Frequency Estimator Performance for a Software-Based Beacon Receiver

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***Abstract*—As propagation terminals have evolved, their design has trended more toward a software-based approach that facilitates convenient adjustment and customization of the receiver algorithms. One potential improvement is the implementation of a frequency estimation algorithm, through which the primary frequency component of the received signal can be estimated with a much greater resolution than with a simple peak search of the FFT spectrum. To select an estimator for usage in a Q/V-band beacon receiver, analysis of six frequency estimators was conducted to characterize their effectiveness as they relate to beacon receiver design.**

I. Introduction

As radio communication links are driven to higher frequencies (e.g. K_a-band and above) by both spectrum congestion and the appeal of higher data rates, the propagation statistics at these frequencies must be known in order to design efficient communication links. Propagation studies are often conducted using beacon receivers, in which a ground terminal monitors the power received from a continuous-wave (CW) beacon on a geostationary satellite. However, due to system electronics and Doppler effects, the received signal will possess some nominal drift in frequency which needs to be accurately determined before a power measurement can be made. Several techniques are available to system designers to track frequency changes in the measured signal, e.g., phase locked loops and multi-sampled FFTs [1]. Herein, we focus on frequency estimation approaches as applied to the GRC-developed beacon receiver and compare the relative performance of each.

II. Frequency Estimators

A. Background

The Fast Fourier Transform (FFT) may be easily applied to measure the received frequency of a beacon signal by finding the peak of the frequency spectrum. However, the resolution of the FFT is defined by f_s/N, where f_s is the sampling frequency of the signal and N is the number of acquired points. Thus, while the actual frequency of the signal may vary continuously, the discrete points of the FFT frequency spectrum are limited to integer multiples of f_s/N (bins). The resolution of the frequency measurement is therefore restricted by the extent to which f_s/N can be minimized. This requires either increasing the number of points collected and/or decreasing the sampling frequency, both of which increase the acquisition time and reduce the rate at which the propagation measurements can be recorded.

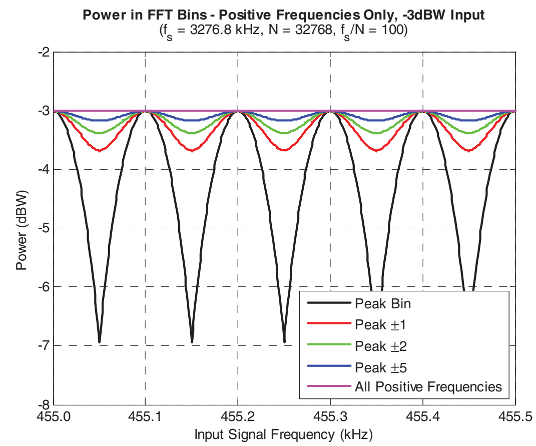

Fig. 1. The scalloping of power observed in the peak bin of the FFT and the reduction in scalloping when summing neighboring bins.

Furthermore, the peak search alone overlooks additional information that is present in the FFT; it is only when the frequency of a signal falls exactly into a bin frequency that the bin will contain the total power of the signal. In all other cases, the power will leak into nearby bins, to an extent governed by the distance of the signal frequency from the nearest bin frequency [2]. When calculating the power of the signal from the peak bin, this introduces a scalloping effect, as shown in Fig. 1, where the power is minimized when the frequency is between bins and maximized when it is near a bin frequency. This effect may be mitigated by summing the peak with several nearby bins, but this does not fully eliminate the scalloping and will effectively decrease the dynamic range of the beacon receiver.

B. Algorithms

A variety of algorithms exist that take advantage of the power in nearby bins to interpolate the frequency when it falls between the bins of the FFT. To characterize the performance of these frequency estimators as pertaining to beacon receiver design, a selection of algorithms were applied to a simulated beacon signal. The frequency estimators considered alongside the FFT peak search were the Buneman [3], Quinn-Fernandes [4], Quinn's First Method [5], Jacobsen's Algorithm [6], and MacLeod's Algorithm [7]. Also considered was a modified version of the Quinn-Fernandes method (coined Quinn-Fernandes-Nessel) that was primed with *a priori* information – namely, a window in which the frequency is known or expected to appear, giving the algorithm a search space smaller

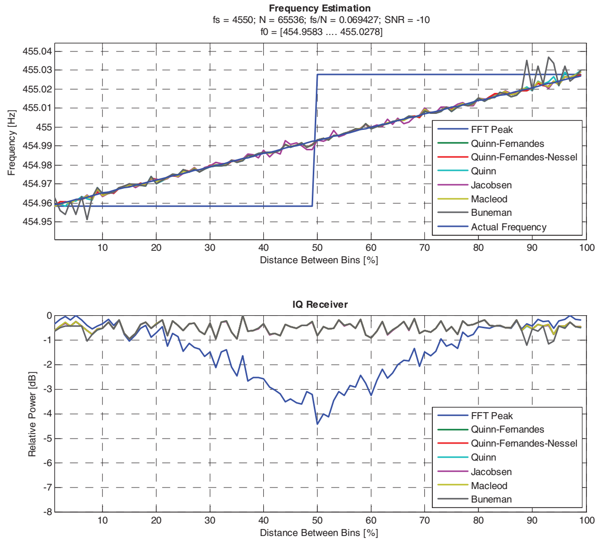

Fig. 2. The frequency estimate of each algorithm (top) and the corresponding power (bottom) for a SNR of -10dB.

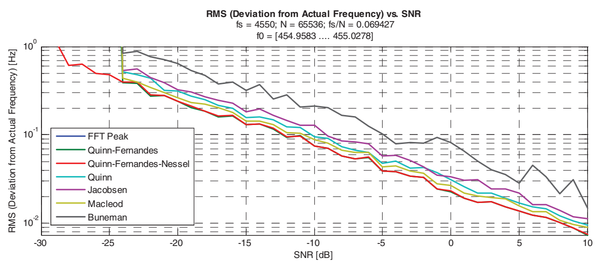

Fig. 3. The RMS error of all six algorithms as the SNR of the input is varied from -30dB to +10dB.

than the entire spectrum of the FFT. Such a modification may be made to any of the algorithms to improve calculation time and performance, but may not always be viable if the expected frequency is not well known.

III. Analysis

To apply the estimators, a noisy sinusoidal beacon signal was simulated and swept from one bin frequency to the next. At each point, the frequency was estimated with each algorithm. The simulation was run with f_s = 4550Hz, N=2^{16}, and f_0 = 454.96 to 455.03 Hz, where f_s is the sampling frequency, N is the number of points, and f_0 is the input frequency. The signal-to-noise ratio (SNR) was also varied to gauge performance in low-power conditions.

At a high SNR (e.g. +10dB), all six estimation algorithms were able to precisely track the frequency of the input signal as it swept from one bin frequency to the next with no discernable difference between them. The error of the FFT peak search, however, was very evident when compared to the estimators. Even with a high SNR, the FFT remains limited in resolution and can only return either the first bin frequency or the second. The scalloping effect is clear in the power measured from the FFT results and non-existent in the power from the estimators.

At a moderate SNR of -10dB, as shown in Fig. 2, the noise is beginning to show in all of the estimates but the algorithms remain significantly more accurate than the FFT. It also becomes apparent that the Buneman algorithm is noisier than the other five estimators when nearer to the bin frequencies.

For a low SNR (e.g. -20dB), the noise is significant, but the estimation algorithms continue to offer a much better approximation than the FFT alone. The Buneman remains noisier than the other algorithms, primarily near to the bin frequencies, but is still functional. As the noise grows, the FFT begins to oscillate between bins as the two neighboring peaks are very similar in magnitude and the noise is significant enough to obscure the true maximum.

To quantify the performance of each algorithm in more detail, the root-mean-square (RMS) error of each algorithm, with respect to the actual input frequency, was calculated as the SNR was varied from -30 to +10 dB. The results are presented in Fig. 3. All six estimators exhibited an exponential increase in RMS error as the SNR decreased. At approximately -24dB SNR, the noise at any point in the spectrum may exceed the peak of the FFT, and most of the methods therefore become unable to track the frequency. Quinn-Fernandes-Nessel manages to survive below this point due to the windowing.

The calculation time of each estimator was also considered. The Buneman was by far the fastest of all algorithms tested, taking an average of 6 ms with 2^{16} points. Quinn-Fernandes was by far the slowest, taking an average 23 ms, although the windowing added to Quinn-Fernandes-Nessel did improve calculation time by 1 ms. The rest of the methods were roughly equivalent, taking an average of 7.5 ms. These figures should be taken in relation to one another, as they will vary with hardware and implementation.

IV. Conclusions

Each of the considered estimators were shown to calculate frequency to within ±1 Hz given an SNR above -24 dB (with f_s=4550; N=2^{16}), and to run within 23 ms as implemented. However, the Buneman was observed to increase in noise near the bin frequencies relative to the other algorithms. Although it remains better than an FFT peak search, it did introduce notable measurement error when implemented in a beacon receiver undergoing laboratory test. The current generation beacon receivers being developed at NASA's Glenn Research Center have thus implemented a selectively windowed Quinn-Fernandes-Nessel approach that applies the windowing criteria when the power level drops below a set threshold. Similar modifications could be made to any of the other methods if operation at lower SNR is needed.

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