Extended Analytic Device Optimization Employing Asymptotic Expansions

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Objective:

Optimal design of thermoelectric couples is investigated analytically for a range of cases using less severe assumptions then the classic solution. Employed in the solution sets are a range of powerful methodologies including asymptotic expansions and Green's Function solutions. Obtained are new dimensionless parameters which should be considered in material selection and device design.

Methodology:

Thermal

$$\frac{\partial}{\partial x}\left[-k_{a,b}\frac{\partial T_{a,b}}{\partial x}\right]+\frac{I_{a,b}\tau_{a,b}}{A_{a,b}}\frac{\partial T_{a,b}}{\partial x}-\frac{{I_{a,b}}^2}{{A_{a,b}}^2\sigma_{a,b}}=\rho_{a,b}c_{p_{a,b}}\frac{\partial T_{a,b}}{\partial t}$$

Electrical

$$\frac{\partial \varphi_{a,b}}{\partial x}=-\alpha_{a,b}\frac{\partial T_{a,b}}{\partial x}-\frac{I_{a,b}}{A_{a,b}\sigma_{a,b}}$$

Ohm's Law

$$\varphi_b(L_b)-\varphi_a(L_a)=IR$$

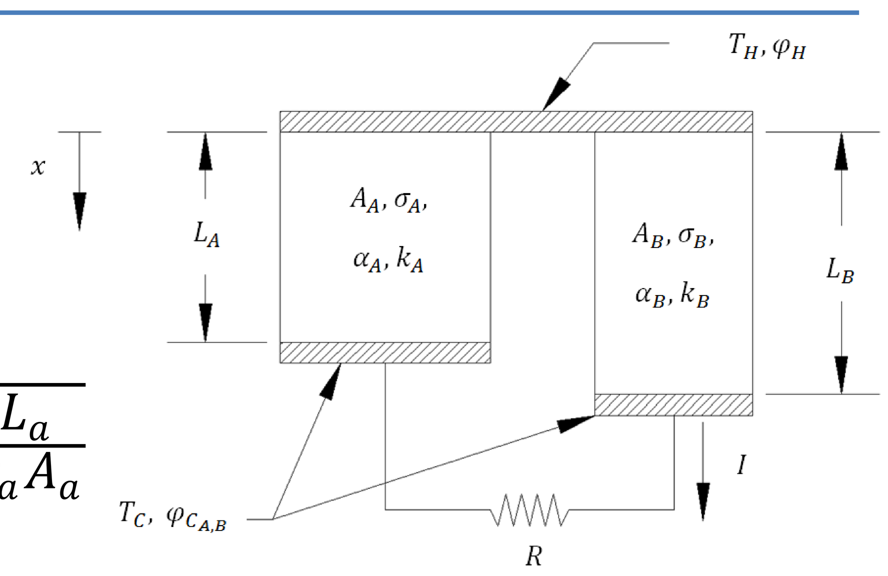
Rectangular Coordinates:

Slenderness ratios:

$$X=\frac{A_b L_a}{A_a L_b}$$

Dimensionless resistance:

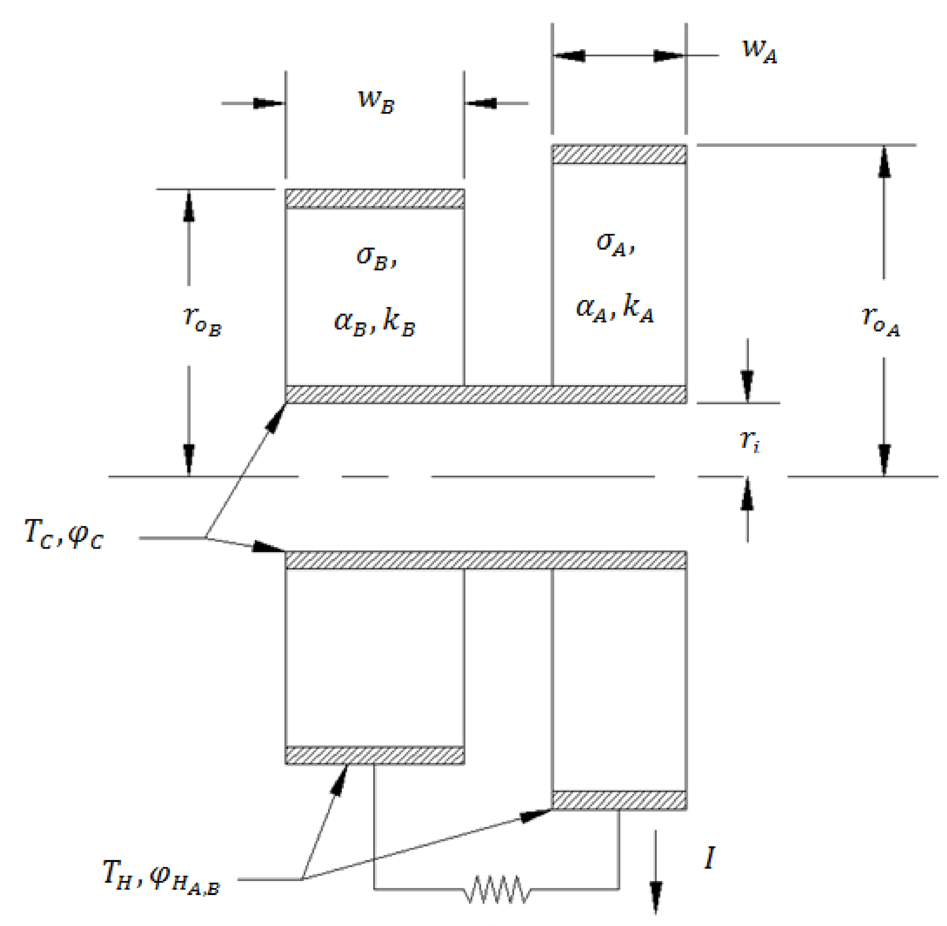
$$Y=\frac{R}{\frac{L_b}{\sigma_b A_b}+\frac{L_a}{\sigma_a A_a}}$$



Cylindrical Coordinates:

$$X=\frac{w_b \ln\left({r_{o,a}}/{r_i}\right)}{w_a \ln\left({r_{o,b}}/{r_i}\right)}$$

$$Y=\frac{R}{\frac{\ln\left({r_{0,b}}/{r_i}\right)}{2\pi\sigma_b w_b}+\frac{\ln\left({r_{o,a}}/{r_i}\right)}{2\pi\sigma_a w_a}}$$



Convection Coefficient Approximation:

- Effective convection coefficient accounts for convection, radiation, and conduction effects between the thermal reservoir and legs lumped into a single parameter.

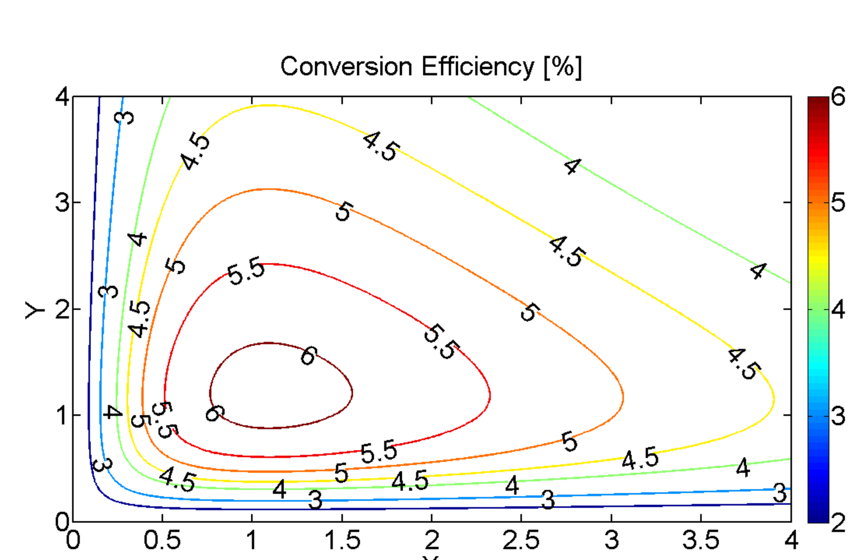
$$h_{eff}^{-1}=h^{-1}+\sum_j\frac{L_j}{k_j}+\frac{1}{\varepsilon\sigma(T_s+T_\infty)(T_s^2+T_\infty^2)}$$

Case Assumptions:

Case	Boundary Condition	Leg Sides	Material Properties	Time Dependence	Material	Temperature [K]
1	Fixed	Insulated	Constant	Steady	Si/Ge RTG	1123/573
2	**Convection**	Insulated	Constant	Steady	Si/Ge RTG	1123/573
3	Fixed	**Convection**	Constant	Steady	Si/Ge RTG	1123/573
4	Fixed	Insulated	**Variable**	Steady	Si/Ge RTG	1123/573
5	Fixed	Insulated	Constant	**Transient**	Si/Ge RTG	1123/573

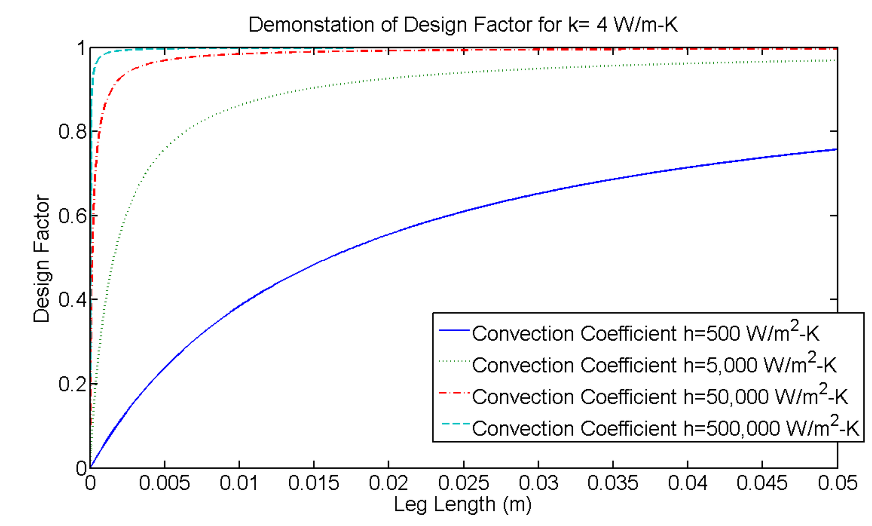
1. Classic Solution:

- The location of the maximum on the conversion efficiency contour plot coincides with the classic solution.
- Presentation of the device design space in this fashion allows for a better understanding of device design.
- Solution stands for both rectangular and cylindrical geometries when the appropriate X and Y values are used.



2. Convection Ends:

- Demonstrates the critical importance of heat exchanger design.
- Design Factor stands as a dimensionless parameter to characterize device design in addition to classic Figure of Merit.
- Efficiency approaches the classic solution as Design Factor approaches unity.



Conversion Efficiency:

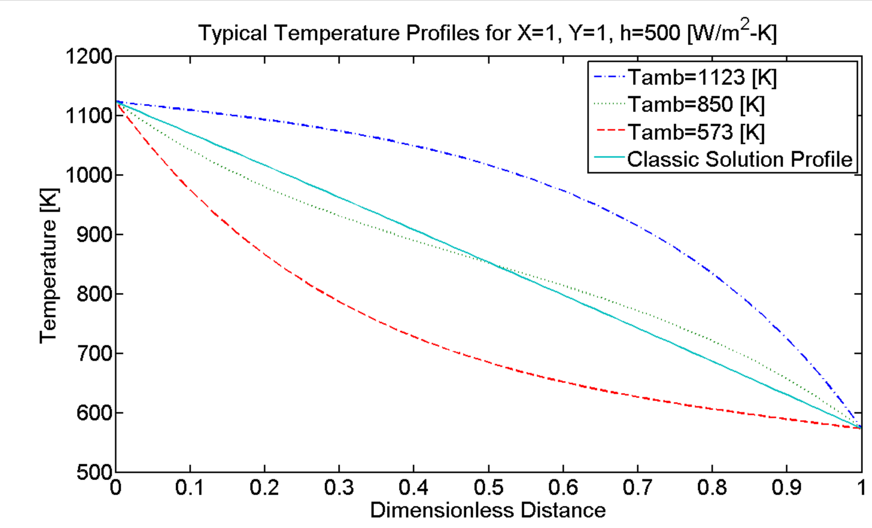
$$\eta=\frac{\eta_{c\infty}Y}{\frac{(1+Y)^2}{T_{\infty h}Z(X,D_b,D_a)}+\frac{(1+Y)(\alpha_b-\alpha_a)}{(D_b\alpha_b-D_a\alpha_a)}\left[1-\frac{\eta_{c\infty}}{2}\left\{1-\frac{D_a+D_b}{2}\right\}\right]-\frac{1}{2}\eta_{c\infty}}$$

New Dimensionless Parameter:

$$\textit{Device Design Factor}: D_{a,b}=\frac{1}{1+\frac{k_{a,b}(h_h+h_c)}{L_{a,b}h_h h_c}}$$

3. Convection Sides:

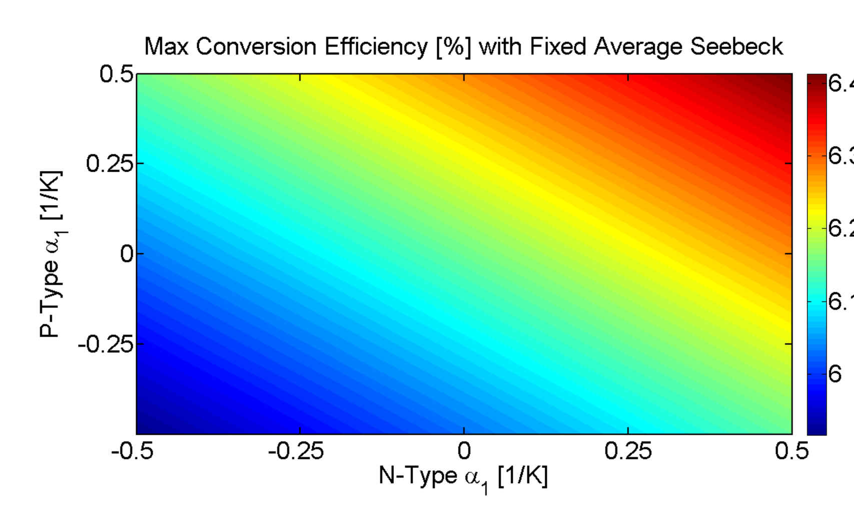
- Demonstrates some of the effects of thermal insulation.
- Accounts for lateral heat transfer in or out of a leg and places a scale of importance on the result of this often neglected heat.
- Leg temperature profiles can be seen to reside near the ambient temperature for the majority of the leg, shown right.
- Analytic solution provides a useful design guideline to achieve maximum conversion efficiency.



$$\textit{Design Guideline}: \frac{PhL^2}{kA}\ll 1$$

4. Variable Property Asymptotics:

- Solution is obtained through straight forward asymptotic expansion.
- Varying properties create non-linear governing equations making exact solutions unreasonable.
- A range of conversion efficiencies exist for a fixed Figure of Merit, shown right.



Asymptotic Expansions:

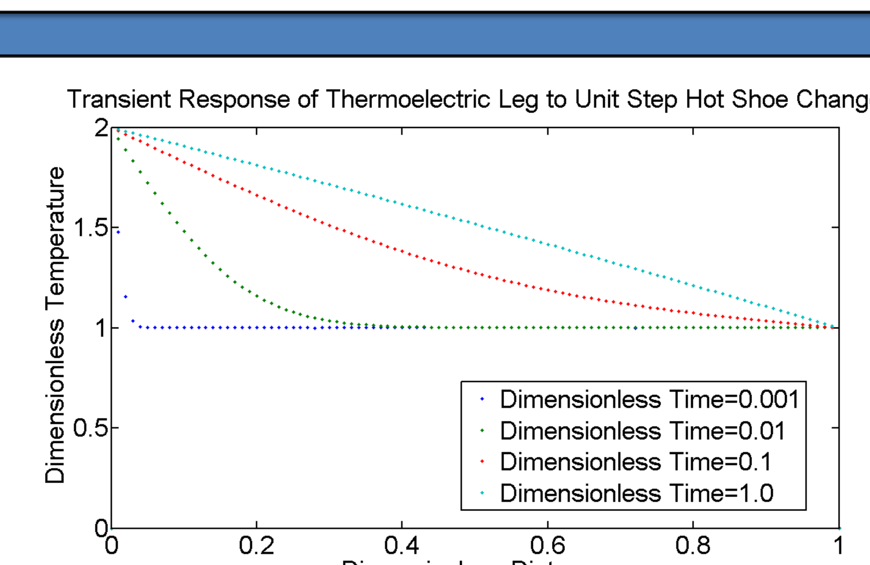
$$\hat{T}=T_0+\epsilon T_1$$

$$\hat{\varphi}=\varphi_0+\epsilon\varphi_1$$

$$\alpha(T)=\tilde{\alpha}\frac{\alpha(T)}{\tilde{\alpha}}=\tilde{\alpha}(\alpha_0+\epsilon\alpha_1 T)$$

5. Transient Green's Function:

- Method solves partial differential system using Green's Function solutions.
- Temperature and voltage profiles are determined as integrals of Green's Functions along with arbitrary initial and boundary conditions.



Green's Function:

$$G(\xi,\tau;x,t)=-2H(t-\tau)\sum_{n=1}^{\infty}e^{n^2\pi^2(\tau-t)}\sin(n\pi x)\sin(n\pi\xi)$$

Solution Scheme:

$$T_{a,b}(x,t)=\textit{Initial}+\textit{Boundary Conditions}+\textit{Joule Heating}$$

Summary:

A set of analytic solutions, well suited for design optimization, have been generated and can serve as both learning tools and numerical benchmark solutions. The cases demonstrate the importance of new design considerations in addition to Figure of Merit.

Case	B.C. Convection [W/m²K]	Leg Side Convection [W/m²K]	Max Efficiency [%]	Xopt	Yopt	Max Power Density [W/m²]
1	∞	0	6.15	1.09	1.22	17,733
2	50,000	0	6.05	1.09	1.22	17,118
2	5,000	0	5.26	1.09	1.23	12,780
2	500	0	2.28	1.09	1.27	2,300
3	∞	5	6.05	1.10	1.21	17,733
3	∞	50	5.33	1.20	1.19	17,733
4	∞	0	6.01	1.16	1.15	17,333