Capillary Flows along Open Channel Conduits: the Open-Star Section

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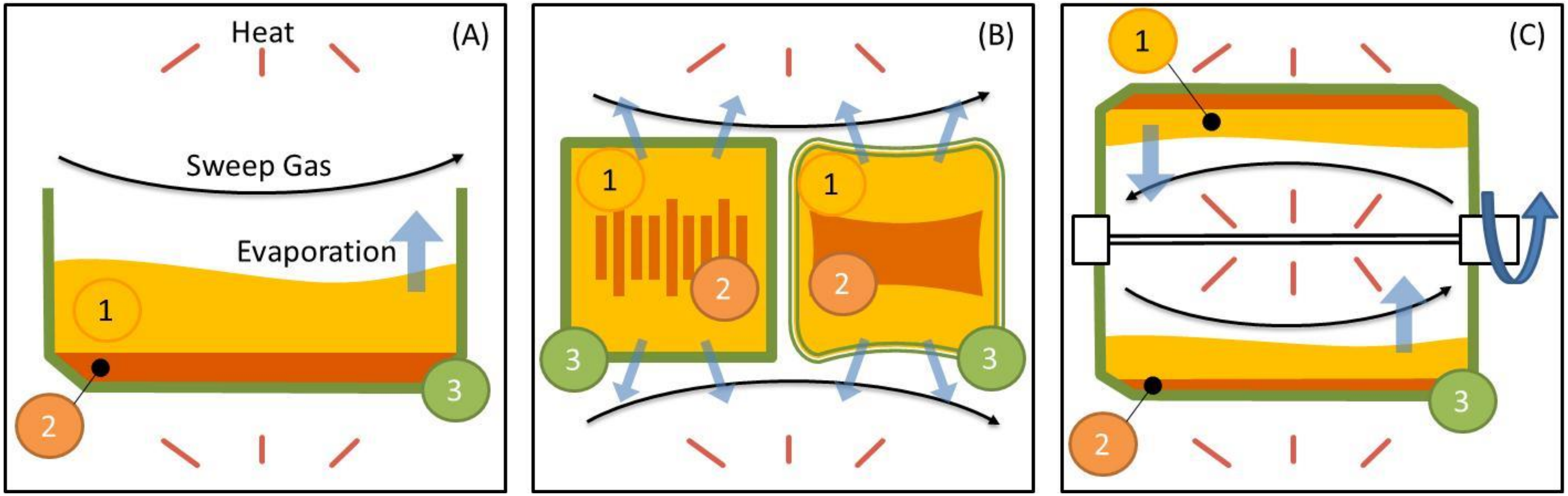
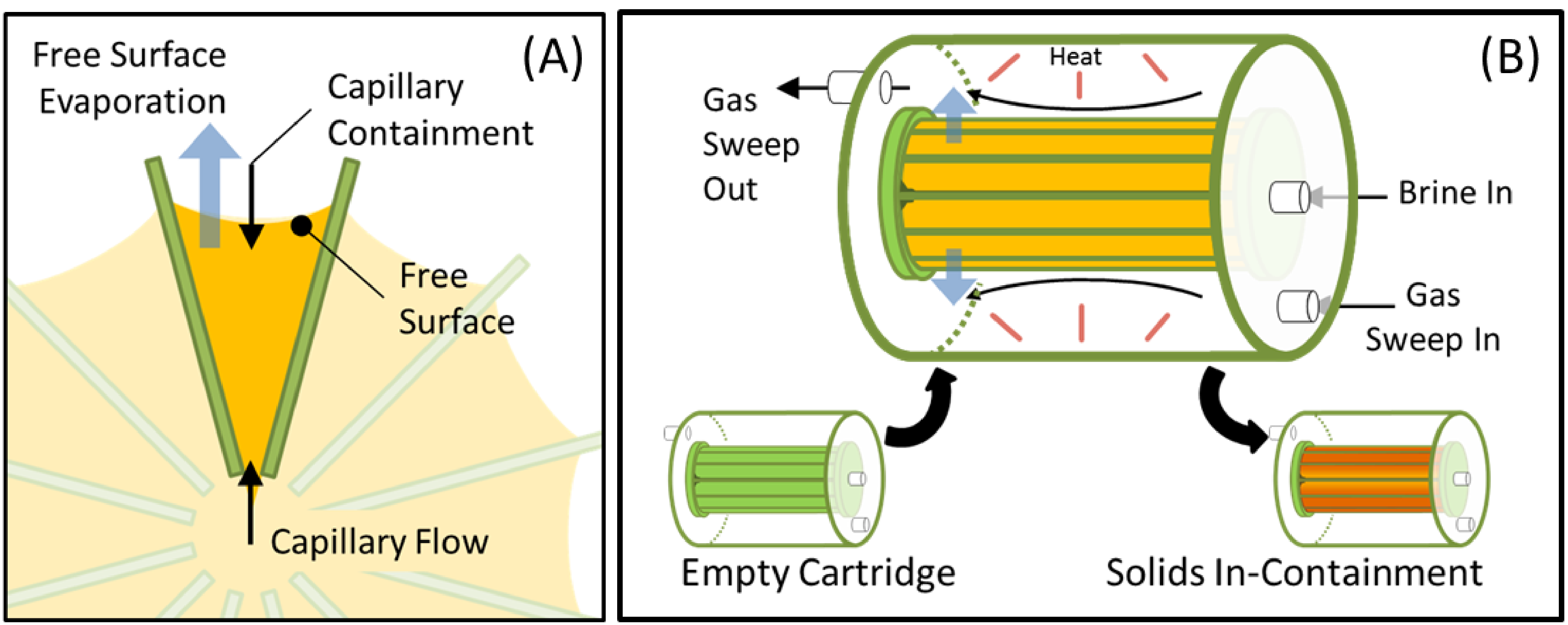
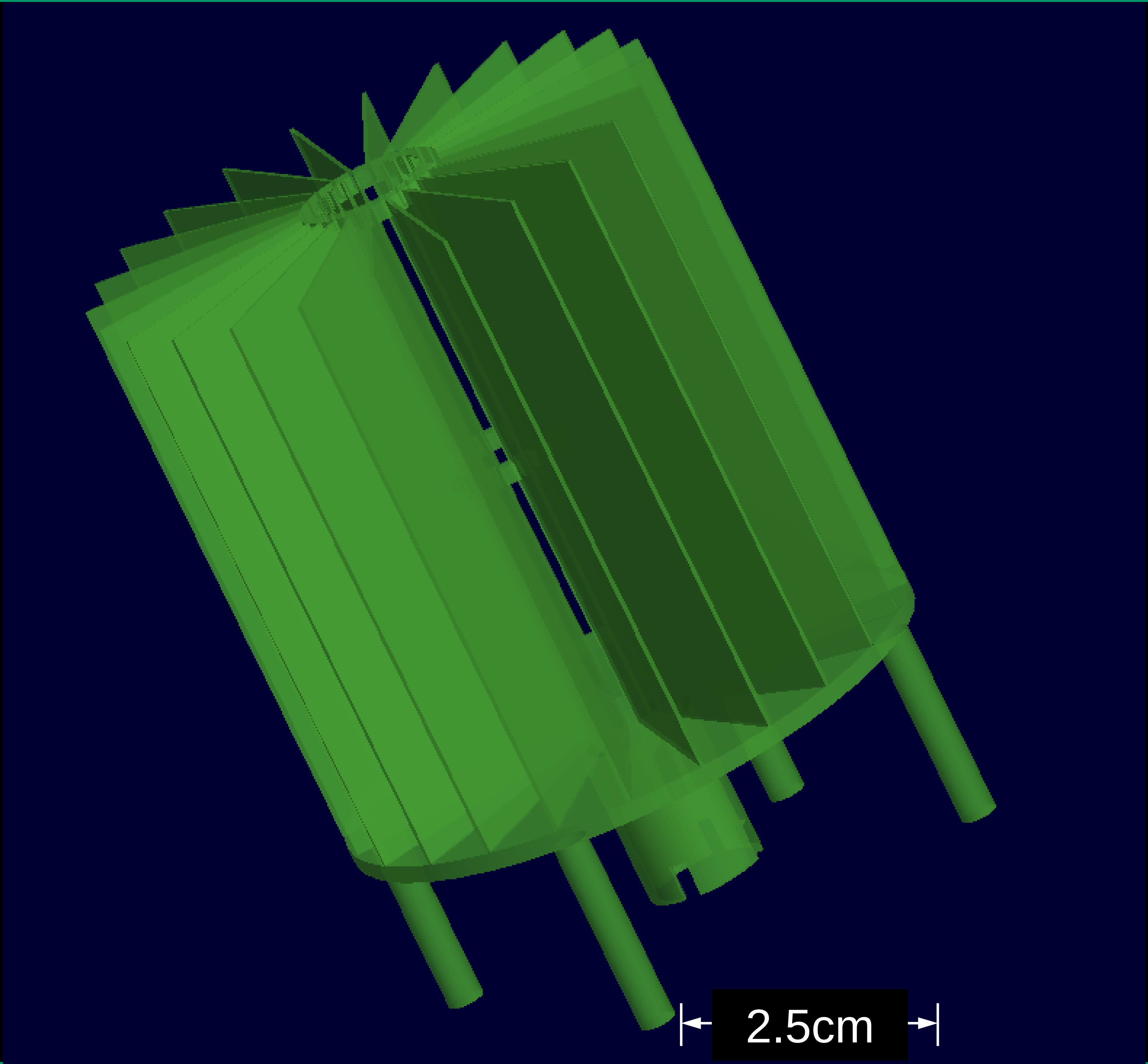
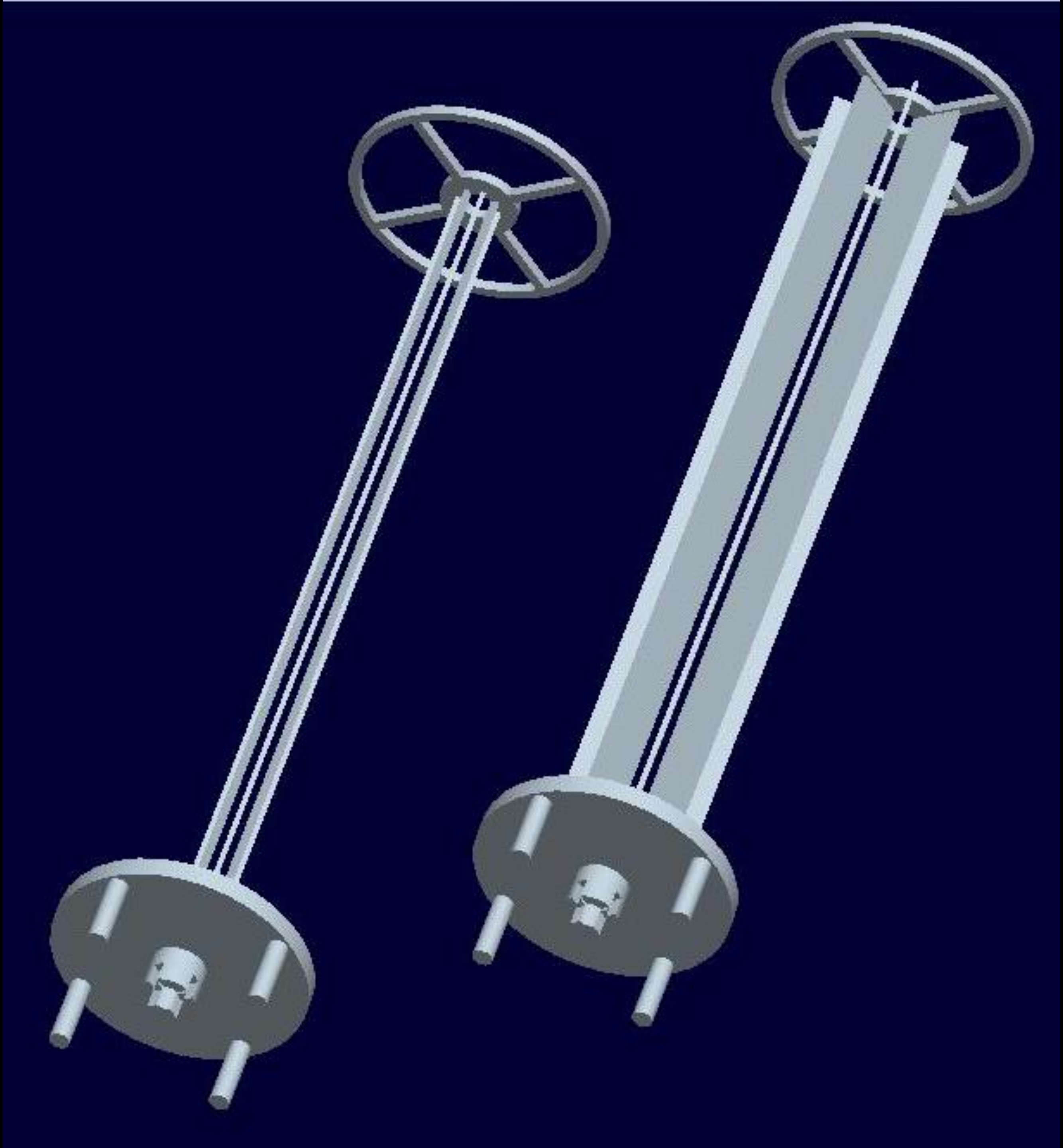
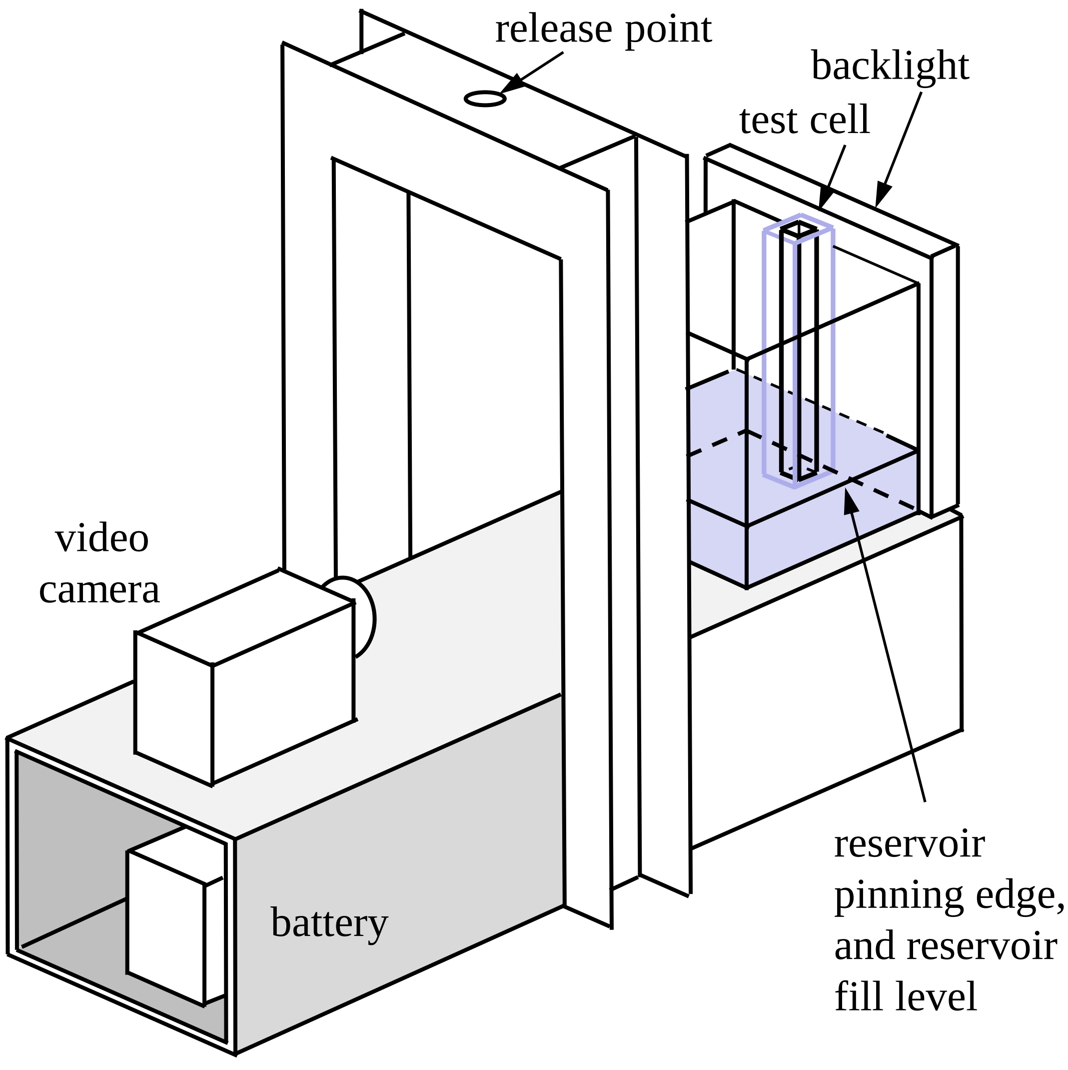
- **a capillary fluidics application aboard spacecraft**
- **the capillary rise problem**
- **‘state of the art’ assessment of the pressure term**
- **a five minute lecture on scaling the problem**

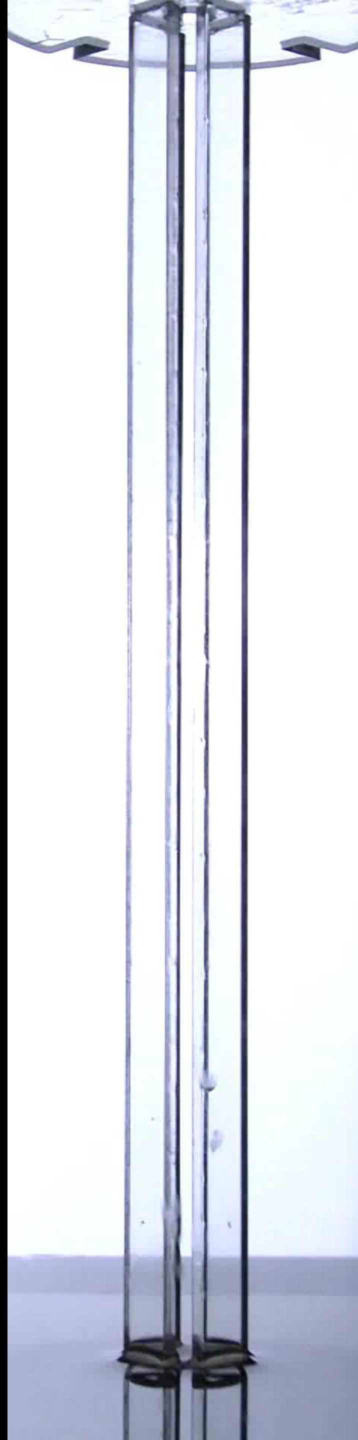

Heat
(A)
Sweep Gas
Evaporation
1
2
3
(B)
1
2
3
1
2
3
(C)
1
2
3

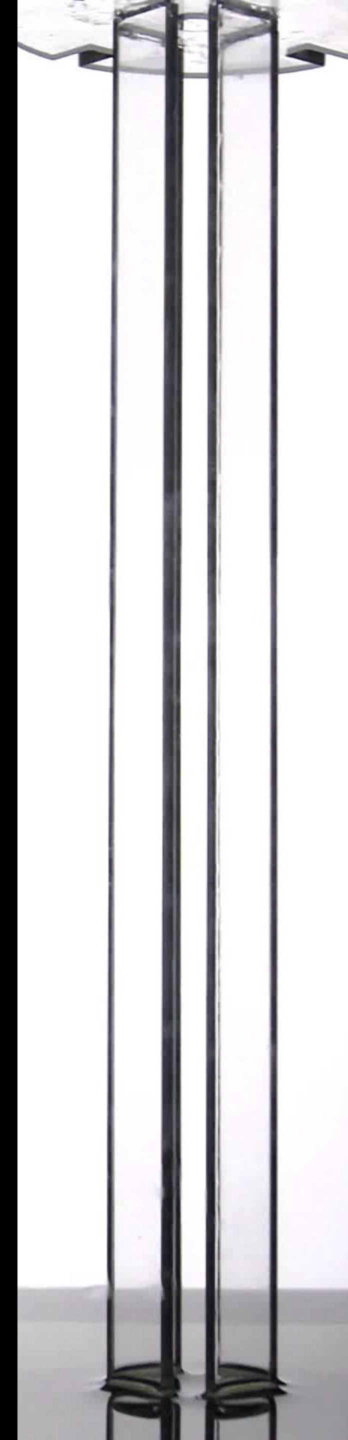

Free Surface
Evaporation
(A)
Capillary
Containment
Free
Surface
Capillary Flow
(B)
Heat
Gas
Sweep
Out
Brine In
Gas
Sweep In
Empty Cartridge
Solids In-Containment

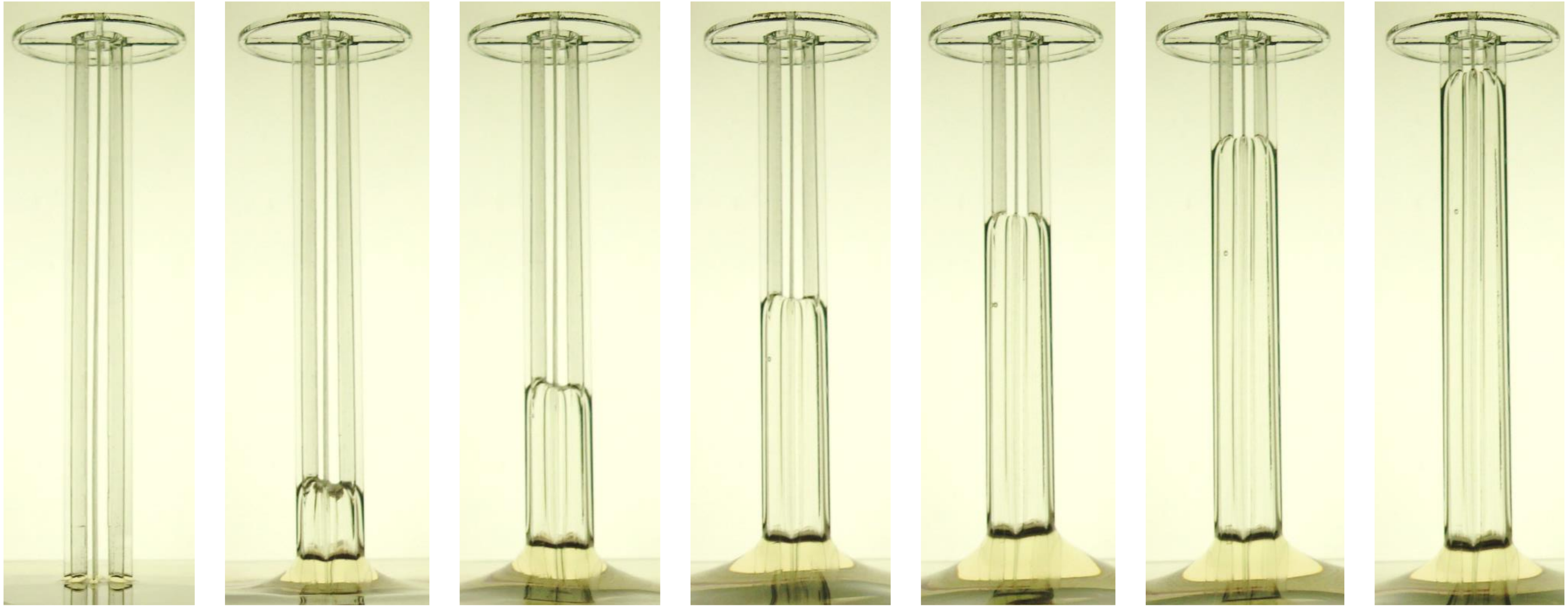

2.5cm

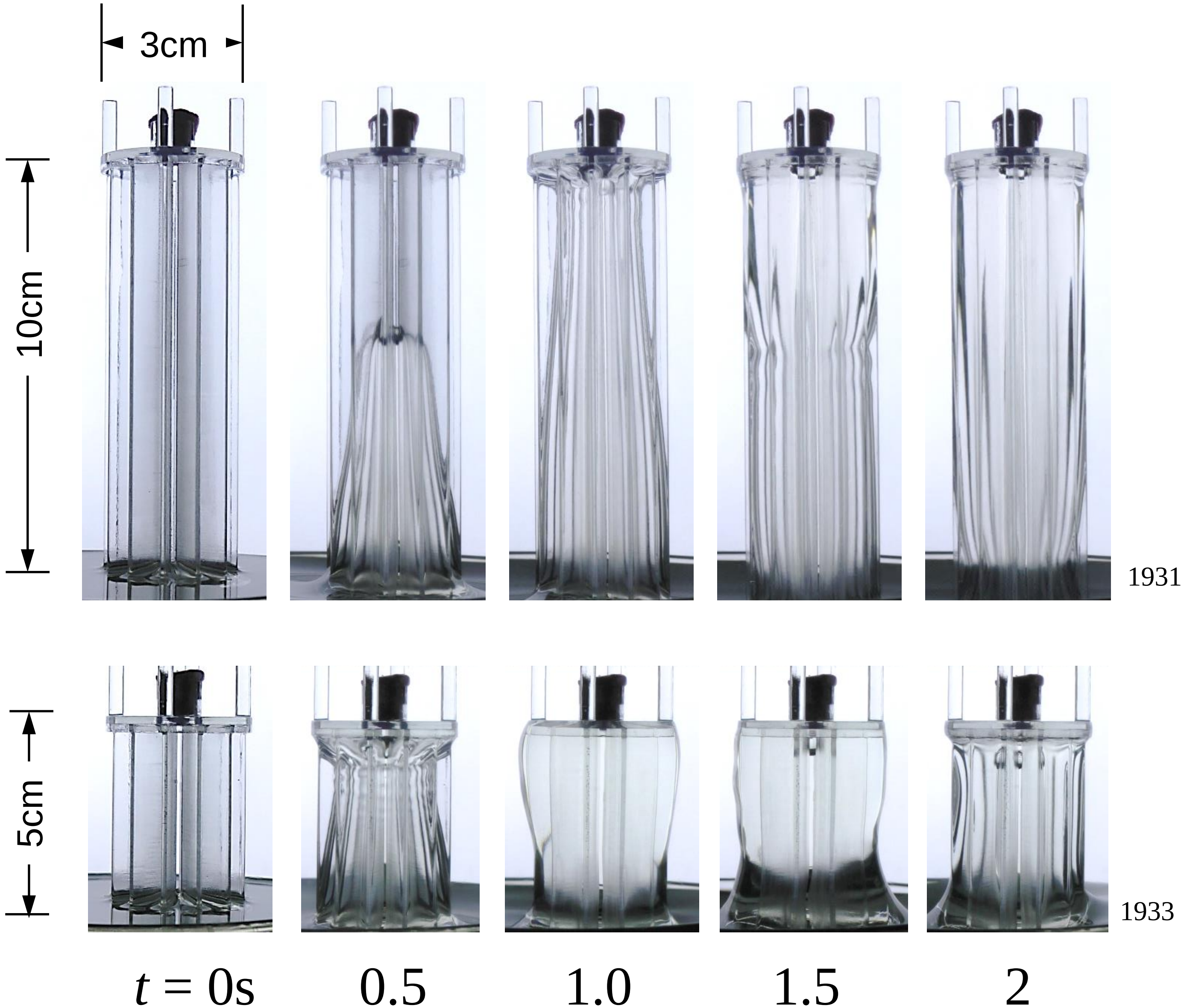


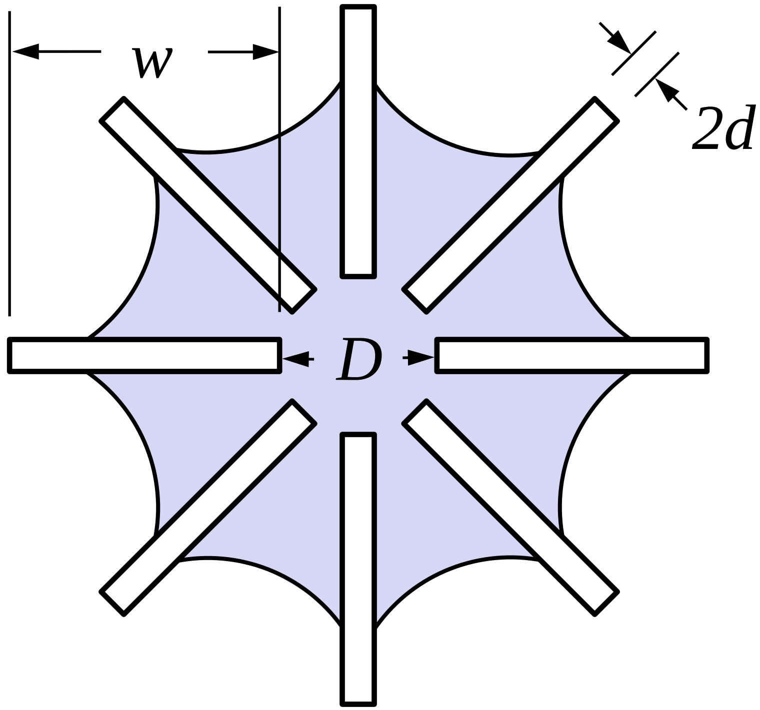

release point
backlight
test cell
video
camera
battery
reservoir
pinning edge,
and reservoir
fill level



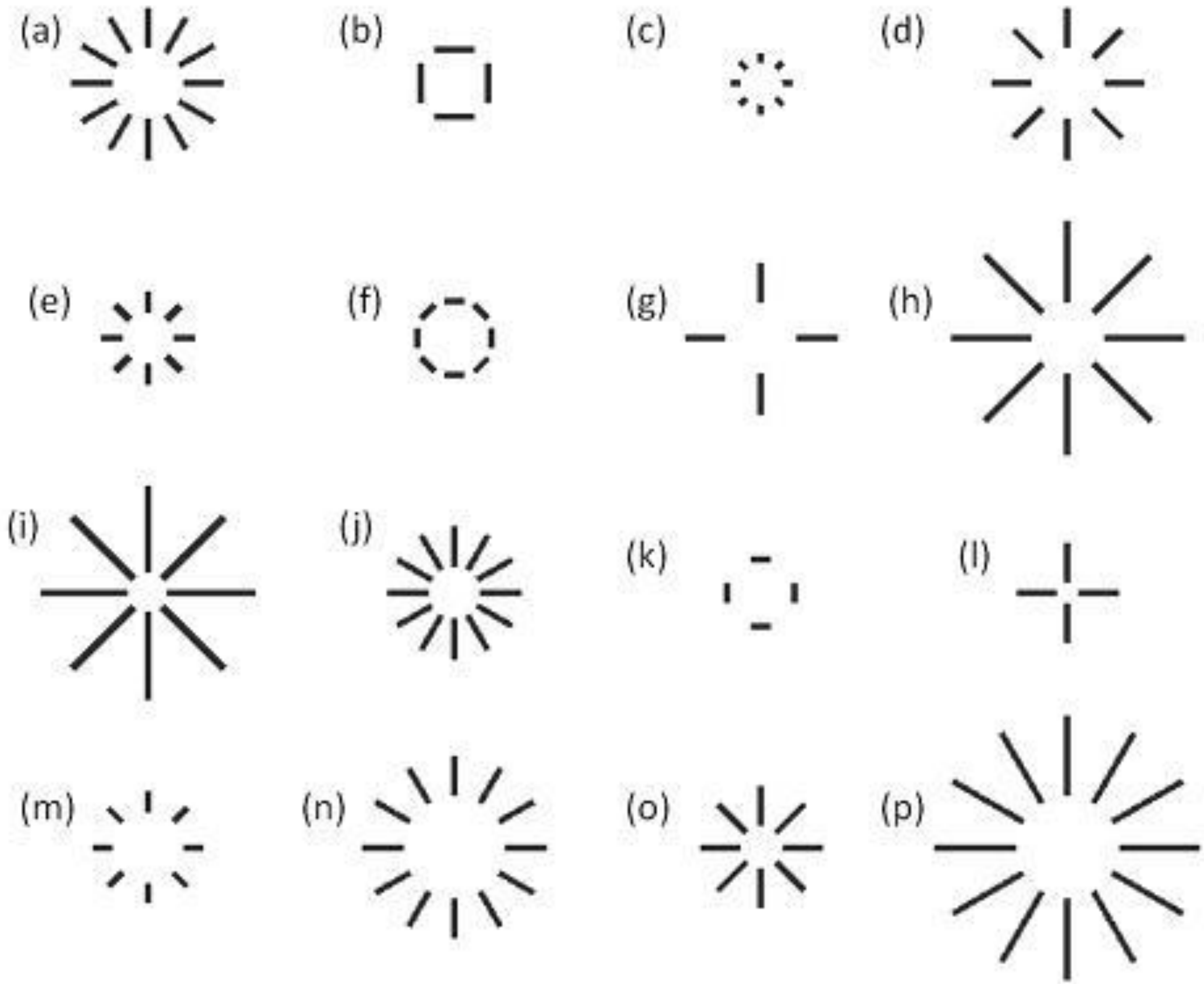


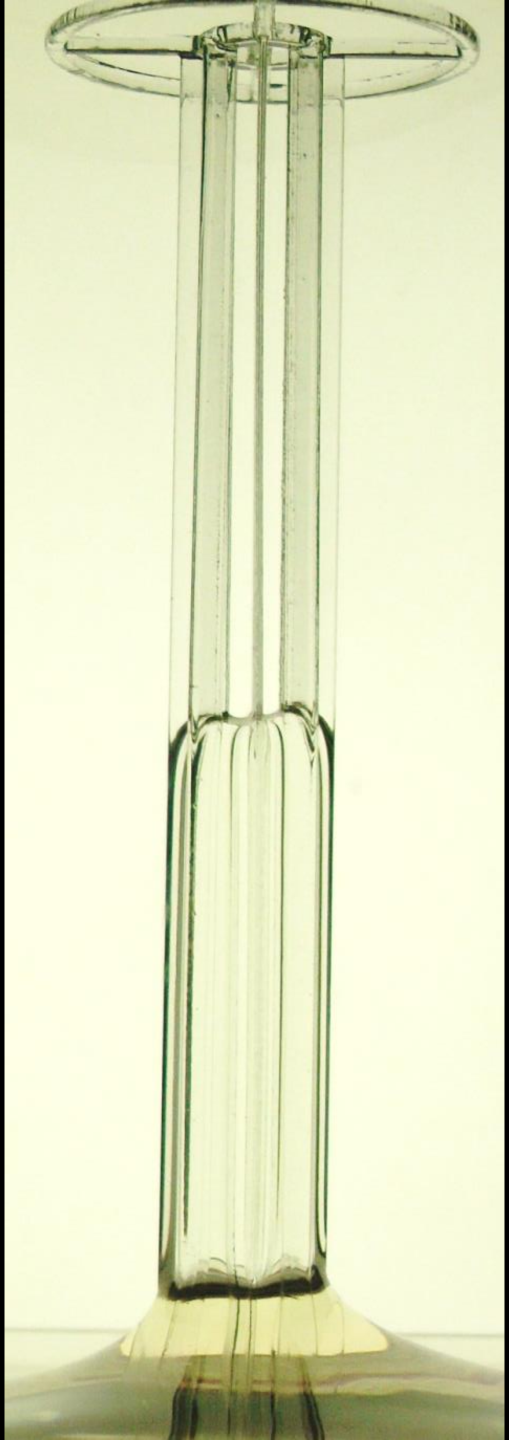



3cm
10cm
5cm
1931
1933
t = 0s
0.5
1.0
1.5
2

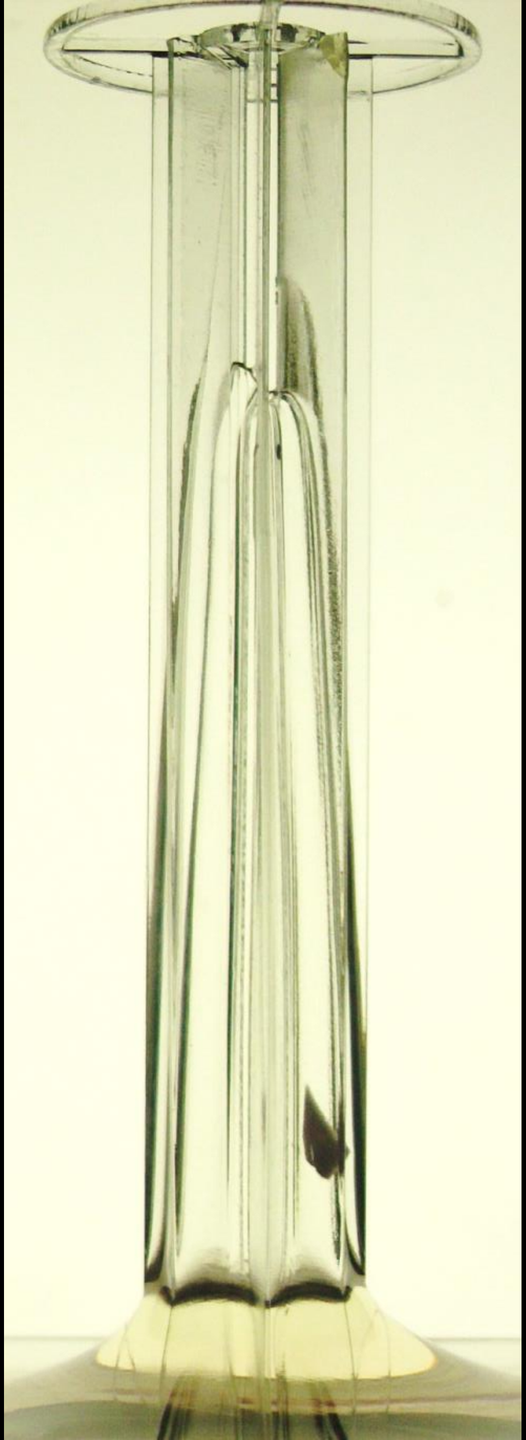


w
2d
D

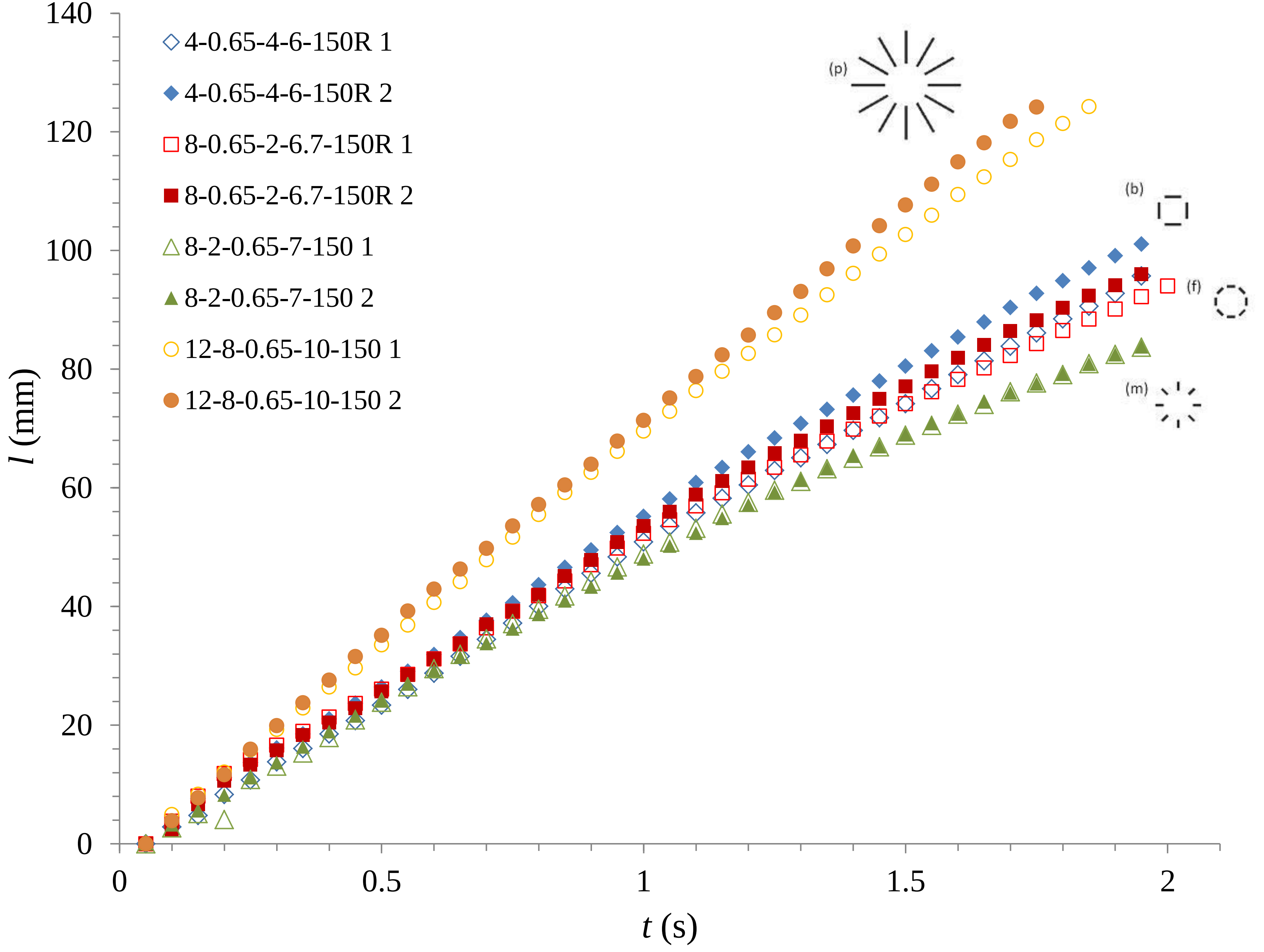

(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)
(k)
(l)
(m)
(n)
(o)
(p)



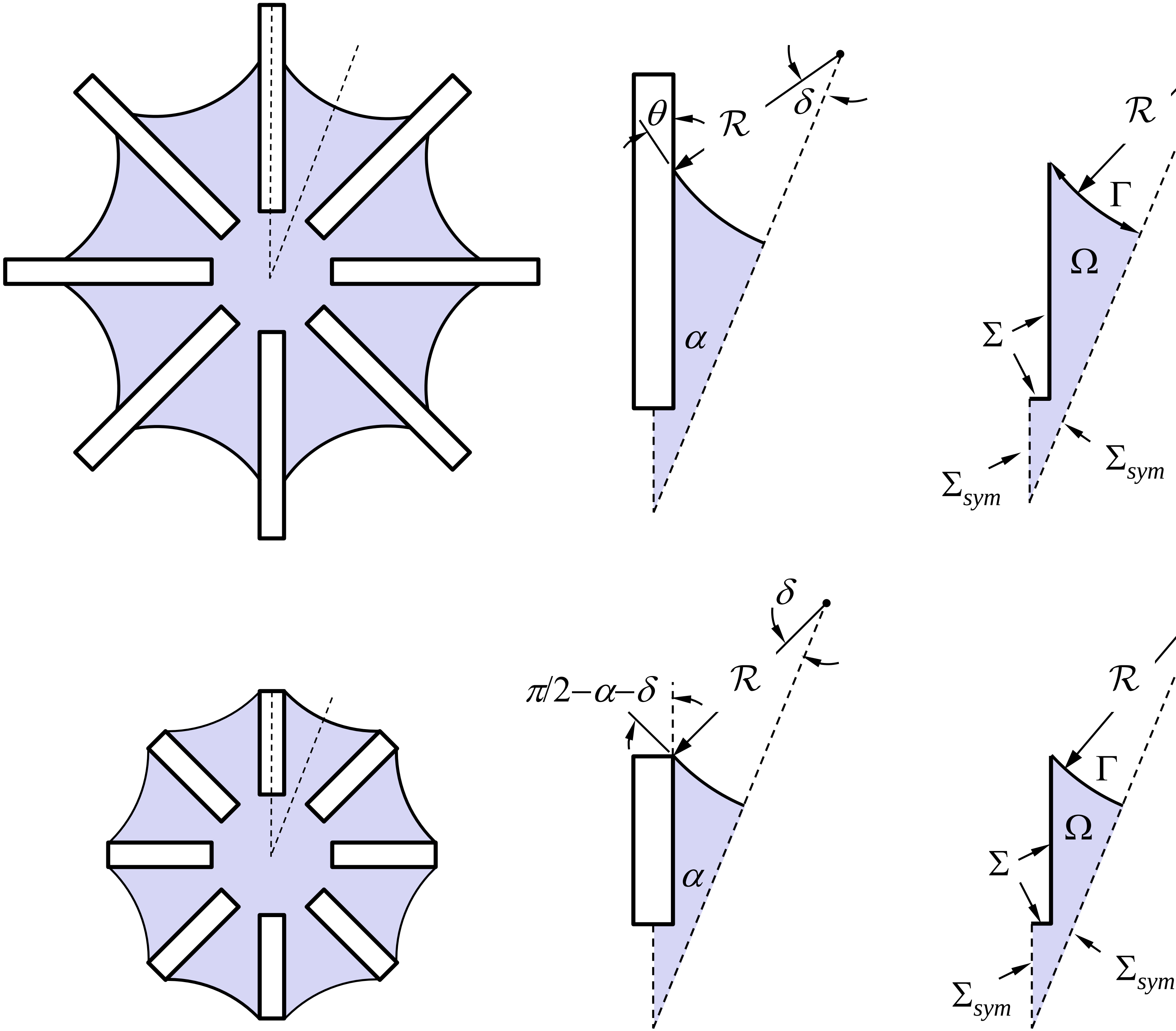
2437



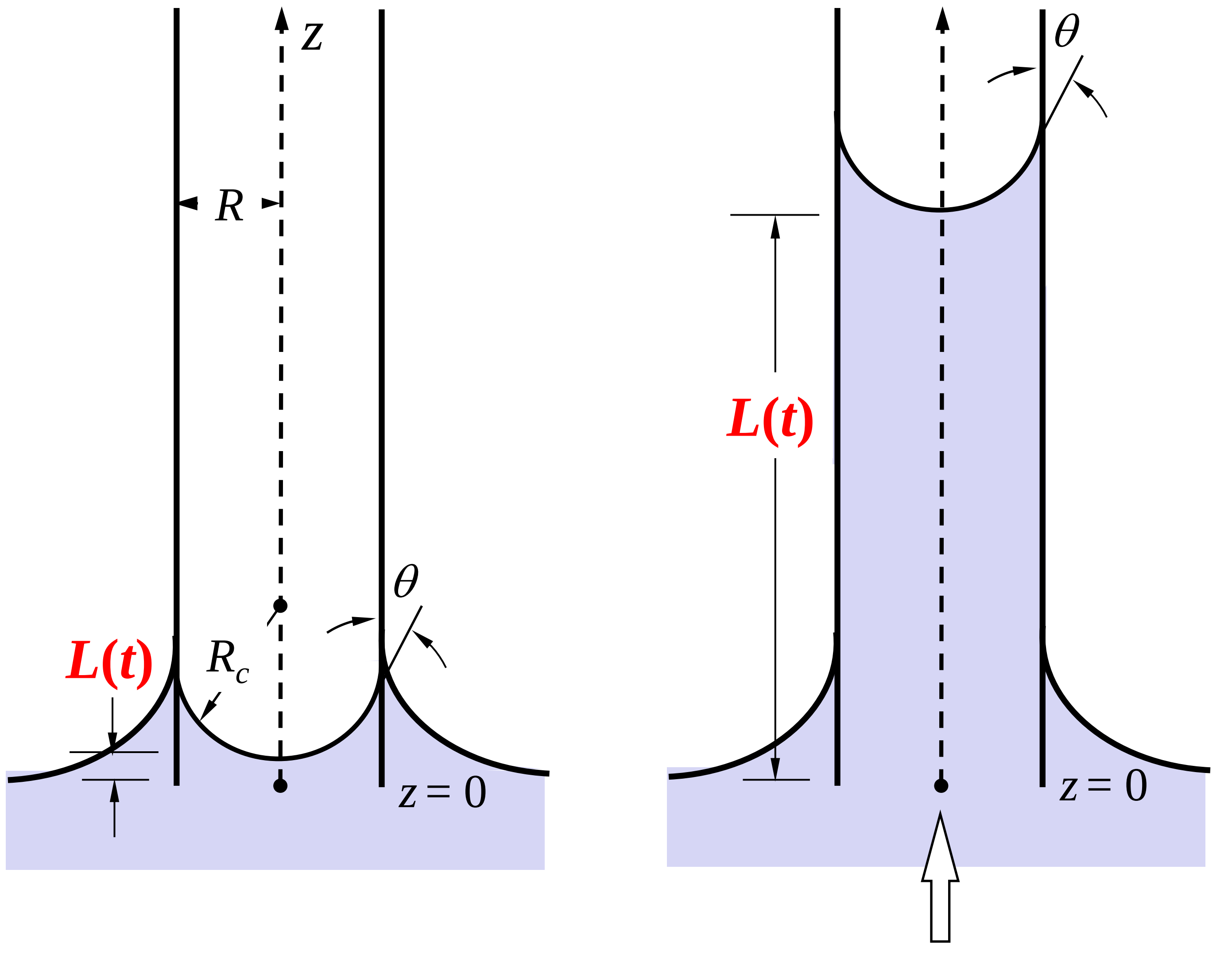
2435



140
120
100
80
60
40
20
0
l (mm)
0
0.5
1
1.5
2
t (s)
4-0.65-4-6-150R 1
4-0.65-4-6-150R 2
8-0.65-2-6.7-150R 1
8-0.65-2-6.7-150R 2
8-2-0.65-7-150 1
8-2-0.65-7-150 2
12-8-0.65-10-150 1
12-8-0.65-10-150 2
(p)
(b)
(f)
(m)



θ
R
δ
α
R
Γ
Ω
Σ
Σsym
Σsym
δ
π/2−α−δ
R
α
R
Γ
Ω
Σ
Σsym
Σsym



z
R
θ
L(t)
R_c
z = 0
θ
L(t)
z = 0

$$\rho\frac{\partial\bar{u}}{\partial t}+\rho(\bar{u}\cdot\nabla)\bar{u}=-\nabla P+\mu\Delta\bar{u}+\rho\bar{g}$$

Ignoring *g*, choose scales for elongating flows

$$P\sim\frac{\sigma}{R_c} \qquad z\sim L(t_s) \qquad \bar{u}\sim W\sim\frac{L}{t_s} \qquad \Delta_s\sim\frac{1}{x_s^2}+\frac{1}{y_s^2}+\frac{1}{z_s^2}$$

$$\frac{\rho L}{t\cdot t_s},\frac{\rho L}{t_s^2}\sim\frac{\sigma}{R_c L},\frac{\mu\Delta_s L}{t_s} \qquad \div\frac{\sigma}{R_c L}$$

$$x_s\sim y_s\sim R$$

$$z_s\gg R$$

$$\frac{\rho R_c L^2}{\sigma t_s^2}\left(\frac{t_s}{t},1\right)\sim 1,\frac{\mu\Delta_s R_c L^2}{\sigma t_s}$$

$$\Delta_s\sim\frac{1}{R^2}$$

Seek 2-time Scaling method

Desire…

$$\frac{t_s}{t}\sim\frac{R}{L} \qquad t\sim\frac{Lt_s}{R}$$

$$L\ll R\ \ldots\ t\ll t_s$$

$$L\sim R\ \ldots\ t\sim t_s$$

$$L\gg R\ \ldots\ t\gg t_s$$

$$\frac{\rho R_c L^2}{\sigma t_s^2}\left(\frac{R}{L}, 1\right) \sim 1, \frac{\mu \Delta_s R_c L^2}{\sigma t_s}$$

Convert to algebraic form

$$\frac{\rho R_c L^2}{\sigma}\left(\frac{R}{L} + 1\right)\frac{1}{t_s^2} + \frac{\mu \Delta_s R_c L^2}{\sigma}\frac{1}{t_s} - 1 = 0$$

Solve for t_s

$$t_s \sim \frac{2\rho}{\mu \Delta_s}\frac{(1 + R/L)}{((1 + 4Su^+)^{1/2} - 1)}$$

$$Su^+ \equiv \frac{\sigma \rho}{\mu^2 \Delta_s^2 R_c L^2}\left(1 + \frac{R}{L}\right)$$

Limits of $L(t_s)\ldots$

$$Su^+ \gg 1, \frac{R}{L} \gg 1; \qquad L \sim \frac{\sigma}{\rho R_c R} t_s^2$$

$$Su^+ \gg 1, \frac{R}{L} \ll 1; \qquad L \sim \left(\frac{\sigma}{\rho R_c}\right)^{1/2} t_s$$

$$Su^+ \ll 1; \qquad L \sim \left(\frac{\sigma}{\mu \Delta_s R_c}\right)^{1/2} t_s^{1/2}$$

$$\rho(C_e R + l_o + l)\frac{d^2 l}{dt^2} + \rho(1 + K/2)\left(\frac{dl}{dt}\right)^2 + \mu\Delta_s\left(l_o + \frac{R}{4} + l\right)\frac{dl}{dt} - \frac{\sigma}{R_c} =$$

$$\frac{F_{Su^+}^2}{mSu^+}\left[\left(\frac{C_e R}{L} + \frac{l_o}{L} + l\right)\frac{d^2 l}{dt^2} + (1 + K/2)\left(\frac{dl}{dt}\right)^2\right] + \frac{F_{Su^+}}{2nSu^+}\left(\frac{l_o}{L} + \frac{R}{4L} + l\right)$$

$$Su^+ \equiv \frac{m}{n^2}\frac{\sigma\rho}{\mu^2\Delta_s^2 R_c L^2}$$

$$F_{Su^+} = (1 + 4Su^+)^{1/2} - 1$$

$$n = \frac{l_o}{L} + \frac{R}{4L} + 1$$

$$m = \frac{C_e R}{L} + \frac{l_o}{L} + 1 + \frac{K}{2}$$

$$t_s \sim \frac{2\rho}{\mu\Delta_s}\frac{m}{nF_{Su^+}}$$

$$\frac{\;}{Su^+}\left[\left(\frac{C_eR}{L}+\frac{l_o}{L}+l\right)\frac{d^2l}{dt^2}+(1+K/2)\left(\frac{dl}{dt}\right)^2\right]+\frac{F_{Su^+}}{2nSu^+}\left(\frac{l_o}{L}+\frac{R}{4L}+l\right)\frac{dl}{dt}$$

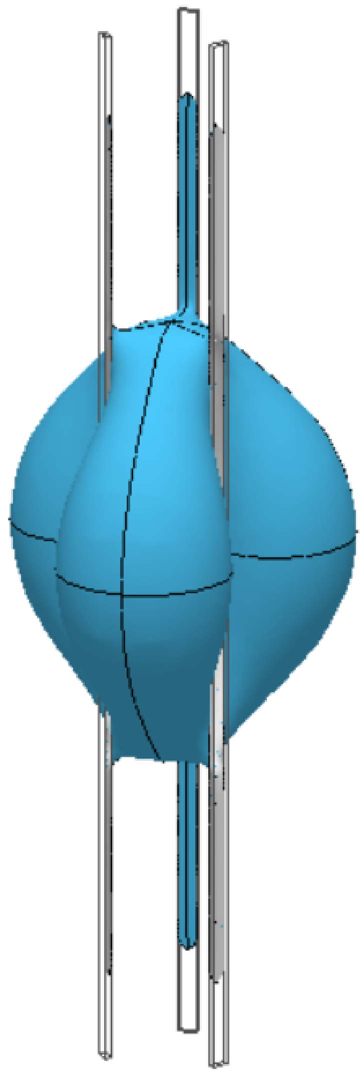
$$t_s=\frac{2\rho}{\mu\Delta_s}\frac{\left(\frac{C_eR}{L}+\frac{l_o}{L}+1+\frac{K}{2}\right)}{((1+4Su^+)^{1/2}-1)\left(\frac{l_o}{L}+\frac{R}{4L}+1\right)}$$

$Su^+\gg 1,\frac{C_eR}{L}+\frac{l_o}{L}\gg 1; k\leq 1;$ $\quad l=\frac{1}{2}t^2$

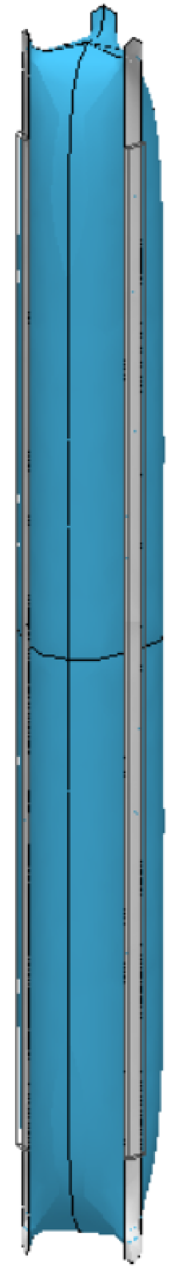
$Su^+\gg 1,\frac{C_eR}{L}+\frac{l_o}{L}\ll 1;$ $\quad l=t$

$Su^+\ll 1,\frac{R}{L}+\frac{l_o}{4L}\ll 1;$ $\quad l=(2t)^{1/2}$

$Su^+\ll 1,\frac{R}{L}+\frac{l_o}{4L}\gg 1;$ $\quad l=t$

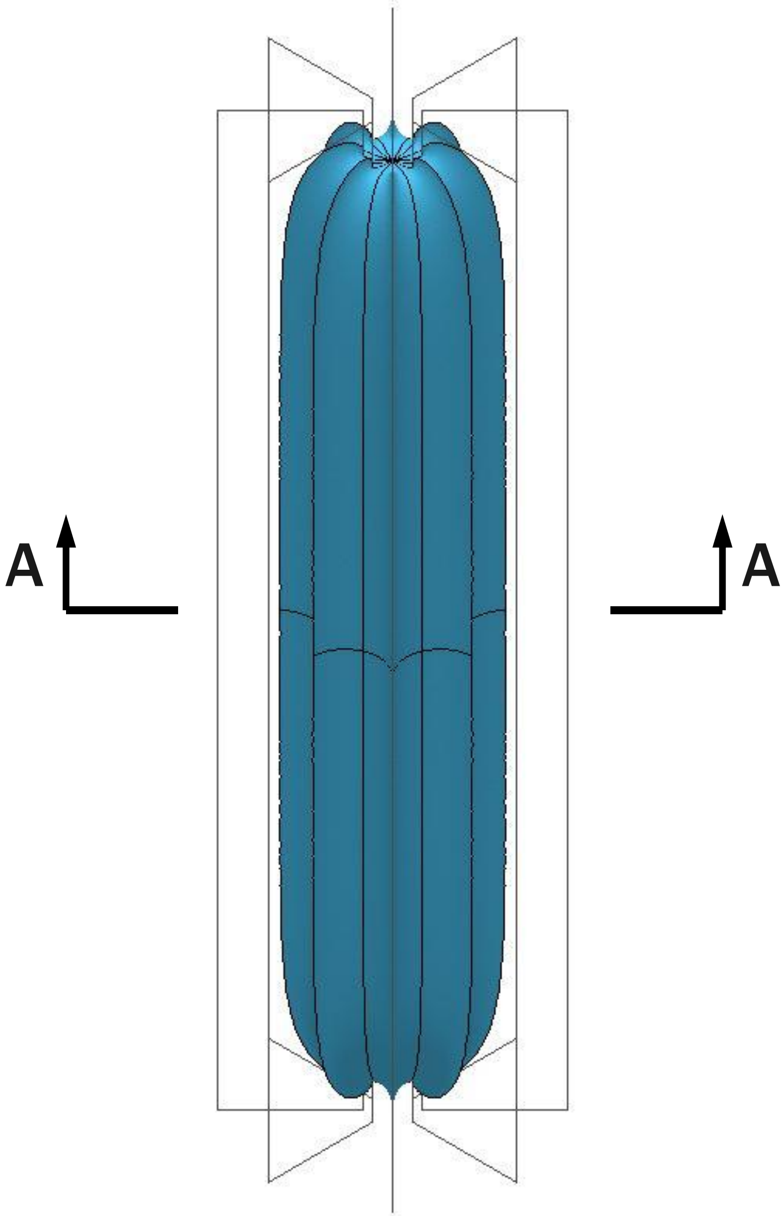


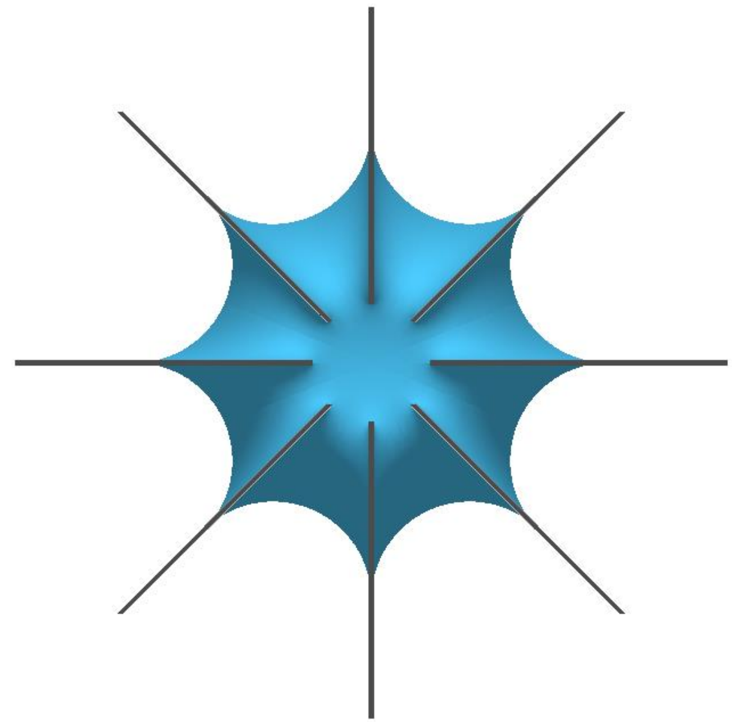
d=0.15



d=0.2

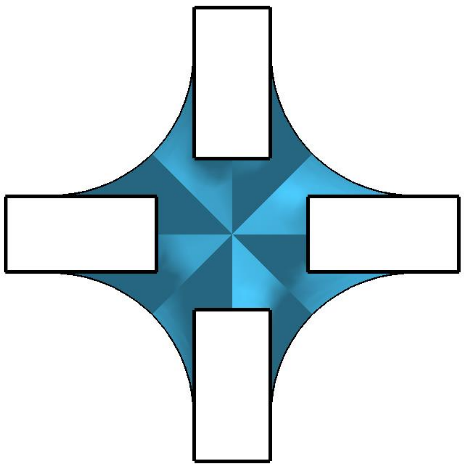
Constant Mean Curvature Surface



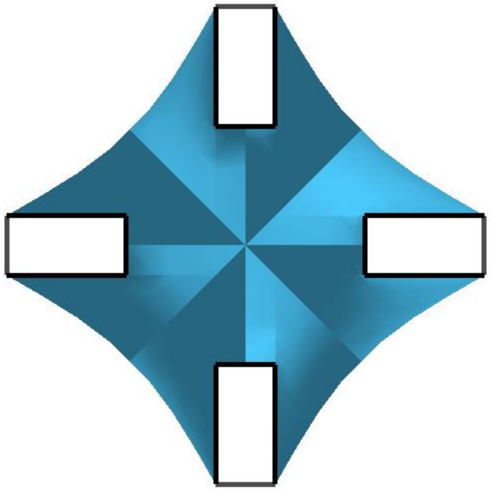


Mid-plane (A-A)

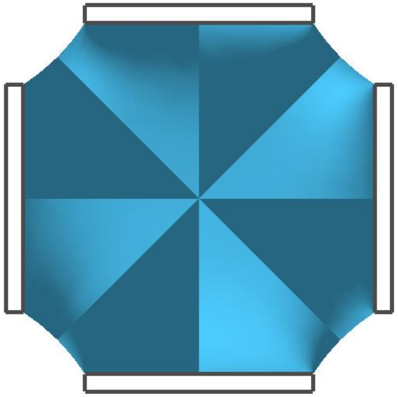
Configurations



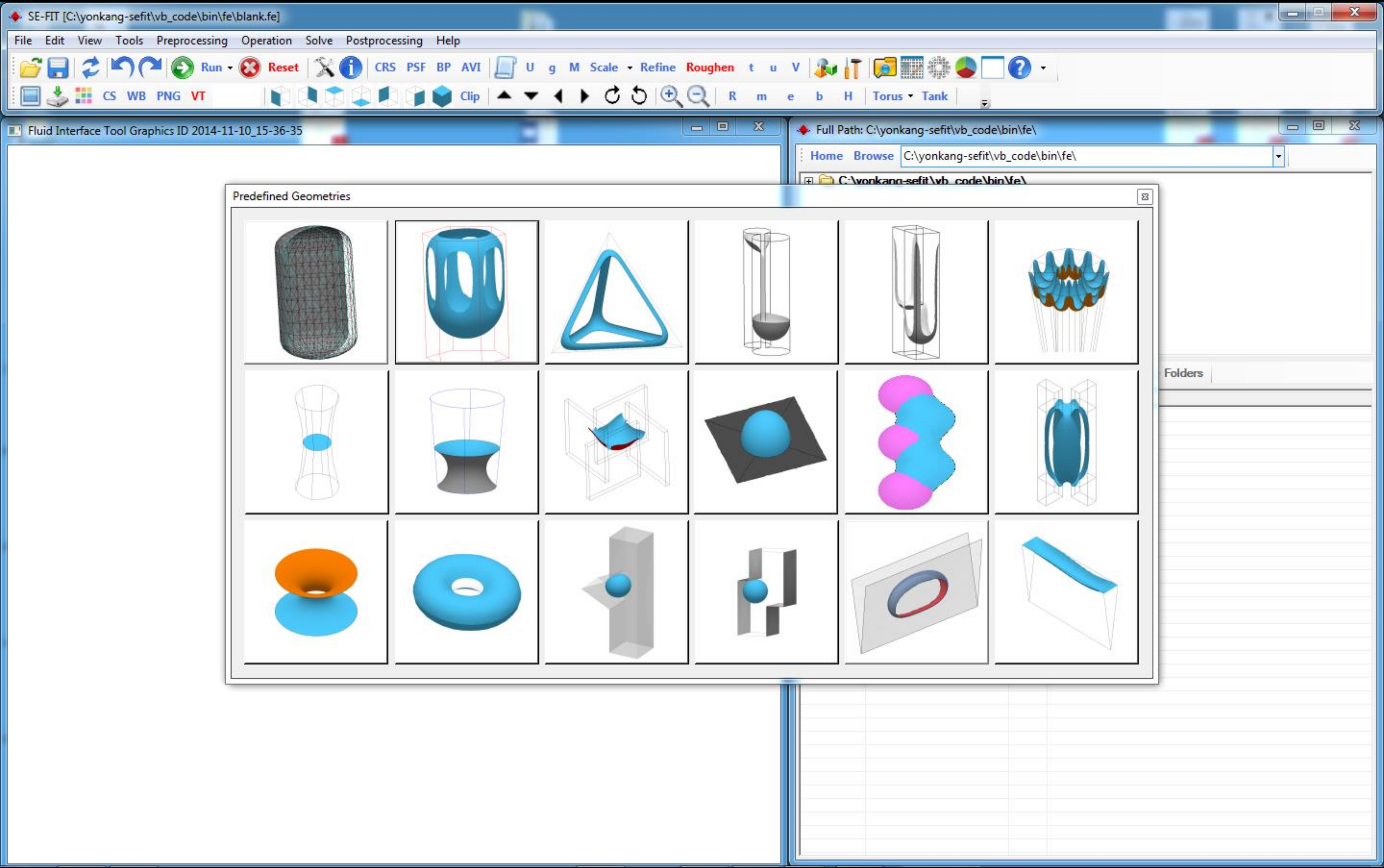
No-pinning



Outer-pinning



Inner-pinning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File Edit View Tools Preprocessing Operation Solve Postprocessing Help
Run Reset CRS PSF BP AVI U g M Scale Refine Roughen t u V
CS WB PNG VT Clip R m e b H Torus Tank
Fluid Interface Tool Graphics ID 2014-11-10_15-36-35
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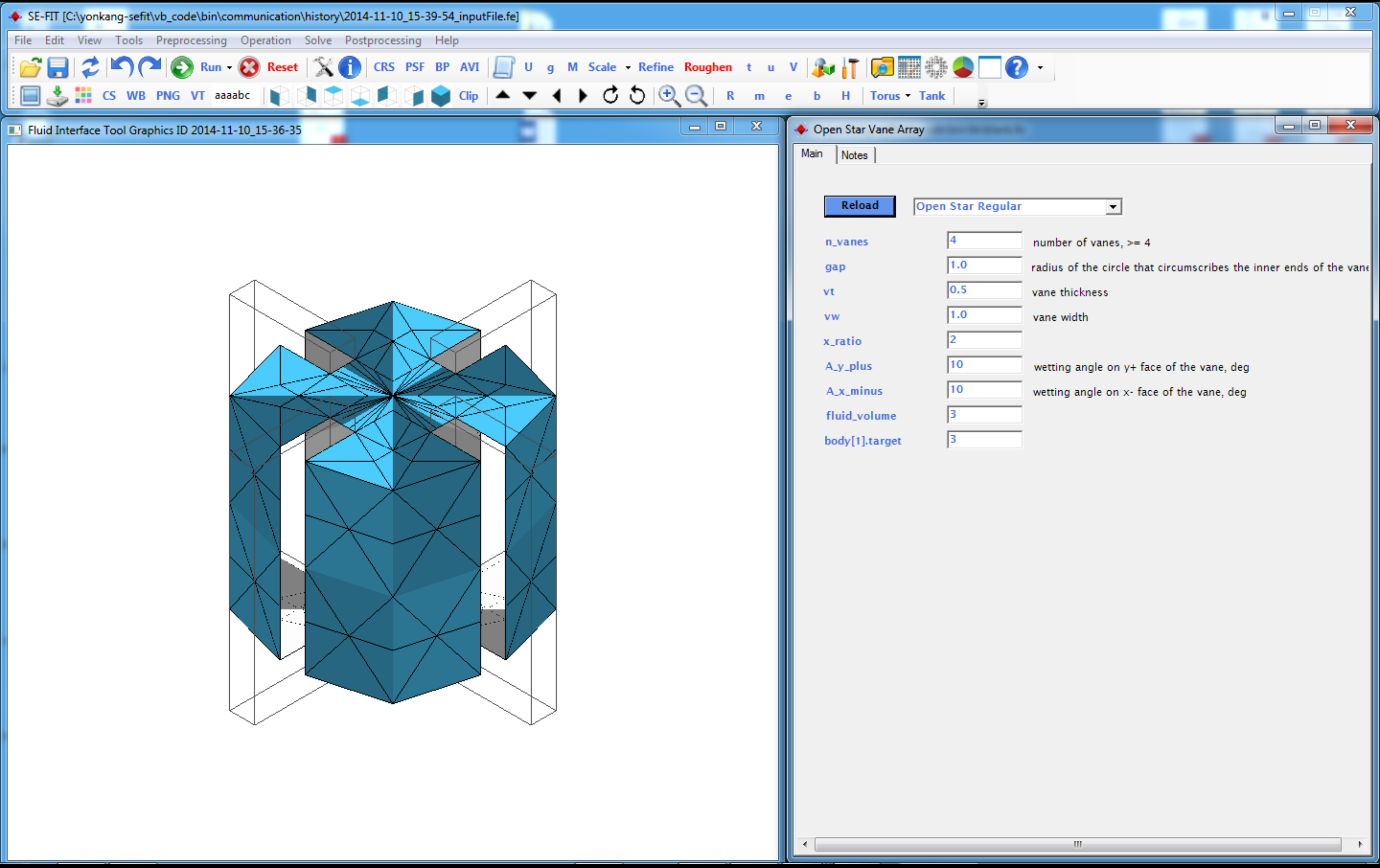
Home Browse C:\yonkang-sefit\vb_code\bin\fe\
Predefined Geometries
Folders



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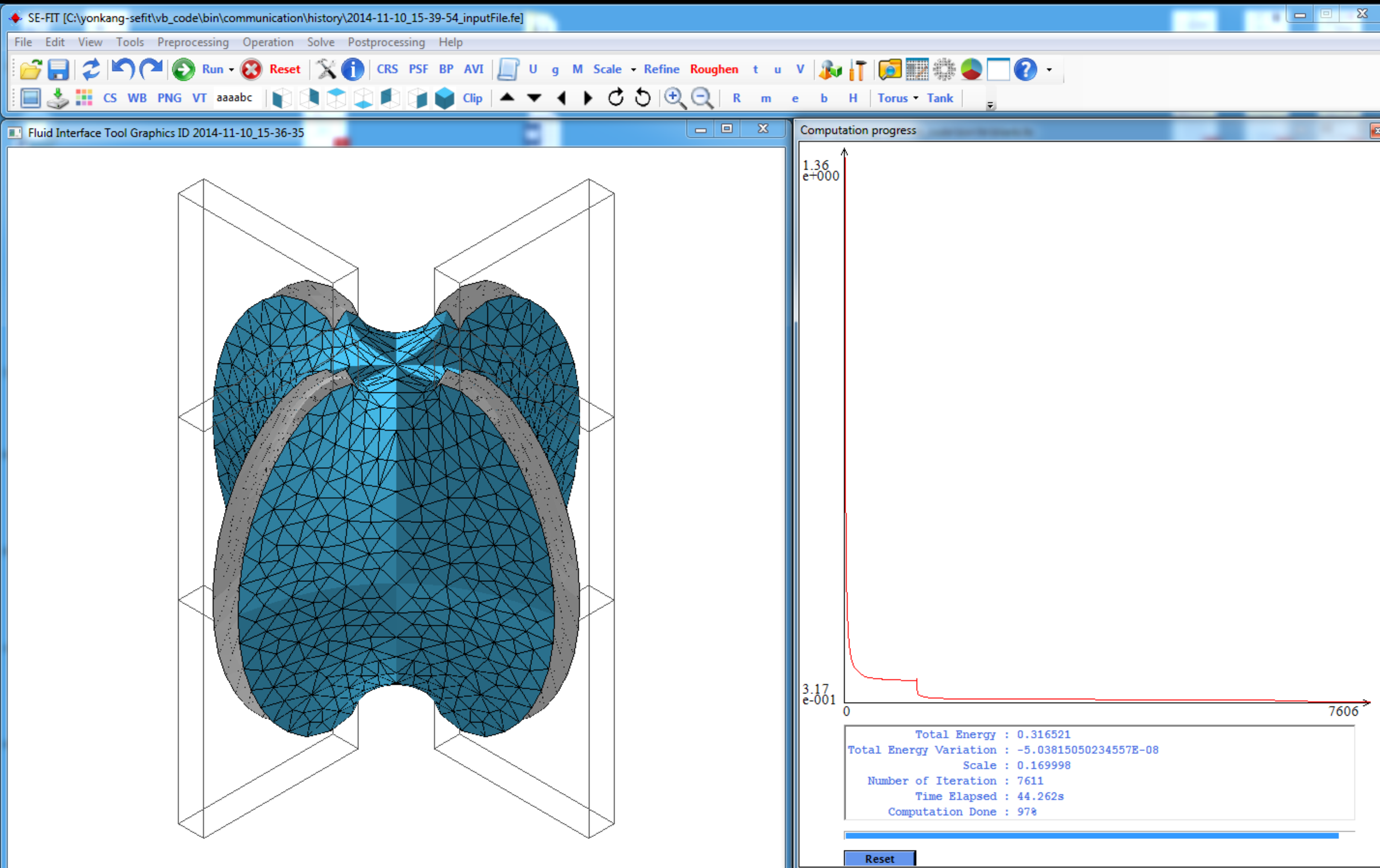
File Edit View Tools Preprocessing Operation Solve Postprocessing Help
Run Reset CRS PSF BP AVI U g M Scale Refine Roughen t u V
CS WB PNG VT aaaabc Clip R m e b H Torus Tank
Fluid Interface Tool Graphics ID 2014-11-10_15-36-35
Open Star Vane Array
Main
Notes
Reload
Open Star Regular
n_vanes
4
number of vanes, >= 4
gap
1.0
radius of the circle that circumscribes the inner ends of the vane
vt
0.5
vane thickness
vw
1.0
vane width
x_ratio
2
A_y_plus
10
wetting angle on y+ face of the vane, deg
A_x_minus
10
wetting angle on x- face of the vane, deg
fluid_volume
3
body[1].target
3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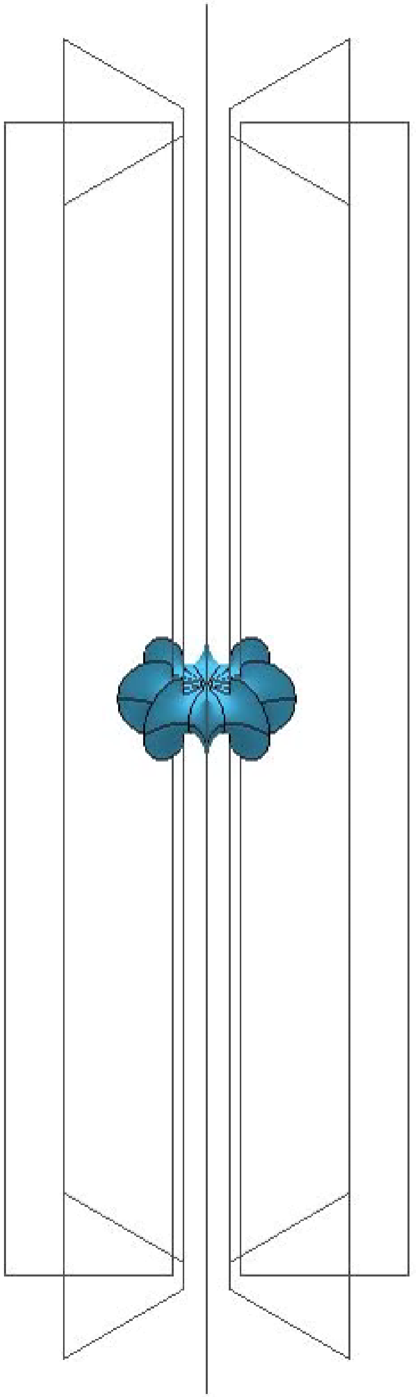
File Edit View Tools Preprocessing Operation Solve Postprocessing Help
Run Reset CRS PSF BP AVI U g M Scale Refine Roughen t u V
CS WB PNG VT aaaabc Clip R m e b H Torus Tank
Fluid Interface Tool Graphics ID 2014-11-10_15-36-35
Computation progress
1.36 e+000
3.17 e-001
0
7606
Total Energy : 0.316521
Total Energy Variation : -5.03815050234557E-08
Scale : 0.169998
Number of Iteration : 7611
Time Elapsed : 44.262s
Computation Done : 97%
Reset

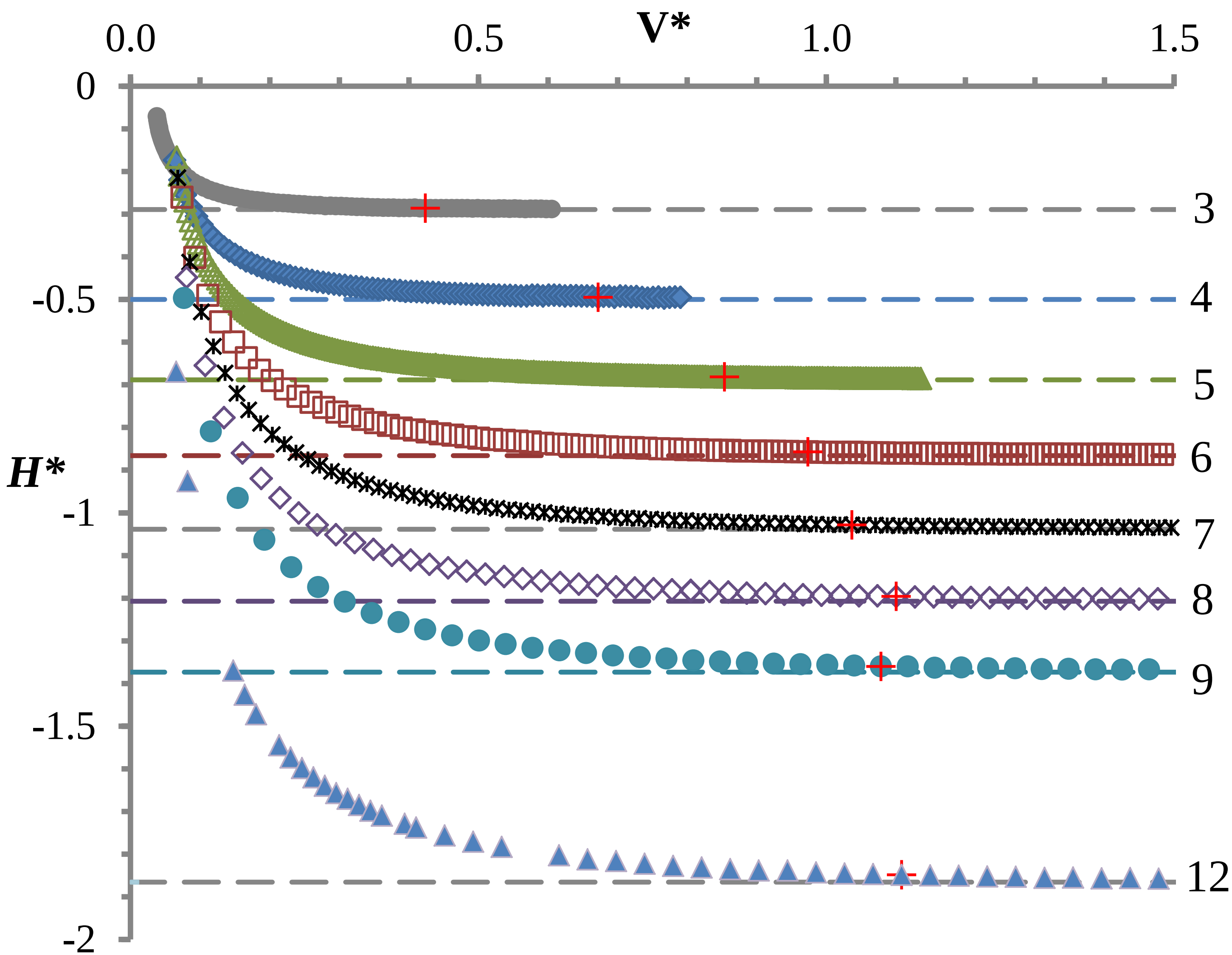
Liquid Drops in the Open-Star Vane Array

- **113 equilibrium surfaces**
- **SE-FIT©*** Parameter Sweep Function

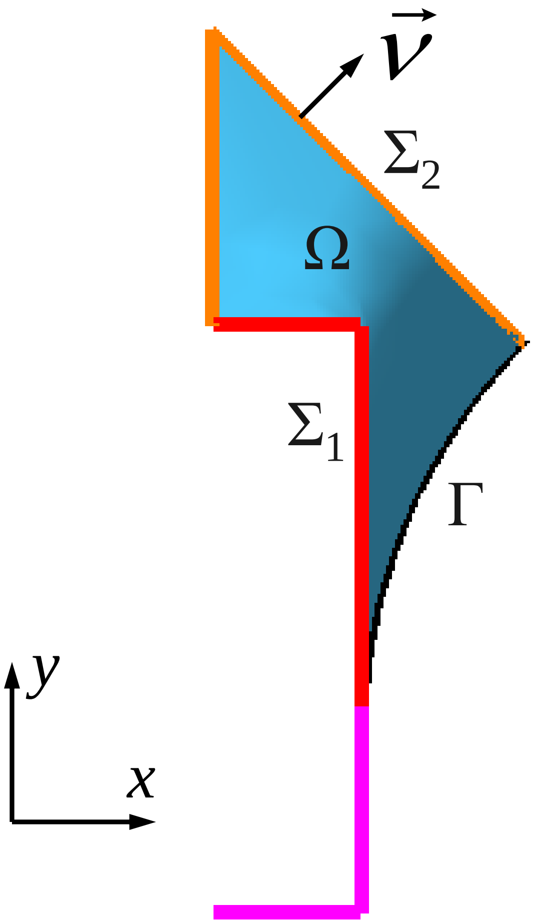


*** Surface Evolver - Fluid Interface Tool (SE-FIT.com)**

No-Pinning



Hybrid Boundary Condition Method



Young-Laplace equation:

$$\nabla \cdot \mathrm{T}u = 2H, \text{ with } \mathrm{T}u = \frac{\nabla u(x,y)}{\sqrt{1+|\nabla u|^2}}$$

B.C. $\vec{v} \cdot \mathrm{T}u = \cos\gamma,$ on Σ_1

$$\vec{v} \cdot \mathrm{T}u = \cos(\pi/2), \text{ on } \Sigma_2$$

$$\vec{v} \cdot \mathrm{T}u = \cos\pi, \text{ on } \Gamma$$

Integrating over Ω

$$(\Sigma_1 \cos\gamma - \Gamma) = 2H\,\Omega$$

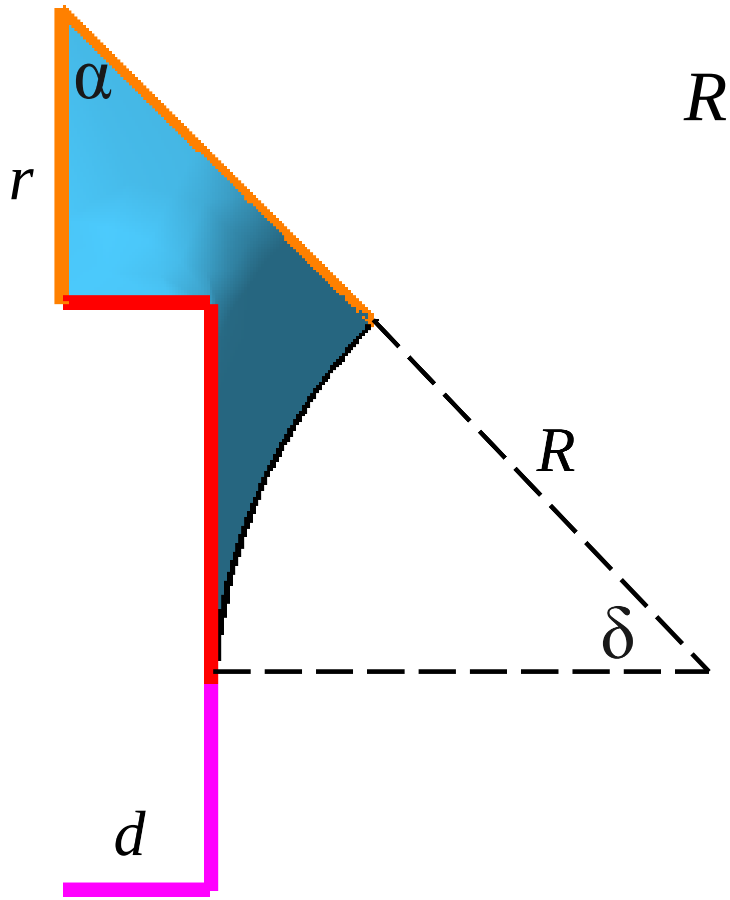
$$\Sigma_1 \cos\gamma - \Gamma > 0 \Rightarrow H > 0, \text{concave}$$

$$\Sigma_1 \cos\gamma - \Gamma = 0 \Rightarrow H = 0, \text{minimal surface}$$

$$\Sigma_1 \cos\gamma - \Gamma < 0 \Rightarrow H < 0, \text{convex}$$

Chen et al, 2012
de Lazzer et al, 1996
Concus & Finn, 1969

Solution: No-Pinning



$$R = \frac{f^2 \lambda}{F_A}\left(-1 \pm \sqrt{1 - \frac{F_A(\lambda + d - r)d}{f^2 \lambda^2}}\right)$$

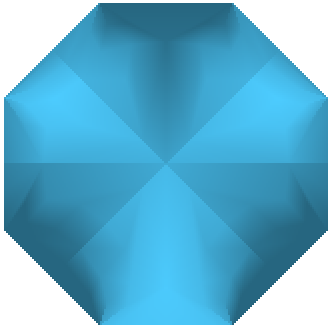
$$f = \frac{\sin\alpha}{\cos\gamma - \sin\alpha}$$

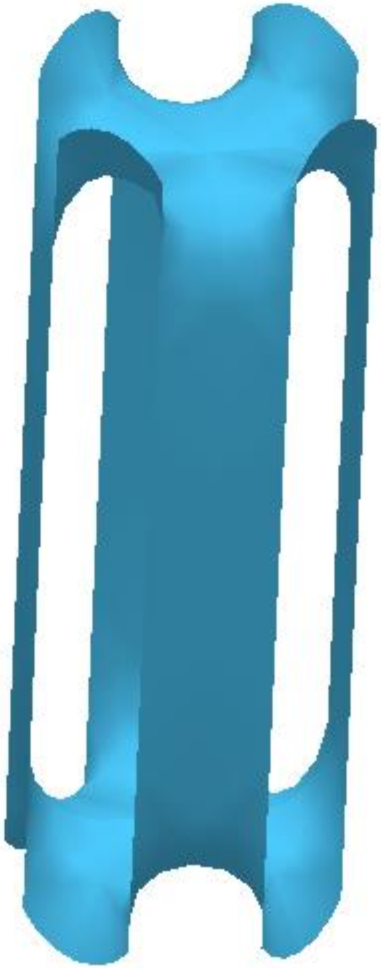
$$F_A = f^2\left(\frac{\cos\gamma \sin\delta}{\sin\alpha} - \delta\right)$$

$$\delta = \frac{\pi}{2} - \alpha - \gamma$$

$$\lambda = \frac{(\cos\alpha + \sin\alpha)d}{\sin\alpha} - r$$

Minimal Surface (Scherk) with 4 Vanes





Thank you!