

**Engine Load Path Calculations – Project Neo**

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# Engine Load Path Calculations – Project Neo

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A mathematical model of the engine and actuator geometry was developed and used to perform a static force analysis of the system with the engine at different pitch and yaw angles. This analysis yielded the direction and magnitude of the reaction forces at the mounting points of the engine and actuators. These data were used to validate the selection of the actuators installed in the system and to design a new spherical joint to mount the engine on the test fixture. To illustrate the motion of the system and to further interest in the project, a functional 3D printed version of the system was made, featuring the full mobility of the real system.

## Nomenclature

$T$  – Thrust output of the rocket engine

$F_g$  – Weight of the engine assembly

### Vertical Model

$\theta$  – Pitch angle of the engine measured in the vertical plane

$R_a$  – Reaction force at the joint of the engine and simplified actuator

$R_e$  – Reaction force at the pivot of the engine

$\delta$  – Angle of reaction force  $R_e$

$\phi$  – Angle of the actuators when moving in the vertical plane

### Horizontal Model

$\lambda$  – Yaw angle of the engine measured in the horizontal plane

$R_{ex}$  – Reaction force at the pivot of the engine in the x direction

$R_{ey}$  – Reaction force at the pivot of the engine in the y direction

$R_{ez}$  – Reaction force at the pivot of the engine in the z direction

$R_{a1}$  – Reaction force at the joint between the engine and the left actuator

$R_{a2}$  – Reaction force at the joint between the engine and the right actuator

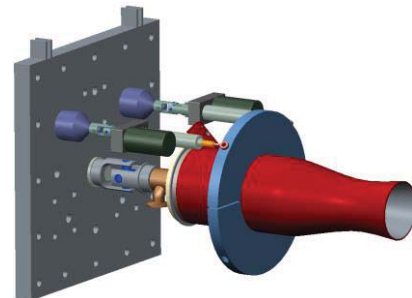
$\phi_1$  – Angle of the left actuator measured from the neutral ( $\lambda=0$ ) position

$\phi_2$  – Angle of the right actuator measured from the neutral ( $\lambda=0$ ) position

## I. Introduction

The engine test skid, which was the focus of my internship, is part of Rocket University's Project Neo (Figure 1). The Neo test skid will be used as part of a Rocket University course on cryogenic liquid methane engines. In addition, the test fixture can be used for projects wanting to measure the thrust output of an engine, to blast surface samples as part of the development of Rocket University curriculum, or. The test fixture could potentially be transported to other centers or to universities to teach about systems engineering via demonstrations of the engine test fixture.

My work focused on adding the ability to change the angle of the engine, known as gimbaling. The work I have



**Figure 1. Isometric View of Engine Assembly.** This model, created in Creo Parametric, shows the engine (red) with mounting ring (blue gray), linear actuators (shades of gray), and the support plate in gray

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done is separated into three major sections: static force analysis of the engine and actuator assembly, designing a spherical joint for mounting the engine, and 3D printing a model of the test skid featuring full mobility of the engine and actuators.

## II. Static Force Analysis

A static force analysis of the engine and actuator assembly was completed to determine: whether the linear actuators currently installed are capable of withstanding the maximum force they will be subject to through the whole pitch and yaw ranges of the engine and to calculate the maximum forces that will be used to design a new main engine support.

To simplify the problem, two major assumptions were made. The first assumption was that the mass of the actuators is negligible and therefore can be ignored. This assumption is acceptable because the weight of the actuators is approximately 2% the weight of the engine assembly and less than 0.001% the thrust of the engine. This turns the actuators into two force members, thus the force transferred through the actuator is along its axis, the direction of which can be found geometrically. The second assumption is that the rate of change of the gimbal angle will be small. The implication of this assumption is that the dynamic forces experienced by the actuators will be only slightly higher than the static forces and, therefore, only a static analysis with a reasonable Factor of Safety is needed to validate the design.

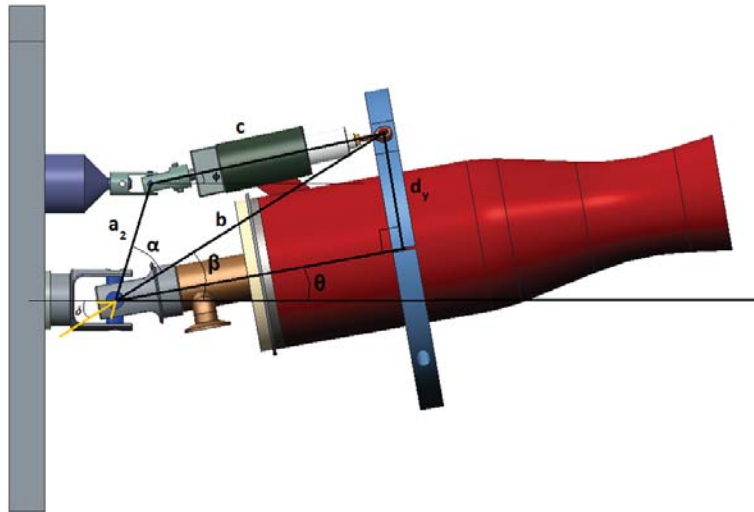
Because the engine assembly is taken apart for cleaning, an exact weight and center of mass cannot be found. The weight of the engine assembly is estimated to be 50 lbf, but for these calculations, 75 lbf was used to fully encompass any error in this estimation. Instead of estimating the center of mass, we used the worst case scenario for these calculations, placing it at the absolute tip of the engine, giving the weight the maximum moment arm to act on. The thrust of the engine used for these calculations was 6000 lbf. Which is the maximum that the test fixture is designed to hold.

After a discussion, we determined that an analysis for all possible combinations of  $\theta$  and  $\lambda$  was not required, but was only needed for the vertical and horizontal planes ( $\theta = 0$  and  $\lambda = 0$ ) to capture the maximum values of the reaction forces. Because of this, the problem was split into two smaller problems, one for the vertical ( $\theta$ ) angles and one for the horizontal ( $\lambda$ ) angles.

### A. Development of Mathematical Model of Geometry

In order to perform a static force analysis, the relationship between the motion of the actuators and the engine needed to be determined. Based on the pre-existing CAD model of the assembly and the physical assembly, I developed two mathematical models in MathCAD that, driven by the pitch or yaw of the engine, calculate the lengths of the actuators and the directions they are pointing. The mathematical models were tested by comparing the calculated lengths and angles from MathCAD with the 3D model in Creo Parametric at different gimbal angles.

The first model developed explored the vertical pitch range (Figure 2). In this model, the actuators act together, simplifying the system. The horizontal model was heavily based on the vertical model because of how the actuators are mounted at a  $45^\circ$  angle from the horizontal. Therefore, the projections of the actuators in the horizontal and vertical planes are the same. The major difference between the models is, to change the gimbal of the engine, one actuator extends while the other contracts when moving horizontally, where the actuators extend or contract together for vertical motion.



**Figure 2. Vertical Engine Gimbal (Side View).** Shown is the geometry used to link the angle of the engine ( $\theta$ ) to the angle of the actuator.

## B. Solving Static Force Problems

Two statics problems were developed and solved, one using the geometry from the vertical model and the other using the geometry from the horizontal model.

### 1. Vertical (pitch) Gimbaling

For the vertical case, the problem simplified to two dimensions because the actuators work in parallel, allowing them to be combined (Figure 2). In this statics problem, I was solving for three unknowns:

- $R_a$  – the reaction force at the joint of the engine and simplified actuator
- $R_e$  – the reaction force at the pivot of the engine
- $\delta$  – the angle of reaction force  $R_e$

To solve for these three unknowns, three equations were used: A sum of forces in the horizontal (x) direction, a sum of forces in the vertical (y) direction, and a sum of moments about the pivot of the engine (Eqs. (1)).

$$\begin{aligned} \sum F_x &= T * \cos(\theta + 180^\circ) + R_e * \cos(\delta) + R_a * \cos(\phi + 180^\circ) = 0 \\ \sum F_y &= T * \sin(\theta + 180^\circ) + R_e * \sin(\delta) + R_a * \sin(\phi + 180^\circ) = 0 \\ \sum M_e &= R_a * \cos(\phi - \alpha + 90^\circ) * a_2 + F_g * CM * \cos(\theta) = 0 \end{aligned} \quad (1)$$

These equations were solved using a solve block in MathCAD. The problem was run for values of  $\theta$ , ranging from  $-10^\circ$  to  $10^\circ$  in increments of one degree. The full results from this problem are available in Appendix B.1.

### 2. Horizontal (Yaw) Gimbaling

The horizontal model is more complex because the actuators cannot be combined; there is a third reaction force that must be solved for. To further complicate the analysis, the forces no longer act in the same 2D plane, increasing the number of unknowns to the five shown below.

- $R_{a1}$  – The reaction force at the joint between the engine and the left actuator
- $R_{a2}$  – The reaction force at the joint between the engine and the right actuator
- $R_{ex}$  – The reaction force at the pivot of the engine in the x direction
- $R_{ey}$  – The reaction force at the pivot of the engine in the y direction
- $R_{ez}$  – The reaction force at the pivot of the engine in the z direction

To solve for these five unknowns, an equal number of equations were needed. A sum of forces in the x, y, and z directions as well as a sum of moments around the y and z axes were used to solve for the forces (Eqs. (2)).

$$\begin{aligned} \sum F_x &= T * \cos(\lambda) + R_{ex} - R_{a1} * \cos(\phi_1) - R_{a2} * \cos(\phi_2) = 0 \\ \sum F_y &= R_{ey} - F_g = 0 \\ \sum F_z &= T * \sin(\lambda) + R_{ez} - R_{a1} * \sin(\phi_1) - R_{a2} * \sin(\phi_2) = 0 \\ \sum M_y &= R_{a1} * \cos(\phi_1 - \alpha + 90^\circ) * a_2 - R_{a2} * \cos(-\phi_2 - \alpha + 90^\circ) = 0 \\ \sum M_z &= +R_{a1} * \cos(\phi_1) * d_y + R_{a2} * \cos(\phi_2) * d_y - F_g * CM * \cos(\lambda) = 0 \end{aligned} \quad (2)$$

These equations were also solved using a solve block in MathCAD. Data were collected by evaluating the solve block at values of  $\lambda$  from  $-10^\circ$  to  $10^\circ$  in increments of one degree. The full results of this problem are available in Appendix B.2

### 3. Results of statics problems

From the two statics problems, the maximum reaction forces that act on the actuators and engine mounting joint were found (Table 1), to find these values from the vertical and horizontal cases, the values of  $\theta$  or  $\lambda$  respectively were varied over the full range of motion of the engine ( $-10^\circ$  to  $10^\circ$ ), in increments of  $1^\circ$  and

**Table 1. Maximum Values of Reaction Forces.** The maximum values of the forces acting of the three connection points of the engine

| Force    | Max value (lbf) | $\theta$   | $\lambda$ |
|----------|-----------------|------------|-----------|
| $R_e$    | 6445.057        | $10^\circ$ | N/A       |
| $R_{a1}$ | 215.833         | $10^\circ$ | N/A       |
| $R_{a2}$ | 215.833         | $10^\circ$ | N/A       |

recorded in excel. The three maxima occur in the vertical plane at the same angle of  $\theta$ . This is due to the geometry of the engine and actuators.

To determine if the actuators are strong enough to hold the engine in place while firing, the maximum force exerted on them from Table 1 must be compared to the maximum rating of the actuators which is 500 lbf. Using this actuator we have a factor of safety of 2.3. With this factor of safety, we are confident they will not fail.

To design the main engine joint, certain maximum forces must be known: the magnitude of reaction force at the joint and the shear and axial components of that reaction force. These values are easily calculated from the results of the statics problems and are shown in Table 2.

**Table 2. Data Needed for Design of Spherical Joint.** These maximum force were needed to design a new main engine joint

| Force | Max value (lbf) | $\theta$   | $\lambda$ |
|-------|-----------------|------------|-----------|
| $R_c$ | 6445.057        | $10^\circ$ | N/A       |
| Shear | 1205.805        | $10^\circ$ | N/A       |
| Axial | 6403.000        | $1^\circ$  | N/A       |

### III. Designing New Spherical Joint

To allow the engine to move while firing, the main solid support needed to be replaced with a joint that allows it to gimbal. This is the final mechanical component needed to allow the engine to gimbal.

#### A. Developing requirements

The requirements for the design of the spherical joint were:

- Allow the engine to gimbal at least ten degrees in all directions
- Withstand all loadings calculated in the static force analysis
- Withstand all weight and thrust of the engine if one or both of the actuators fail
- Maintain the same spacing as the current U-joint (so that we don't have to redesign the rest of the system)
- Meet all above requirements with a minimum factor of safety of 1.5

Additional non critical requirements were also taken into consideration in the design of this system, such as:

- Cost
  - Parts and raw materials
  - Labor to manufacture custom parts
- Ease of manufacturing
- Able to be installed and removed easily

#### B. Selection of Tie Rod End

Whatever solution we used as the main engine joint, it would need to withstand both axial and shear forces. We researched several possible solutions: universal joints, spherical bearings, and tie rod ends. Universal joints were dismissed because they are generally designed for transmitting rotational energy and aren't typically rated for axial loading. The universal joints that we did find with a high enough axial load rating were beyond our budget. Standard spherical bearings were dismissed because they didn't offer the needed load capacity. A thrust rated tie rod end which is a type specially designed spherical joint, were selected because they offer the needed axial and shear strength. Maintaining a factor of safety greater than 1.5 (Table 3).

**Table 3. Tie Rod Factor of Safety.** Comparison of the maximum calculated loads and the maximum loads the chosen joint can handle.

|             | Maximum (lbf) | Joint Capacity (lbf) | Factor of Safety |
|-------------|---------------|----------------------|------------------|
| Axial Force | 6403.000      | 9891                 | 1.545            |
| Shear Force | 1205.805      | 3596                 | 2.982            |

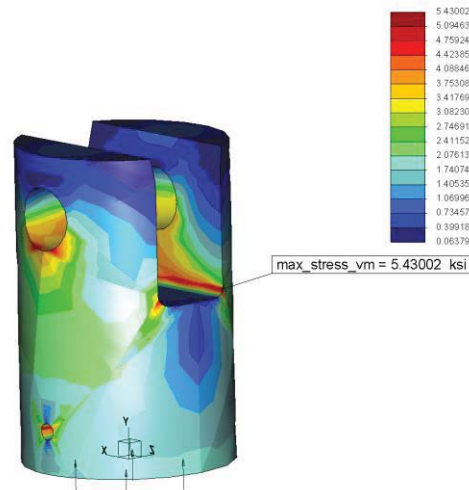
#### C. Design and Analysis of Coupler

To connect the selected tie rod end to the engine, a coupler was needed. The coupler needed to withstand the maximum value of  $R_c$  (6445.057 lbf). No consideration of axial vs. shear forces is needed because the coupler will move with the engine, so the path of the force does not change with  $\theta$  or  $\lambda$ . A design for the coupler was modeled in Creo. To verify that the design is capable of withstanding the maximum loading, Creo Simulate was used to perform Finite Element Analysis (FEA) on the coupler (Figure 3). Using the Maximum von Mises stress found by Creo Simulate, the material needed could then be specified.

The exact specifications of the steels available to manufacture this part are not known; therefore, the maximum stress will be compared to minimum possible, the yield strength of the weakest common steel, which in tension is 25 ksi<sup>2</sup>. This leaves a factor of safety of 4.6. This factor of safety was deemed acceptable, and the design is ready for manufacturing.

#### D. Future Manufacturing and Installation

With the design of the joint complete, it can be put into production at any time. The tie rod end will be ordered and the coupler will be machined when the team is ready to test gimbaling the engine while it is firing.

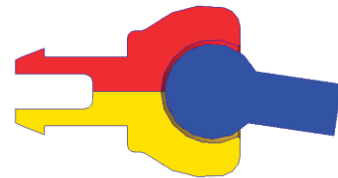


**Figure 3. Von Mises Stress.** Von Mises stress in the coupler, used to determine material strength needed

### IV. 3D Printed Model of Test fixture

I was offered the opportunity to learn about 3D printing during my internship. This knowledge was used and expanded on by redesigning the Neo test fixture to be printed on the MakerBot Replicator 2 belonging to the SwampWorks Lab.

The 3D printed test fixture model will include the ability to gimbal over the same range of angles as the real test fixture. To make this happen, all moving assemblies must be mobile. The three sub-assemblies that do move are the two actuators and the main engine joint; however, they cannot be directly scaled down because the actuators are electromechanical devices which can't be 3D printed and the main joint would be too fragile if it was printed. Because of this, a redesign of these components was necessary. The other components of the 3D printable test fixture were then designed around the needs of the redesigned components.

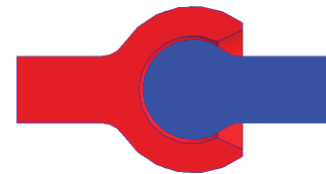


**Figure 4. Section view of first joint concept.** In red and yellow are the two symmetrical halves of the cup and in blue is ball end.

#### A. Design of Printable Spherical Joint

The first version of spherical joint was designed to prevent the cup of the joint from cracking when inserting the ball into the cup of the joint. This was accomplished by printing the cup as two separate halves. The two halves were to be held together by inserting their common “leg” into a hole and inserting a wedge to hold them in place (Figure 4).

The second revision also circumvented inserting the ball into the cup, this time by printing the cup as one part and printing the ball already inside it (Figure 5). Special spacing was needed to allow for the support structure and for error in the print so that the ball and cup would be able to move as a separate piece; the CAD model and printed joint are shown in Figure 5. The design shown allows for twenty five degrees of motion off of the central axis in all directions. This iteration greatly simplified the assembly of the system as the joint is made all at once.



**Figure 5. Section view of second joint concept and printed model.** (Top) In red the single piece cup end and in blue, the ball end (Bottom) Completed print of ball joint

#### B. Heat Set Threaded Inserts Replacing Actuators

To allow the printed model to move, we needed to replace the linear actuators with 3D printable parts that could still move the engine through its full motion. Inspired by the lead screw design of the actuators, we explored replacing the actuator with a screw. There were several challenges that came along with this, primarily that printed parts are not very strong between layers. Because of this, they can't be used to make very fine screw threads because the threads will quickly strip. Also, repeatedly driving a metal screw into the plastic would quickly wear through the soft plastic threads, rendering

them useless.

The screws needed to be able to be driven in and out repeatedly to demonstrate the function of the assembly. To allow for this to happen, I determined that both the external and internal threads needed to be metal.

The first design concept involved imbedding a hex nut inside the printed part by pausing the print part way through, installing the nut, and then resuming the print. This was abandoned because the size of the nut would result in very little material transferring the load around the nut and this process could easily cause defects in the print. The second design concept used heat set threaded inserts. They were chosen based on positive results from a test done by Bill Morris for the blog *I Heart Robotics* on the axial load capabilities of these inserts when installed in 3D printed thermoplastics<sup>1</sup>. A test block was printed to determine optimal hole design and practice setting the inserts (Figure 6). A team of two pulled on the inserts in an attempt to roughly determine the force needed to pull the insert out of the plastic. By hand, they were unable to remove the insert. The force generated in this experiment was orders of magnitude greater than the force that the parts will experience; therefore, we concluded they will not fail in use.



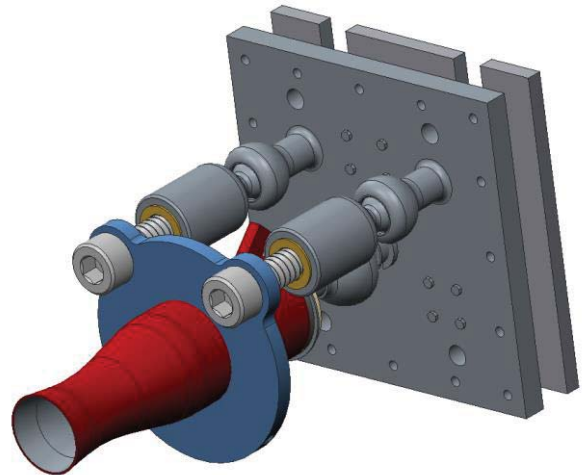
**Figure 6. Heat Set Inserts.** *Four inserts installed in test block and one insert not installed.*

### C. Simplification and Scaling of Parts and Sub-Assemblies

The test fixture has a large number of small intricate parts that will not be printable when scaled down for printing. Because of this, many of the parts had to be eliminated to proceed with printing. The main components that remained after the simplification are: the skid, strong back, engine + actuators, and uni-strut electrical support system. Parts that formed rigid bodies were combined into a single printed part where convenient to allow for fast and simple printing. In addition, the connection points between the different printed parts had to be redesigned for printing.

### D. Completion of Printable Model

The first revision of the design for the printable model has been finished (Figure 7). It utilizes the spherical joint design and the threaded inserts mentioned above. During the remaining two weeks of my internship, the design will be finished and the model will be printed.



**Figure 7. 3D Printable Design.** *The design of the assembly had to be changed to allow for printing.*

## V. Conclusion

The static force analysis yield results that were used to design a main engine joint capable of withstanding the thrust of the engine. A design for a 3D printed model of the test fixture is completed and will be completed and printed prior to the end of this internship.

During this internship, I gained a great deal of practical engineering experience, utilizing and expanding my knowledge of statics, mechanics of materials, machine design, and 3D printing. It also granted me the opportunity to experience what it would be like to work for NASA, a career path I plan on pursuing.

## Appendix

### A. Mathematical Geometry Models

The mathematical models used to define the geometry of the engine and actuators are provided for reference.

1. Vertical Geometry Model

## Geometry Definition Engine + Actuators

**when looking from the left side: x is to the right, y is up, and z is coming out toward you**

Looking at the arms from the side of the test stand. Projection on the xy-plane

$$\theta := 10 \text{ deg}$$

Gimbal angle

Negative angles tilt the engine down

$$\alpha := 73.1355 \text{ deg}$$

Projection on yz-plane of angle from centerline of engine at  $\Theta=0$  deg gimbal to a line through the center of the two universal joints. From Creo

$$d := 7 \text{ in}$$

Distance from centerline of engine to actuator mount on ring. From Creo

$$d_y := d \cdot \cos(45 \text{ deg}) = 4.95 \text{ in}$$

$$\beta := \text{atan2}(12.5 \text{ in}, d_y) = 21.603 \text{ deg}$$

Fixed angle from centerline of engine to line from universal joint to actuator mount on ring

$$b := \sqrt{(12.5 \text{ in})^2 + (d_y)^2} = 13.444 \text{ in}$$

Distance from center of universal joint to actuator mount on ring projected on z-plane. From Creo

$$C := \alpha - \beta - \theta = 41.533 \text{ deg}$$

Angle across from side c

$$c := \sqrt{a_2^2 + b^2 - 2 \cdot a_2 \cdot b \cdot \cos(C)} = 10.168 \text{ in}$$

Distance from center of actuator universal joint to actuator mount on ring

$$-11 \text{ in} + c = -0.832 \text{ in}$$

This gives the displacement from the centered zero position ( $\Theta=0$ ) of the linear actuator as shown in the model

Numbers < 0 indicate that actuator has contracted  
Numbers > 0 indicate that actuator has extended

$$B := \text{acos}\left(\frac{-b^2 + a_2^2 + c^2}{2 \cdot a_2 \cdot c}\right) = 118.757 \text{ deg}$$

Angle across from side b

$$B_0 := 106.864489888306 \text{ deg}$$

value of B at  $\Theta=0$  deg gimbal

$$\phi := B - B_0 = 11.892 \text{ deg}$$

"Gimbal" angle on actuator. is accurate to approx. 13th decimal place.

## Force and Moment Balance

### *U-Joint Reactions*

$$T := 6000 \text{ lbf}$$

Thrust of engine

$$CM := 26.5 \text{ in}$$

Distance from engine u-joint along centerline of the engine

$$F_g := -75 \text{ lbf}$$

Force due to gravity acting on the engine assembly

$$\psi := \phi + (90 \text{ deg} - \alpha) = 28.757 \text{ deg}$$

$\psi$  is the angle between the reaction force  $R_a$  and a line perpendicular to the moment arm from center of engine u-joint

$$\text{ORIGIN} := 1$$

Starts matrices/vectors at 1 not 0

sum of moments taken about center of the engine u-joint

|                                                                                                                                                                                                             |                                                                                                                      |                      |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------|----------------------|
| <div style="display: flex; flex-direction: column; align-items: center;"> <div style="margin-bottom: 10px;">Guess Values</div> <div style="margin-bottom: 10px;">Constraints</div> <div>Solver</div> </div> | $R_e := 2000 \text{ lbf} \quad R_a := 0 \text{ lbf} \quad \delta := 0 \text{ deg}$                                   |                      |
|                                                                                                                                                                                                             | Delta is the angle of the reaction force at the engine mount                                                         |                      |
|                                                                                                                                                                                                             | $T \cdot \cos(\theta + 180 \text{ deg}) + R_e \cdot \cos(\delta) + R_a \cdot \cos(\phi + 180 \text{ deg}) = 0$       |                      |
|                                                                                                                                                                                                             | $T \cdot \sin(\theta + 180 \text{ deg}) + R_e \cdot \sin(\delta) + R_a \cdot \sin(\phi + 180 \text{ deg}) + F_g = 0$ |                      |
| $R_a \cdot \cos(\psi) \cdot a_2 + F_g \cdot CM \cdot \cos(\theta) = 0$                                                                                                                                      |                                                                                                                      |                      |
| $y := \text{find}(R_e, R_a, \delta) = \begin{bmatrix} 6445.057 \text{ lbf} \\ 431.666 \text{ lbf} \\ 0.188 \end{bmatrix}$                                                                                   |                                                                                                                      | Re<br>Ra<br>$\delta$ |

$$\delta := y_3 = 10.78335 \text{ deg}$$

value of  $\delta$  converted to degrees

## 2. Horizontal Geometry Model

## Geometry Definition Engine + Actuators

**when looking from the side: x is to the right, y is up, and z is coming out toward you**

Looking at the arms from the side of the test stand. Projection on the xy-plane

$$\lambda := 10 \text{ deg}$$

Gimbal angle

Negative angles tilt the engine to the right if you are looking down the nozzle of the engine

$$\alpha := 73.1355 \text{ deg}$$

Projection on yz-plane of angle from centerline of engine at =0 deg gimbal to a line through the center of the two universal joints. From Creo

$$a := 7.15902 \text{ in}$$

Distance from center line of engine mount to center line of actuator mount. From Creo

$$\gamma := 43.7411 \text{ deg}$$

When looking down the nozzle of the engine, this is the angle from the horizontal (z-axis) to a line from the center of the engine mount to the center of the mount for the actuator. From Creo

$$a_2 := a \cdot \cos(\gamma) = 5.172 \text{ in}$$

Projection of 'a' in the xz-plane

$$d := 7 \text{ in}$$

Distance from centerline of engine to actuator mount on ring. From Creo

$$d_y := d \cdot \cos(45 \text{ deg}) = 4.95 \text{ in}$$

$$\beta := \text{atan2}(12.5 \text{ in}, d_y) = 21.603 \text{ deg}$$

Fixed angle from centerline of engine to line from universal joint to actuator mount on ring

$$b := \sqrt{(12.5 \text{ in})^2 + (d_y)^2} = 13.444 \text{ in}$$

Distance from center of universal joint to actuator mount on ring projected on z-plane. From Creo

### Left Actuator

$$C_1 := \alpha - \beta - \lambda = 41.533 \text{ deg}$$

Angle across from side c

$$c_1 := \sqrt{a_2^2 + b^2 - 2 \cdot a_2 \cdot b \cdot \cos(C_1)} = 10.168 \text{ in}$$

Distance from center of actuator universal joint to actuator mount on ring

$$-11 \text{ in} + c_1 = -0.832 \text{ in}$$

This gives the displacement from the centered zero position ( $\lambda=0$ ) of the linear actuator as shown in the model

Numbers < 0 indicate that actuator has contracted  
Numbers > 0 indicate that actuator has extended

$$B := \arccos\left(\frac{-b^2 + a_2^2 + c_1^2}{2 \cdot a_2 \cdot c_1}\right) = 118.757 \text{ deg}$$

Angle across from side b

$$B_0 := 106.864489888306 \text{ deg}$$

Value of B at  $\lambda=0$  deg gimbal

$$\phi_1 := B - B_0 = 11.892 \text{ deg}$$

"Gimbal" angle on actuator. is accurate to approx. 13th decimal place.

### Right Actuator

$$C_2 := \alpha - \beta + \lambda = 61.533 \text{ deg}$$

Angle across from side c

$$c_2 := \sqrt{a_2^2 + b^2 - 2 \cdot a_2 \cdot b \cdot \cos(C_2)} = 11.883 \text{ in}$$

Distance from center of actuator universal joint to actuator mount on ring

$$-11 \text{ in} + c_2 = 0.883 \text{ in}$$

This gives the displacement from the centered zero position ( $\lambda=0$ ) of the linear actuator as shown in the model

Numbers < 0 indicate that actuator has contracted  
Numbers > 0 indicate that actuator has extended

$$B := \arccos\left(\frac{-b^2 + a_2^2 + c_2^2}{2 \cdot a_2 \cdot c_2}\right) = 95.971 \text{ deg}$$

Angle across from side b

$$B_0 := 106.864489888306 \text{ deg}$$

Value of B at  $\lambda=0$  deg gimbal

$$\phi_2 := -B + B_0 = 10.894 \text{ deg}$$

"Gimbal" angle on actuator. is accurate to approx. 13th decimal place.

## Force and Moment Balance

### U-Joint Reactions

$$T := -6000 \text{ lbf}$$

Thrust of engine

$$CM := 26.5 \text{ in}$$

Distance from engine u-joint along the centerline of the engine

$$F_g := 75 \text{ lbf}$$

Force due to gravity acting on the engine assembly

$$\psi_1 := \phi_1 + (90 \text{ deg} - \alpha) = 28.757 \text{ deg}$$

$\psi_1$  is the angle between the reaction force  $R_{a1}$  and a line perpendicular to the moment arm from center of engine u-joint

$$\psi_2 := -\phi_2 + (90 \text{ deg} - \alpha) = 5.971 \text{ deg}$$

$\psi_2$  like wise with  $R_{a2}$

$$\text{ORIGIN} := 1$$

Starts matrices/vectors at 1 not 0

$$T_x := T \cdot \cos(\lambda) = -5908.847 \text{ lbf}$$

$$T_z := T \cdot \sin(\lambda) = -1.042 \cdot 10^3 \text{ lbf}$$

sum of moments taken about center of the engine u-joint

|              |                                                                                                                                                         |                           |                                                |
|--------------|---------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------|------------------------------------------------|
| Guess Values | $R_{ex} := 2000 \text{ lbf}$                                                                                                                            | $R_{ey} := 0 \text{ lbf}$ | $R_{ez} := 100 \text{ lbf}$                    |
|              | $R_{a1} := 0 \text{ lbf}$                                                                                                                               | $R_{a2} := 0 \text{ lbf}$ |                                                |
| Constraints  | $T_x + R_{ex} - R_{a1} \cdot \cos(\phi_1) - R_{a2} \cdot \cos(\phi_2) = 0$                                                                              |                           | force in X                                     |
|              | $T_z + R_{ez} - R_{a1} \cdot \sin(\phi_1) - R_{a2} \cdot \sin(\phi_2) = 0$                                                                              |                           | force in Z                                     |
|              | $R_{ey} - F_g = 0$                                                                                                                                      |                           | force in Y                                     |
|              | $R_{a1} \cdot \cos(\psi_1) \cdot a_2 - R_{a2} \cdot \cos(\psi_2) \cdot a_2 = 0$                                                                         |                           | moments around Y                               |
|              | $R_{a1} \cdot \cos(\phi_1) \cdot d_y + R_{a2} \cdot \cos(\phi_2) \cdot d_y - F_g \cdot CM \cdot \cos(\lambda) = 0$                                      |                           | moments around Z                               |
| Solver       | $y := \text{find}(R_{ex}, R_{ey}, R_{ez}, R_{a1}, R_{a2}) = \begin{bmatrix} 6304.282 \\ 75 \\ 1121.799 \\ 214.432 \\ 189.012 \end{bmatrix} \text{ lbf}$ |                           | Rex<br>Rey<br>Rez<br>Ra1 (left)<br>Ra2 (right) |

**B. Results from Static Force Analysis**

All values are in lbf unless otherwise noted. White columns are values taken directly from the static force analysis; shaded columns are values that were calculated using the data from the white columns. Maximum values are labeled with an asterisk and in bold in all columns where they are calculated.

*1. Results from Vertical Statics Problem*

| <b>Vertical</b>       |                  |             |                  |                  |         |                     |
|-----------------------|------------------|-------------|------------------|------------------|---------|---------------------|
| Pitch, $\theta$ (deg) | $R_e$            | Delta (deg) | Axial Force      | Shear Force      | $R_a$   | $R_{a1}$ & $R_{a2}$ |
| -10                   | 6367.785         | -9.389      | 6282.479         | -1038.818        | 380.493 | 190.247             |
| -9                    | 6370.996         | -8.384      | 6302.910         | -928.934         | 382.403 | 191.202             |
| -8                    | 6374.240         | -7.380      | 6321.436         | -818.767         | 384.341 | 192.171             |
| -7                    | 6377.521         | -6.375      | 6338.085         | -708.130         | 386.312 | 193.156             |
| -6                    | 6380.845         | -5.369      | 6352.851         | -597.053         | 388.323 | 194.162             |
| -5                    | 6384.215         | -4.364      | 6365.706         | -485.791         | 390.377 | 195.189             |
| -4                    | 6387.639         | -3.358      | 6376.672         | -374.153         | 392.482 | 196.241             |
| -3                    | 6391.120         | -2.351      | 6385.740         | -262.171         | 394.643 | 197.322             |
| -2                    | 6394.666         | -1.344      | 6392.907         | -149.987         | 396.868 | 198.434             |
| -1                    | 6398.282         | -0.337      | 6398.171         | -37.633          | 399.163 | 199.582             |
| 0                     | 6401.975         | 0.671       | 6401.536         | 75.000           | 401.535 | 200.768             |
| 1                     | 6405.753         | 1.680       | <b>*6403.000</b> | 187.800          | 403.994 | 201.997             |
| 2                     | 6409.623         | 2.689       | 6402.565         | 300.705          | 406.547 | 203.274             |
| 3                     | 6413.593         | 3.698       | 6400.239         | 413.661          | 409.205 | 204.603             |
| 4                     | 6417.673         | 4.708       | 6396.019         | 526.748          | 411.976 | 205.988             |
| 5                     | 6421.873         | 5.719       | 6389.909         | 639.938          | 414.872 | 207.436             |
| 6                     | 6426.202         | 6.730       | 6381.922         | 753.091          | 417.904 | 208.952             |
| 7                     | 6430.672         | 7.742       | 6372.055         | 866.292          | 421.086 | 210.543             |
| 8                     | 6435.295         | 8.755       | 6360.312         | 979.514          | 424.43  | 212.215             |
| 9                     | 6440.086         | 9.769       | 6346.704         | 1092.730         | 427.951 | 213.976             |
| 10                    | <b>*6445.057</b> | 10.783      | 6331.255         | <b>*1205.805</b> | 431.666 | <b>*215.833</b>     |
| Maximum Values        | 6445.057         | N/A         | 6403.000         | 1205.805         | N/A     | 215.833             |

## 2. Results from Horizontal Statics Problem

| Horizontal           |          |                        |          |           |                  |                 |                 |
|----------------------|----------|------------------------|----------|-----------|------------------|-----------------|-----------------|
| Yaw, $\lambda$ (deg) | $R_c$    | $R_{ex}$ – Axial Force | $R_{ey}$ | $R_{ez}$  | Shear Force      | $R_{a1}$        | $R_{a2}$        |
| -10                  | 6403.751 | 6304.282               | 75       | -1121.799 | 1124.303         | 189.012         | <b>*214.432</b> |
| -9                   | 6403.398 | 6322.722               | 75       | -1010.476 | 1013.256         | 190.203         | 212.858         |
| -8                   | 6403.088 | 6339.236               | 75       | -898.885  | 902.008          | 191.383         | 211.345         |
| -7                   | 6399.202 | 6353.820               | 75       | -757.053  | 760.759          | 192.553         | 209.886         |
| -6                   | 6402.591 | 6366.467               | 75       | -675.011  | 679.165          | 193.718         | 208.475         |
| -5                   | 6402.400 | 6377.176               | 75       | -562.787  | 567.762          | 194.880         | 207.109         |
| -4                   | 6402.246 | 6385.942               | 75       | -450.411  | 456.613          | 196.043         | 205.781         |
| -3                   | 6402.127 | 6392.763               | 75       | -337.915  | 346.138          | 197.210         | 204.487         |
| -2                   | 6402.042 | 6397.636               | 75       | -225.327  | 237.481          | 198.384         | 203.223         |
| -1                   | 6401.992 | 6400.561               | 75       | -112.679  | 135.357          | 199.569         | 201.985         |
| 0                    | 6401.975 | <b>*6401.536</b>       | 75       | 0.000     | 75.000           | 200.768         | 200.768         |
| 1                    | 6401.992 | 6400.561               | 75       | 112.679   | 135.357          | 201.985         | 199.569         |
| 2                    | 6402.042 | 6397.636               | 75       | 225.327   | 237.481          | 203.223         | 198.384         |
| 3                    | 6402.127 | 6392.763               | 75       | 337.915   | 346.138          | 204.487         | 197.210         |
| 4                    | 6402.246 | 6385.942               | 75       | 450.411   | 456.613          | 205.781         | 196.043         |
| 5                    | 6402.400 | 6377.176               | 75       | 562.787   | 567.762          | 207.109         | 194.880         |
| 6                    | 6402.591 | 6366.467               | 75       | 675.011   | 679.165          | 208.475         | 193.718         |
| 7                    | 6399.202 | 6353.820               | 75       | 757.053   | 760.759          | 209.886         | 192.553         |
| 8                    | 6403.088 | 6339.236               | 75       | 898.885   | 902.008          | 211.345         | 191.383         |
| 9                    | 6403.398 | 6322.722               | 75       | 1010.480  | 1013.260         | 212.858         | 190.203         |
| 10                   | 6403.751 | 6304.282               | 75       | 1121.800  | <b>*1124.304</b> | <b>*214.432</b> | 189.012         |
| Maximum Values       | 6403.751 | 6401.536               | N/A      | N/A       | 1124.304         | 214.432         | 214.432         |

## 3. Maximum Values from Both Models

| Overall Maximum Values |             |             |          |          |
|------------------------|-------------|-------------|----------|----------|
| $R_c$                  | Axial Force | Shear Force | $R_{a1}$ | $R_{a2}$ |
| 6445.057               | 6403.000    | 1205.805    | 215.833  | 215.833  |

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## References

<sup>1</sup>Bill Morris, "Fasteners for 3D Printing: Threaded Inserts Part 3 of n", *I Heart Robotics* [blog], URL: <http://www.iheartrobotics.com/2012/05/fasteners-for-3d-printing-threaded.html> [cited 23 July 2014]

<sup>2</sup>Joseph R. Davis, *Carbon and Alloy Steels*, ASM Specialty Handbook, ASM International, Materials Park, OH, 1996 pp. 51.