Design, Analysis, and On-Sun Evaluation of Reflective Strips Under Controlled Buckling

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Solar concentrators are envisioned for use in a variety of space-based applications, including applications involving in situ resource utilization. Identifying solar concentrators that minimize mass and cost are of great interest, especially since launch cost is driven in part by the mass of the payload. Concentrators must also be able to survive the wide temperature excursions on the lunar surface. Identifying smart structures which compensate for changes in concentrator geometry brought about by temperature extremes are of interest. Some applications may benefit from the ability to change the concentrator's focal pattern at will. This paper addresses a method of designing a single reflective strip to produce a desired focal pattern through the use of controlled buckling. Small variations in the cross section over the length of the reflective strip influence the distribution of light in the focal region. A finite element method of analysis is utilized here which calculates the curve produced for a given strip cross section and axial load. Varying axial force and strip cross section over the length of the reflective strip provide a means of optimizing ray convergence in the focal region. Careful selection of a tapered cross section yields a reflective strip that approximates a parabola. An array of reflective strips under controlled buckling produces a light weight concentrator and adjustments in the compression of individual strips provide a means of compensating for temperature excursions or changing the focal pattern at will.

Nomenclature

cm	=	centimeter
FEM	=	Finite Element Method
GPa	=	gigapascal
kg	=	kilogram
m	=	meter
nm	=	nanometer
W	=	watt

I. Introduction and Background

THE concept of utilizing solar concentrators for a variety of space exploration applications has been well documented.[1] Solar concentrators provide a means of concentrating the sun's energy, typically achieving high process heating temperatures at a relatively sharp focal point. An ideal solar concentrator is parabolic in

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shape, causing all reflected light to converge on a single point.[2] The shape can be formed from a single 3-dimensional piece, a series of spherical sections placed in the approximate shape of a parabola or offset parabola, or a series of parabolic strips each directed at a common focal region.[3] The parabolic strip serves as the starting point for discussion here. A new approach, one of controlling the geometry of a 2-dimensional reflective strip by applying an axial force, was first proposed by White.[4] A series of strips under compression would comprise a complete solar concentrator. Controlling geometry by buckling offers two advantages. First, there is the prospect of creating a light weight solar concentrator with inexpensive materials and an inexpensive manufacturing process. Second, careful control of the axial force provides a means of compensating for thermally induced distortions or controlling the concentrator's focal pattern at will.

In this study, the mechanical and reflective qualities of a buckled strip under axial loading are considered in depth. Ordinarily, an axially loaded member is considered to have failed when subjected to a load greater than its buckling limit, and the nature of the resulting curve has been the subject of much mathematical study. The curve is called an elastica, and a pure elastica serves as the starting point for discussion here.[5] Typically, the term elastica refers to the deflected shape of a strip having uniform cross-sectional geometry. A finite element method (FEM) of analysis of the strain energy can be utilized to model the shape of the strip under an axial load. Minimizing the summation of strain energies leads to a FEM solution for an elastica and defines the shape of the strip under a particular axial load and displacement. The novel approach presented here is the use of FEM minimization to guide the selection of the cross sectional geometry of individual elements enabling the tailoring of the shape upon buckling. Selecting unique thicknesses per node results in a tapered thickness, which leads to an elastica of parabolic shape. Selecting varying material properties per node leads to the same result.

Another major departure from previous work is to consider utilizing an inexpensively manufactured surface finish to minimize cost. In essence, the surface quality of a solar concentrator needs to be good enough to direct sunlight into the aperture of a receiver or to impinge on the surface providing process heating. Grinding and polishing of large components akin to telescope manufacturing is time consuming and prohibitively expensive. Other solutions to developing a mirror finish have also been explored, including the casting of polymer matrix composites onto a polished mold, the application of leveling coatings, diamond turning, and the use of mirror-like appliqués.[1,6-7] Aluminum strips diamond turned to a mirror finish, tapered in thickness, and placed under axial load until buckled to the shape of a parabola may prove to be an inexpensive means of creating a light weight solar concentrator.

To evaluate the problems and prospects of utilizing elasticas as a building block for strip concentrators, a research effort was initiated to model the elastica and its reflections. Results from the modeling effort were utilized to design and build an elastica strip concentrator. The concentrator comprises eleven tapered elastica strips and a frame to achieve buckling under compression. On-sun testing of the concentrator was utilized to evaluate performance.

II. Modeling and On-Sun Testing

The premise for modeling is based on calculating the buckled strip's curvature utilizing an energy minimization method. Taking into account each element's geometry and the applied load, the sum of all element strain energies is calculated. Excel© contains a tool called "Solver" that works to optimize the value in a selected cell based on varying the values in other specified cells. In this case, the cell that is optimized contains the total value of all element strain energies. The cells that are varied contain the "x" and "y" displacements and tangent slopes of each node, subject to user-input constraints. The iterative process continues until an acceptably low value of total strain energy is reached.[8]

A. Numerical Study: A Single Strip 152 cm in Length

The spreadsheet allows the user to define the number of nodes for which displacements and tangent slopes are calculated. Twenty nodes were selected for this study. The spreadsheet accepts input for the strip's geometry, including unbuckled length, cross section dimensions, and modulus of elasticity, and allows the user to apply an axial load at the strip's endpoint. From displacements and tangent slopes, the following additional calculations were performed. First, the direction of reflected light from each node was calculated based on each node's tangent slope. Second, for each reflected light ray from half of the buckled strip, a point was charted at its intersection with a user-specified distance from the buckled strip's midpoint. Third, a series of calculations were performed to determine and chart the distribution of reflected light intensity at the user-specified focal line. Each node is assumed to have a total reflected intensity area of one unit. The location of each ray impinging on the focal line was calculated, and the intensity for that node's reflected light contribution was accumulated.

Finally, the elastica shape was compared to a parabola superimposed on the data and optimized utilizing a least squares standard error of estimate analysis.[9]

The test case was run with the following reflective strip properties and loading conditions. The relaxed length of the strip was 152.4 cm, parsed into twenty elements, each 7.62 cm in length (in the x-direction). The cross section dimensions were 2.54 cm wide (in the y-direction) by 0.318 cm thick (in the z-direction). The modulus of elasticity selected was representative of aluminum, at 68.9 GPa. One endpoint was constrained against "x" and "y" translation and was free to rotate. The other endpoint was constrained against "y" translation and was free to rotate and translate in the "x" direction. The load was applied in the negative "x" direction at a value of 2.04 kg. The user-specified focal line was set at 247.9 cm from the buckled strip's midpoint.

A parabola was created for comparison, derived from Equation (1),

$$x = a + by + cy^2 \tag{1}$$

where a, b, and c were constants selected to minimize the least squares standard error of estimate between the as-modeled elastica and the parabola. The constants a, b, and c were 59.7, 0, and -0.00011, respectively.

Fig. 1 summarizes the uniformily thick strip displacement as-modeled, the parabola utilized for comparison, the parabola's focus, and the half-curve reflected rays. When the entire buckled stip is considered, Fig. 1 would also contain a mirror image of the half-curve reflected rays across the buckled strip midpoint. One reflected ray, the reflected ray from Node 5, passes through the focus of the parabola. Other reflected rays either pass the centerline before or after the focus. The innovation for tailoring the elastica to the shape of a parabola is tailoring the strain energies in the summation of strain energies, achieved by tailoring node thickness. Thickness is inversely proportional to strain energy and strain energy is proportional to bending moment. Increasing thickness decreases bending. Decreasing thickness increases bending.

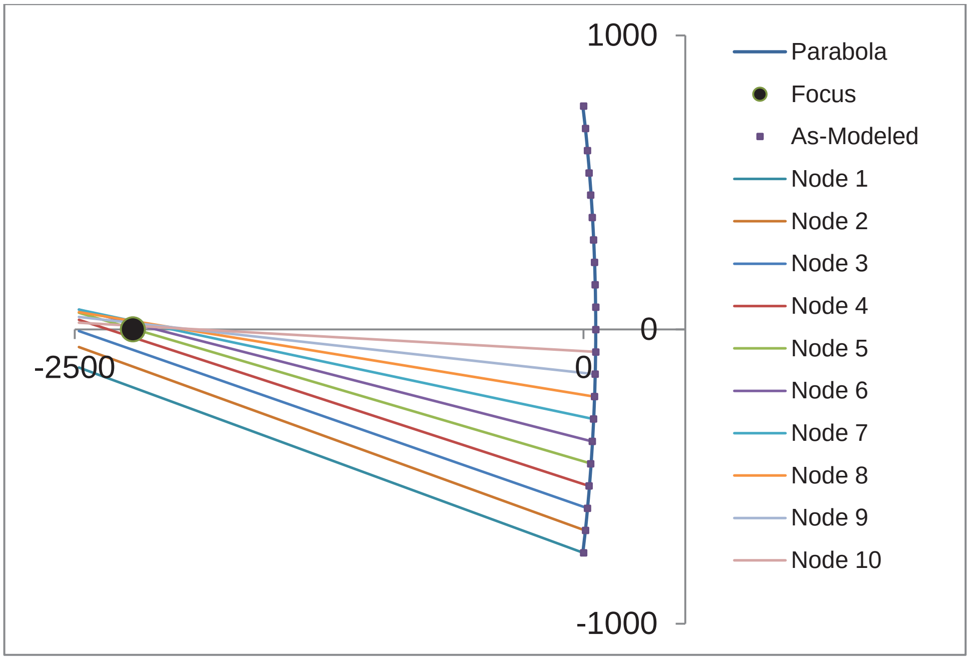

Fig. 1. Displacement and Half-Curve Reflected Rays Compared to a Parabola.

Fig. 2 is a summary of intensities of the strip and parabola from Fig. 1 at the user specified focal line. Unit intensity was allowed to impinge on the elastica and the distribution of reflected light was parsed into a histogram at the user-specified focal line. Each node had a total reflected intensity of one unit. The location of each ray impinging on the focal line was calculated, and the intensity for that node's reflected light contribution was accumulated. The same process was repeated for accumulating intensity from the parabola used for comparison. For both the elastica and the parabola, the total intensity under the curve is 20 units.

The lesson learned from this numerical study was that to convert the elastica in Fig. 1 to a shape approaching a parabola, the strain energies contributed from those nodes having reflected rays passing the centerline before the focus must be reduced because too much bending is occurring. The strain energies contributed from those nodes having reflected rays passing the centerline after the focus must be enhanced because insufficient bending is occurring. In other words, the central nodes having reflected rays passing the centerline before the focus must be thickened and the outer nodes having reflected rays passing the centerline after the focus must be thinned.

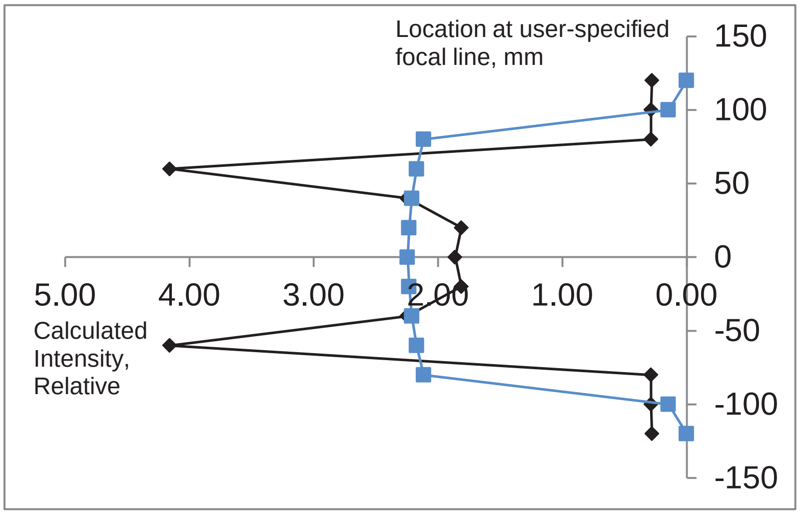

Fig. 2. Reflected Light Intensity Distribution for Elastica (black) and Parabola (blue).

B. Test Article Selected for Manufacture

The next step in the development process was to identify the size of an elastica concentrator to manufacture, one suitable for mounting on a sun tracker that was available from another project, along with a functional framework capable of holding a copper calorimeter at the focus to evaluate performance. Though diamond turning would have been ideal for the reflective surface, the choice was made to utilize aluminum stock manufactured with a number 8 reflective finish. This compromise was brought about by limited funding for elastica concentrator procurement. The size selected for the concentrator was 33 cm x 34 cm, made from eleven elasticas each 2.5 cm wide and 33 cm long as shown in Fig. 3. Stock aluminum of ample thickness was utilized in order to accommodate removal of material from the non-reflective side via careful milling operations.

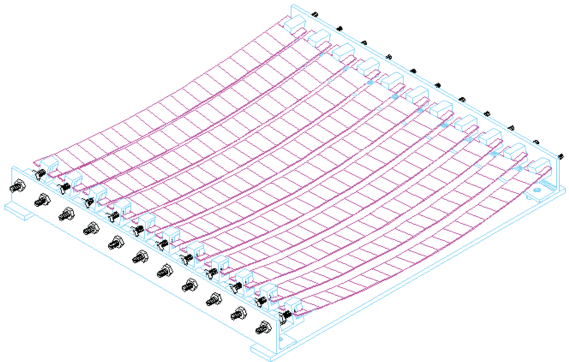

Fig. 3. Artist’s Concept of the Elastica Array Concentrator.

The Excel model was exercised utilizing the lessons learned from the numerical study. The node pattern in Fig. 4 was generated and submitted for milling.

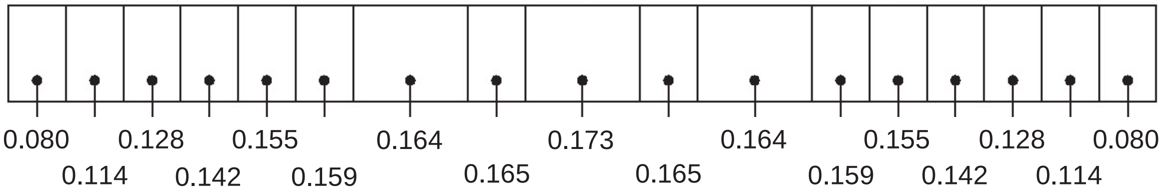

Fig. 4. Node Pattern and Thickness Values (in cm).

The tailored elastica strips were designed to form a parabolic shape under buckling. For this to occur, the strips must be loaded axially, and it was critical that the ends were free to rotate in the direction of buckling. If the ends were constrained to any significant degree, it would adversely affect the desired deflected shape. In addition, the strip must be maintained at a desired amount of lateral rotation relative to the overall strip assembly, necessary because as strips are located further from the assembly's center, they must be rotated so that the reflected light is directed back to a concentrated point above the assembly center.

As shown in Fig. 5., each strip's end was slotted with a width that was slightly larger than the width of the block to which it was mated. This helped prevent the sides of the slot from rubbing against the block, which would restrain the strip's free rotation. The inside face of the strip's slot fits into a slightly wider slot across the block. This method of assembly contained the strip between the end blocks and forced it to buckle in the desired direction, while maximizing its ability to buckle without constraints.

Also note that the end blocks were free to rotate laterally relative to the strip and were fastened securely at a degree of rotation that focused the reflected light as desired. The blocks were also adjustable along the direction of buckling, so that the desired amount of deflection could be imposed on the strip. Locking nuts anchored the buckling setting.

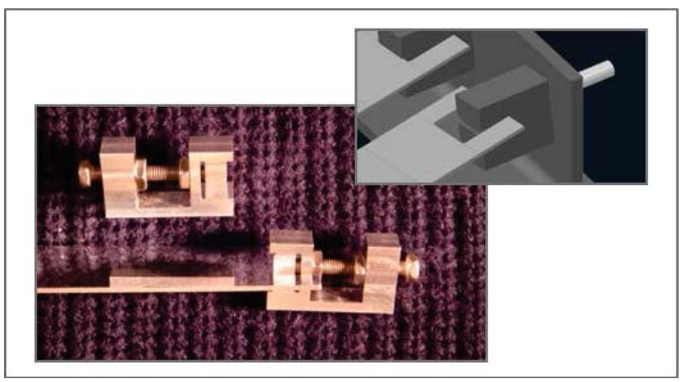

Fig. 5. Strip End Detail

The Excel© spreadsheet that was utilized to design the elastica also provides theoretical values of deflection and reflection based on the model dimensions. In general, the fabrication process utilized to make the elasticas resulted in dimensions that were slightly above their specified values. Table 1 summarizes the as-manufactured thicknesses from one strip. The fabricated strip was 0.005 to 0.015 cm thicker than intended, which in turn caused a slightly different deflection. Table 1 also shows that applying a slightly greater axial force served to compensate for the errors in manufacturing. In the future, tighter tolerances could be achieved by selecting a milling facility with better equipment.

C. Predicted Performance

The plan for evaluating concentrator performance was based on measuring the energy captured by an insulated copper calorimeter divided by the energy impinging on the concentrator surface. Sun tracking was required in order to maintain optical alighnment with the sun over the time that calorimeter measurements were being made. Estimates of the various efficiencies throughout the system, multiplied together, generated an overall predicted efficiency value. The list of efficiencies includes:

- an estimate of the fraction of light successfully entering the calorimeter, i.e. the reflectivity and specularity of the #8 aluminum finish

- an estimate of the concentrator area not shadowed by the calorimeter
- an estimate of the energy lost from the calorimeter, via radiation out of the opening

For the first efficiency estimate, a Devices and Services Model 15R portable specular reflectometer was utilized to estimate the reflectivity and specularity of the #8 aluminum finish. The instrument was equipped with a red light emitting diode peaking at 660 nm and a variable receiver aperture with settings of 15, 25, and 46 milliradians. Though the red light from the diode is only a fraction of the solar spectrum, it was considered here, for estimating purposes, to be a fair approximation to the solar spectrum. The largest cone angle of the instrument most closely matched the geometry of the distance from the elastica surface to the calorimeter and the calorimeter diameter. A value of 0.74 was obtained from a sample of the #8 aluminum finish, as summarized in Table 2.

Table 1. Comparison of Design and As-Manufactured Dimensions.

			Design with 6.35 kg	Actual with 7.41 kg
Element	Design Thickness (cm)	Actual Thickness (cm)	y-Deflection (cm)	y-Deflection (cm)
1	0.080	0.086	0	0
2	0.114	0.124	0.26	0.26
3	0.128	0.137	0.50	0.49
4	0.142	0.152	0.71	0.70
5	0.155	0.163	0.89	0.88
6	0.159	0.168	1.04	1.04
7	0.164	0.170	1.17	1.17
8	0.164	0.170	1.27	1.27
9	0.165	0.170	1.34	1.34
10	0.173	0.178	1.39	1.38
11	0.173	0.178	1.40	1.39
12	0.165	0.180	1.39	1.38
13	0.164	0.170	1.34	1.33
14	0.164	0.170	1.27	1.26
15	0.159	0.168	1.17	1.15
16	0.155	0.168	1.04	1.02
17	0.142	0.152	0.89	0.87
18	0.128	0.137	0.71	0.69
19	0.114	0.124	0.50	0.48
20	0.080	0.094	0.26	0.25

Concentrator area not shadowed by the calorimeter was estimated by calculating the shadowing based on calorimeter outside diameter (and a small contribution from the I-beam holding the calorimeter in place) divided by the rectanglular area of the elastica array (less the openings between the aluminum strips), subtracted from unity. A value of 0.75 was obtained, as summarized in Table 2.

The Steffan-Boltzman equation was used to estimate the energy lost from the calorimeter via radiation, utilizing both room temperature and the maximum temperature observed, 100 °C. The loss due to radiation was estimated to be 1.3 watts, yielding an efficiency factor of 0.98 based on the available power impinging on the concentrator, as summarized in Table 2.

Multiplying the efficiency estimates suggests that the accumulated efficiency of the as-built system ought to be about 54%.

Table 2. Estimated Efficiency Values

Item	Efficiency estimate
Reflectivity & Specularity of the #8 aluminum finish	0.74
Shadowing of the concentrator	0.75
Radiant heat losses	0.98
Accumulated efficiency:	0.54

D. On-sun Testing with a Calorimeter, Observed Performance.

The sky conditions during the late September test were clear, with some upper level haze. The test took place around 2 pm. Care was taken ahead of time to adjust first the angle and then the compression of the individual mirrors while on sun utilizing each mirror's adjustable end block, with the goal of focusing each strip of light onto a single band at the opening of the calorimeter. Adjusting the angle was done with ease. It was also easy to see the light converge onto the single band upon adjusting the compression. However, two concerns developed from the process, first, the range available for compression from the end block design was limited and second, setting the locking nuts was a challenge.

Light that successfully entered the calorimeter was absorbed by the black inner walls and heated a copper cylinder of known mass. The sun tracking hardware also included a calibrated solar cell at the base of a diffuse light-blocking rectangular tube, thermocouples, and a data logger. To measure efficiency, the solar concentrator was placed on sun, the tracking mechanism was activated, and the time-varying temperature of the copper along with its mass was used to provide a measure of the solar energy collected by the system. The calibrated solar cell provided a measure of incident solar energy impinging on the system. A photograph of the elastica concentrator mounted on the sun tracking hardware is shown in Figure 6.

Figure 7 shows the test results. The average solar flux was 773 W/m^2, the total solar power on the mirror (minus the shadowed area) was 44.9 W, the power to the calorimeter was 19.7 W, and the overall measured efficiency was 43.8%. Operationally, the calorimeter heated up uniformly, as indicated by a similar rise and a close range in temperature values for the eight thermocouples monitoring the calorimeter.

The discrepancy between predicted and observed performance is best described by the observation that not all of the light reflected from the mirrors entered the calorimeter. Figure 8 shows that about 10% of the light impinged on the outside of the calorimeter. Use of a diamond turned finish should improve efficiency by reducing the amount of light impinging on the outside of the calorimeter.

Fig. 6. Elastica Concentrator on Sun-Tracking Hardware.

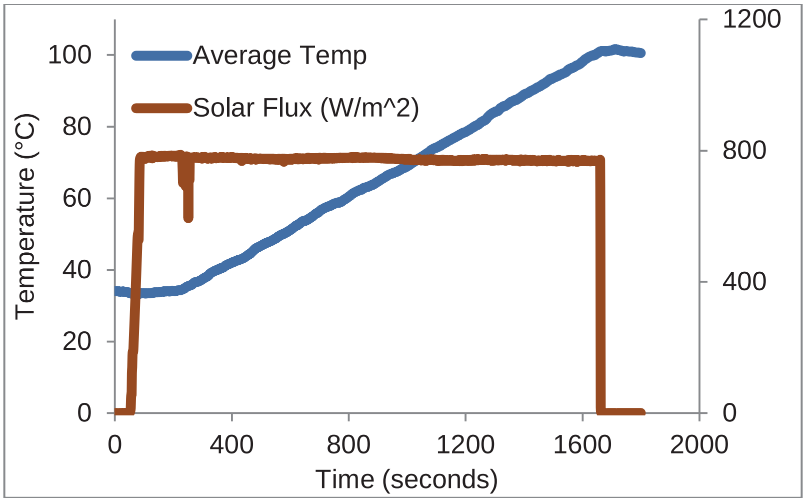

Fig. 7 Average Thermocouple Temperatures and Measured Solar Flux

Fig. 8. The copper calorimeter, instrumented with thermocouples, enclosed in insulation, and held in place by a cover, under concentrated sunlight.

III.   Future Work

To minimize mass, a novel variation on the elastica concentrator would use piano wire to aid in compression. Rather than housing and compressing the elasticas in a rectangular frame, future work may be linked to evaluating the elastica performance while held in compression by piano wire. In an array of wires, each wire would serve as a means to place an individual elastica under compression and to rotate the elastica in order to direct the sunlight to a focus. In the aggregate the bulky rectangular frame could be replaced with a frame to hold the wires, one of considerably less mass.

IV.   Conclusion

This paper addresses a three phase effort to develop an elastica concentrator. First, the notion of utilizing a FEM analysis of the strain energy to predict elastica shape was introduced, emphasizing that minimizing the summation of strain energies leads to a solution that defines the shape of a strip under a particular axial load and displacement. Second, a test article was manufactured by exercising our model in order to identify cross sectional thickness of numerous nodes, and removing the prescribed amount of material from the back side of eleven aluminum strips having a #8 finish. The milling process left a slight excess of material, however, increasing axial load compensated for the milling error. Finally, the elastica concentrator performance was evaluated on-sun using a sun tracker equipped with an existing calorimeter and calibrated solar cell. Predicted performance, based on reflectivity, specularity, shadowing, and energy loss suggested an efficiency of about 54%. During on-sun testing, the concentrator heated the calorimeter uniformly and exhibited an efficiency of about 44%. More light was lost to the outside of the calorimeter than anticipated. Minimizing shadowing from the device at the focus undergoing process heating and improving the finish of the elastica surface should improve performance.

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