An analysis method for superconducting resonator parameter extraction with complex baseline removal

Giuseppe Cataldo

NASA Goddard Space Flight Center, Greenbelt, MD 20771

Introduction

A new semi-empirical model is proposed for **extracting the quality (Q) factors of arrays of superconducting microwave kinetic inductance detectors (MKIDs)**. The determination of the total, internal and coupling Q factors enables the computation of the loss in the superconducting transmission lines. **The method used allows the simultaneous analysis of multiple interacting discrete resonators** with the presence of a complex spectral baseline arising from reflections in the system. The baseline removal allows an unbiased estimate of the device response as measured in a cryogenic instrumentation setting.

MKIDs measure the change in kinetic inductance caused by the absorption of photons in a thin strip of superconducting material. The variation in kinetic inductance can be measured via the change in resonant frequency of a microwave resonator, in which the inductance is combined with a capacitor [1].

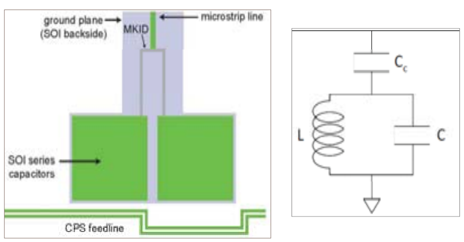


Figure 1. (Left) Conceptual sketch of an ultrasensitive MKID. The sub-millimeter/far-IR radiation arrives through a superconducting microstrip line, with thin crystalline silicon serving as the microstrip dielectric, fabricated using a silicon-on-insulator (SOI) wafer. The MKID sections absorb the incident radiation and at microwave frequencies behave as inductors, dominated by the kinetic inductance. Coupled with a parallel-plate SOI capacitor, it forms a microwave resonator. (Right) A lumped-element representation of a MKID, where L, C, and C_c represent the inductor, capacitor and coupling capacitor, respectively.

Problem statement

The resonators' asymmetric line shape is due to **impedance mismatches in the feedline** originating from:

- wire-bond connections,
- sample mount imperfections,
- the transmission line geometry.

These asymmetries do not allow the resonators to be reproduced by the classical Maxwell-Helmholtz-Drude dispersion model [2-7].

Objectives

1. Extract the Q factors of multiple interacting discrete resonators by:
   - Fitting the baseline and removing it from the transmission measured data.
   - Analyzing the data with the removed baseline through a physical model capable of representing the interaction of the different resonators.
2. Verify and validate the model through an ABCD-matrix approach, which takes into account the main elements of the actual device and is capable of reproducing the transmission measurements within a certain accuracy level.

Baseline removal and resonator model

A schematic of the device under test is shown below.

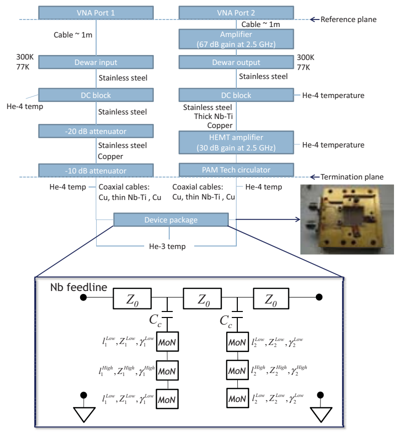
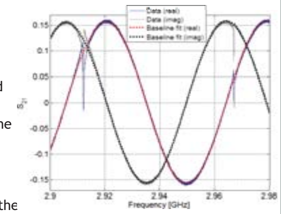


Figure 2. The device package was cooled down to ~330 mK in a Helium-3 bath. It consists of two molybdenum nitride (MoN) stepped impedance resonators and a niobium (Nb) feedline. A series of amplifiers, attenuators and cables connect the device under test to a vector network analyzer (VNA) used to measure the transmission coefficient, S_{21}, between the two ports.

Figure 3. The dewar is shown open with all the cables, attenuators and amplifiers before the measurement run at the NASA Goddard Space Flight Center.

Baseline removal

1. The complex baseline was reproduced through a fit of a 4-term Fourier series to the measured data:
$$S_{21,bas} = \sum_{j=1}^{4} a_j \exp\left[i\left(b_j \omega + c_j\right)\right]$$
with a reduced $\chi^2 = 0.9975$.
2. It was then removed from the measured data to eliminate the influence of reflections and move the reference plane to the input of the package.



Data normalization

The baseline fit enabled the estimation of the unknown gain, which is set by forcing the Smith chart amplitude to be equal to 1 far removed from the resonators.

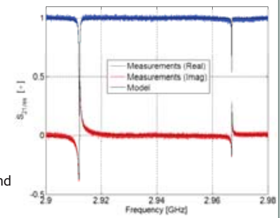
Resonator model

The resonators were modeled as follows:

$$S_{21,res} = 1 + \sum_{j=1}^{2} \frac{A_{1,j} + A_{2,j}x + A_{3,j}x^2 + \ldots}{1 + ix + A_{4,j}x^2 + \ldots},$$

$$x = Q_{tot,j}\left(\frac{\omega}{\omega_{0,j}} - \frac{\omega_{0,j}}{\omega}\right), \quad Q_{tot,j} = \frac{\omega_{0,j}}{\gamma_j}.$$

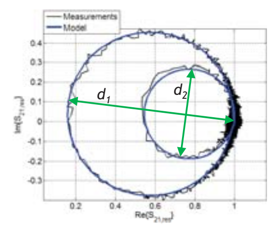
This functional form allows a **realizable, causal transmission-line representation** and enables one to extract the total Q, $Q_{tot,j}$, as well as the resonance frequency, $\omega_{0,j}$, and width, γ_j, of each resonator.



Internal and coupling Qs

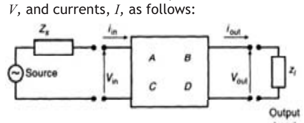
$$Q_{int,j} = \frac{Q_{tot,j}}{1 - d_j}, \quad Q_{c,j} = \frac{Q_{tot,j}}{d_j}$$

i	Q_{tot}	Q_{int}	Q_c
1	21,628	78,491	29,855
2	60,115	109,987	132,578



Resonator model validation through the ABCD-matrix method

The ABCD matrix for a 2-port network is defined in terms of the total voltages, V, and currents, I, as follows:



$$\begin{bmatrix} V_{in} \\ I_{in} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{out} \\ I_{out} \end{bmatrix}$$

The scattering parameter S_{21} is related to the ABCD parameters as:

$$S_{21} = \frac{2Z_l}{AZ_l + B + CZ_lZ_s + DZ_s},$$ with Z_s and Z_l the source and load impedances.

The ABCD matrix of a cascade connection of multiple networks is equal to the product of the ABCD matrices of the individual transmission line elements.

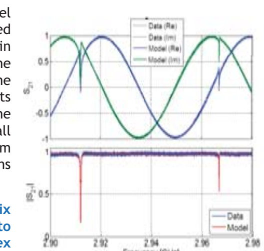
For a transmission line, $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & iZ_0 \sinh(\gamma l) \\ i \sinh(\gamma l)/Z_0 & \cosh(\gamma l) \end{bmatrix}$,

where Z_0 is the transmission line characteristic impedance and $\gamma = \sqrt{\varepsilon_{eff}\mu_{eff}} \cdot \omega / c$ its propagation constant. Both are a function of the effective dielectric permeability and permittivity, frequency and speed of light.

29 transmission line elements were used to model the device response below the termination plane (see Fig. 2). Their impedances and physical lengths were measured, while their effective dielectric permeabilities and permittivities were simulated through HFSS. For simplicity, the two coupling capacitors, C_c, were modeled as lumped elements.

The transmission line model recovers the observed baseline response to within a 5% accuracy from the measured transmission line coaxial cable lengths. Its limitations lie in the difficulty of determining all the reflections in the system generated by imperfections between the connections.

Ultimately, **this ABCD-matrix method can be used to determine the complex phase velocity and the kinetic inductance fraction of the materials [8].**



Conclusions

- A semi-empirical model was developed to calibrate the transmission response of MKIDs at cryogenic temperatures. The model executes the following steps:
  1. Fits and removes the baseline amplitude and adjusts the phase to move the reference plane to the package input.
  2. Estimates the unknown gain by normalizing the measured data. This is realized by enforcing a unit-amplitude Smith chart response away from the resonator positions.
  3. Models the resonators and their coupling structures with a realizable, causal transmission-line representation.
  - From the residuals, an accuracy of 2% was inferred.
  - The total, internal and coupling Q factors for each resonator were then computed from the calibrated data.
- An ABCD-matrix approach was employed to validate the semi-empirical model and can be used to study the material properties of the transmission line of the device under test.
  1. This allows one to compute the complex phase velocity and kinetic inductance fraction from the transmission line material properties and geometry.
  2. Validation of the semi-empirical model was achieved within ~5% due to our limited knowledge of the detailed reflections occurring in the system.

References

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Acknowledgments

I would like to acknowledge the support of my colleagues at the NASA Goddard Space Flight Center, namely E. J. Wollack, S. H. Moseley, E. Barrentine, K. U-Yen and A. D. Brown, as well as Prof. J. A. Hoffman, my academic advisor at the Massachusetts Institute of Technology.

Contacts

Giuseppe.Cataldo@NASA.gov