Pairs of degenerate eigenvalues
  - Non-unique eigenvectors



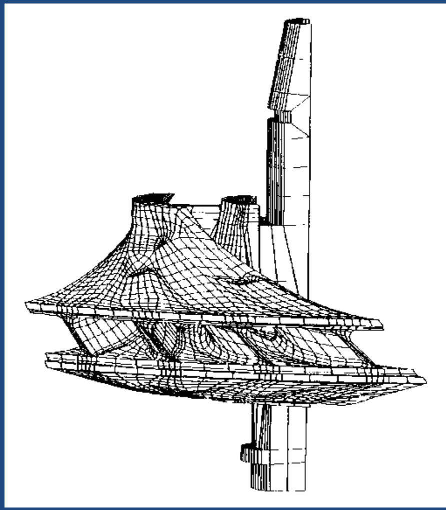
- The last family of modes:
  - Only exist if N is even
  - Has unique eigenvalues & eigenvectors.



Turbomachinery Component Example

SSME Fuel Turbopump 3rd Stage Impeller

- Cyclically Symmetric Structure with N = 6
- Modes and Frequencies calculated for all harmonics



Mode	Natural Frequencies (Hz)			
	Harmonic 0	Harmonic 1	Harmonic 2	Harmonic 3
1	3,710	4,210	5,180	4,900
2	8,360	4,210	5,180	6,530
3	9,780	7,190	5,750	6,850
4	10,100	7,190	5,750	10,400
5	10,500	8,740	7,900	11,500
6	11,200	8,740	7,900	13,200
7	12,100	9,670	10,000	14,300
8	13,500	9,670	10,000	14,900
9	16,200	11,200	11,500	15,200
10	17,500	11,200	11,500	15,900
11	17,800	12,500	13,300	16,000
12	18,000	12,500	13,300	16,500
13	18,600	13,000	14,000	18,900
14	18,900	13,000	14,000	19,300
15	19,500	14,700	14,400	19,600
16	19,800	14,700	14,400	19,900
17	20,300	15,400	15,400	21,400
18	21,000	15,400	15,400	21,500
19	21,300	16,200	15,500	22,200
20	21,700	16,200	15,500	23,200

Implications of Cyclic Symmetry - Generalized Force

- Generalized (or Modal) Force defined as

$$\{\mathfrak{F}\}_m = \{\Phi\}_m^T \{F\}.$$

- This is just the dot product of each mode with the excitation force vector and means that the response is directly proportional to the similarity of the spatial shape of each mode with the spatial shape of the force.

- For pure harmonic waves, the "Orthogonality Principle" states

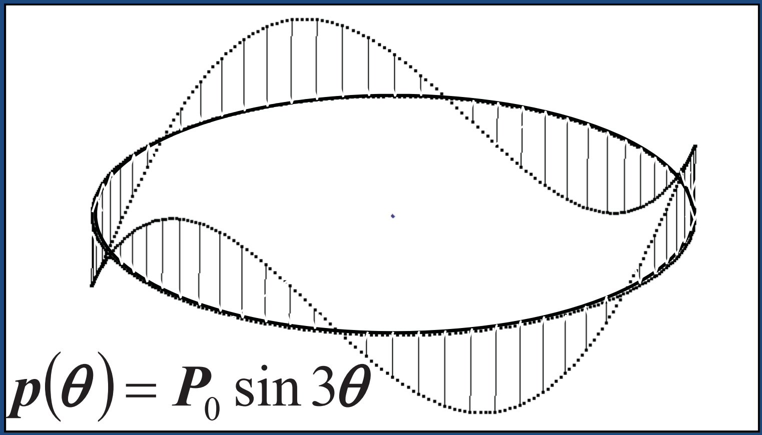
$$\int_{-\pi}^{\pi} \sin(n\theta)\sin(m\theta) = \begin{cases} \pi \text{ when n=m} \\ 0 \text{ otherwise} \end{cases}$$

- Think of the {P} as a continuous function, and the force the same way.
  - Then the Dot Product is the same as an integration of the product of the two functions.
  - So this says that the only non-zero result of an excitation wave shaped like a Sine and a mode shaped like a Sine is for the components of those waves that have the same wave number!

Have to determine Nodal Diameter of Modes to identify Resonance

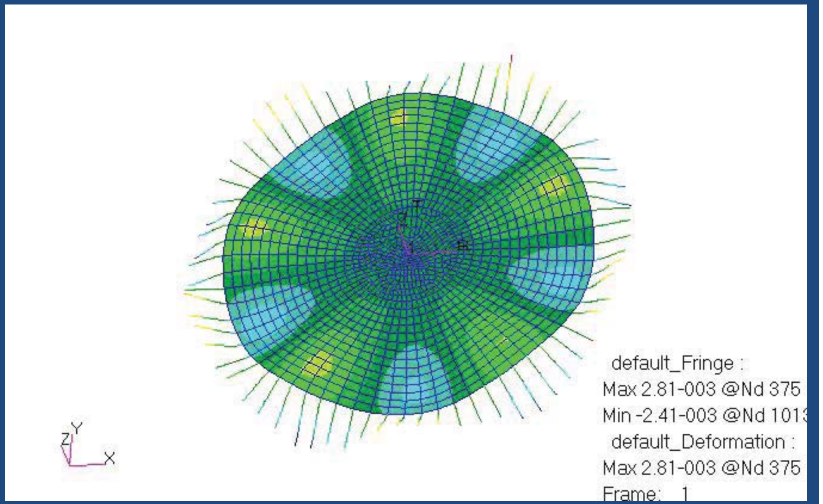
5ND Traveling Wave will excite a 5ND mode, while a 93 might (next section), but a 3 won't!

93 stationary 2[nd] stage stator vanes.



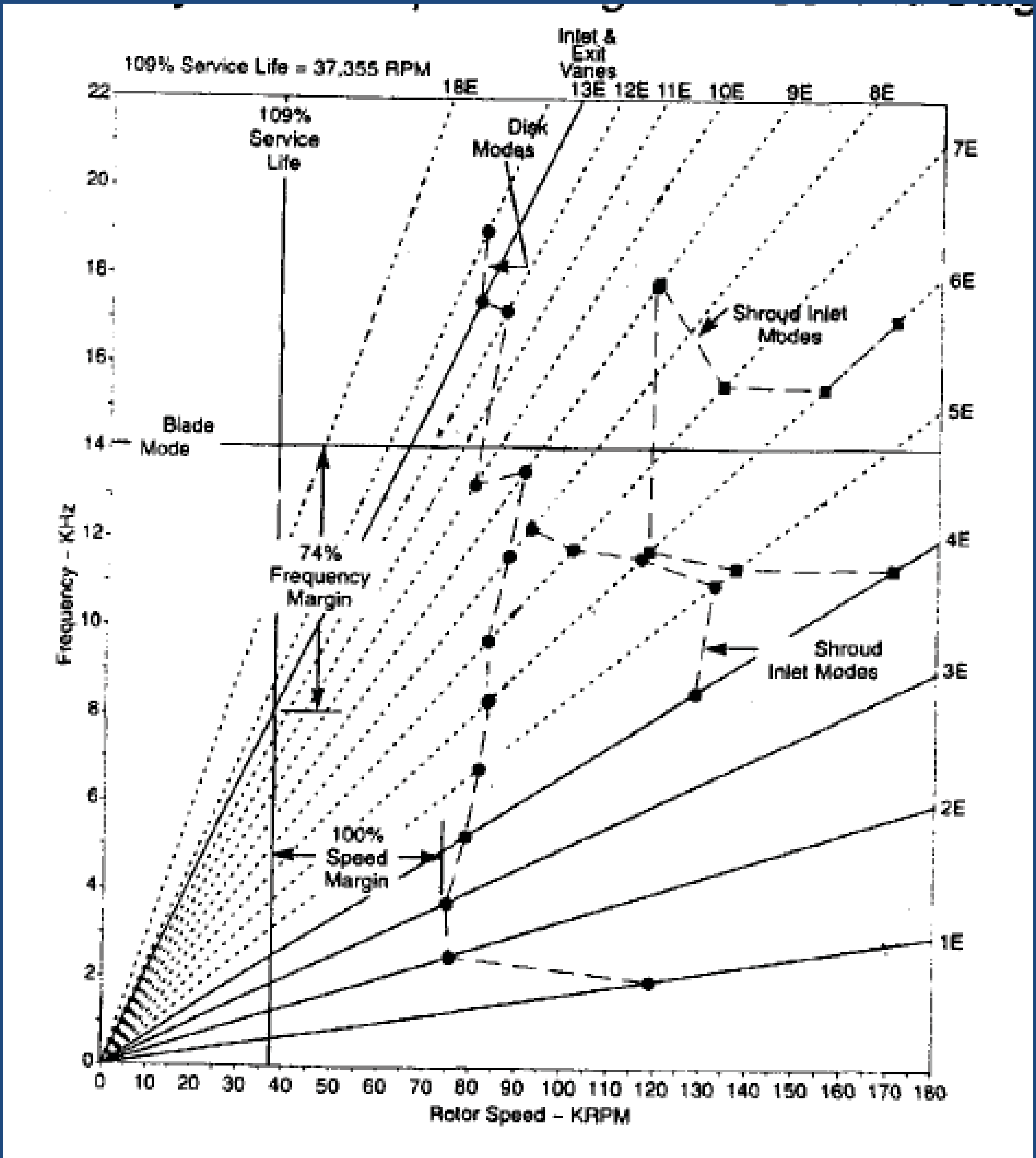
5ND mode of Impeller (modal test using holography)

5ND Mode of Bladed-Disc



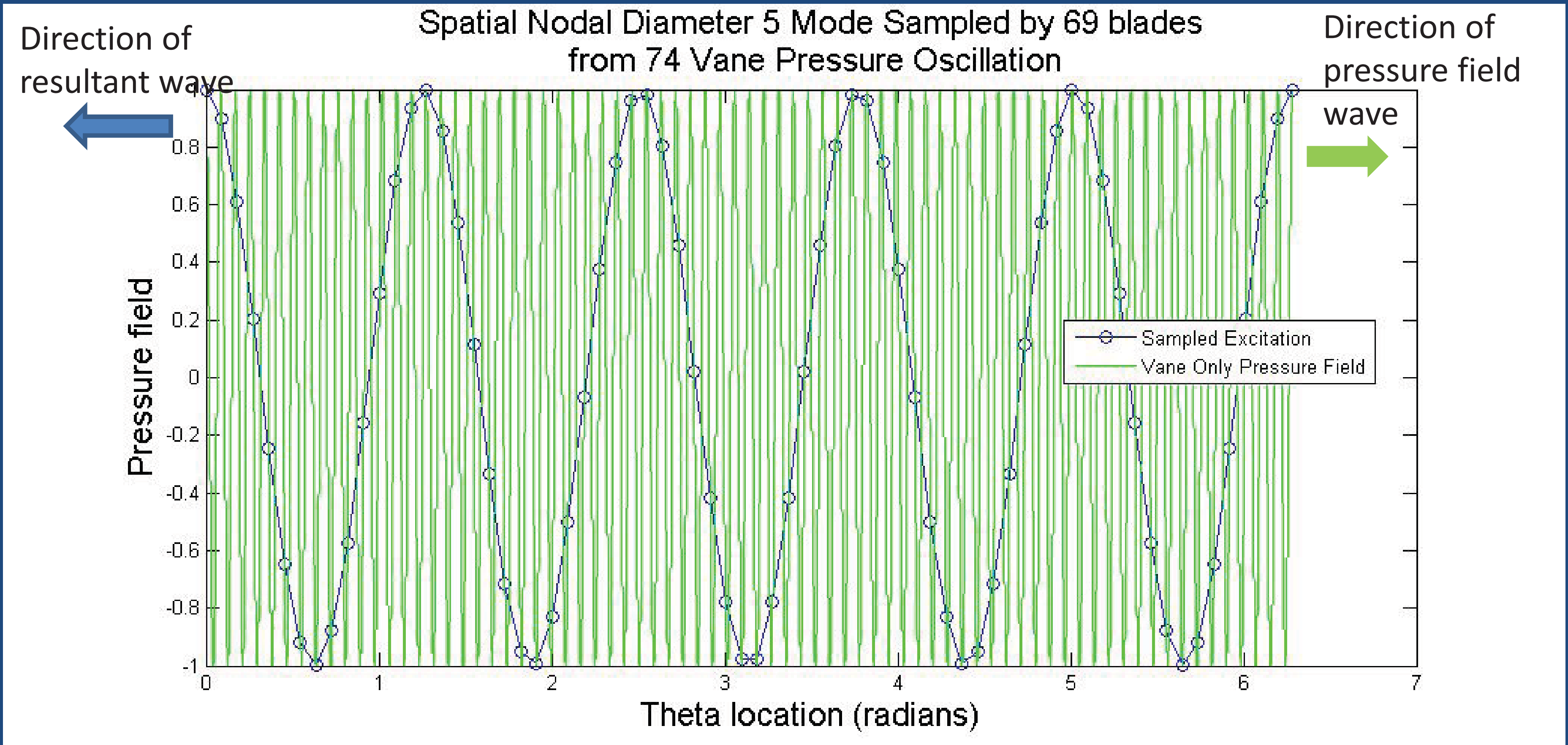
Impeller Campbell Diagram

- “Triple Crossover Points” (speed, ω, and ND) needed for resonance of pure shroud (disk) modes.
- ND mode at exact spatial number of distortions in excitation.



"Blade/Vane" Interaction causes ND excitation

- Sampling by discrete number of points on structure of pressure oscillation results in spatial Nodal Diameter excitation at the difference of the two counts.
- E.g., a 74 wave number pressure field (coming from 74 vanes), exciting 69 blades results in a Nodal Diameter mode of 69-74=-5, where sign indicates direction of traveling 5ND wave (*plot courtesy Anton Gagne*).



Tyler–Sofrin Blade–Vane Interaction Charts

- Chart identifies Nodal Diameter families that can be excited
- All modes in Campbell have to be from these families
  - E.g., Nodal Diameter 5, blade mode 3 (torsion)

Upstream Nozzle Multiples	37	74	111	148		Downstream Stator Multiples	57	114	171	228
Blade multiples						Blade multiples				
69	32	-5	N/A	N/A		69	12	N/A	N/A	N/A
138	N/A	N/A	27	-10		138	N/A	24	-33	N/A
207	N/A	N/A	N/A	N/A		207	N/A	N/A	N/A	-21

- Temporal Frequency of Excitation is at the engine order of the distortion.
- Much of chart is marked “N/A – not applicable” because…..
  - Highest number of ND waves in a cyclic symmetric structure is N/2 or (N/2)-1

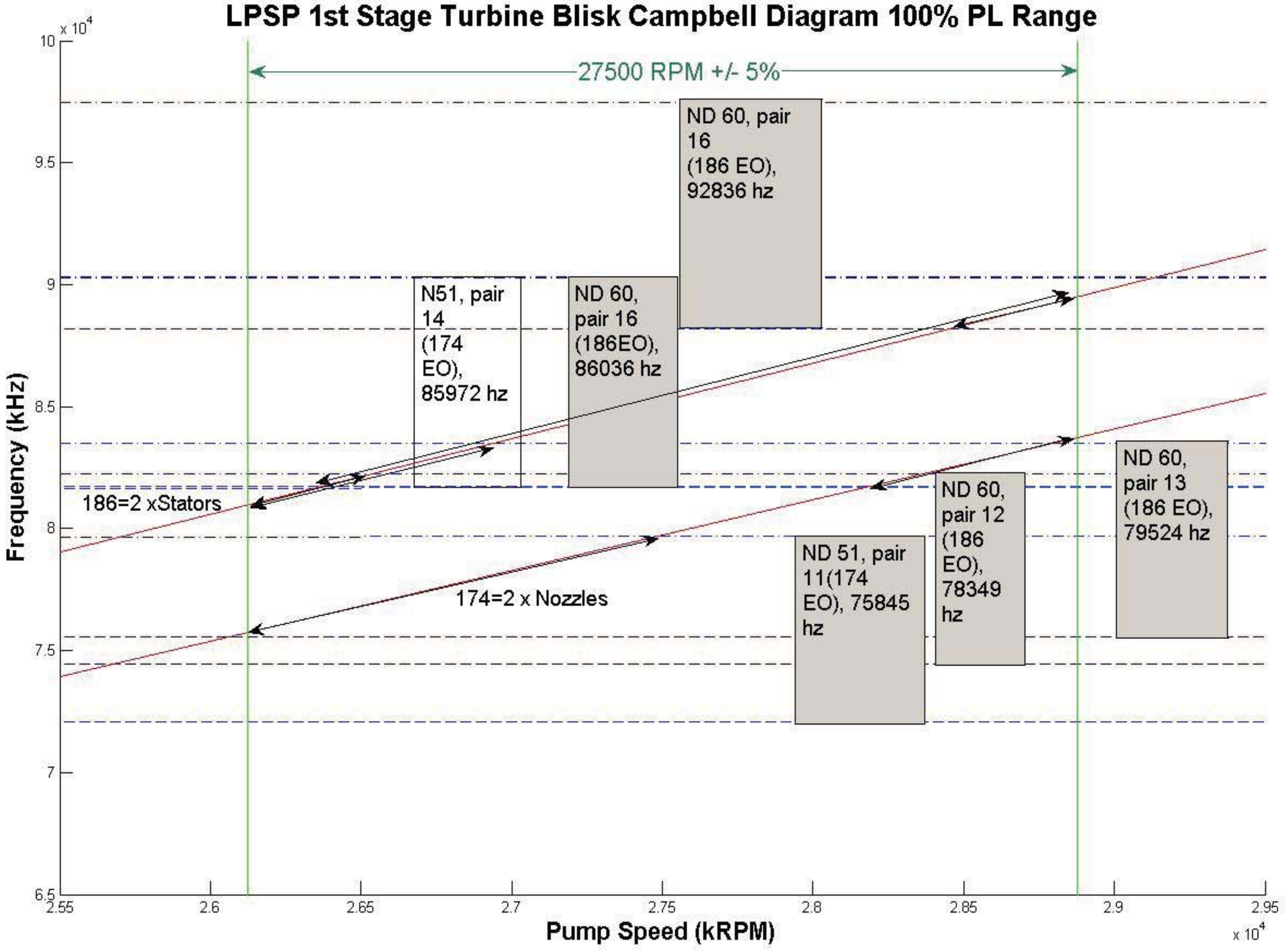
Example – LPSP Turbine Blisk Aliasing Tables, Non-Problematic Modal Evaluation

- 123 blades allow (123-1)/2=61 Nodal Diameters

	1st Stage Nozzles			2nd Stage Stators			Exit Guide Vanes		
	87	174	261	93	186	279	97	194	291
blades									
123	**36**	**51**	138	**30**	63	156	**26**	71	168
246	159	72	**15**	153	**60**	**33**	149	**52**	**45**
369	282	195	108	276	183	90	272	175	78
0	87	174	261	93	186	279	97	194	291

- Many modes in these ND's, including crossings judged non-problematic
  - 15ND, 33ND, and 45ND crossing modes eliminated due to probable low 3X forcing function, high frequency
  - 52ND modes extremely complicated, high frequency
  - Many modes eliminated due to non-adjacency of forcing function
- No 1-6ND modes have crossings with appropriate forcing function.

Turbine Blisk Campbell for Problematic Modes in 100% PL Range

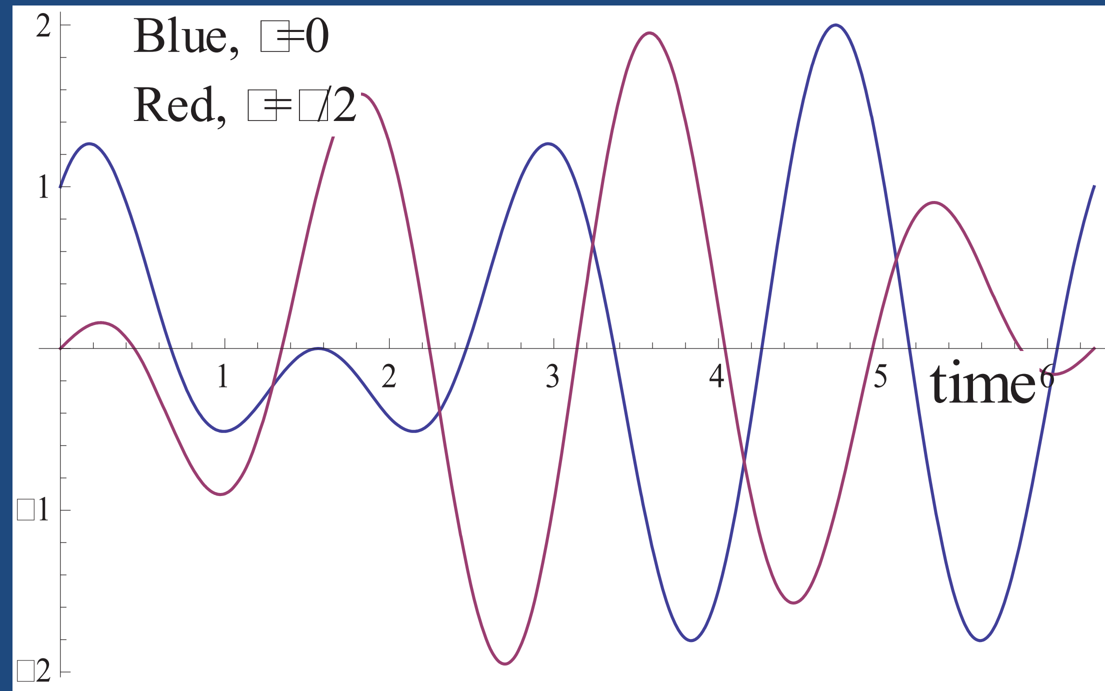


Turbine Blisk Problematic Modes Possible Resolutions

		Natural Frequencies				
ND	mode pair number	70% PL	100% PL	mode shape description	excitation source and order	potential solution
36	5	**28955.47**	28964.54	1st Torsion NOTE 12/31/13- 2ND blade -are nozzles problem?	87=1 x 1st st upstream nozzles	change # nozzles to 89 to move lower bound of 70% above mode.
51	9	**58040.23**	58048.62	1st blade chordwise 2nd bending	174= 2 x 1st st upstream nozzles	change # nozzles to 89 to move lower bound of 70% above mode.
51	11	75845.08	**75878.04**	1st blade spanwise TE 2 wave	174= 2 x 1st st upstream nozzles	CFD to determine magnitude of 2x
60	12	78335.00	**78349.16**	2nd blade bending	186 = 2 x upstream 2nd st stator	CFD to determine magnitude of 2x
60	13	79511.67	**79524.37**	2nd blade ?	186 = 2 x upstream 2nd st stator	CFD to determine magnitude of 2x
51	14	85944.41	**85972.20**	1st blade TE spanwise 1 wave	174= 2 x 1st st upstream nozzles	CFD to determine magnitude of 2x
60	14	86007.76	**86035.52**	1st blade TE 1.5 wave	186 = 2 x downstream 2nd st stator	CFD to determine magnitude of 2x
60	16	92827.38	**92835.74**	1st blade TE 0.5 wave	186 = 2 x downstream 2nd st stator	change # stators to 92, mode will be above range

Spatial Fourier Analysis helpful to identify ND number of both excitation and modes

$$s = \mathrm{Sin}[3t + 2\theta] + \mathrm{Cos}[4t + 3\theta]$$



First perform Temporal Fourier Analysis, using Complex Form

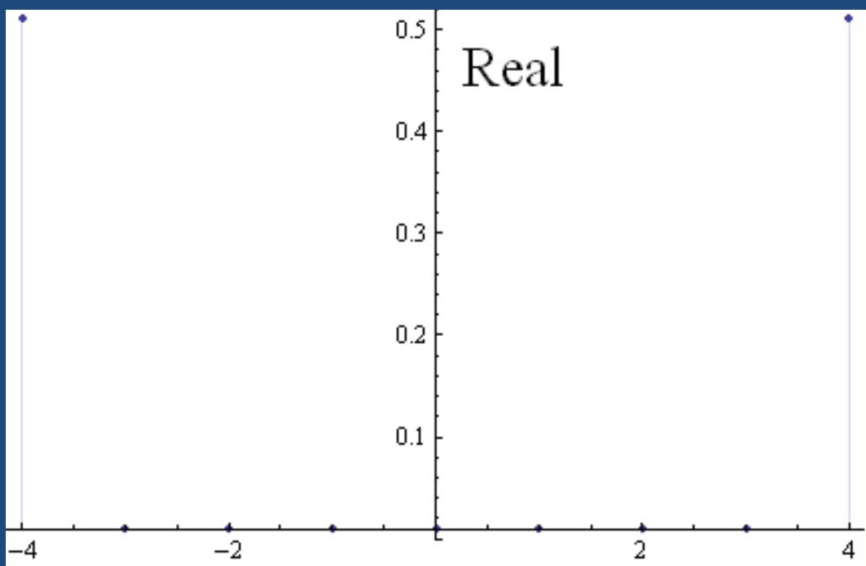
$$p(t) = \sum_{n=-\infty}^{\infty} [c_n e^{in\Omega_1 t}]$$

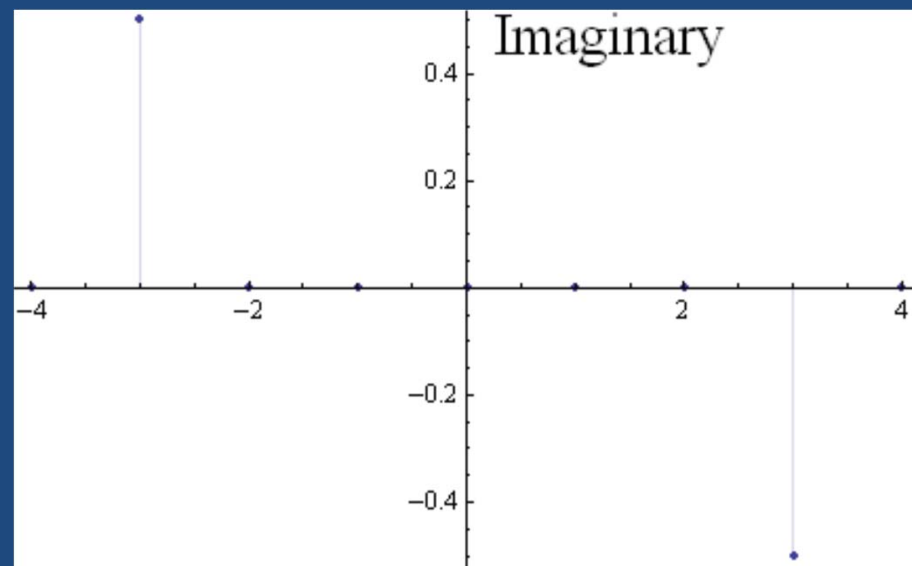
where

$$c_n = \frac{1}{T_1} \int_0^T p(t) e^{in\Omega_1 t} dt$$

Rules relating Complex to Harmonic Fourier Coefficients

1) Since $e^{ix}=\cos(x)+I\sin(x)$, imaginary part of coefficients is sine, and real part is cosine;
2) Since cosine is a symmetric function, cosine amplitude= Amp(Re (-n)) + Amp (Re(+n);
3) Since sine is antisymmetric, the sine amplitude = Amp (Im (-n) - Amp (Im(+n)).





- So we have
  - Amplitude of cos(4t)=0.5+.05 =1.

  and
  - Amplitude of sin(3t)= 0.5- (-0.5)=1

Spatial Decomposition of Each Temporal Fourier Component

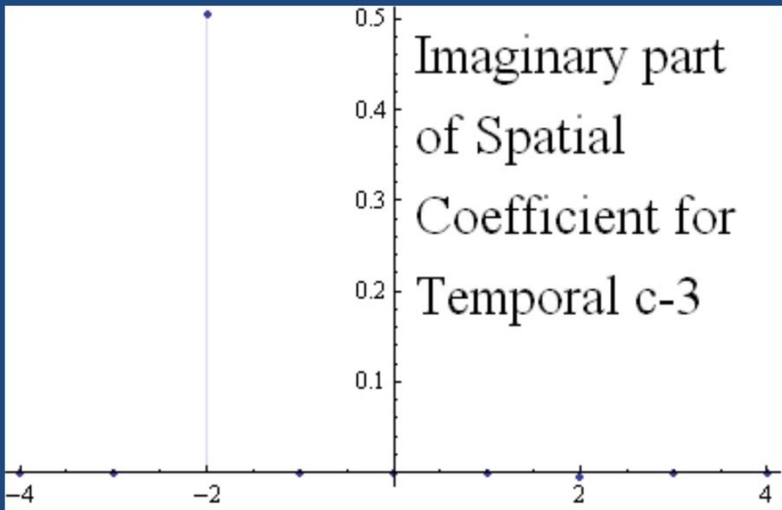
- Now look at obtaining spatial components of the temporal fourier components for every dof; i.e, the spatial components at each frequency, or “temporal bin”.

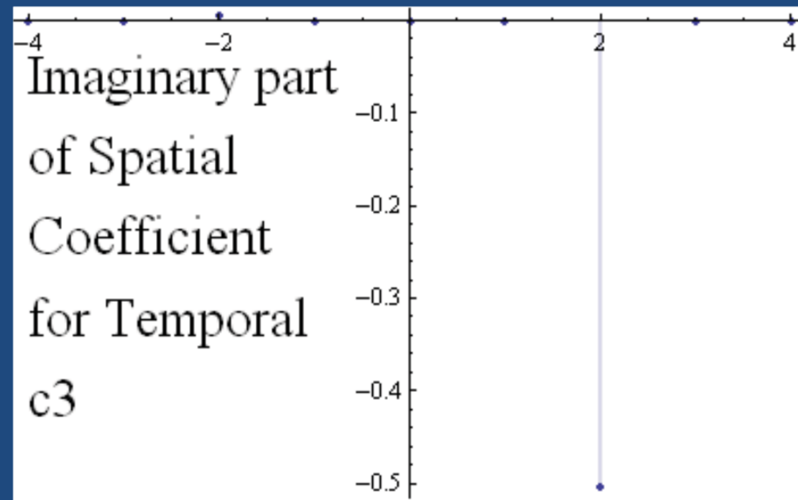
$$p(t) = \sum_{n=-\infty}^{\infty} [c_n e^{in\Omega_1 \theta}]$$

where

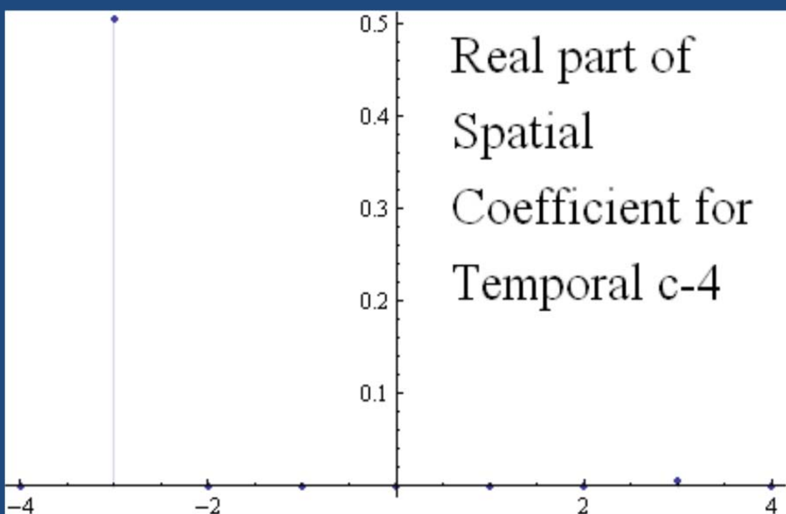
$$c_n = \frac{1}{2\pi} \int_0^{2\pi} p(t) e^{in\Omega_1 \theta} d\theta$$

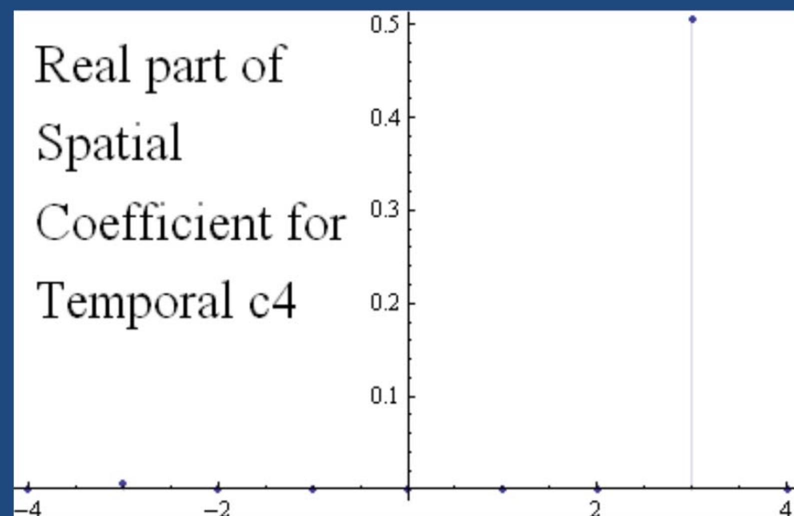
$$s = \mathrm{Sin}[3t + 2\theta] + \mathrm{Cos}[4t + 3\theta]$$





For Sin(3t), |sin(2θ)|= .5- (-.5)=1

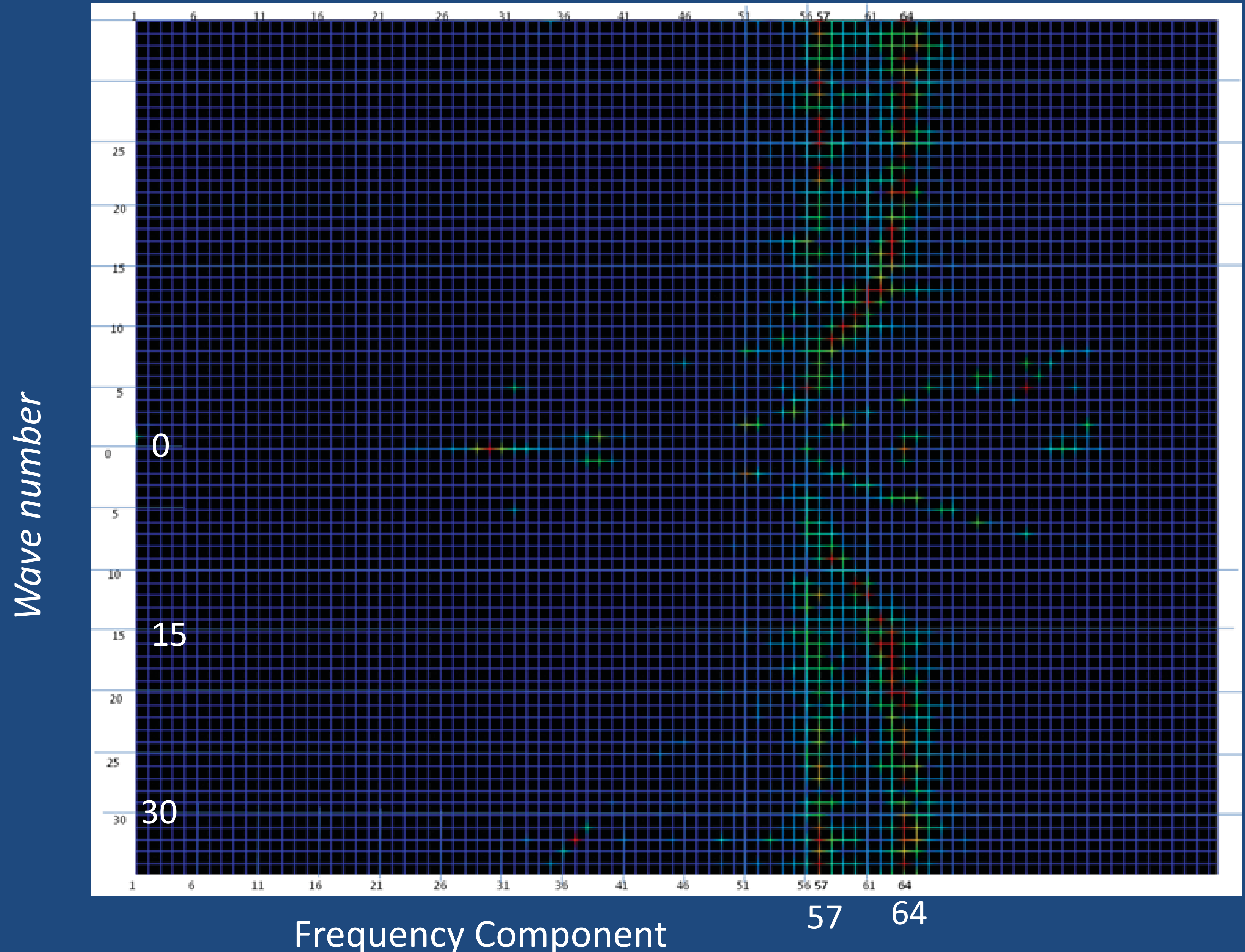




For Cos(4t), |cos(3θ)|= .5+ .5=1

- Sign of the θ parts indicate the direction of the spatially traveling waves.

2-D Fourier Transform Shows Spatial Complexity of J2X turbine flow field and response, used in evaluation of forced response methodologies



Conclusion

- Structural Dynamic Analysis of Turbomachinery is critical aspect of design, development, test, and failure analysis of Rocket Engines.
- Process of Analysis starts consists of modeling, modal analysis and characterization, comparison with excitation field, and forced response analysis if necessary.
- Thorough understanding of Fourier Analysis, Vibration Theory, Finite Element Analysis critical.
- Knowledge of Turbomachinery Design and Fluid Dynamics very useful.

Merci Beaucoup!