REGIONAL SPHERICAL HARMONIC MAGNETIC MODELING FROM NEAR-SURFACE AND SATELLITE-ALTITUDE ANOMLAIES

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Abstract

The compiled near-surface data and satellite crustal magnetic measured data are modeled with a regionally concentrated spherical harmonic presentation technique over Australia and Antarctica. Global crustal magnetic anomaly studies have used a spherical harmonic analysis to represent the Earth's magnetic crustal field. This global approach, however is best applied where the data are uniformly distributed over the entire Earth. Satellite observations generally meet this requirement, but unequally distributed data cannot be easily adapted in global modeling. Even for the satellite observations, due to the errors spread over the globe, data smoothing is inevitable in the global spherical harmonic presentations. In addition, global high-resolution modeling requires a great number of global spherical harmonic coefficients for the regional presentation of crustal magnetic anomalies, whereas a lesser number of localized spherical coefficients will satisfy. We compared methods in both global and regional approaches and for a case where the errors were propagated outside the region of interest. For observations from the upcoming Swarm constellation, the regional modeling will allow the production a lesser number of spherical coefficients that are relevant to the region of interest.

Introduction

Studies on geomagnetism have used a spherical harmonic analysis as a tool of representing Earth's magnetic fields both of internal and of external. Therefore, geomagnetic features have been modeled by spherical harmonics in a global scale. However, this global approach is best applied where the data are uniformly distributed over the entire Earth such as polar-orbiting satellite measurements. Until now, these satellite observations are good for regional or larger-scale studies due to the limited resolutions at such altitudes. For the detailed features of the magnetic anomalies, unequally distributed data measured at the near-surface are useful but cannot be easily adopted in global modeling. A spherical cap analysis (Haines, 1985) has been used as an alternative in regional modeling for geopotential fields and recently this technique was well updated by Thebault et al. (2006) and Thebault (2008). In this study, we introduce a more straightforward regional analysis technique as a magnetic field modeling that has been effectively implemented to model satellite gravity field data for small cap regions of the Earth and Moon (Han et al.,2008). We tested to compare between the methods in both global and regional approaches and a case that the errors are propagated outside the region of interest in modeling in a different way. A high-resolution, regional modeling is also attempted with Australian gridded data grids by a lesser number of spherical coefficients that are relevant to the region of interest.

Theory of Spherical Slepian basis functions

Slepian (1983) showed that optimally concentrated, band-limited and time-limited signal is considered by maximizing the energy inside the region (or interval) to the whole area. This concentration problem turned out to be the association of the eigenvalues and eigenvectors of an integral matrix form of spherical harmonics.. Such basis functions are orthogonal each other and satisfy the same differential equation and boundary condition as the spherical harmonics on a sphere. These are in real the linear combinations of conventional spherical harmonic functions only concentrating the area of interest.

Given a band-limited signals $f(\theta,\varphi)$ on a sphere in terms of the usual Spherical Harmonics with a relevant set of coefficients, these signals also can be represented by a set of Slepian basis functions and expansion coefficients, respectively, as follows:

$$f(\theta,\varphi)=\sum_{n=0}^{N}\sum_{m=-n}^{n}g_{nm}Y_{nm}(\theta,\varphi)=\sum_{k=1}^{(N+1)^2}c_k s_k(\theta,\varphi) \qquad (1)$$

where θ and φ are colatitude and longitude of the sphere, respectively. $Y_{nm}(\theta,\varphi)$ is spherical harmonic function of degree *n* and order *m*, g_{nm} is the SH coefficient of the same degree and order, respectively. These signals can also be represented by the *k*-th Slepian function, s_k and its expansion coefficients, c_k, respectively. *N* is the highest degree of spherical harmonic expansion used. The *k*-th Slepian function is the expression of linear combination of spherical harmonic functions so that it can be rewritten as;

$$s_k(\theta,\varphi)=\sum_{n=0}^{N}\sum_{m=-n}^{n}s_{k,nm}Y_{nm}(\theta,\varphi) \qquad (2)$$

The new spherical harmonic coefficients, $s_{k,lm}$ of new basis function, $s_k(\theta,\varphi)$ is now determined by maximizing the energy concentration over the area of interest to the energy over the entire sphere, which is as follows:

$$\lambda_k=\int_0^{2\pi}\int_0^{\theta_0}s_k^2(\theta,\varphi)\sin\theta d\theta d\varphi \Big/ \int_0^{2\pi}\int_0^{\pi}s_k^2(\theta,\varphi)\sin\theta d\theta d\varphi \qquad (3)$$

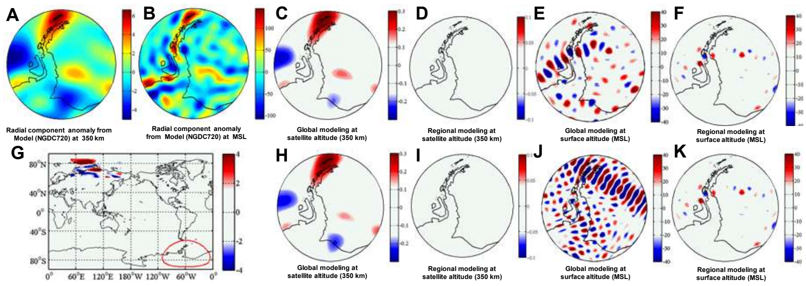
where λ_k is the ratio of energy concentrated within the spherical cap (θ_o) compared to the entire sphere . Grunbaum *et al.* (1982) and Simons and Dahlen (2006) showed that the solution to this problem turned out an eigen problem in a spectral domain. With using (2) and orthonormality property, this can be straightforwardly expressed as:

$$\sum_{n=0}^{N}\sum_{m=-n}^{n}D_{nmn'm'}s_{k,nm}=\lambda_k s_{k,n'm'} \qquad (4)$$

$$D_{nmn'm'}=\int_0^{2\pi}\int_0^{\theta_0}Y_{nm}(\theta,\varphi)Y_{n'm'}(\theta',\varphi')\sin\theta d\theta d\varphi \qquad (5)$$

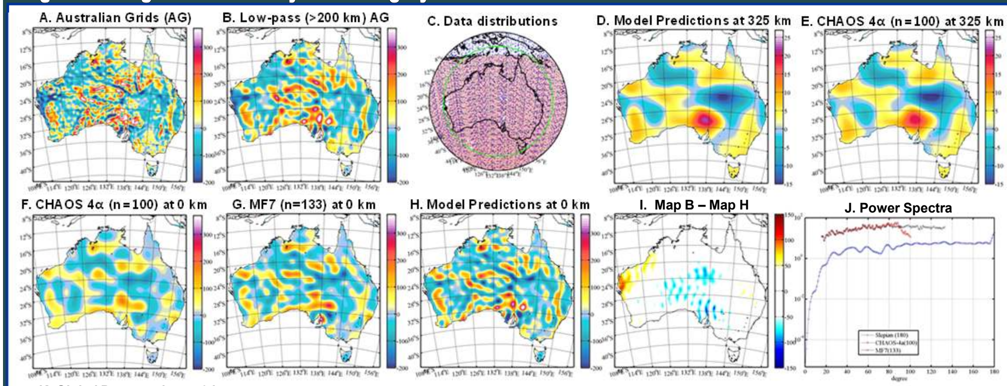
The λ_k and $s_{k,lm}$ should be determined by the eigenvalues and eigenvectors of $D_{nm,n'm'}$ with the geometric bounds of the area of interest (or radius of spherical cap) . This area can also have irregular bounds on the sphere such as Australia or Africa continents. It is noted that the total number of $(N+1)^2$ eigenvalues and eigenvectors exist but the necessary ones to optimally represent the maximum energy of signals over the area of interest are much fewer. That is, some of the largest eigenvalues (close to 1) and the corresponding eigenvectors are chosen to use as a set of basis functions concentrated within the area of interest.

Advantages of Regional over Global: Truncation errors and error distributions

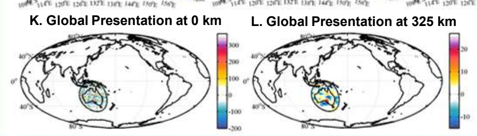


We made to compare between the methods in the global and regional approaches. First, we synthesized the anomaly features over the Weddell Sea area from the global spherical harmonic model (NGDC720, Maus, 2010) using the degrees of 16 to 150 at the satellite altitude (~350 km) (**Map A**) and the surface level (**Map B**). Then, using the global SH approach, we modeled the degree up to 120 only. The differences between the synthesized and the modeled at 350 km (**Map C**) and at the surface (**Map E**), respectively. However, using the regional (i.e., Slepian) approach, we better modeled with only 339 coefficients out of 14,640 (i.e., the total number of usual global SH coefficients) and showed the differences at the 350 km (**Map D**) and at the surface (**Map F**). The bottom-rowed images are made to test the error propagation outside the region of interest. For the global modeling, we used all global no matter how different the error budgets are distributed geographically. We assumed some errors in the satellite data over the Arctic regions (**Map G**), and modeled the anomalies over the Weddell Sea area with the same model parameters. The resulting differences between the different modeling approaches, are seen in **Map H** at the satellite altitude in a global, and in **Map I** in a regional approach. The anomaly differences are more significant at the surface level such as **Maps J** (global) and **K** (regional).

Regional magnetic anomaly modeling by localized basis functions over Australia

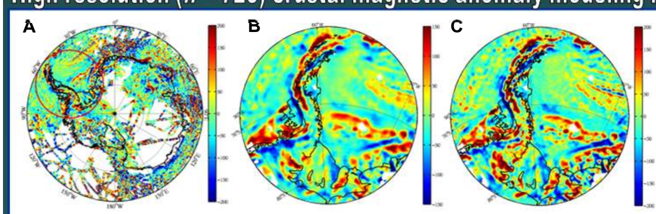


Australian aeromagnetic grids with a 0.05°. Interval (**Map A**) was low-pass filtered with 200 km WL (**Map B**), resampled at 0.5°. CHAMP vector data were selected for the use of the global model, *CHAOS-4α* (N. Olsen). All data are shown in **Map C** [AG:7891(red), CHAMP:6072(blue)]. With two data sets, Slepian SH modeling was done over the cap area of radius 25° centered at 25°S,135°E with 1600 Slepian coefficients (which is equivalent to the degree of 180 in global SH modeling). The scalar crustal anomaly predictions at 325 km altitude (**Map D**) and surface level (**Map H**), respectively, are present.



For the comparison, we mapped the scalar magnetic anomalies from *CHAOS-4α* model with degree 100 at the same altitudes (**Map E** at 325 km, **Map F** at the surface). Also, the scalar crustal anomaly predictions from *MF7* (Maus et al.) up to 133 at the surface level is shown in **Map G**. The differences between the observations and the Slepian model predictions at the surface level are also drawn in **Map I**. Power spectra among the global models and Slepian-converted to the global SH are in **Map J**. As shown, better representations in amplitude and more detailed features at the surface level are seen in Map H. **Maps K** and **L** are the presentations of Slepian model into the global scale at both altitudes. Black circled lines are the same as the boundary of **Map C**.

High resolution (n = 720) crustal magnetic anomaly modeling in the Antarctic region (Weddell Sea area)



A. Newly compiled ADMAP data with some (not all) newly-added survey tracks. A red circle delineated the area for high-resolution modeling in this study.

B. The predictions from NGDC720 (Maus, 2010) over the same area. As the statistics reads, the energy and the wavelengths compared to the localized modeling outputs are significantly different. It infers that the high-resolution localized modeling will properly represent the anomaly characteristics.

Data sets	Min. Max (nT)	Mean (nT)	Std.Dev. (nT)	C.C.
I. Input	-636,683	2.4	83.1	0.99
II. Predicted	-641,658	2.15	82.4	
III. NGDC(16-720)	-305,359	-0.3	53.2	0.77
Differences (**I** - **II**)	-89,93	0.2	12.3	
Differences (**II** - **III**)	-374,470	2.5	53.5	

C. Predicted anomaly estimates over the Weddell Sea area. The degree 720 cut-off corresponds to an angular wavelength of 30 arc minutes, providing a 15 arc minute model resolution. The input data were first low-pass filtered long than 50 km WL and resampled at the grid of 12.5 km. Total points of input data in this cap (radius is 11.5°, centered at 71.5°S, 57°W) was amounted to 34,754 and the number of Slepian coefficients used for this modeling was 5,578. Note that the number of gauss coefficients to represent the degree up to 720 (from 16) for global expansions is 518,145.

Acknowledgements

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