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Interpolation Method needed for Numerical Uncertainty Analysis of Computational Fluid Dynamics

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Problem

- Using Computational Fluid Dynamics (CFD) to predict a flow field is an approximation to the exact problem and uncertainties exist.
- There is a method to approximate the errors in CFD via Richardson's Extrapolation.
  - This method is based off of progressive grid refinement.
- Unless using a Structured Grid with every other point, some interpolation method must be used.

Summary of Richardson's Extrapolation

- Navier Stokes Equations
  - 2nd order, non-homogeneous, non-linear partial differential equations
- Richardson's Extrapolation is used to produce 4th order accurate solution from separate 2nd order accurate Navier Stokes Solutions

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Summary of Richardson's Extrapolation

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- ASME V&V 20-2009 Outlines a 5-step Procedure to Richardson's Extrapolation using Roache's (1998) Grid Convergence Index (GCI) Method
- Assumptions
  1. Three discrete solutions are in the asymptotic range
  2. Meshes have a uniform spacing over the domain
  3. Meshes are related through systematic refinement
  4. Solutions are smooth
  5. Other sources of numerical error are small

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ASME V&V 20-2009 5-step Procedure to Richardson's Extrapolation

- 5 Step Procedure for Uncertainty Estimation
  - Step 1: Representative Grid Size

$$h = \left[\left(\sum_{i=1}^{N} \Delta V_i \right) / N \right]^{1/3}$$

where
N = total number of cells used for the computations
ΔV_i = volume of the i^{th} cell [4]

$$h_1 < h_2 < h_3$$

ASME V&V 20-2009 5-step Procedure to Richardson's Extrapolation

– Step 2: Select 3 significantly (r>1.3) different grid sizes

$$r_{21} = h_2/h_1$$
$$r_{32} = h_3/h_2$$

– Use CFD Simulation to analyze key variables, φ

$$\varepsilon_{32} = \varphi_3 - \varphi_2$$

$$\varepsilon_{21} = \varphi_2 - \varphi_1$$

ASME V&V 20-2009 5-step Procedure to Richardson's Extrapolation

– Step 3: Calculate observed order, p

$$p = \left[1/\ln(r_{21})\right]\left[\ln\left|\varepsilon_{32}/\varepsilon_{21}\right| + q(p)\right]$$

$$q(p) = \ln\left(\frac{r_{21}^{p} - s}{r_{32}^{p} - s}\right)$$

$$s = 1 \cdot \mathrm{sign}(\varepsilon_{32}/\varepsilon_{21})$$

ASME V&V 20-2009 5-step Procedure to Richardson's Extrapolation

– Step 4: Calculate extrapolated values

$$\varphi_{\text{ext}}^{21} = (r_{21}^{p}\varphi_1 - \varphi_2)/(r_{21}^{p} - 1)$$

$$e_a^{21} = \left|\frac{\varphi_1 - \varphi_2}{\varphi_1}\right|$$

ASME V&V 20-2009 5-step Procedure to Richardson's Extrapolation

– Step 5: Calculate Fine Grid Convergence Index & Numerical Uncertainty

$$\mathrm{GCI}_{\mathrm{fine}}^{21} = \frac{Fs \cdot e_a^{21}}{r_{21}^{p} - 1}$$

The Factor of Safety, $Fs = 1.25$

– Assumption that the distribution is Gaussian about the fine grid, 90% Confidence

$$U_{num} = GCI/1.65$$

Solver Interpolation

- FLUENT
  - Includes a Mesh-to-Mesh Interpolation
  - Performs a zeroth-order (nearest neighbor) interpolation
  - Designed for initial conditions from a previous solution
- OPENFOAM
  - Mapfields fuction interpolation
  - Used for initialization of a solution from a previous model
- Using these ‘zeroth-order’ interpolation schemes is not sufficient for comparing errors from the mesh

Matlab Interpolation Schemes

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- Matlab
  - High level language used for numerical computations
- CFD data is in various forms
  - 1D, 2D, 3D, uniform, non-uniform
  - Generic Scheme is sought for all CFD data

	Matlab Function		
Interpolation Method	interp1	interp2	interp3
'nearest' – Nearest neighbor interpolation	X	X	X
'linear' – Linear interpolation (default)	X	X	X
'spline' – Cubic spline interpolation	X	X	X
'pchip' – Piecewise cubic Hermite interpolation	X		
'cubic'	X	X (uniformly-spaced only)	X (uniformly-spaced only)
'v5cubic' – cubic interpolation used in Matlab 5	X		

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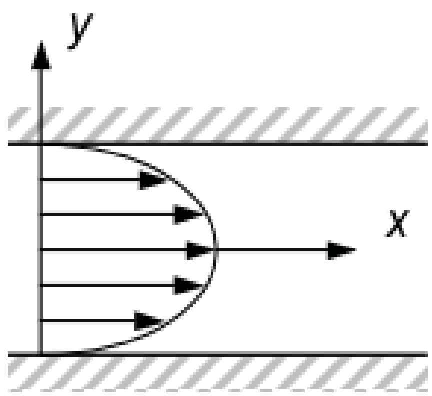
Example Problem

- Fully developed flow between parallel plates
  - Exact Solution to Navier Stokes
  - Provide a good example of errors that can be induced from interpolation

$$\bar{V} = -\frac{1}{12\mu}\left(\frac{\partial P}{\partial x}\right)a^2$$

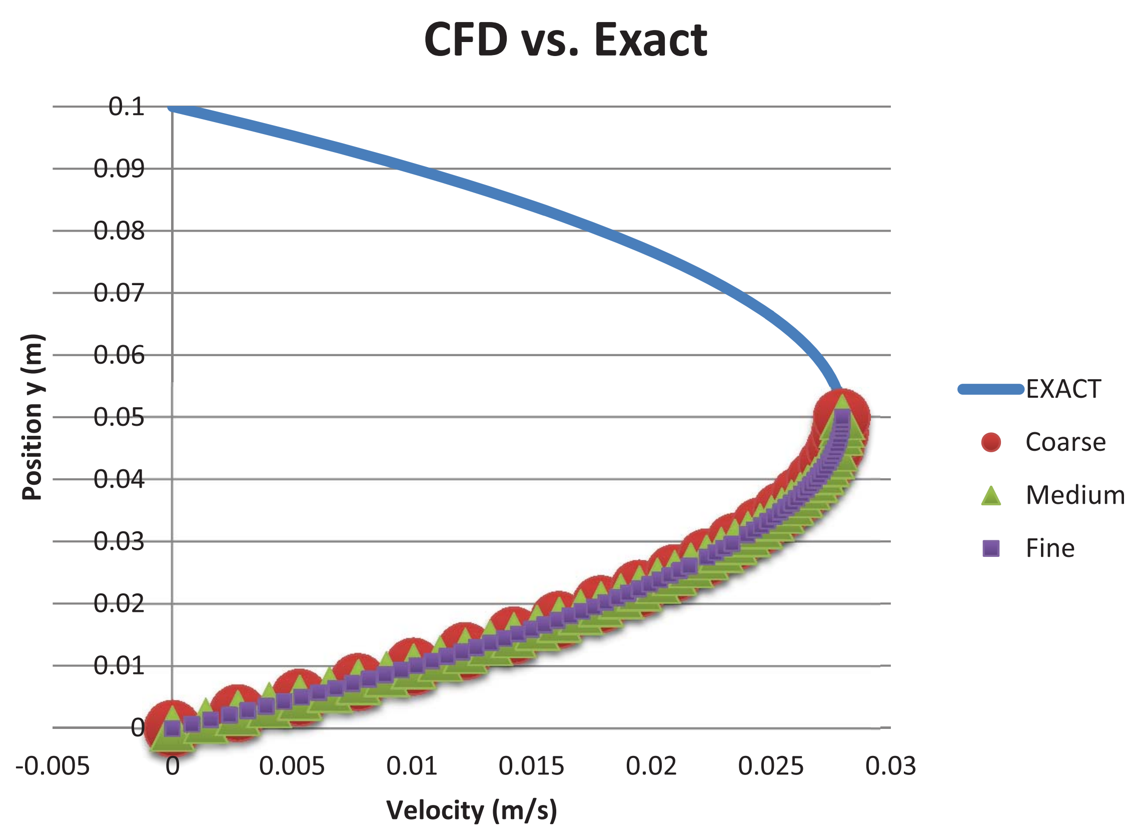


$$u = \frac{a^2}{2\mu}\left(\frac{\partial P}{\partial x}\right)\left[\left(\frac{y}{a}\right)^2 - \left(\frac{y}{a}\right)\right]$$

a (m)	0.1
rho (kg/m3)	1.225
mu (Ns/m2)	0.00001789
dp/dx (N/m3)	-0.004

Example Problem

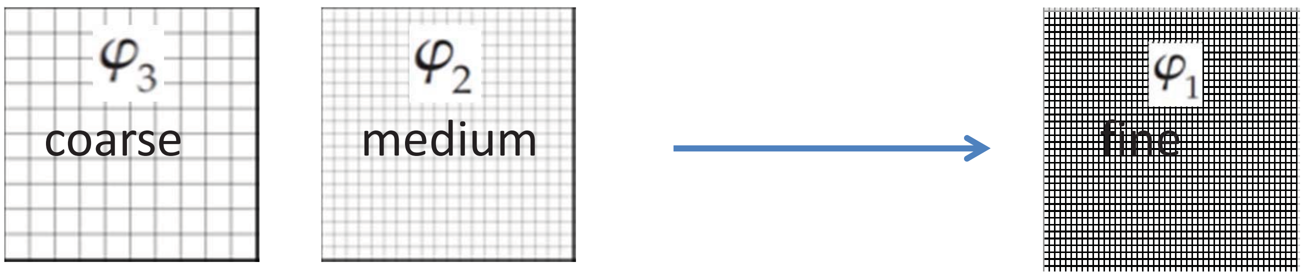
- Constructed a CFD Model in FLUENT
  - 3 Grids
    - Coarse, 7,140 Cells
    - Medium, 14,186 Cells
    - Fine, 24,780 Cells



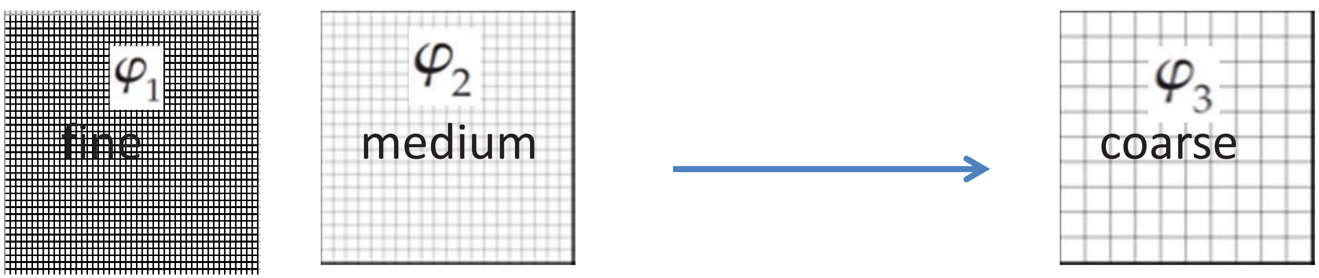
Example Problem

$$\varepsilon_{21} = \varphi_2 - \varphi_1$$
$$\varepsilon_{32} = \varphi_3 - \varphi_2$$

- Interpolation Direction?
  1. Interpolate Coarse and Medium Mesh -> Fine

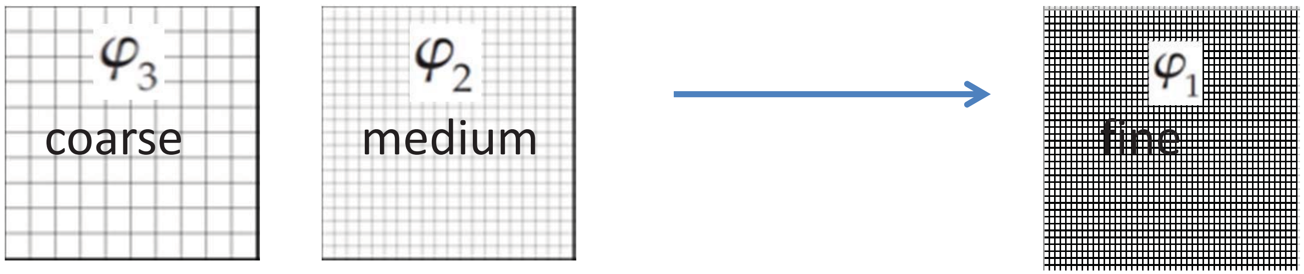


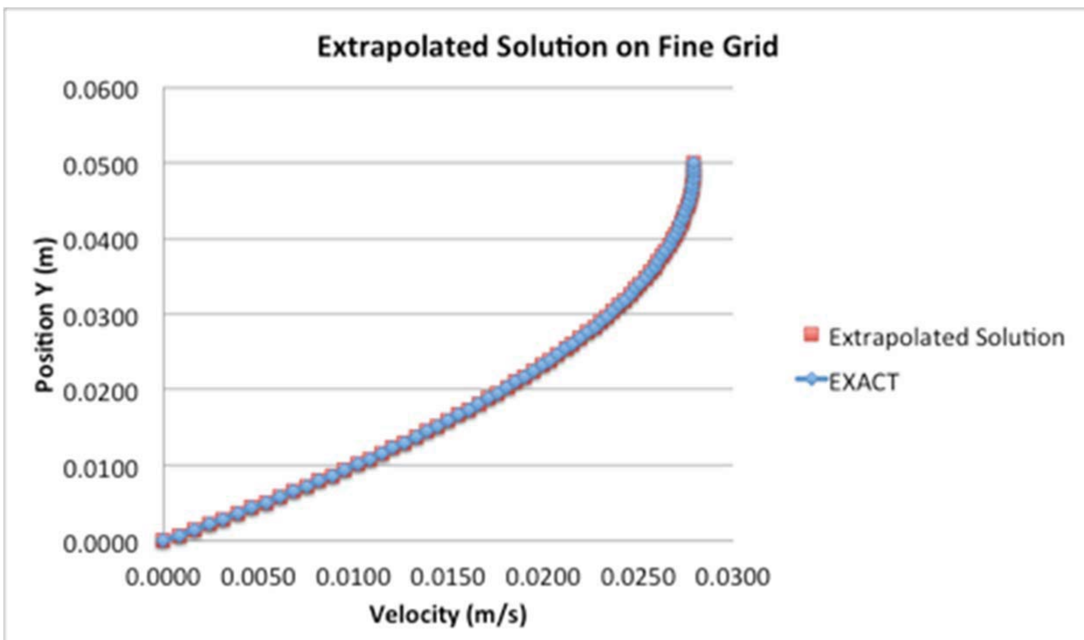
  1. Interpolate Medium and Fine Mesh -> Coarse



Example Problem

1. Linearly Interpolate Coarse and Medium Mesh -> Fine

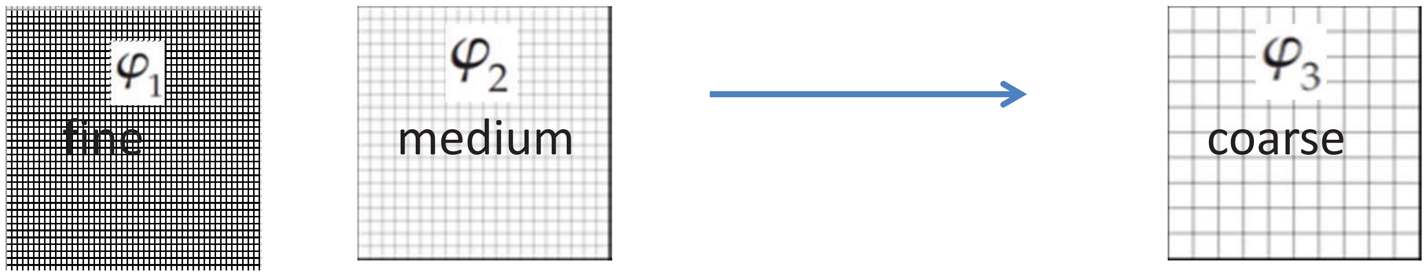


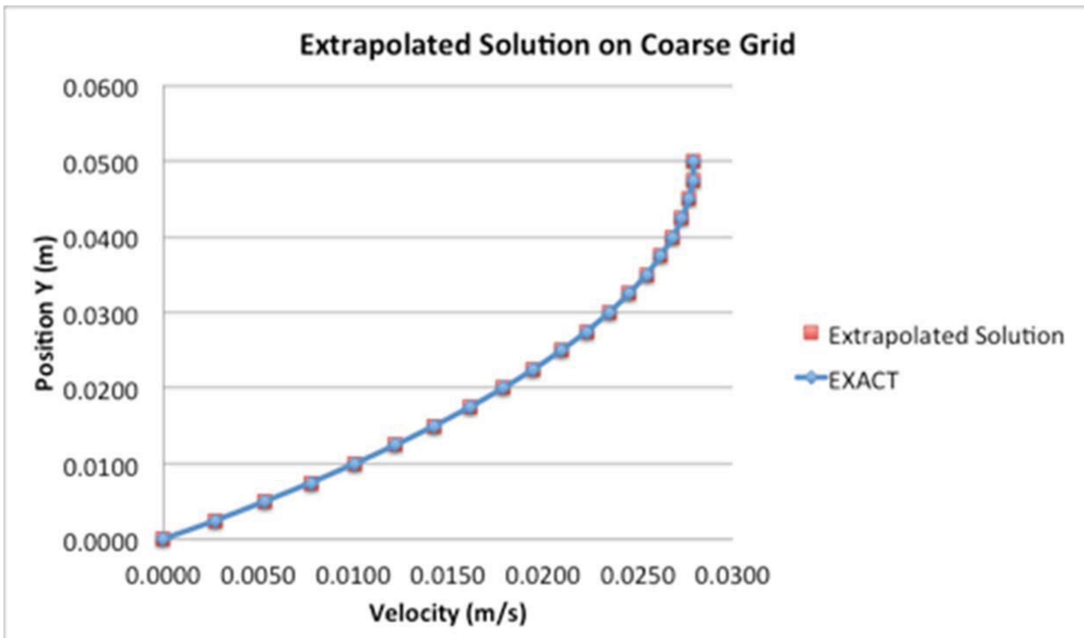


Max % Error Extrapolated Values	Average % Error Extrapolated Values
0.8950	0.0596

Example Problem

2. Linearly Interpolate Fine and Medium Mesh -> Coarse





Max % Error Extrapolated Values	Average % Error Extrapolated Values
0.0792	0.0175

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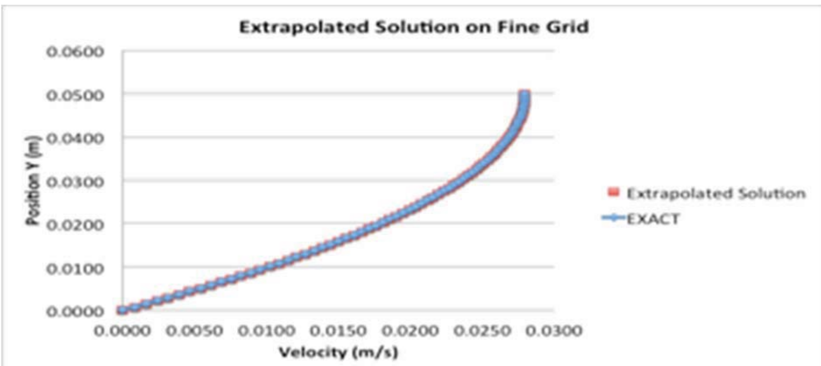
Example Problem

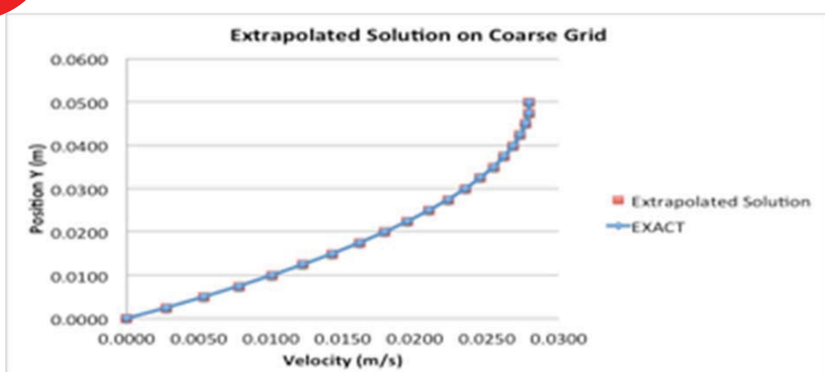
- Interpolation Direction?

1. Interpolate Coarse and Medium Mesh -> Fine



Max % Error Extrapolated Values	Average % Error Extrapolated Values
0.8950	0.0596

1. Interpolate Medium and Fine Mesh -> Coarse



Max % Error Extrapolated Values	Average % Error Extrapolated Values
0.0792	0.0175

Example Problem

– Interpolating to the coarse grid was selected
– Other interpolation methods
  - "nearest" – Fluent's Mesh-to-Mesh
  - "linear" – Matlab

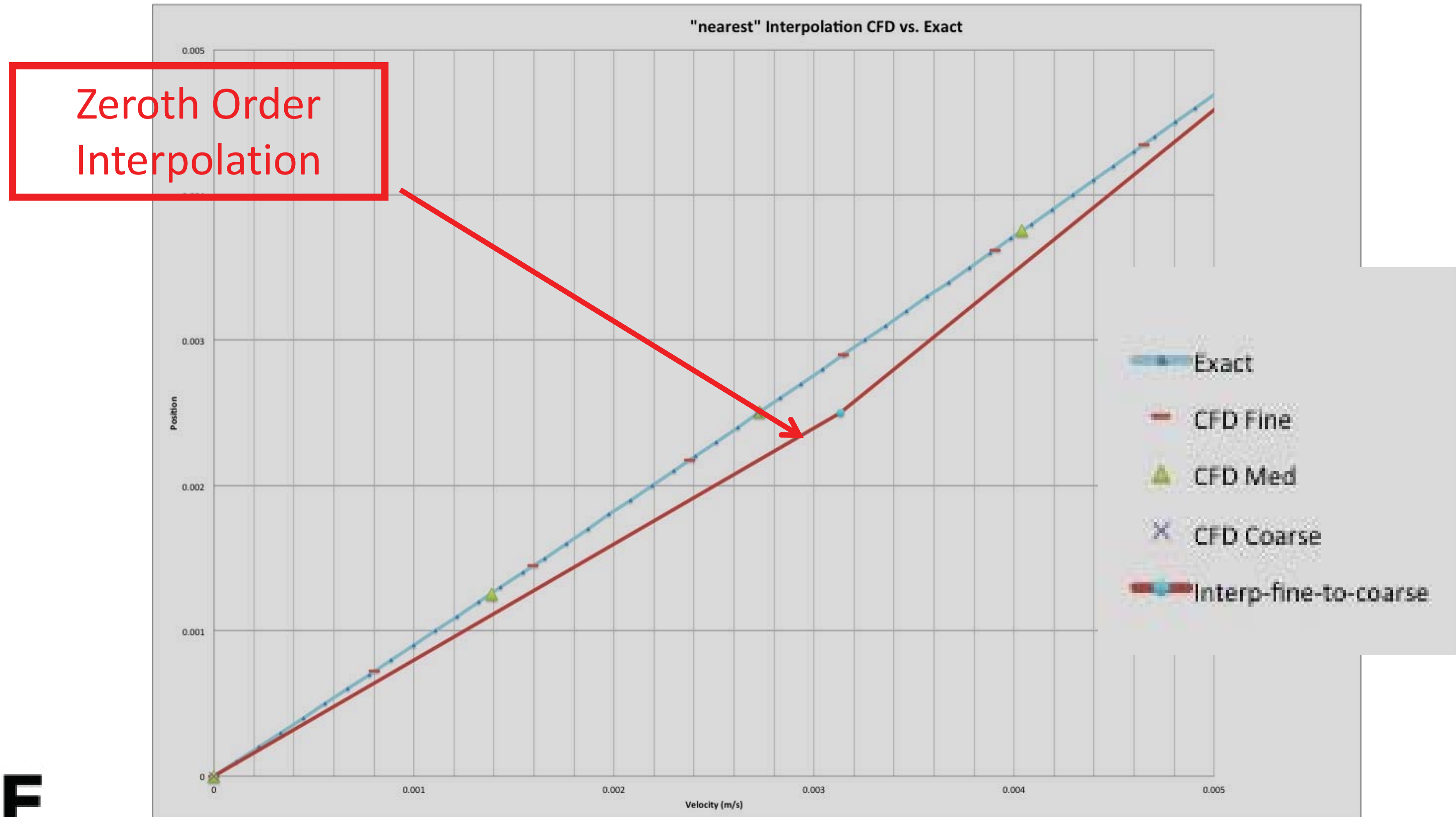
    yfi = interp1(fine(:,2),fine(:,1),coarse(:,2),'linear')

  - "cubic" – Matlab

    yfi = interp1(fine(:,2),fine(:,1),coarse(:,2),'cubic')

Example Problem

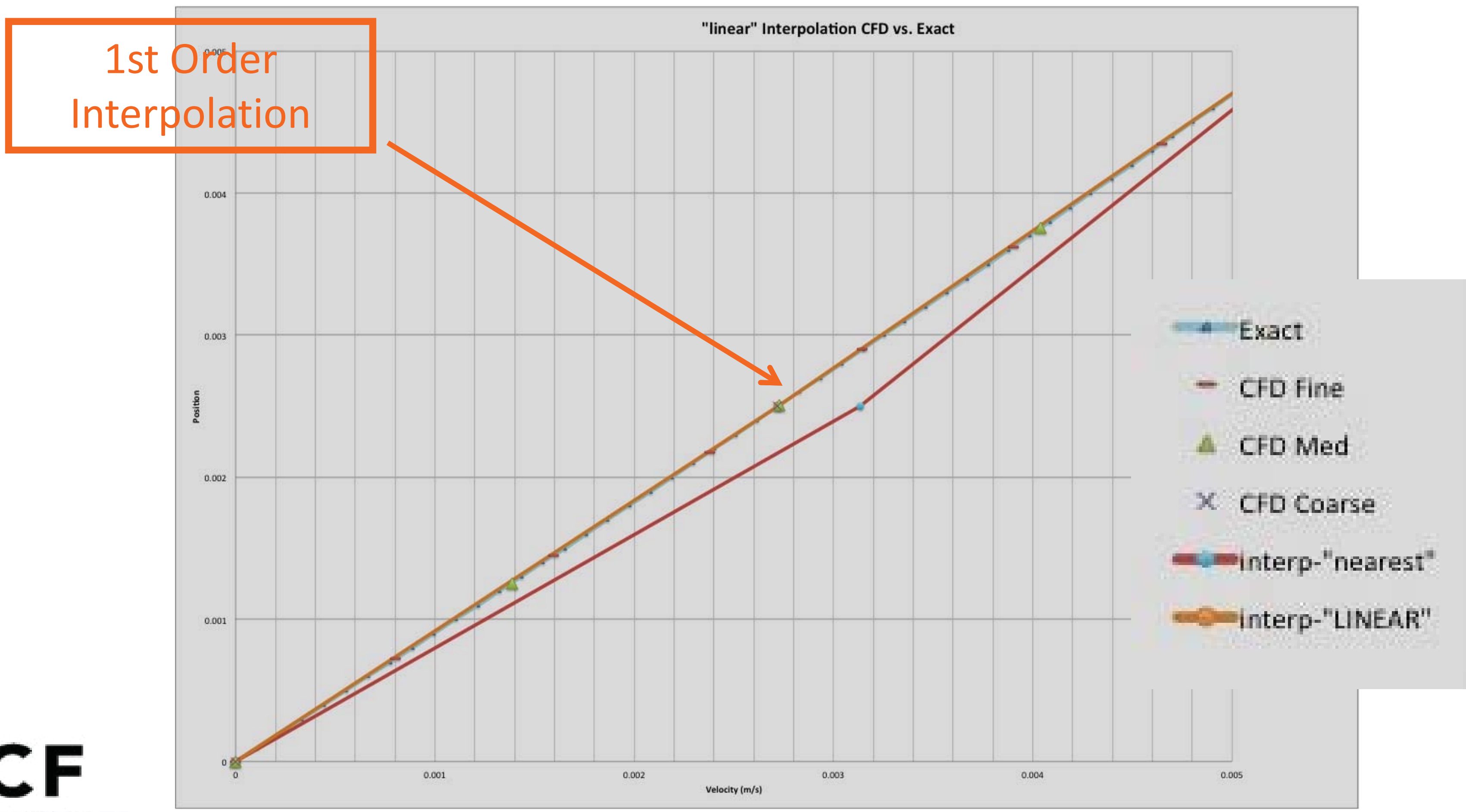
- “nearest” – Fluent’s Mesh-to-Mesh



Example Problem

- “linear” – Matlab

yfi = interp1(fine(:,2),fine(:,1),coarse(:,2),'linear')

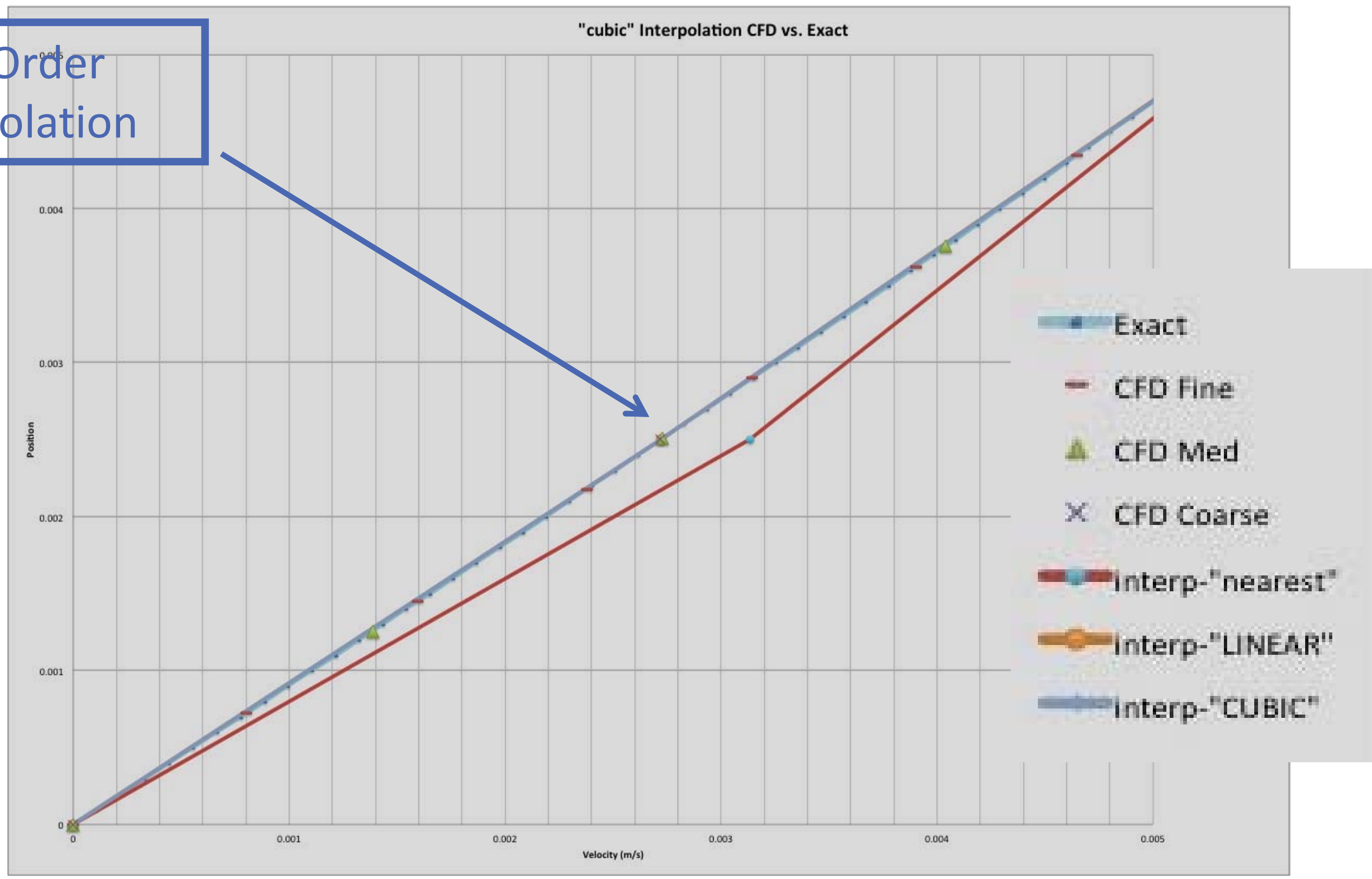


Example Problem

- “cubic” – Matlab

```
yfi = interp1(fine(:,2),fine(:,1),coarse(:,2),'cubic')
```

3rd Order Interpolation



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Matlab Interpolation Schemes

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- Extending the Interpolation Schemes to 2D and 3D
  - Interp2 and Interp3 Matlab Functions
    - Require use of MeshGrid
      - Transforms the domain of vectors into arrays
      - For Meshes in the 4 million to 8 million Cell Range
        - Error "Maximum variable size allowed by program is exceeded"
  - Griddata Function
    - Nearest, Linear, Natural, Cubic, and v4
    - Nearest, Linear, and Natural are the only options available in 2D and 3D
- The only options available for 1D, 2D, and 3D
  - Interp1 and Griddata – 'nearest' and 'linear'

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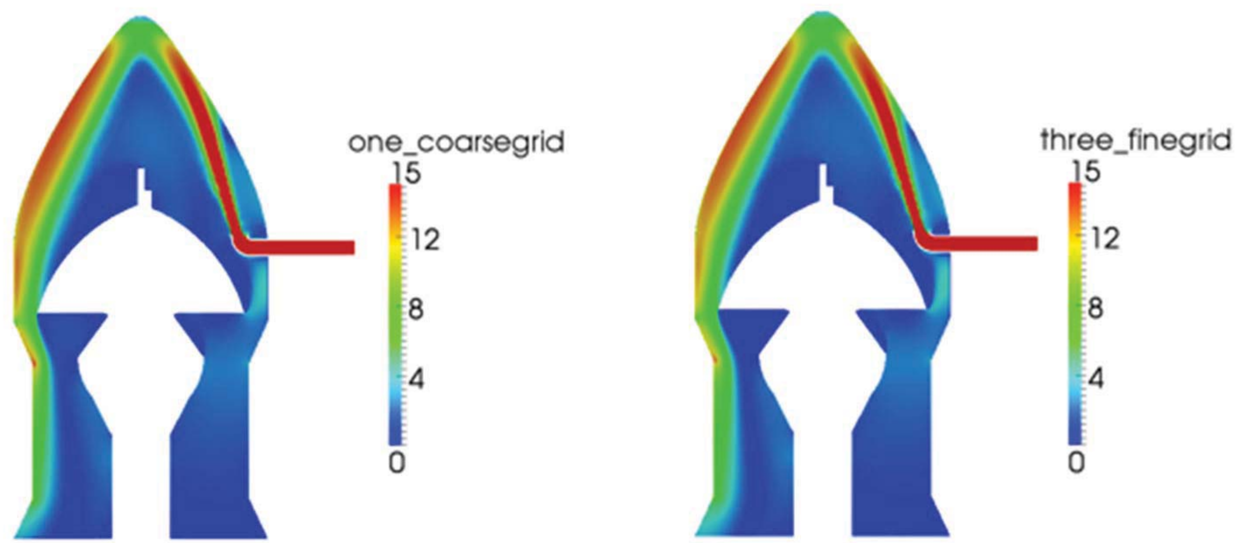
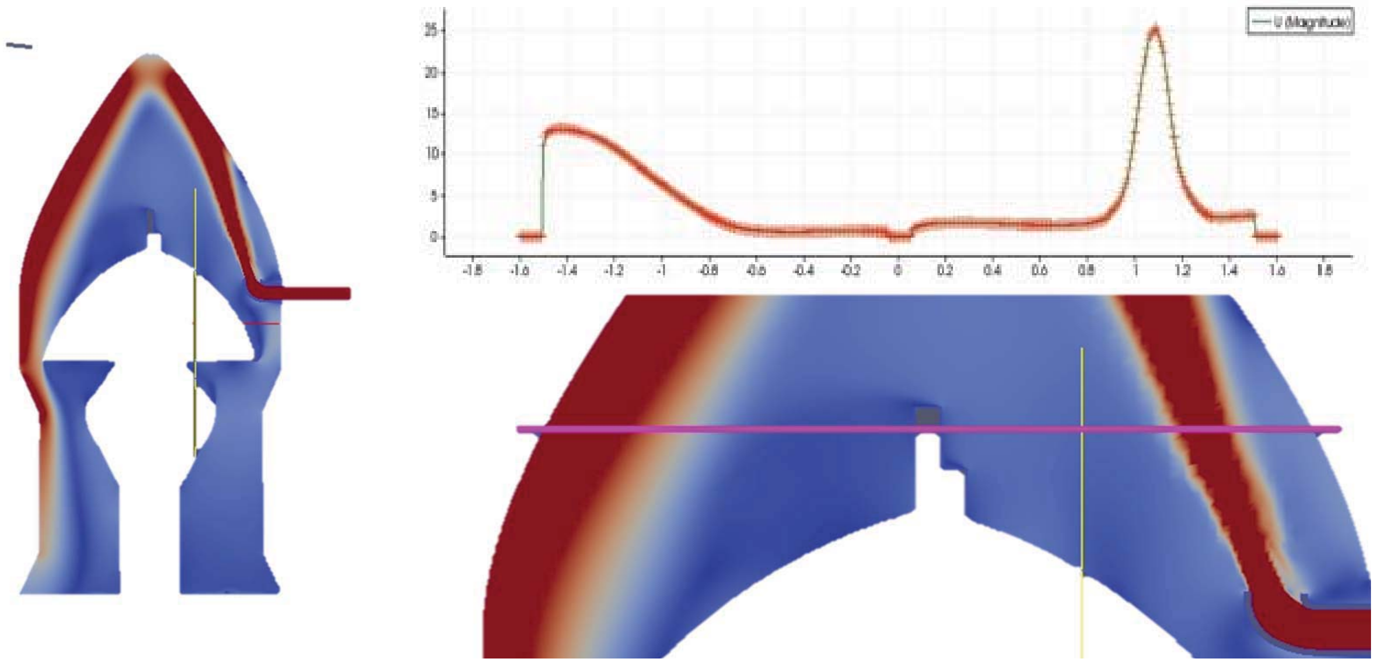
3D Example

- Airflow around encapsulated spacecraft
  - Matlab griddata ‘linear’ option used
  - Interpolating Fine and Medium Grid onto Coarse Grid



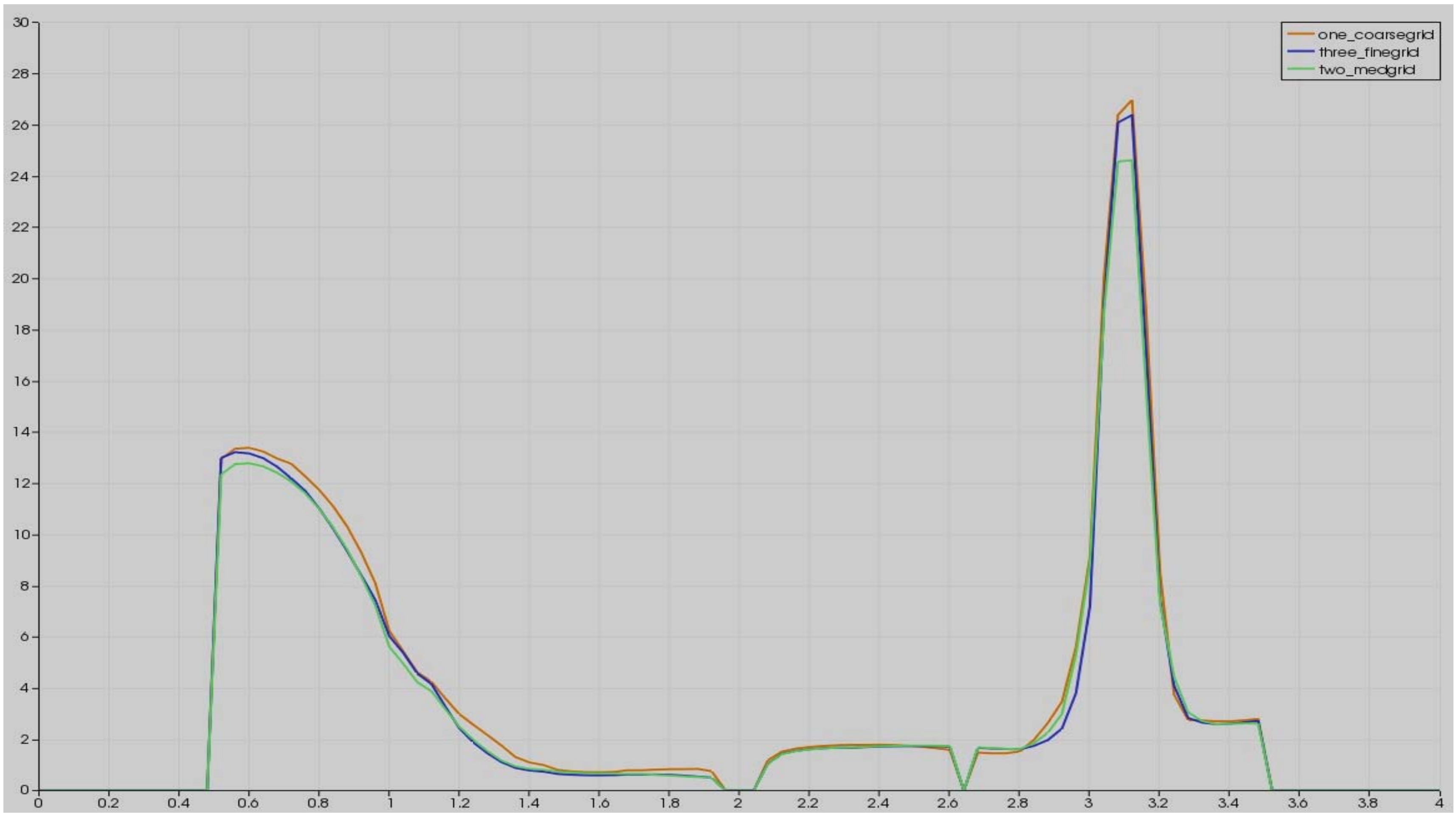
3D Example

- Comparing using a Line Plot



3D Example

- Comparing using a Line Plot



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Conclusion / Recommendation

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- By comparing the interpolation schemes in one, two, and three dimensions and investigating the options that are readily available in Matlab
  - Recommend the "linear" option be used when comparing the error or uncertainty due to the grid
    - interp1 or griddata Matlab commands
- If coarse grid has the level of detail required
  - Recommend interpolating from the fine and medium grids onto the coarse grid
    - Lower Error in the Extrapolated Solution
    - Smaller Data Set

– Future Work include higher order interpolation schemes in 3D (Radial Basis Function Interpolation, 4th order)

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