Some Interesting Applications of Probabilistic Techniques in Structural Dynamic Analysis of Rocket Engines

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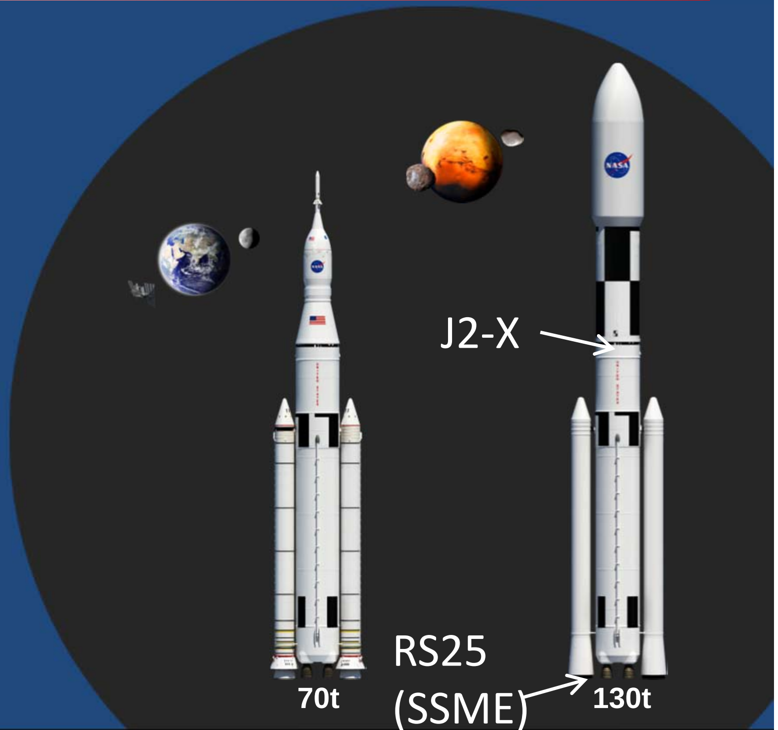
Travelling To and Through Space

Space Launch System (SLS) – America’s Heavy-lift Rocket

- Provides initial lift capacity of 70 metric tons (t), evolving to 130 t
- Carries the Orion Multi-Purpose Crew Vehicle (MPCV) and significant science payloads
- Supports national and international missions beyond Earth’s orbit, such as near-Earth asteroids and Mars




Solid Rocket Booster Test


Friction Stir Welding for Core Stage


Shell Buckling Structural Test


MPCV Stage Adapter Assembly


Selective Laser Melting Engine Parts


RS-25 (SSME) Core Stage Engines in Inventory

“We’re not dead yet!”

Agenda

- Introduction
- Prediction of Probability of Failure of Turbine Blades during Testing*
- Combination of Random and Harmonic Loads in Structures¥
- Accounting for Speed Variation (Dither) of Turbomachinery in Analysis#
- Conclusion

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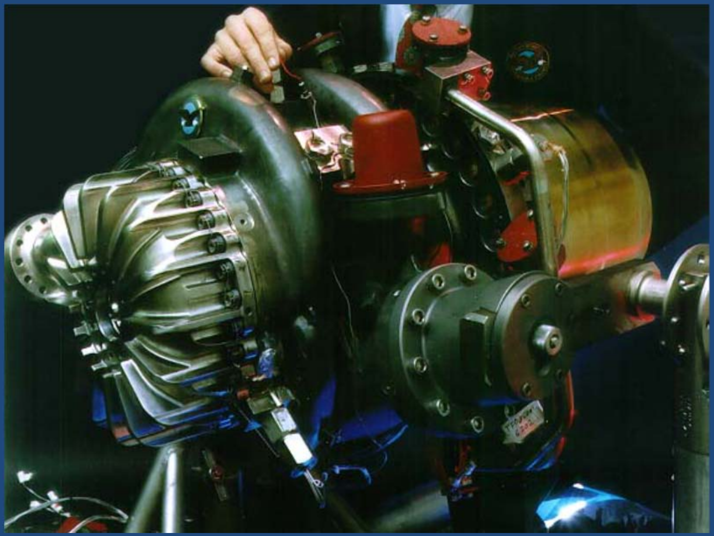
*__Probabilistic Methods to Determine Resonance Risk and Damping for Rocket Turbine Blades__
Andrew M. Brown, Michael DeHaye, Steven DeLessio, *Journal of Propulsion and Power*, 2013, Vol.29: 1367-1373, 10.2514/1.B3483

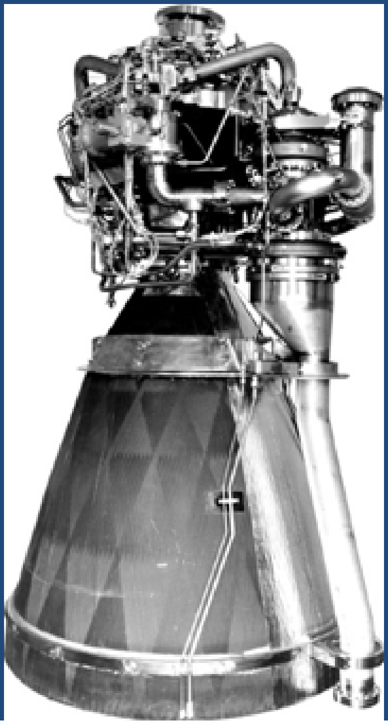
¥**Combining Loads from Random and Harmonic Excitation Using the Monte Carlo Technique**
Andrew M. Brown, *Journal of Spacecraft and Rockets*, Vol. 37, No. 4 (2000), pp. 541-543. doi: 10.2514/2.3599
also full details in NASA/TP—2003–212257.

#**Implementation of Speed Variation in the Structural Dynamic Assessment of Turbomachinery Flow Path Components**
Andrew M. Brown, R. Benjamin Davis and Michael K. DeHaye
J. Eng. Gas Turbines Power 135(10), 102503 (Aug 30, 2013) Paper No: GTP-13-1206; doi: 10.1115/1.4024960

How turbomachinery is used in Rocket Engines

- Liquid Fuel (LH2, Kerosene) and Oxidizer (LO2) are stored in Fuel tanks at a few atmospheres.
- Turbines, driven by hot gas created by mini-combustors, tied with shaft to pump, which sucks in propellants and increases their pressures to several hundred atm.
- High pressure propellants sent to Combustion chamber, which ignites mixture with injectors
- Very hot gas directed to converging/diverging Nozzle to increase flow to very high velocity for thrust.

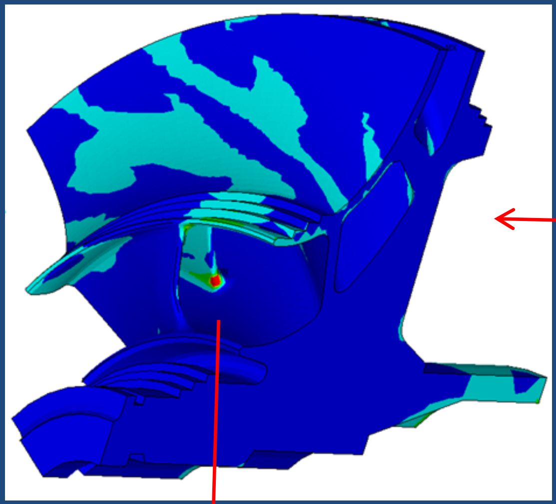


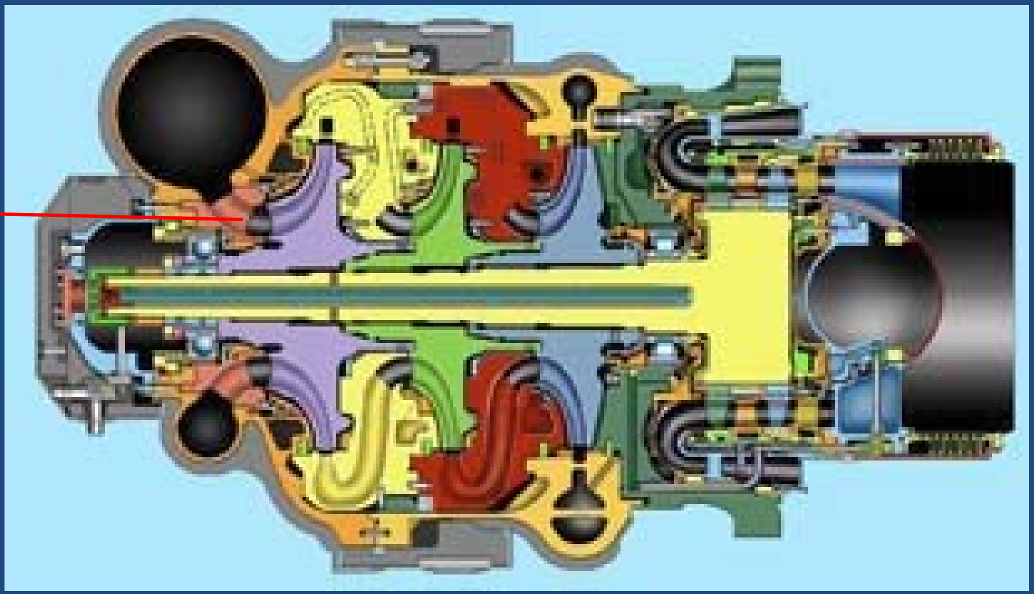


MSFC Fastrac engine

Motivation: Avoid High Cycle Fatigue Cracking in Turbomachinery

- Cracks found during ground-test program stop engine development
  - If cracks propagates, it could liberate a piece, which at very high rotational speeds could be catastrophic (i.e., engine will explode).





- In J2-X Rocket Engine program, became apparent that turbine blade external damper (needed to show deterministic design good) behind schedule.
- Identified probabilistic analysis as method to quantify risk during individual tests in series.

Prediction of Probability of Failure of Turbine Blades during Testing – Motivation

- Standard blade forced-response analysis process recognizes uncertainty in material properties and in prediction of natural frequencies.
- For J-2X clear that other non-deterministic variables (damping, mistuning) also important.
- Needed to properly assess risk of blade failure using actual non-deterministic nature of these rv’s rather than using deterministic design values.
- Substantial research and application in literature of probabilistic methods to turbomachinery issues
  - Much of effort (“top down”) calculates reliability by comparison to measured reliability of sub-systems on similar engines - Packard, ’02.
  - Crack growth characterization in probabilistic FEA (“bottoms-up) - Petrov, ’08.
- OBJECTIVE -calculate probability of failure using closed-form finite life solutions in terms of these 4 non-deterministic variables and peak FEA-derived stress state.
- Answer 1) What is P_f during a specific test series?

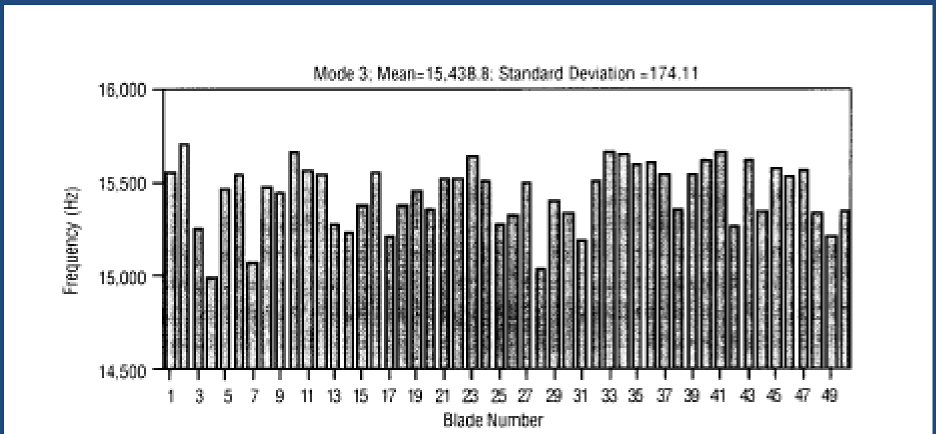
  2) If previous analysis showed low safety factors, why didn’t it fail?

Input Variables & Assumptions

A. Brown
MSFC Propulsion
Structural Dynamics

- Variation of *Natural Frequency* fn typically accounted for using rule-of-thumb +/-10% in frequency response analysis.

- Here, data from previous engine programs show distribution is somewhat Gaussian with a 3σ variation of +/- 5% (ρ=1.67%).



Input Variables & Assumptions

- In gas-generator rocket engine cycle flow rate, turbine efficiency determines rotor rotational rate, so *Operating Speed* is random variable.
- For the engine balance used here, resulting operating speed distribution is

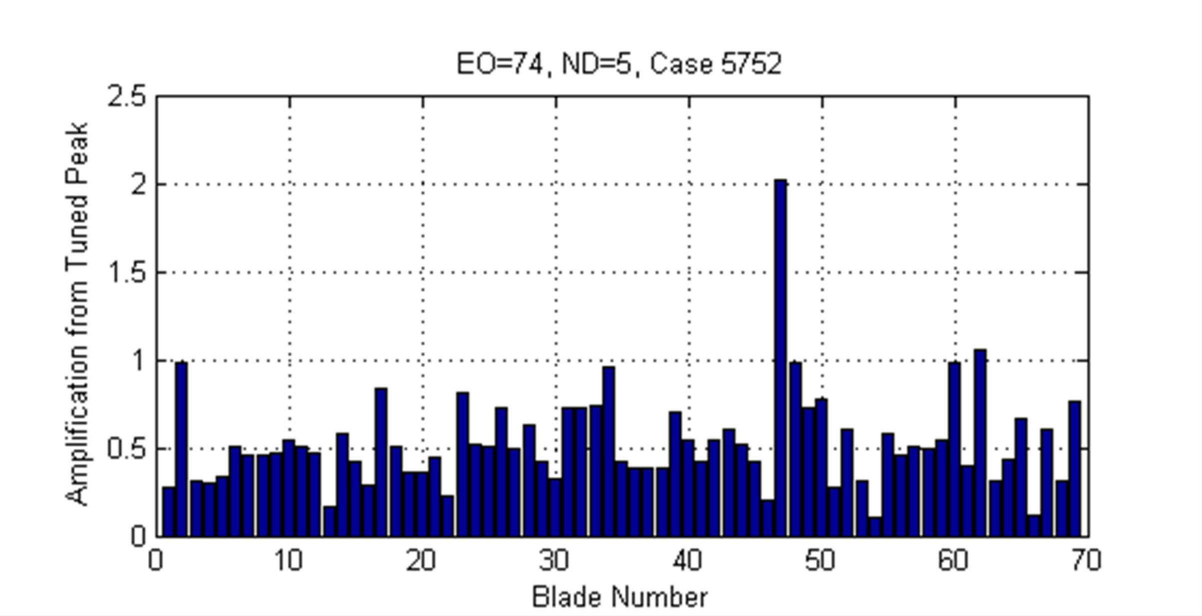
$$\text{Speed} \sim N(\mu=30{,}635 \text{ rpm}, \sigma=307.7 \text{ rpm})$$

$\Rightarrow$ +/- 3σ range is 6%

- Exception is in "Powerpack" testing, where turbopumps are isolated and rotational speed is controlled.

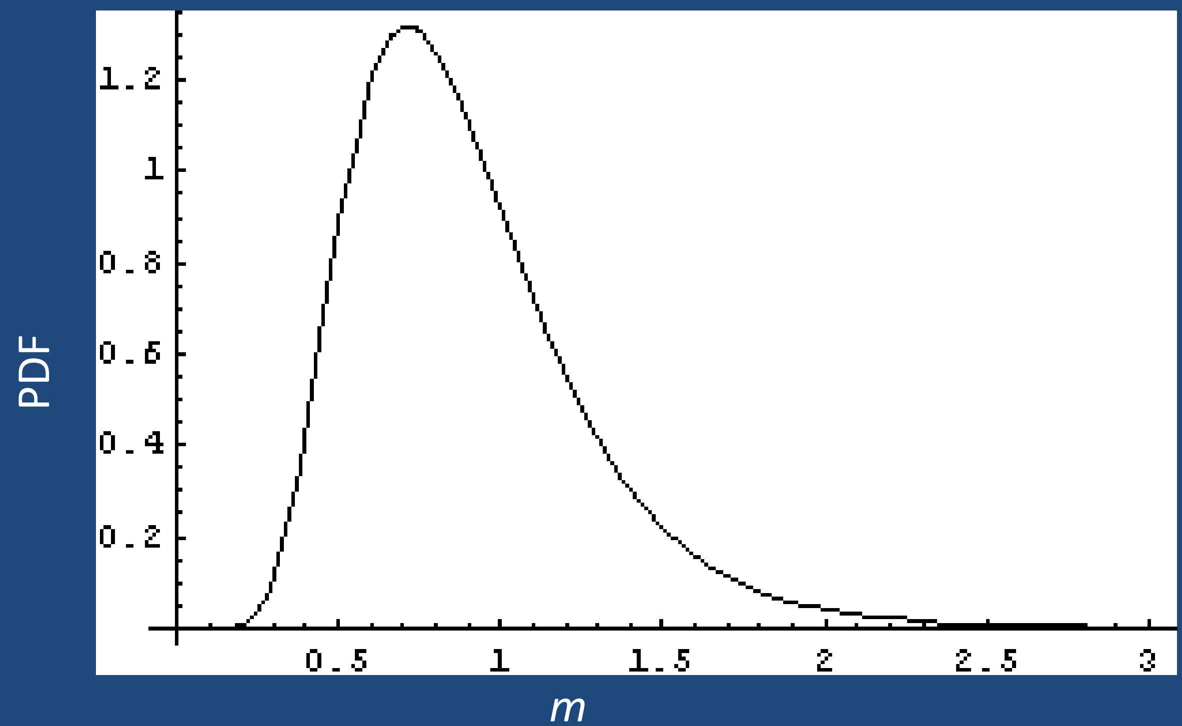
Input RV's – Mistuning Background

- Imperfectly cyclically-symmetric (*mistuned*) bladed-disks exhibit warping of nodal-diameter modes and amplification of peak response compared to perfect cyclic-symmetric (*tuned*) disk.
- Effects of mistuning non-deterministic since every build will be different.
- J-2X is one of first rocket turbopumps developed since practical methods developed to predict statistics of mistuning amplification value ***m***.
- Analysis performed ("SNM method") to develop statistics of *m* for 3 of J-2X problematic modes.



Statistics of Amplification due to Mistuning

- For 69 blade-disks, stats developed for entire-blade population (690,000) and max-responding blade per bladed-disk population (10,000) for 3 different modes.
- Debate → concensus: for probabilistic analysis, use mean value of 0.9 with Lognormal fit.

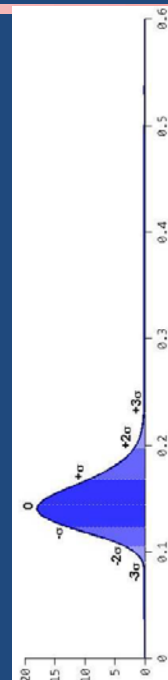


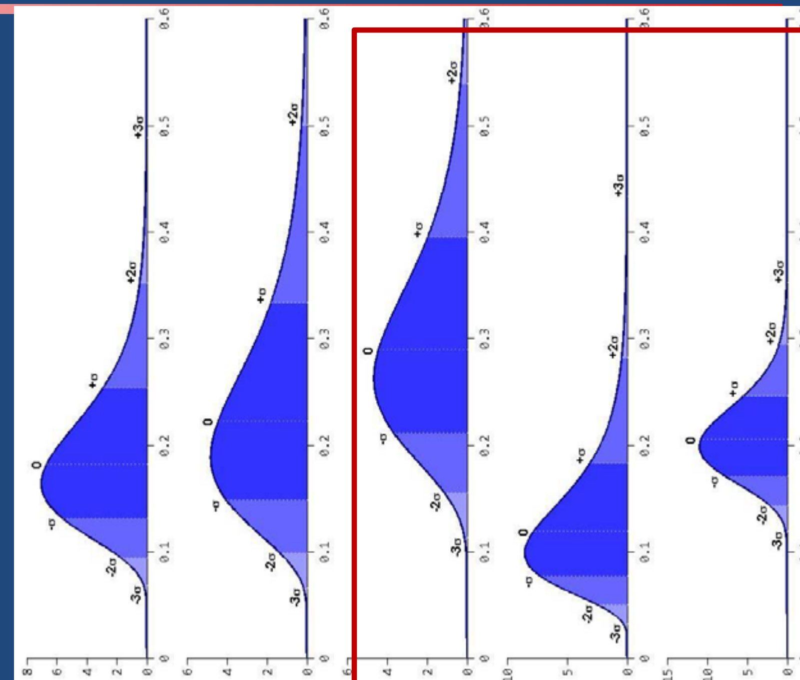
Input RV's – Damping

- Damping is critical parameter for forced response prediction, so "whirligig" test program used to obtain data.
- Whirligig was mechanically-driven rotor with similar bladed-disk (J2-S) with similar dampers excited by pressurized orifice plate to simulate blade excitation.
- Key assumption is that this reflects true configuration.
- SDOF Curve fit technique applied to selected top-responding blades to derive damping from response.

Damping Results from Whirligig

- Data shows wide-variation in damping, but reasonable population (15-20 acceptable samples) for characterization of mean and type.
- Lognormal distribution fits obtained for each mode.





	Nodal Diameter	5	5	5	5	5	5	5	5	5	5
	Mode	3	4	5	6	7	8	9	10	11	12
	Samples	18	17		17		14	12	8	16	20
Amp	**Mean**	**15.6**	**7.8**		**20.7**		**18.9**	**13.5**	**6.0**	**43.5**	**15.4**
	Sigma	**3.2**	**1.9**		**9.2**		**18.6**	**8.4**	**0.9**	**17.7**	**3.2**
	Min	9.9	5.0		7.4		5.4	6.1	5.0	23.8	12.4
	Max	20.3	11.2		35.4		54.2	33.6	7.7	87.7	24.1
Freq	**Mean**	**10967**	**13831**		**23068**		**28867**	**30588**	**32998**	**34643**	**37191**
	Sigma	**17**	**69**		**282**		**345**	**211**	**256**	**220**	**132**
	Min	10936	13695		22921		28446	30165	32497	34357	37056
	Max	10997	13908		23816		29662	30907	33311	35013	37346
Zeta	**Mean**	**0.404**	**0.702**		**0.146**		**0.193**	**0.242**	**0.304**	**0.131**	**0.209**
	Sigma	**0.103**	**0.163**		**0.023**		**0.065**	**0.102**	**0.097**	**0.059**	**0.038**
	Min	0.314	0.520		0.106		0.116	0.139	0.162	0.078	0.153
	Max	0.720	0.976		0.191		0.348	0.450	0.423	0.325	0.293
	LogNormal Dist.:										
	0σ Equivalent	0.391	0.684		0.144		0.183	0.223	0.290	0.119	0.206
	-σ Equivalent	0.305	0.544		0.123		0.132	0.149	0.212	0.078	0.172
	-2σ Equivalent	0.237	0.433		0.105		0.095	0.099	0.155	0.051	0.143
	-3σ Equivalent	0.184	0.343		0.090		0.068	0.066	0.113	0.033	0.119

Probabilistic Analysis

- First, determine Stress state (Sa, Sm) of problem location from finite element frequency response analysis at resonance (w. ζ=.0025).
- Then, for a sample taken from distributions of all random variables (ie, Monte Carlo analysis), calculate Equivalent Alternating Stress A_{eq}:

$$A_{eq} = \frac{S_a * FAF * m * \frac{.0025}{0.01\zeta} * \sqrt{\frac{\frac{1}{((1-(\frac{speedrpm}{fnrpm})^2)^2 + (\frac{2(0.01)\zeta * speedrpm}{fnrpm})^2}}{\frac{1}{2(0.01)\zeta}}}}{1 - \frac{S_m}{F_{tu}}}$$

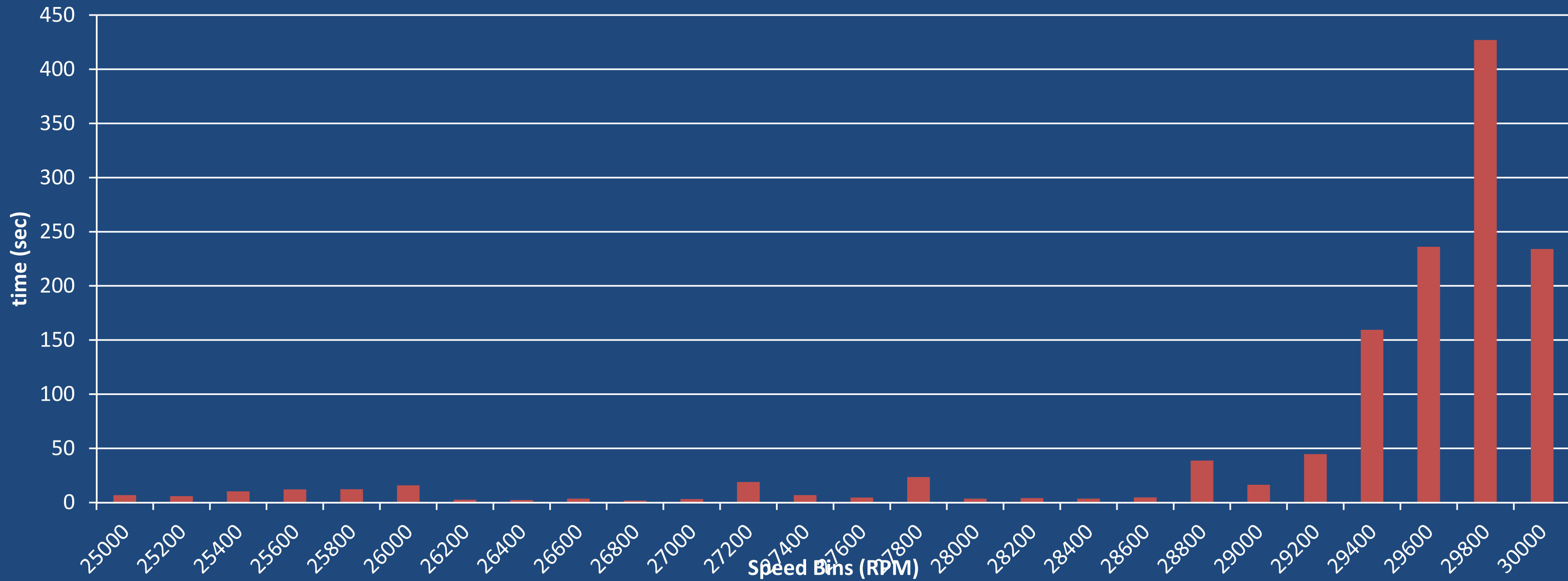
- Nominal HCF cycle count data ("s-n curves") ->

$$N_{fail} = 10^{\left(-9.2461 \times Log_{10}(A_{eq}) + 20.672\right)}$$

- Finally, failure using "finite life approach" is $\Phi = \frac{N_{accum}}{N_{fail}} > 1$

Probability of Failure using Damage Fraction

- N_{accum} is excitation frequency * time at that frequency.
  - Speeds in test series recorded in 80 rpm wide bins, calculate incremental damage fraction within each *i*'th bin.



$$\Phi_i = \int_0^{time_i} d\Phi = \int_0^{time_i} \frac{\Omega_i}{N_{fail}} dt \Rightarrow \Phi_{total} = \sum_{i=1}^{\text{number of bins}} \Phi_i$$

$$p_f = p(\Phi > 1) = \frac{\# samples \ \Phi > 1}{total \ \# samples}$$

Technique Verification

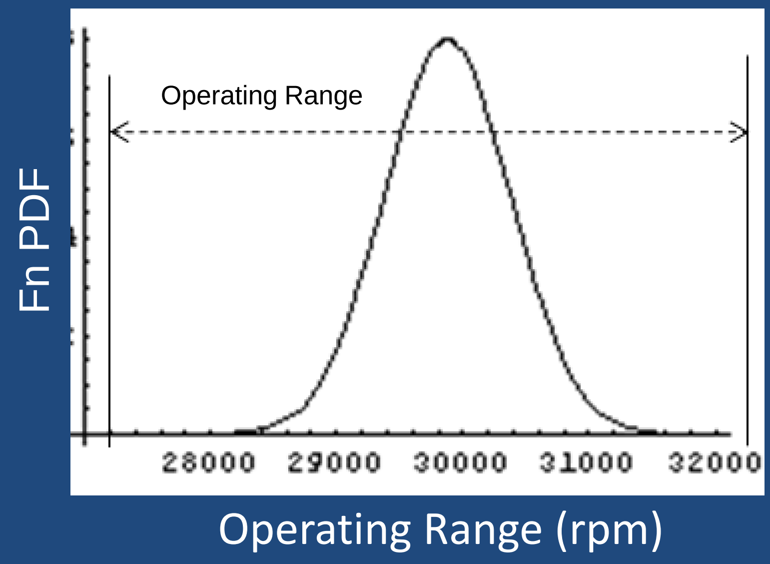
- To "verify" technique, p_f was calculated for tests that had already taken place, assuming both that the speeds are "post-priori" known and "a-priori" unknown.
- Deterministic analysis indicated Safety Factor < 1 for mode 14 in ND 5 family
  - $fn \sim N(36851\ hz, 615\ hz)$, $\zeta \sim LN(0.304\%, .097\%)$.
- Results for these technique verifications were reasonable
  - for a single hot-fire test, p_f only 1% (specifically because of a low probability of resonance) , so fact that blade did <u>*not*</u> crack should be expected.

J-2X Powerpack II A-priori Analysis

- Speed mean controllable, enabling engineering team to make assumptions of 1200s total run time, 4 dwells of 100s, 20 dwells of 30s, ramping from 26902 rpm-31200 rpm at continuous rate of 20 rpm/s (during ramps).

$$\boxed{P_f = 33.8\%}$$

- Explanation for results is extensive overlap of fn distribution over operating range (ensuring resonance) , and lower damping of problematic mode.



- Test Results – dampers not put in, extra precautions taken, blade did not fail

  "Statisticians are never wrong, they are only unlucky"

J-2X Engine 10001 A-Priori Analysis

- Equally important to assess P_f for first full-scale engine test to determine if external blade dampers required.
- In this test speed will resolve to a single value within distribution following Speed ~ N(30635 rpm, 307 rpm).
- Time of operation given as 550 s.
- Single dwell formulation relatively simple, enables large (100,000) sample MC run.
- P_f = 1.06%, very low because of low probability of resonance itself, which was independently calculated (using only rv's speed and natural frequency) to be $P_{resonance}$ = 3.1%.

Sources of Error and Conclusions

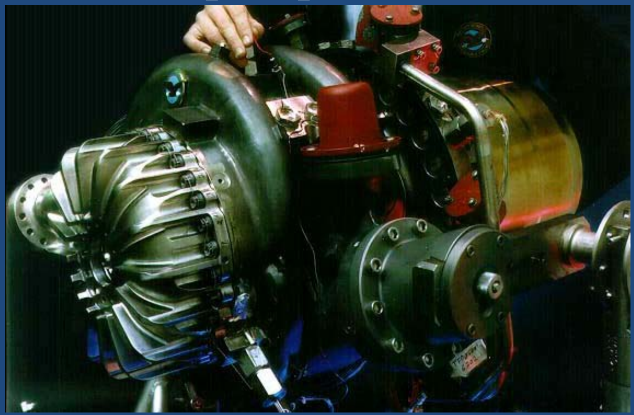
- Error:
  - Some non-deterministic input variables assumed to be deterministic.
  - Mistuning and Damping assumed to be independent and they probably are not; unknown effect on results.
  - Response away from resonance approximated by SDOF curve fit.
- Framework procedure established for quantifying risk of turbine blade failure due to resonance.
- Probabilistic analysis enable first-time use of statistical distributions of most of random variables, including Natural Frequency, Operational Speed, Mistuning, and Damping.
- Results very useful for project decision-making during development phase.
- Framework also applied to a number of other J2X turbopump dynamics issues.
  - Used to determine appropriate deterministic value of damping to use for design for specific reliability goals.
  - Design of test series to put equivalent damage on pump inducer blade as it would experience if it were at resonance (worst case), given that the fn is actually non-deterministic.

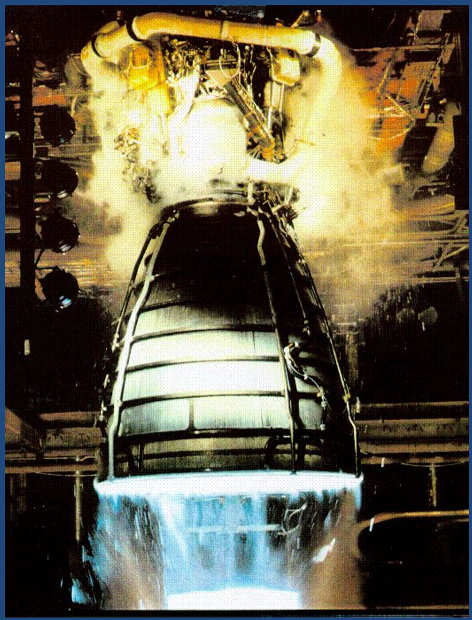
Combination of Random and Harmonic Loads in Structures – Introduction

- Many structural components are in an environment with both random and harmonic loads.

  - Rocket Engines

Turbopump-harmonic





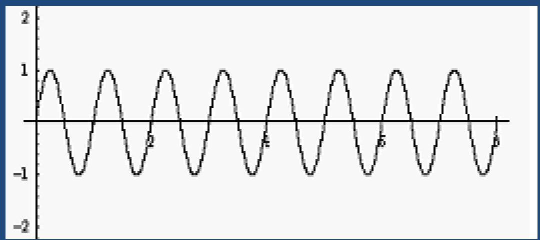
Combustion-random

- Each type first calculated individually.
- Results of analyses then combined for use by stress in both ultimate/yield analysis and HCF analysis.

- Frequency response analysis to generate harmonic load first calculated

e.g., 1 lb Sine Amplitude load at 1 hz

lb

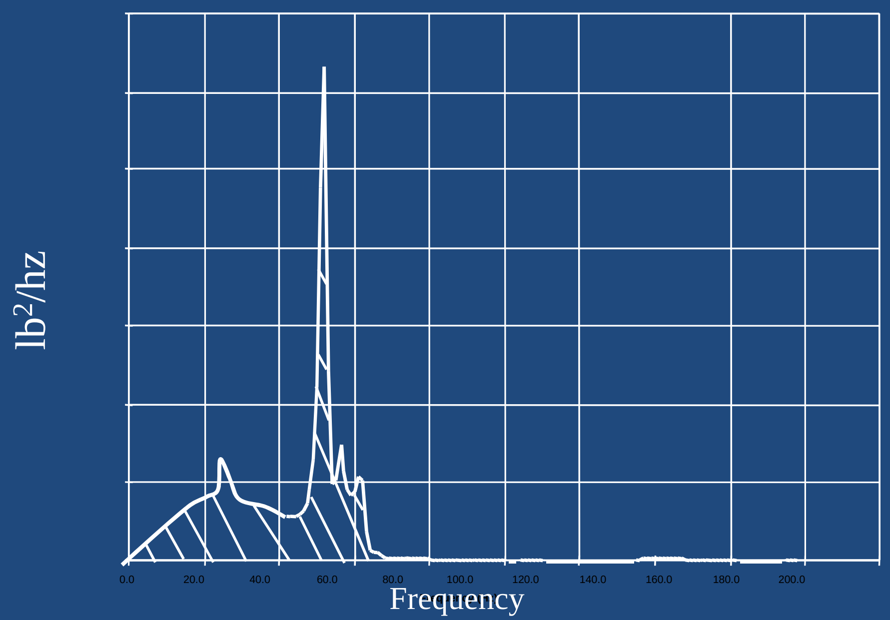


t (sec)

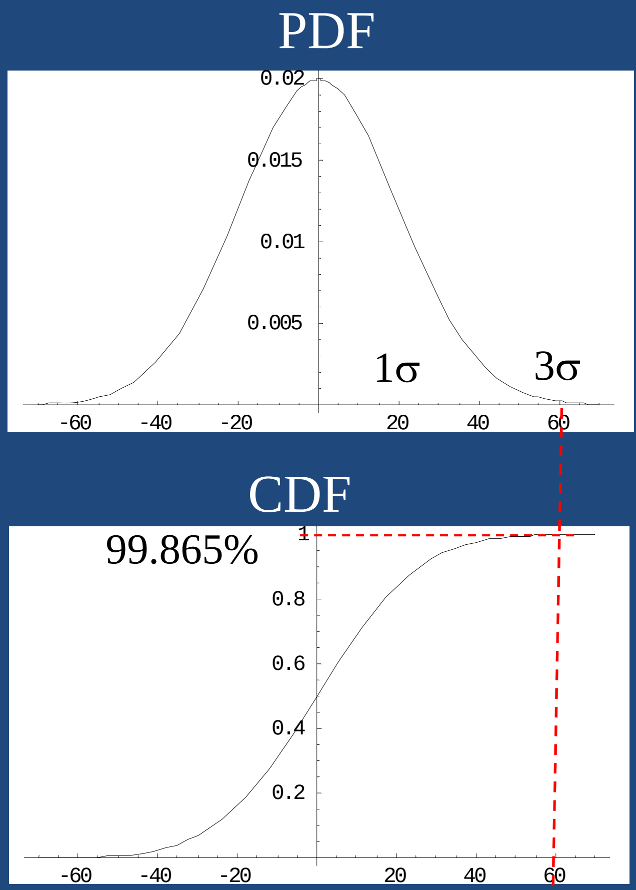
Random Loads

- PSD's of accelerations at different zones in engine defined and applied as base drive random analysis.

Typical Random Response Analysis Result



Assumed Gaussian Distribution



Mean Square Φ^2 = Area under random response curve = 400 lb²

$$\boxed{1\sigma = RMS = \sqrt{\Phi^2} = 20lb}$$

- Loads extremely sensitive to probability level chosen (or actually obtained) because of flatness of Cumulative Distribution Function at tails.

Loads Combination Equations

- Extensive, difficult research into reducing each load type individually; however, little thought into how load is combined.
- Main goal of methods is to estimate an "equivalent 3σ" design load;
  - 3σ is traditionally used for pure random loading, i.e., the load that exceeds 99.865% of the occurrences.
- "<u>Standard Method</u>" used in SSME:

$$\text{design load} = A_{\sin} + 3\sigma_{ran}$$

- "<u>3*ssMS</u>" Method: $$\text{design load} = 3\sqrt{(\sigma_{\sin})^2 + (\sigma_{random})^2}$$
- Both techniques exceed 99.865% by definition, not tied to a specific probability.
- "<u>Peak</u>" method proposed by Steinberg, adopted initially by engine contractors.

$$design\ load = \sqrt{(A_{\sin})^2 + (3\sigma_{random})^2}$$

Typical MC-1 Engine Load Set

Glue Bracket 3 GB-3	Shear 1 (lbs)	Shear 2 (lbs)	Axial (lbs)	Bending 1 (in-lbs)	Bending 2 (in-lbs)	Torque (in-lbs)
Sine X	97	7	0	3	78	72
Sine Y	91	7	0	3	98	70
Sine Z	119	5	0	2	78	52
Sine Peak (RSS)	**178**	**11**	**0**	**5**	**148**	**113**
3 sig Random X	450	113	0	16	25	1475
3 sig Random Y	781	66	0	9	41	828
3 sig Random Z	155	1	0	4	1101	6
Random Peak (RSS)	**915**	**130**	**0**	**19**	**1102**	**1692**
Stringer Bracket 3 (Lower Support) **SB-6**						
Sine X	18	8	11	8	17	2
Sine Y	12	4	10	7	11	1
Sine Z	11	12	8	3	28	3
Sine Peak (RSS)	**24**	**15**	**17**	**11**	**34**	**4**
3 sig Random X	35	333	6	85	1349	52
3 sig Random Y	60	192	10	145	775	29
3 sig Random Z	12	1	11	83	6	0
Random Peak (RSS)	**70**	**384**	**16**	**187**	**1556**	**59**
Stringer Bracket 3 (Upper Support) **SB-5**						
Sine X	59	7	21	81	9	21
Sine Y	58	5	21	80	6	26
Sine Z	43	4	16	59	5	25
Sine Peak (RSS)	**93**	**9**	**34**	**129**	**12**	**42**
3 sig Random X	44	447	117	93	1557	69
3 sig Random Y	76	256	202	160	893	38
3 sig Random Z	139	2	1002	322	4	0
Random Peak (RSS)	**165**	**515**	**1029**	**371**	**1795**	**79**

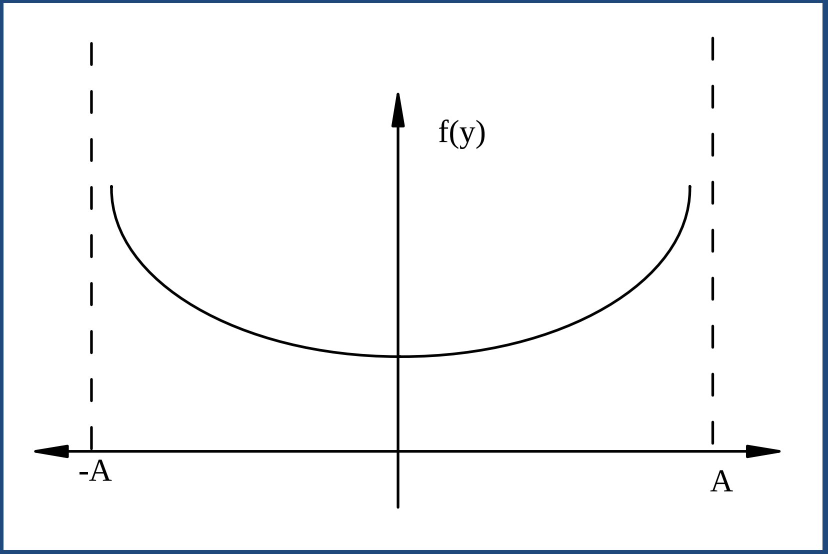
Loads Combination using PDF's

- Harmonic signal can be defined as stationary random process when combined with an independent Gaussian process since phase relationship with random signal is random.
- Define harmonic signal as

$$y_{\sin} = A\sin(\omega t + \varphi)$$

- Then PDF of sine distribution is

$$f(y) = \frac{1}{\pi A \sqrt{1 - \left(\frac{y}{A}\right)^2}}$$



“Exact” Solution Now Easily Obtained

- Create and Integrate Joint PDF of Normal and Sine Distributions to obtain CDF of design load z:

$$CDF(z) = \int_{-A}^{A} \left(\frac{1}{\sigma_{ran}\sqrt{2\pi}} \int_{-\infty}^{z-y} \exp\left(\frac{-\left(\frac{x}{\sigma_{ran}}\right)^2}{2} \right) dx \right) \frac{1}{\pi A \sqrt{1-\left(\frac{y}{A}\right)^2}} dy$$

- *Mathematica®* can perform not only integration, but also inverse:
  - Given a load (e.g. calculated using “standard” method) calculate exact reliability level.
  - Given a desired reliability level, solve for corresponding load.

- Developed Excel Macro:
  - easily integrates into existing loads calculation spreadsheets
  - Accesses *Mathematica®* to perform inverse-integration to obtain design load corresponding to 99.865% reliability
  - returns value seamlessly into spreadsheet.

Loads Combination using Monte Carlo

- Gaussian random vector using random analysis results (σ_{ran}) first simulated:

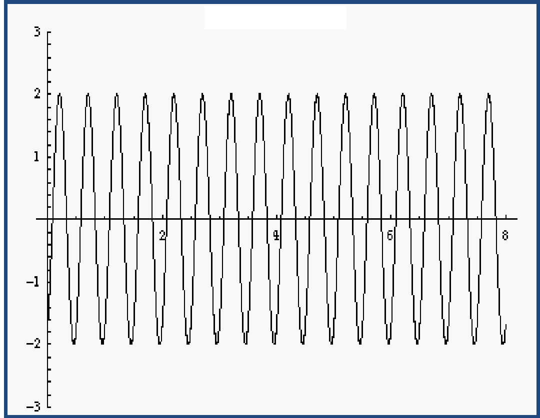
$$\{r\} \sim N(0.0, \sigma_{random})$$

- Independent sine vector generated using harmonic analysis results (A_i):
  - Create uniform distribution $\{x\}_i \sim U(0,1)$
  - Generate sine distribution $\{y\}_i = A_i \sin(2 \pi \{x\}_i)$
- Vectors of same length added to form total response:

$$\{z\} = \{r\} + \{y\}$$

- CDF calculated for {z}, 99.865% (or any other desired level) selected.
- Excel Macro created to perform Monte Carlo Simulation to obtain design load corresponding to 99.865% reliability (within Excel).
  - Less than a minute for 400,000 samples.

Example



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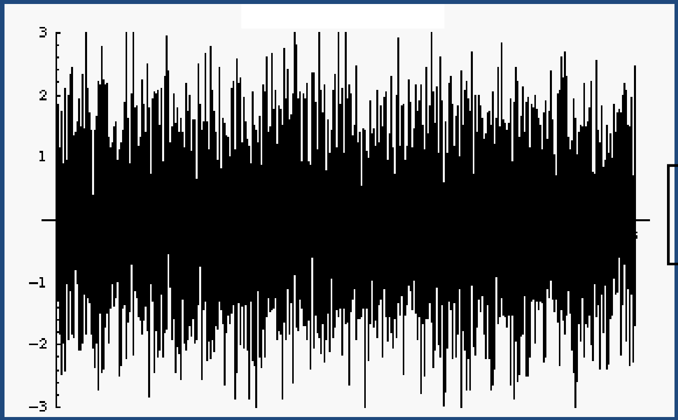

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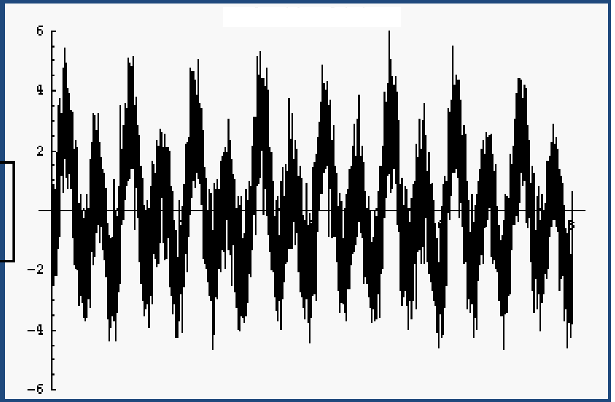


$y = 20\sin\omega t$

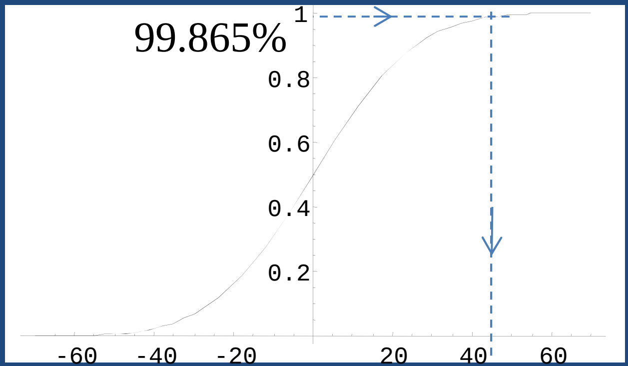
$r = N(\mu=0, \sigma=10)$

CDF

Microsoft Excel Macro

$$F(z) = \int_{-A}^{A} \left(\frac{1}{\sigma_{ran}\sqrt{2\pi}} \int_{-\infty}^{z-y} \exp\left(\frac{-\left(\frac{x}{\sigma_{ran}}\right)^2}{2} \right) dx \right) \frac{1}{\pi A \sqrt{1-\left(\frac{y}{A}\right)^2}}\, dy$$

Integral of Joint PDF



Standard method $\rightarrow$ 50
SRSS $\longrightarrow$ 51.96
Peak $\longrightarrow$ 36.05

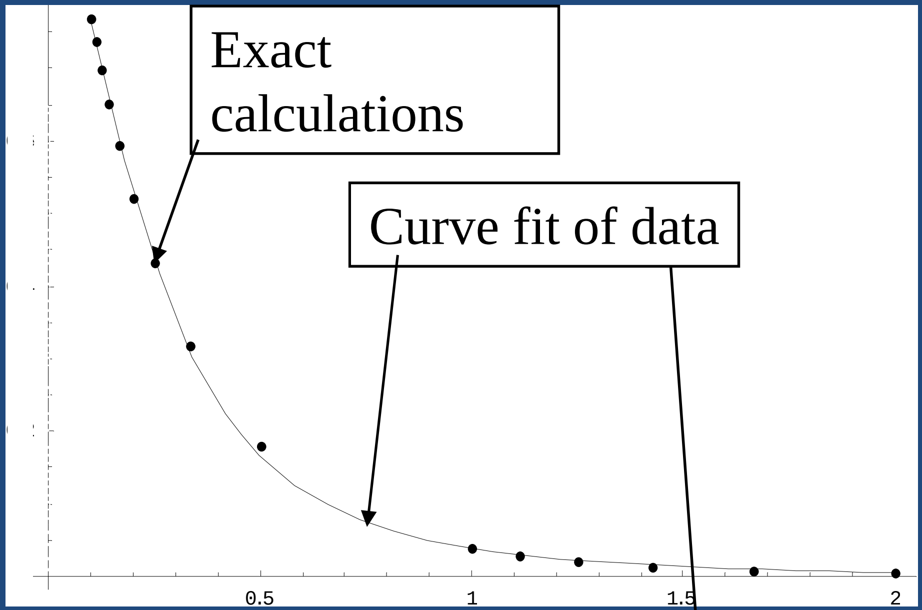
Design Load = 44.07

Evaluation and Comparison of Methods

Amp. Sine	1 σ random	Integration method for 99.865% (baseline)	MC 99.860 (400,000 samples)	standard method - A + 3σ	% over-shoot from baseline	CDF value from integration	3*ssMS	% over-shoot	CDF value from integration	"peak" method - ss(3sig,A)	% over-shoot	CDF value from integration
10	5	22.034	22.031	25	13.5%	99.970%	25.981	17.9%	99.990%	18.028	-18.2%	93.930%
5	5	17.668	17.653	20	13.2%	99.957%	18.371	4.0%	99.912%	15.811	-10.5%	94.896%
5	20	60.915	60.888	65	6.7%	99.919%	60.930	0.03%	99.865%	60.208	-1.2%	95.758%
26	4	34.760	34.772	38	9.3%	99.983%	56.445	62.4%	100.000%	28.636	-17.6%	94.291%
97	14.67	129.081	129.195	141.01	9.2%	99.986%	210.422	63.0%	100.000%	106.517	-17.5%	94.316%
50	98.7	313.047	313.422	346.1	10.6%	99.951%	314.524	0.5%	99.871%	300.292	-4.1%	95.534%
64	109.33	352.240	353.079	391.99	11.3%	99.955%	354.978	0.8%	99.875%	334.176	-5.1%	95.443%

- MC closely agrees with Integration method
- Two generally accepted methods always above 99.965%.
- “Peak” method underpredicts “3σ” value

Curve Fit of Overshoot of 3*ssMS Method over CDF of 99.865%



Ratio $x = \sigma_{ran}/A_{sin}$

$$\text{overshoot} = 0.0323928 e^{-x}\left(-\frac{0.00257298}{x^5}+\frac{0.0722376}{x^4}-\frac{0.715841}{x^3}+\frac{2.64516}{x^2}+\frac{1.24289}{x}\right)$$

$$\text{design load} = \frac{3\sqrt{\left(\frac{A_{sin}}{\sqrt{2}}\right)^2+\sigma_{ran}^2}}{1+\text{overshoot}}.$$

- Similar equation derived for "Equiv. 2σ" (97.725%, research suggests more appropriate for HCF)

Conclusions

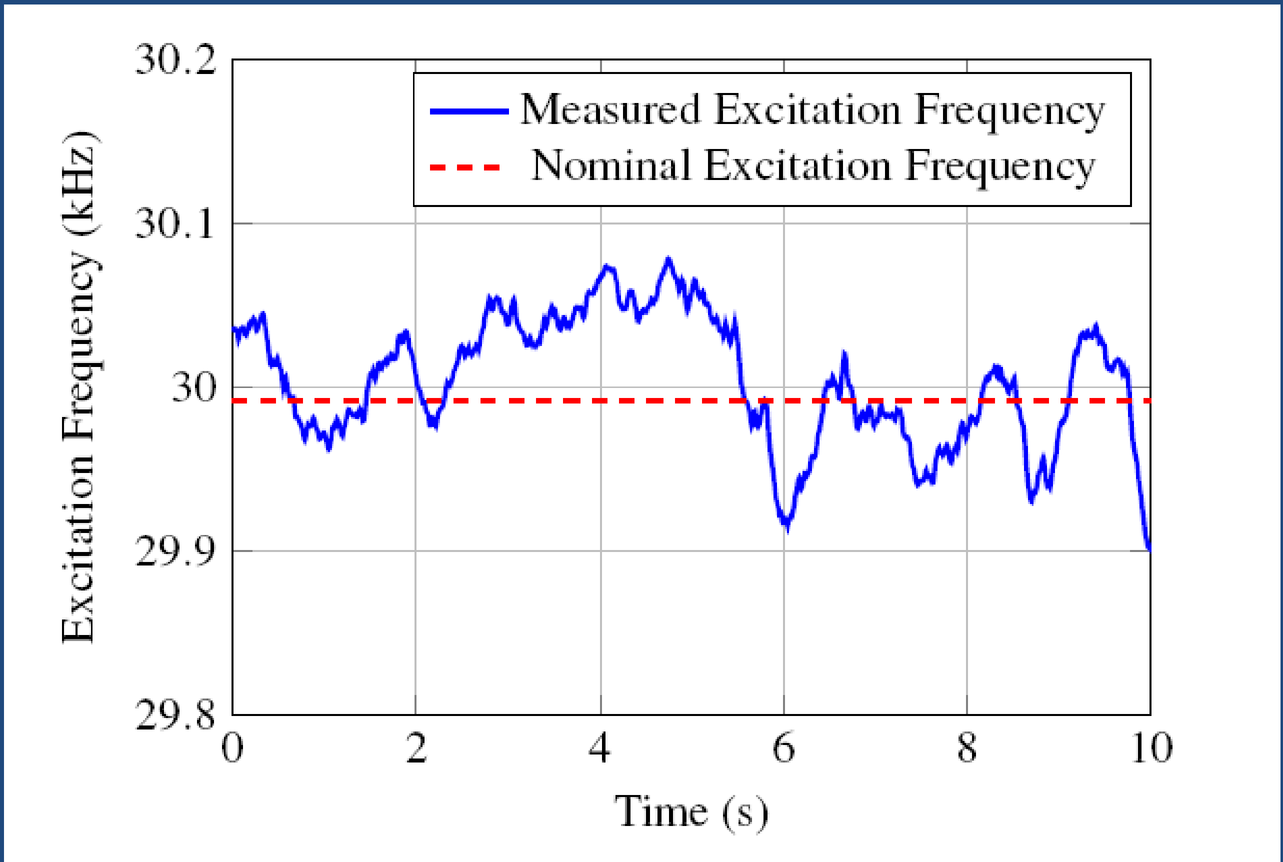
- Probability Values calculated, compared, & evaluated for several industry-proposed methods for combining random and harmonic loads.
- Two new excel macros written to calculate combined load for any specific probability level.
- Closed form Curve fits generated for widely used 3σ and 2σ probability levels.
- For design of lightweight aerospace components, obtaining accurate, reproducible, statistically meaningful answer critical.

Accounting for Speed Variation (Dither) of Turbomachinery in Analysis – Introduction

- Structural (S_{ult} & HCF) assessment critical for turbomachinery flow path components undergoing possible resonance.
- Resonance generally avoided, but impossible for higher modes found with modern analysis, especially with wide speed ranges.
  - J2-X Fuel Pump turbine stator operates from 26Krpm-34Krpm; 69N forcing excites modes 10-18 between 30KHz-40Khz.
- Criteria triggers forced response analysis at worst case resonant condition.
- Finite life analysis, where actual fatigue damage during operational time is calculated, frequently used if endurance limit criteria violated.

Many Turbopumps "Dither"

- May be beneficial to incorporate fact that real turbopumps dither about a nominal mean speed. (*separate from uncertainty in mean speed itself*)



J2-X Powerpack Adjusted Speed Trace

- During time speed is not exactly at natural frequency, damage accumulation is significantly reduced.

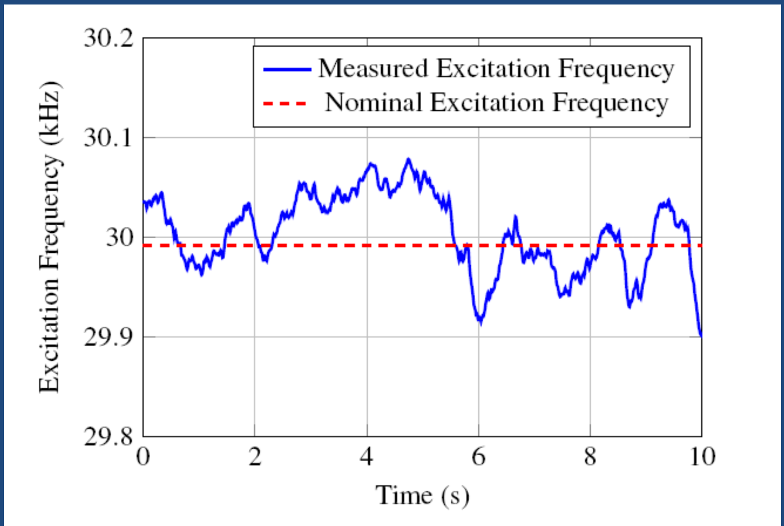
Literature, Purpose

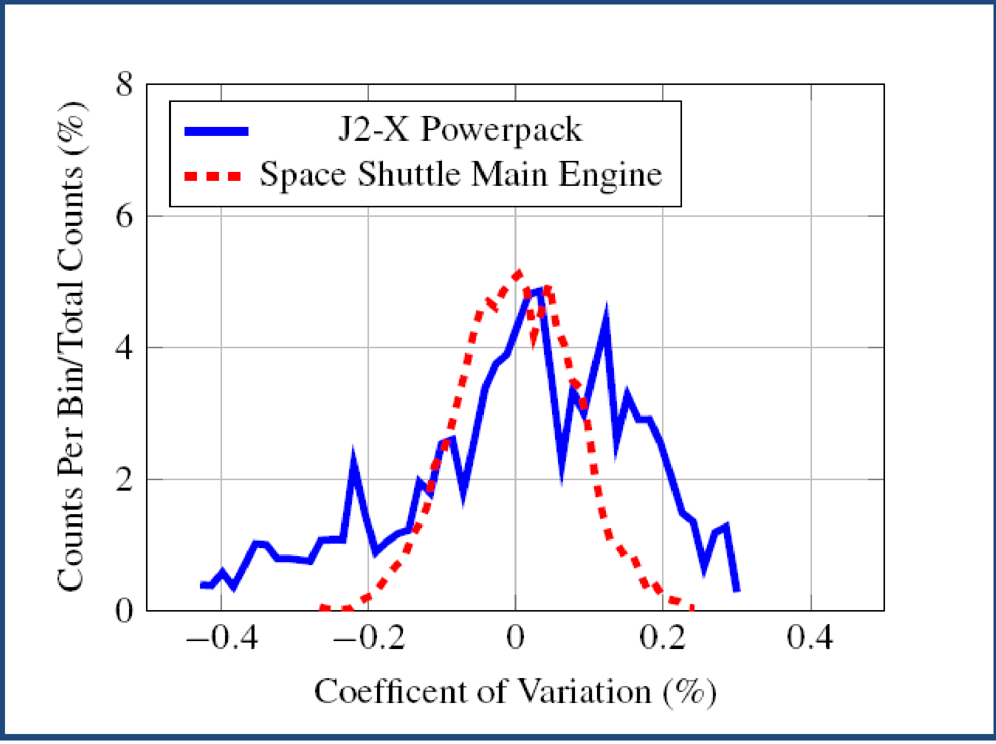
- Initial studies of response of systems with time varying excitation frequency Ω by Lewis- 1932, Cronin- 1965.
- Lollack, 2002, defined reduction in peak response for monotically varying Ω, useful for defining rate of sine-sweep tests.
- Henson, 2008, studied harmonically varying Ω.
- For rocket engines, Ω varies non-deterministically. Motivated previous work by authors (2010) that developed numerical approach for calculating response and general sensitivities.
- Unacceptable HCF factor for J2-X stator resonant 30Khz mode prompted need for practical technique.

- Purpose of this research
  - *to develop practical design techniques that account for excitation frequency stochasticity in the fatigue life of turbomachinery components.*

Excitation Data

- Taken from hot-fire testing of J2-X and SSME.
- Ω = engine speed (hz)*[forcing pressure distortions/Rev] (FPR).
- Since purpose is to examine fatigue life at resonance, actual mean speed adjusted to natural frequency for analysis.
- Histograms for two different engines show ~ Gaussian distribution of speed.



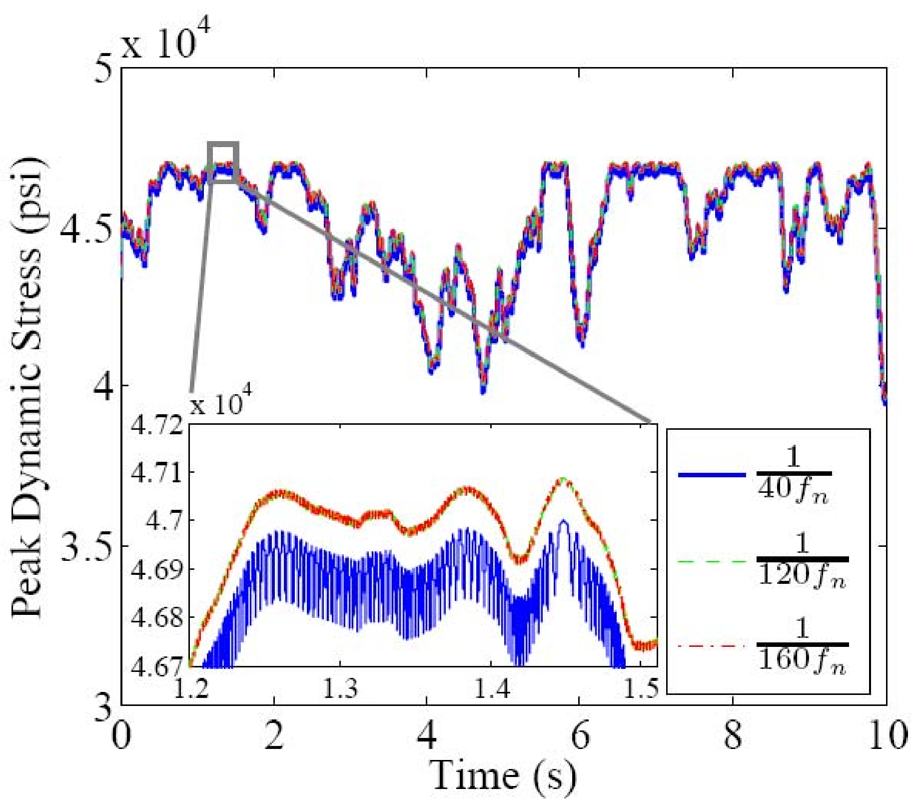


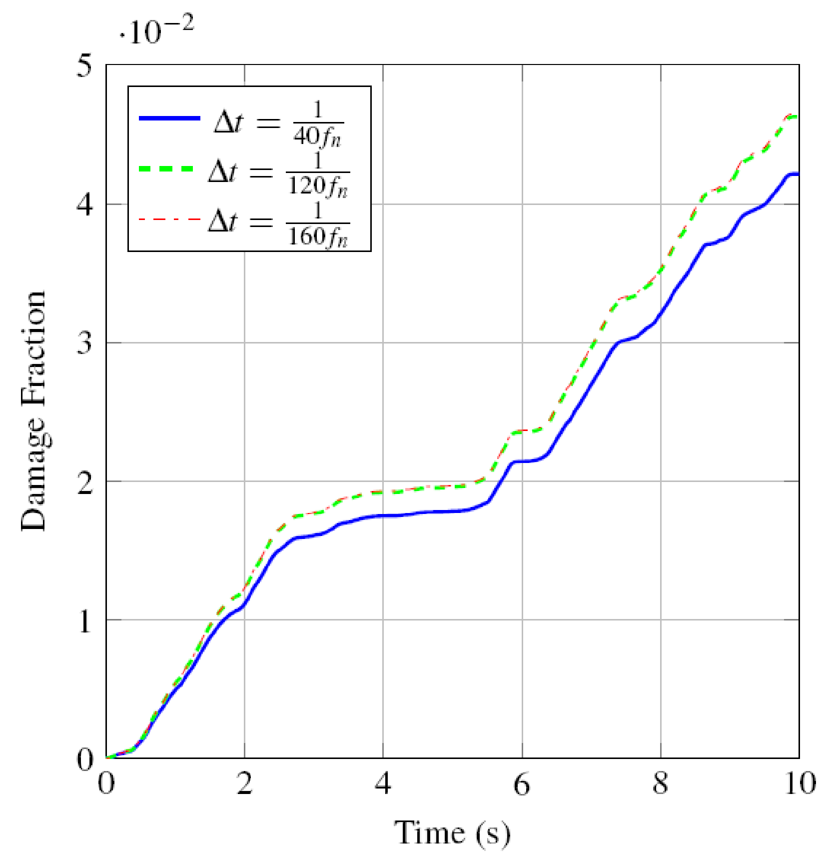
Theoretical Basis, Numerical Transient Solution

- SDOF EoM
$$\ddot{x} + 2\varsigma\omega\dot{x} + \omega^2 x = \frac{f(t)}{m}$$
where
$$f(t) = A\sin(\phi(t))$$
- Ω is derivative of ϕ(t), constant in classical vibration analysis. For specified time-varying Ω,
$$\phi(t) = \int_0^t \Omega(\tau)d\tau$$
- Calculate A necessary to generate peak resonant value of σ_{alt} previously obtained by FEA,
$$\sigma_{alt} \equiv x = \frac{A}{\omega^2 2\zeta}$$
- Now can solve for σ_{alt} in EOM with using numerical Runge-Kutte procedure implemented in Matlab; agrees with Lollack's results for linearly varying Ω.
- Finally, Calculate damage fraction Φ using Miner's rule, $\Phi = \sum_{i=1}^{K} \frac{n}{N}$, which becomes
$$\Phi(t) = \int_0^t \frac{\Omega(\tau)}{N(\tau)} d\tau$$

Convergence of Time Step in Transient Solution

- Applied deterministic speed variation from specific hot-fire test.
- Time histories of Peak Dynamic Stress and Damage Fraction generated.
- Convergence studies performed → $\Delta t = 1/120 f_n$.





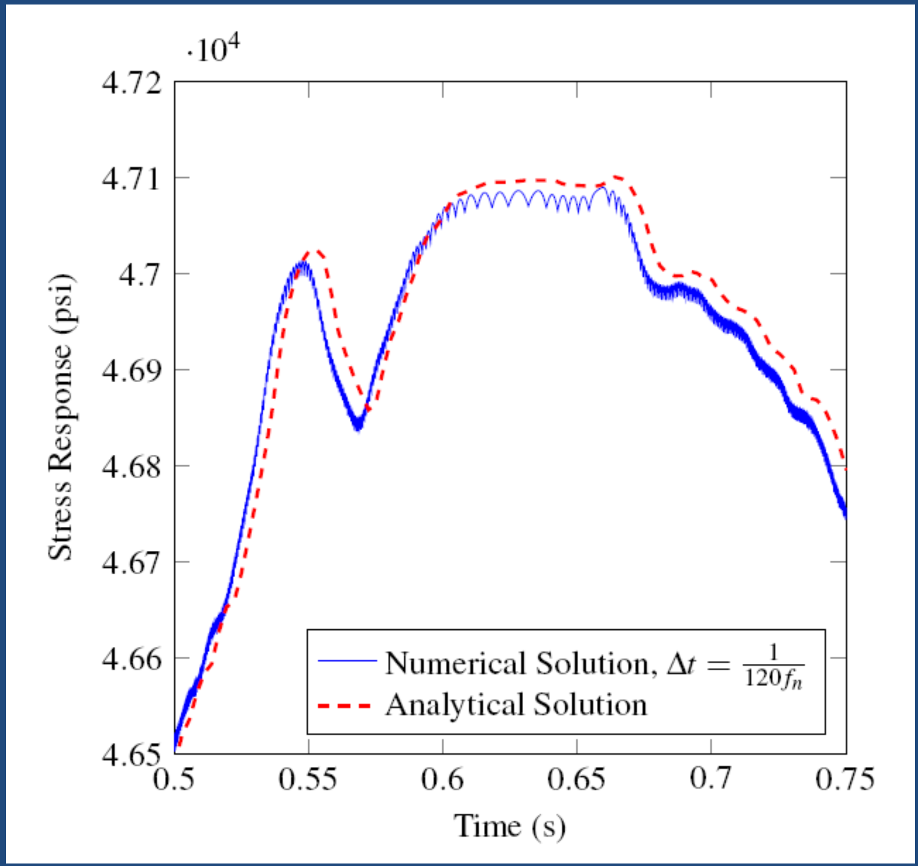
Analytical Solution

- Hypothesis from previous work that if $f_n \gg \frac{d(speed)}{dt}$, then closed-form (computationally fast) standard analytical equation for SDOF steady-state response would be accurate.

$$x_{steady-state} = \frac{A/\omega^2}{\sqrt{\left(1-\left(\frac{\Omega}{\omega}\right)^2\right)^2 - \left(2\zeta\frac{\Omega}{\omega}\right)^2}}$$

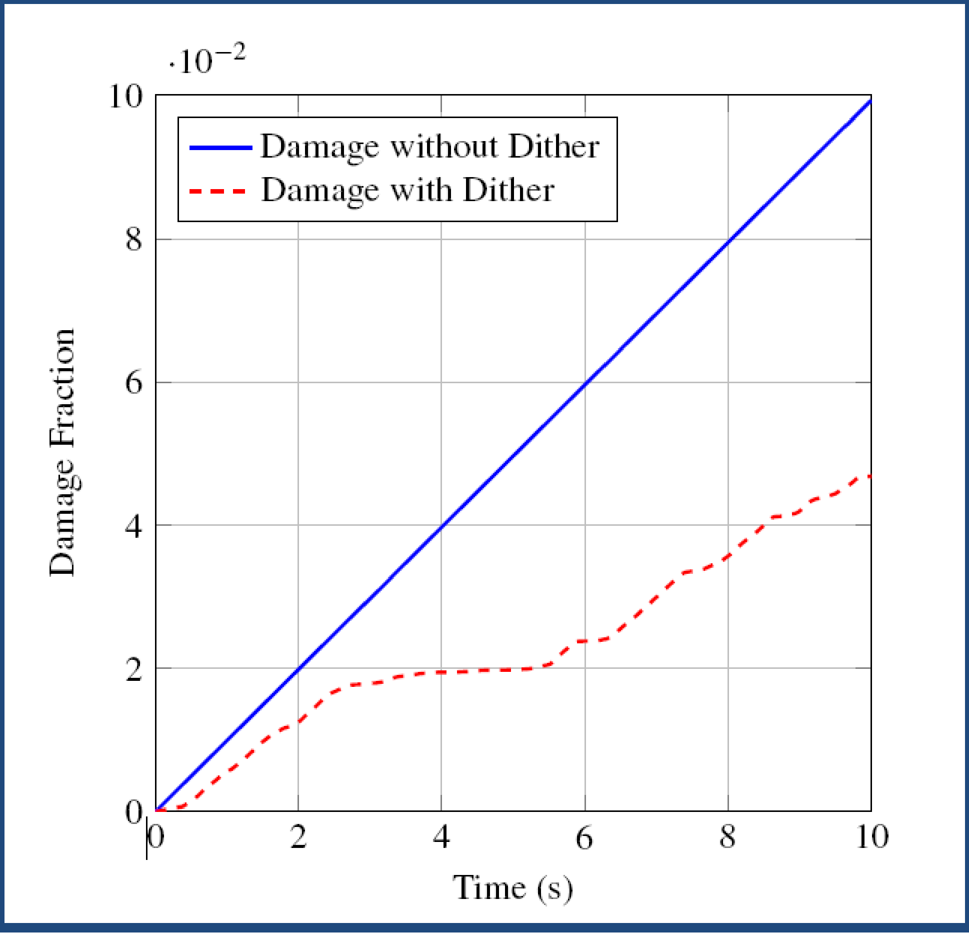
- Validation by comparing response with numerical transient solution.

Stress Response Zoom-In



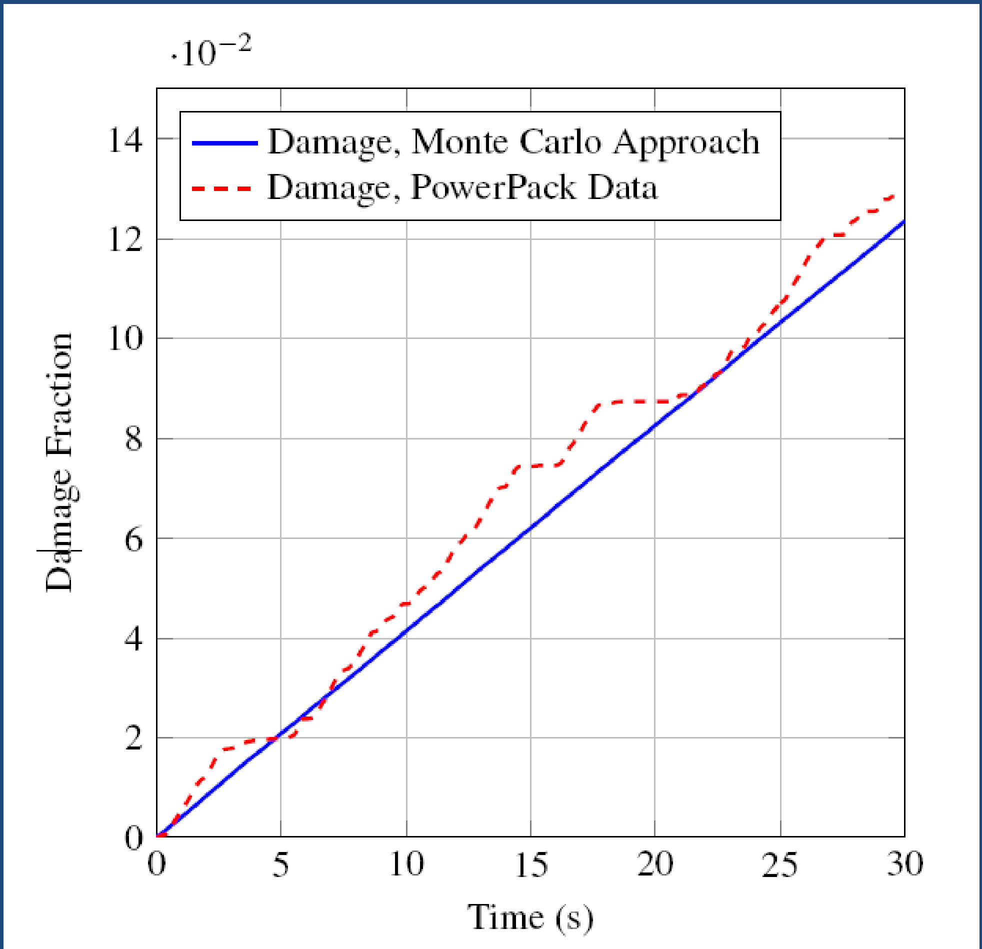
“Dither Life Ratio” for Specified Excitation History

- Calculation of damage performed considering dither for specific 10 sec. window.
- Damage calculation assuming constant resonant excitation → 2.135 times more damage, call it “Dither Life Ratio”.



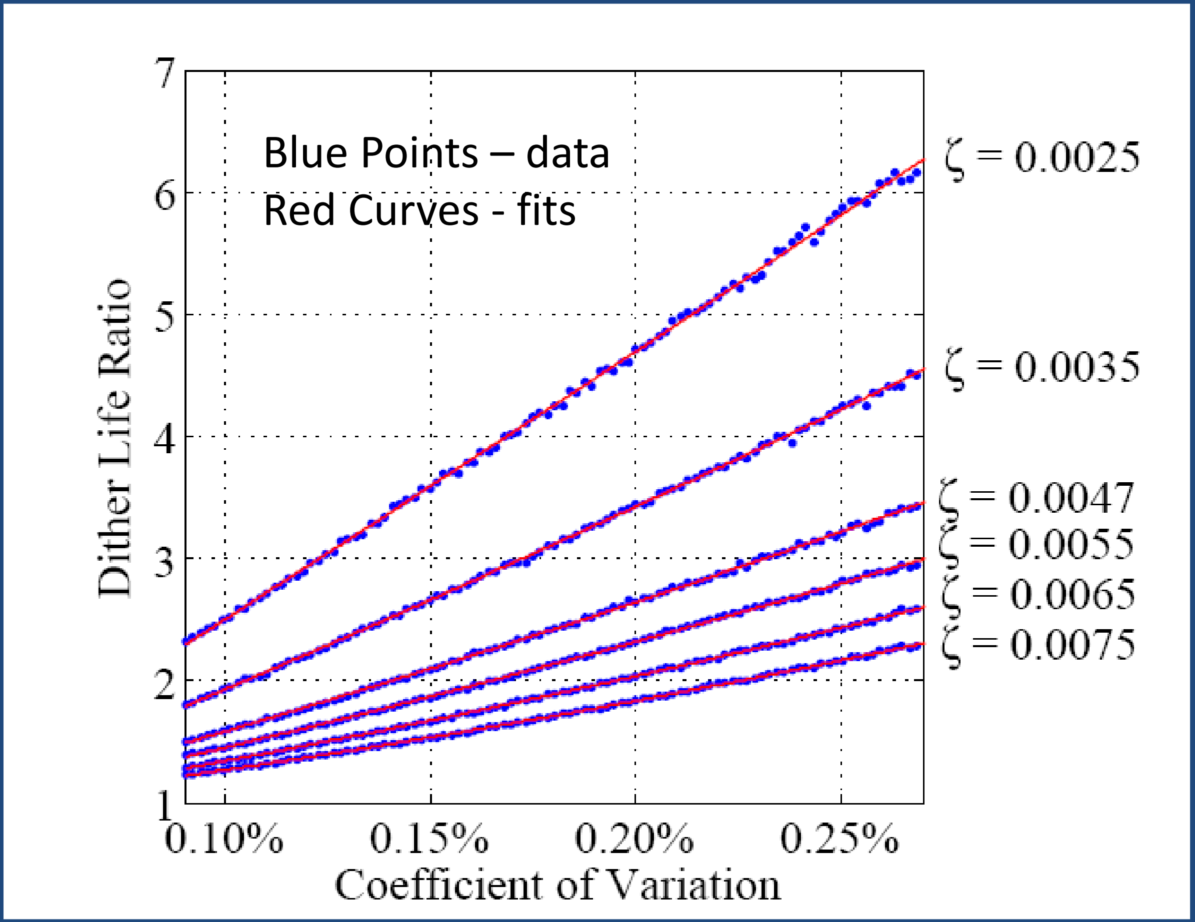
Monte Carlo for Unknown Frequency History

- During design phase, actual speed time histories unknown, but statistics from similar engines known.
- Prompted development of Monte Carlo method using rapid analytical solution.
- Speed vector created using Normal distribution.
- Powerpack data → std dev =38.6 hz (cov=0.129%).
- MC results linear because rate of change of frequency variation *not* correct (and very high), but damage accumulation is accurate on the average.



Sensitivity of DLR to speed COV and ζ

- Accuracy of Monte Carlo technique with analytical solution allows comprehensive sensitivity study to key parameters
- Results: Larger for high COV for speed, since more time spent off-resonance.
  - Larger for small ζ, since peaks are sharper and time spent off-resonance will have less response.



Conclusions

- Numerical and Analytical methods developed to determine damage accumulation in specific engine components when speed variation included.
- Dither Life Ratio shown to be well over factor of 2 for specific example.
- Steady-State assumption shown to be accurate for most turbopump cases, allowing rapid calculation of DLR.
- If hot-fire speed data unknown, Monte Carlo method developed that uses speed statistics for similar engines.
- Application of techniques allow analyst to reduce both uncertainty and excess conservatism.
- High values of DLR could allow previously unacceptable part to pass HCF criteria without redesign.
- Given benefit and ease of implementation, recommend that any finite life turbomachine component analysis adopt these techniques.