A One-Dimensional Global-Scaling Erosive Burning Model Informed by Blowing Wall Turbulence

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This paper makes no attempt to comprehensively review erosive burning models or the data collected in pursuit of them; the interested reader could begin with Landsbaum[1] for a historical summary. However, a discussion and comparison to recent work by McDonald[2] and Rettenmaier and Heister[3] will be included, along with data generated by Strand, et. al.[4,5,6,7,8]. Suffice it to say that the search for a way to predict erosive burning in any size motor with formulas cleanly applicable to a typical 1D ballistics analysis has been long thwarted. Some models were based on testing that failed to adequately simulate the solid rocket motor environment. In most cases, no real-time burn rate measurement was available. Two popular models, even when calibrated to recent motor-like real-time burn rate data obtained by Furfaro[9], were shown by McMillin[10] to be inadequate at modeling erosive burning in the Space Shuttle Reusable Solid Rocket Motor (RSRM), the Space Launch Systems' Five-Segment RSRM (RSRMV), and the five-segment Engineering Test Motor (ETM)-3.

Subsequently to the data cited from Strand and Furfaro, additional motors of the same kind as Furfaro's were tested with RSRMV propellant, utilizing 7 segments per motor and 3 throat sizes[11]. By measuring propellant web thickness with ultrasonic gages, the burn rate was determined at cross-flow Mach numbers up to Mach 0.8. Furthermore, because of the different throat sizes in otherwise identical motors, this provides a unique look at the effect of pressure and base burn rate on the erosive response.

Figure 1 shows example of the data pertaining to the high Mach motor, where the port area is initially less than the throat area. The burn rate data was processed using a smoothing method developed to reduce the noise without too severely introducing end effects that limit the range of useful data. Then, an empirical ballistics scheme was used to estimate the flow condition based on the burn rate measurements and pressure measured between each segment.
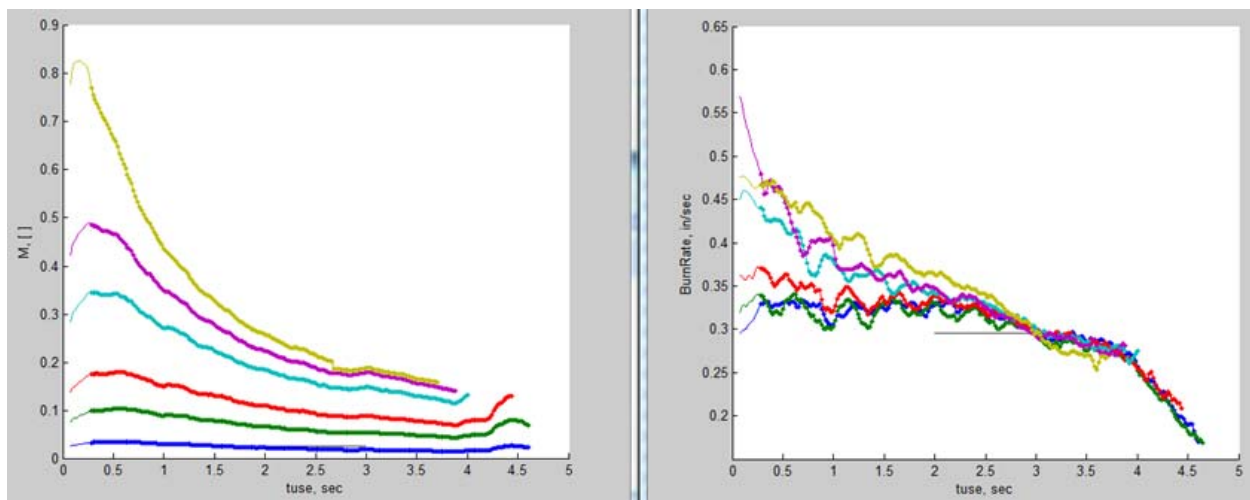
Figure 1: Analyzed Mach # and Processed Burn Rate for the Six Measured Segments of the High Mach Test Motor

Figure 2 shows the impact of the differing range of test condition on the measurement results. In this limited case, mass flux appears more capable of predicting erosive burning at the different pressure conditions. However, the mass flux parameter does not include any information about motor scale,

thus making it inadequate for building a predictive model. Once more data is added at multiple burn rates and scales, Figure 3, its generality is further questioned.
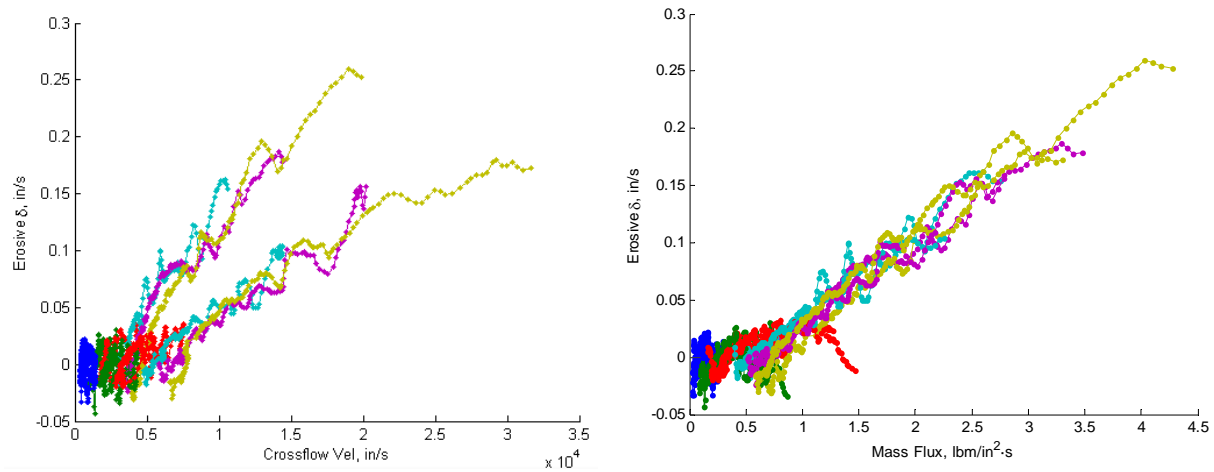

Figure 2: Erosive Contribution to Burn Rate, Plotted Against Velocity, and Against Mass Flux
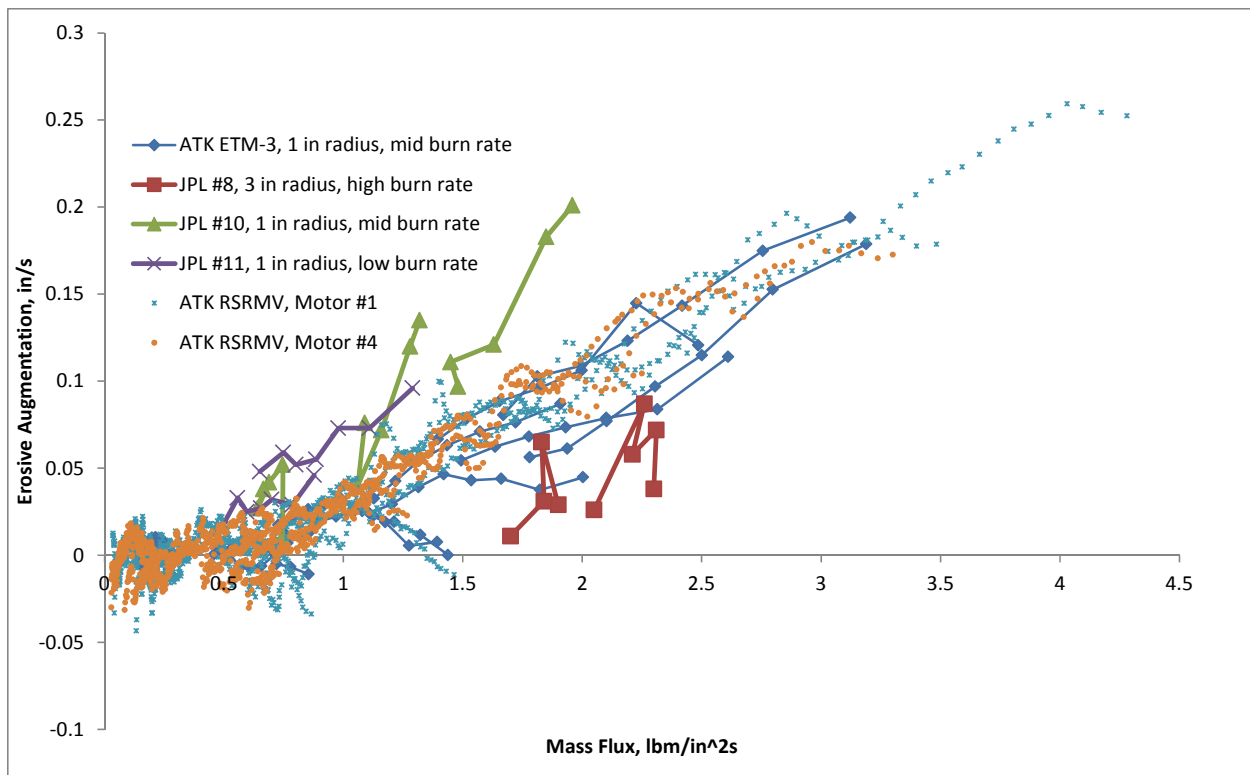

Figure 3: Erosive Behavior of Similar Propellants From Different Test Series

Though some of the JPL motors were tested at different initial port diameters of 2, 4, and 6 inches, and the latest data spans a range from 2 to 4.8 inches diameter, determining solely on these datasets how erosive burning scales with diameter cannot be reliably undertaken. This is especially true if one desires

a model capable of predicting motors with initial diameters as high as 60 inches. One must resort to fluid flow theory and semi-empirical turbulent modeling to derive a suitable parameter.

It is thought that erosive burning response can be directly related to shear stress. McDonald[12] made a valiant attempt at this using the smooth-wall turbulent Prandtl equation to link one-dimensional velocity, U_{1D}, to shear velocity, u*, thus incorporating the motor diameter through the Reynolds Number's appearance in the Prantl equation. However, it is questionable how well the Prantl equation, derived for smooth, non-blowing wall pipes, applies to solid rocket motor flow.

Instead, a turbulent flow function based on blowing wall flows was sought. The literature contain a couple significant sets[13,14] of blowing wall flat-plate data, which demonstrate a similarity relationship between the ratio of friction coefficients with and without blowing, C_f and C_{f0}, respectively, and the blowing parameter, B_f. The blowing parameter is a function of flow condition and friction coefficient, as in Equation (1), where v_w is the velocity of flow entering at the wall.

(1) $\quad B_f = \dfrac{v_w}{U_{1D}} \dfrac{1}{C_f/2}$

The relationship above can be solved implicitly for both C_f/C_{f0} and B_f, once the data have been fit with a suitable curve. Though this data was obtained for flat plate flow, because the effect on friction factor was presented as a ratio, perhaps it could still apply in a pipe-like solid rocket flow, if the base C_{f0} were drawn from pipe theory instead.

Though the Prandtl equation could be a C_{f0} relationship, it appears that solid rocket flow has more to do with fully rough than fully smooth flow. It is reasonable to assume that dimensional roughness is on the order of large AP particle size. Even assuming one-fourth of the ammonium perchlorate (AP) size as the defining roughness level, solid rocket flow when erosive burning is expected, at any but the smallest velocities, can be expected to be fully rough. For the range of Reynolds numbers of interest, Equation (2) is an adequate correlation to within 2%, where D is the diameter and k_s is the equivalent roughness. For now, k_s is evaluated as equal to the AP particle diameter, but could instead be considered some constant multiple of it.

(2) $\quad \dfrac{C_{f0}}{2} = \dfrac{1.325/8}{\left(ln\left(\frac{1}{3.7}\frac{k_S}{D}\right)\right)}$

Figure 4 shows these few witnesses to effect of blowing on turbulent friction coefficient. "Simpson Data" and the "Simpson Fit" were both listed in his thesis[13]. "PMK Fit of Data <5" is the correlation listed in Pimenta, Moffat and Kays[14]; their data span was only from 1 to 5 on the $1+B_f$ axis. The "Boardman/Lees" curve is Boardman[15] quoting a 1958 study by L. Lees, for $1+B_f$ values from 5 to 100. All three of these curve fits appear to be combinations of convenient non-dimensional parameters, not equations rigorously derived from the underlying fluid mechanics. So it seems reasonable that a different relationship could be useful even in extrapolation if: 1. It is consistent with the data, knowing that all the data has some error associated with it, 2. It correctly predicts the extreme values of $1+B_f = 1$ and $1+B_f \rightarrow \infty$, and 3. It

is simple enough to solve for C_f and B_f, so that erosive burning data can be compared to these values or combinations thereof for building a 1D ballistics correlation. For condition 2, at $1+B_f = 1$, there is no blowing, so the ratio C_f/C_{f0} must equal 1; as it approaches ∞, there is no significant crossflow relative to the strong blowing, and C_f/C_{f0} approaches 0. A function that meets all three criteria is shown in Equation (3).

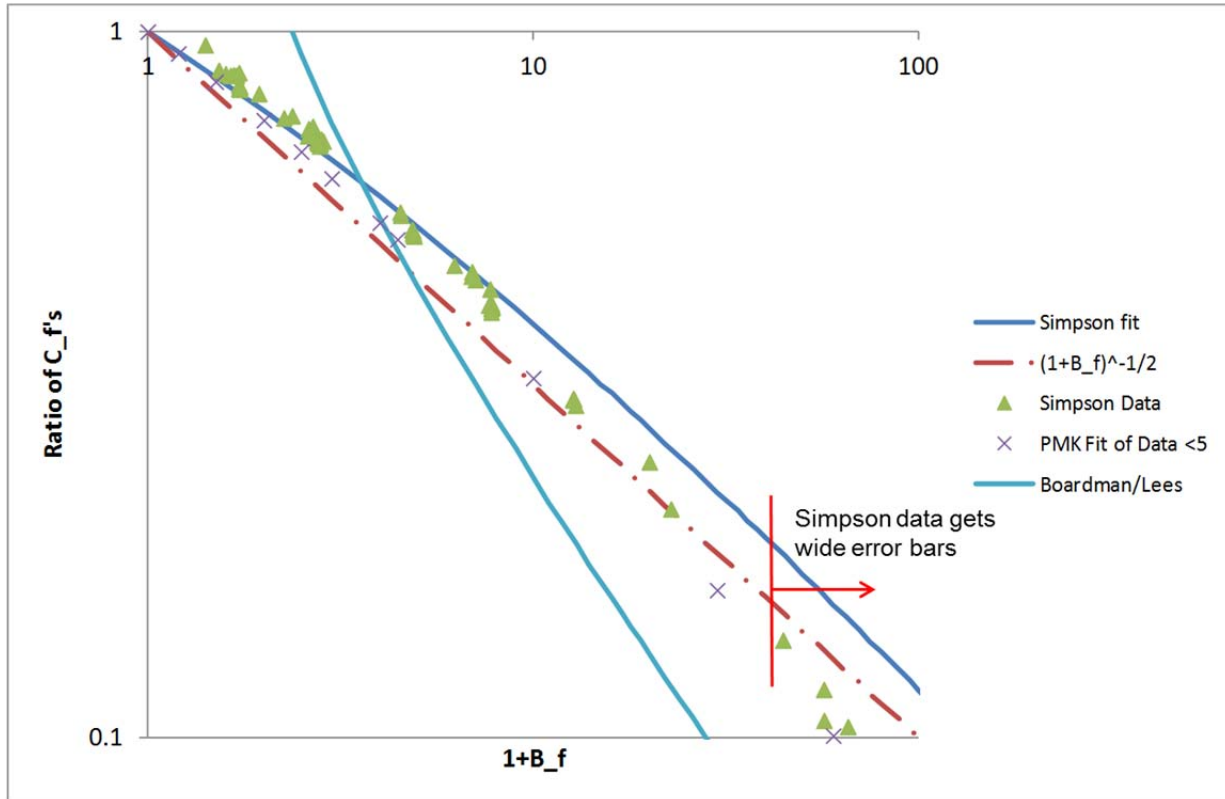$$(3) \quad \frac{C_f}{C_{f0}} = \frac{1}{(1+B_f)^{1/2}}$$

Figure 4: Blowing Wall Turbulence Results and Fits

Combining Equations (1) and (3) allows solving for C_f explicitly in Equation (4).

$$(4) \quad \frac{C_f}{C_{f0}} = \frac{2}{\frac{v_w}{U_{1D}C_{f0}/2} + \sqrt{\left(\frac{v_w}{U_{1D}C_{f0}/2}\right)^2 + 4}}$$

Now the C_f can be calculated for every data point in the erosive burning test sets. Two related parameters appear potentially useful, as shown for now for a subset of the data in Figure 5.

$$\sqrt{\frac{C_f}{2}}\frac{U_{1D}}{v_w}$$

$$\left(\frac{1}{1+v_0^+ U_{1D}^+}\right)^{1/2} = \frac{1}{(1+B_f)^{1/2}} = \frac{C_f}{C_{f0}}$$
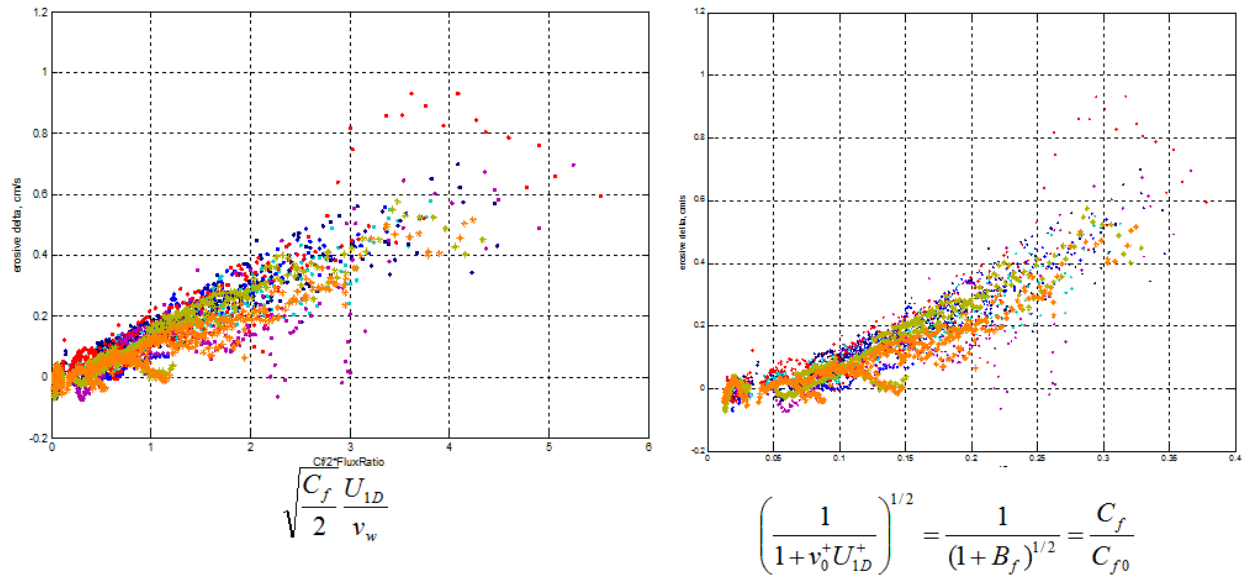
Figure 5: Erosive Burning Correlates to Two Friction Coefficient Parameters

This correlation will be expanded upon and rigorously computed using all the data for the final paper. For now, initial correlation estimates, when used to predict large motors ETM-3 and RSRMV, indicate pressure increases of low-to-mid 10s of psi, consistent with the motor data. Other popular models, by contrast, unless modified with a parameter specific to the large diameter scale, predict hundreds of psi, or none at all. The final paper will include relative pressure results based on predictions and reconstructions of these motors' performance using the developed model.

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