

Acoustics of Jet Surface Interaction Scrubbing Noise

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Motivation

Interaction of jet exhaust with nearby solid surfaces:

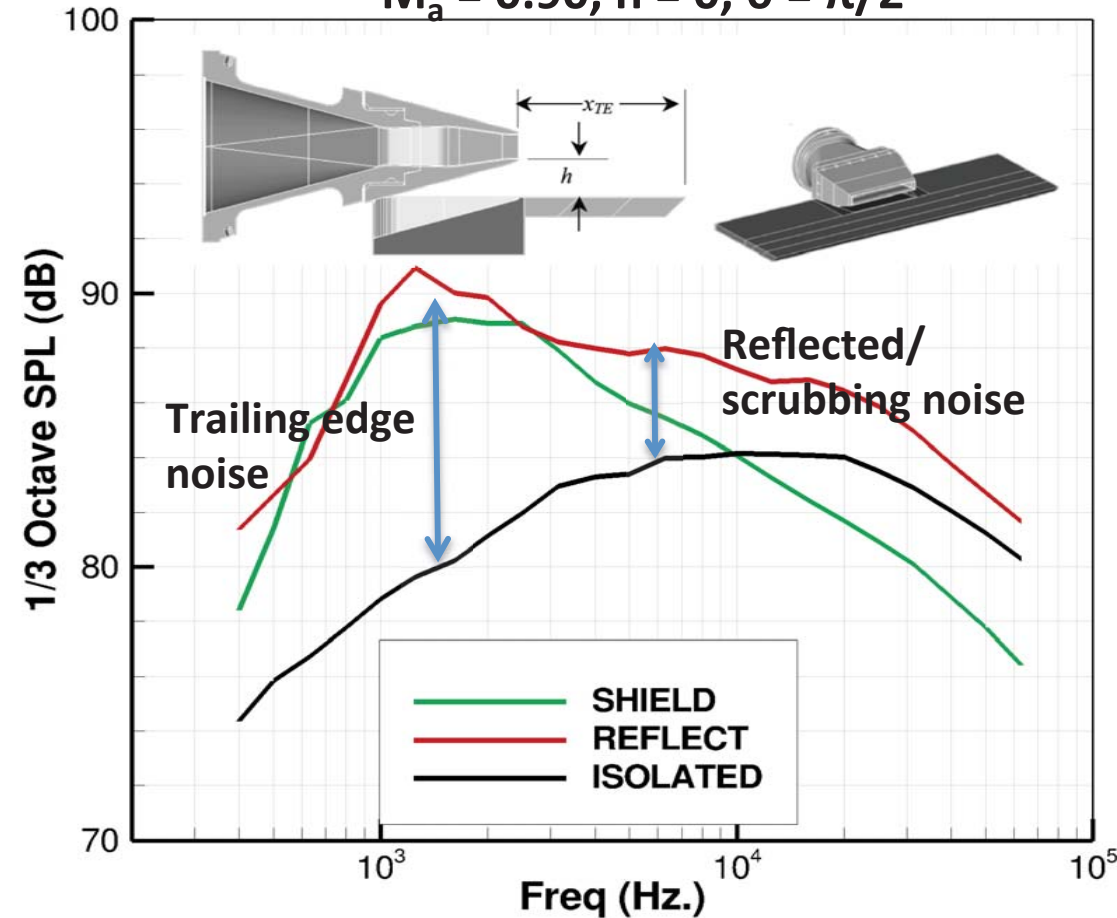
- Hybrid Wing Body (HWB) concepts
- High aspect ratio rectangular exhaust with extended beveled surfaces
- Over the wing engine mount
- Nearby structural components could provide noise shielding
- They could also produce new sources of sound



Measurements*

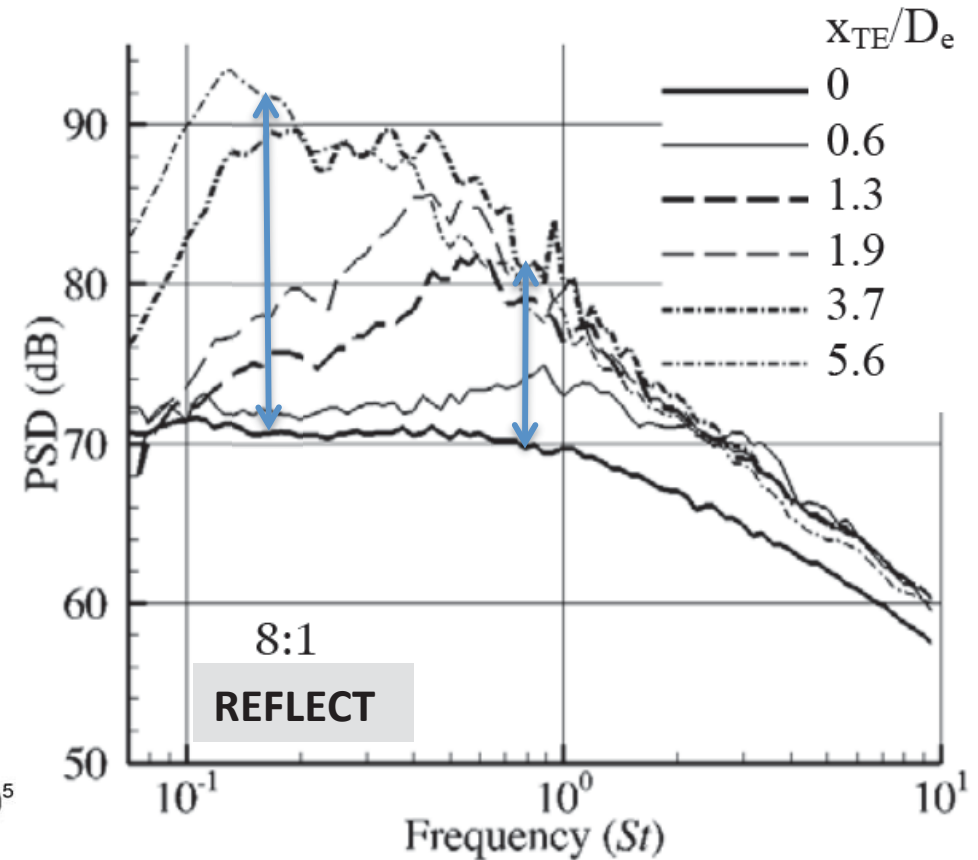
Rectangular Jet (AR = 8)

$M_a = 0.90, h = 0, \theta = \pi/2$



$X_{TE} / D_e = 5.6, D_e = 2.14''$

$M_a = 0.50, h/D_e = 0.45, \theta = \pi/2$

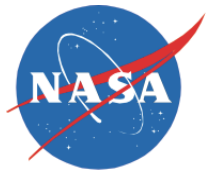


* James Bridges, AIAA-2014-0876



Outline

- Governing Equations
- Propagation GF Applicable to High-AR Rectangular Jet
- A Parametric Study of the GF
 - Frequency
 - Temperature
 - Source Location
 - Directivity/ Flight Effect
 - Wall Impedance Effect
 - Reflected / Isolated Jet



Scrubbing Noise

- NS Equations \rightarrow (Mean Flow + Linear Eqs. for Fluctuations)
- Locally Parallel Mean Flow
- Compressible
- Constant Static Pressure
- Ideal Gas Law

Variable density inhomogeneous PB eq.

$$L\pi' = \Gamma, \quad \pi' \simeq \frac{p'(\vec{x}, t)}{\gamma \bar{p}}$$
$$L \equiv D \left(D^2 - \frac{\partial}{\partial x_j} \left(c^2 \frac{\partial}{\partial x_j} \right) \right) + 2c^2 \frac{\partial U}{\partial x_j} \frac{\partial^2}{\partial x_1 \partial x_j}, \quad D \equiv \frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}$$

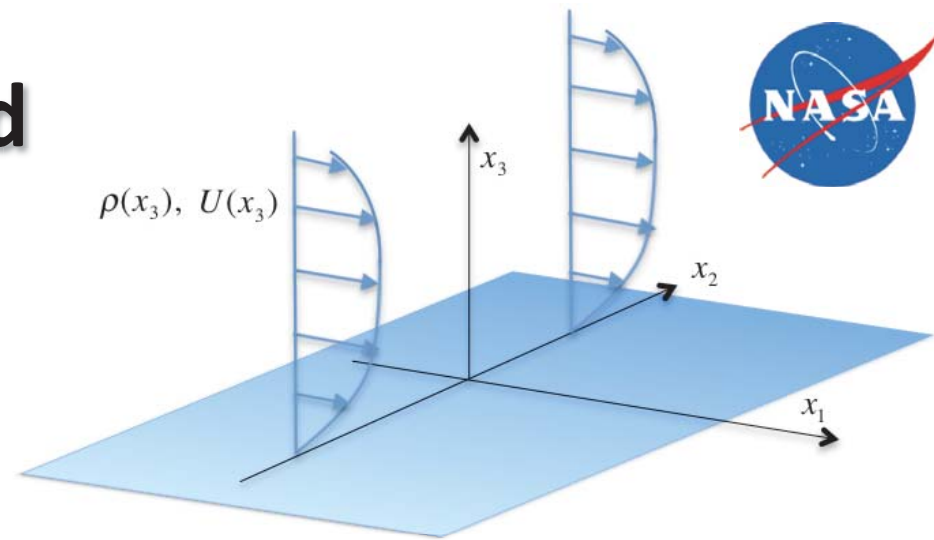
- Source term Γ is defined according to the generalized Acoustic Analogy, (Goldstein 2010)

Green's Function Method



$$\pi'(\vec{x}, t) = \int_{\vec{y}} \int_{\tau} G(\vec{x}, t; \vec{y}, \tau) \Gamma(\vec{y}, \tau) d\tau d\vec{y}$$

$$LG(\vec{x}, t; \vec{y}, \tau) = \delta(\vec{x} - \vec{y}) \delta(t - \tau)$$



- **Wetted side of the plate only**
(scattered noise component discussed by Goldstein *et al*, 2013)

Transform: $(x_1, x_2, t) \rightarrow (k_1, k_2, \omega)$

$$G(\vec{x}, t; \vec{y}, \tau) \rightarrow \hat{G}(\vec{k}_t, x_3; y_3, \omega) \quad \vec{k}_t \equiv (k_1, k_2)$$

$$\frac{\partial^2 \hat{G}}{\partial x_3^2} + \left(\frac{(c^2)'}{c^2} - \frac{2k_1 U'}{-\omega + k_1 U} \right) \frac{\partial \hat{G}}{\partial x_3} + \left(\frac{(-\omega + k_1 U)^2}{c^2} - k_1^2 - k_2^2 \right) \hat{G} = \frac{i}{(2\pi)^3} \frac{\delta(x_3 - y_3)}{c^2 (-\omega + k_1 U)}$$

- **Far-field spectral density**

$$\overline{p^2}(\vec{x}, \omega) = \int_{\vec{y}} \int_{\vec{\xi}} \int_{\tau=-\infty}^{\infty} \mathbf{G}^*(\vec{x}, \vec{y} - \vec{\xi}/2; \omega) \mathbf{G}(\vec{x}, \vec{y} + \vec{\xi}/2; \omega) q(\vec{y}, \vec{\xi}, \tau) e^{i\omega\tau} d\tau d\vec{\xi} d\vec{y}$$

GF Method (cont'd)

- The Eq is re-arranged into a self-adjoint 2nd order ODE
- Two linearly independent solutions $V_j(\vec{k}_t, x_3, \omega)$, $j = 1, 2$

$$V_j'' + f(\vec{k}_t, x_3, \omega)V_j = 0$$

$$\begin{array}{l|l}
 \text{IVP} & V_1(x_3) = 1 \\
 x_3 = 0 & \frac{\partial V_1(x_3)}{\partial x_3} - \psi V_1(x_3) = 0, \quad \psi(k_1, \omega, \bar{Z}) = \left(\frac{i\kappa_o}{\bar{Z}} \frac{c_\infty^2}{c^2(0)} + \frac{c'(0)}{c(0)} + \frac{k_1}{\omega} U'(0) \right) \\
 & \bar{Z}(\omega) = \frac{(\hat{p}' / \hat{v}'_3)}{\rho_\infty c_\infty} \\
 \text{BVP} & V_2(x_3) = 1, \quad x_3 = 0 \\
 & \frac{\partial V_2(x_3)}{\partial x_3} + i\chi_\infty V_2 = 0, \quad x_3 \rightarrow \infty
 \end{array}$$

- Radiation condition

$$\chi_\infty^2 \equiv (-\kappa_o + k_1 M_\infty)^2 - k_1^2 - k_2^2 > 0, \quad \kappa_o = \omega / c_o$$

GF Method (cont'd)



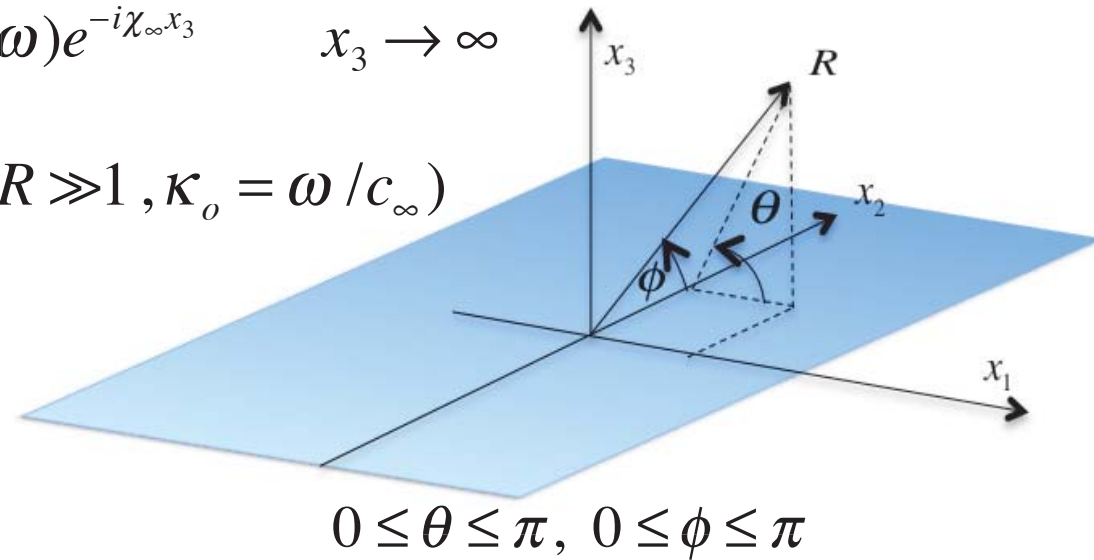
$$\mathbf{G}(\vec{x}, \vec{y}; \omega) = \frac{i}{(2\pi)^3} \frac{1}{c(y_3)c(x_3)} \int_{k_1} \int_{k_2} \frac{-\omega + k_1 U(x_3)}{(-\omega + k_1 U(y_3))^2} \frac{b_2 V_1(\vec{k}_t, y_3, \omega)}{W_o(\vec{k}_t, \omega, \vec{Z})} e^{i\Theta(\vec{k}_t, \vec{x}, \omega)} dk_1 dk_2$$

$$\Theta(\vec{k}_t, \vec{x}, \omega) = k_1(x_1 - y_1) + k_2(x_2 - y_2) - \chi_\infty x_3$$

$$V_2(\vec{k}_t, x_3, \omega) = b_2(\vec{k}_t, \omega) e^{-i\chi_\infty x_3} \quad x_3 \rightarrow \infty$$

- Stationary Phase solution ($\kappa_o R \gg 1$, $\kappa_o = \omega / c_\infty$)

$$\vec{k}_t^s = \kappa_o (\sin \phi^s \cos \theta^s, \cos \phi^s)$$



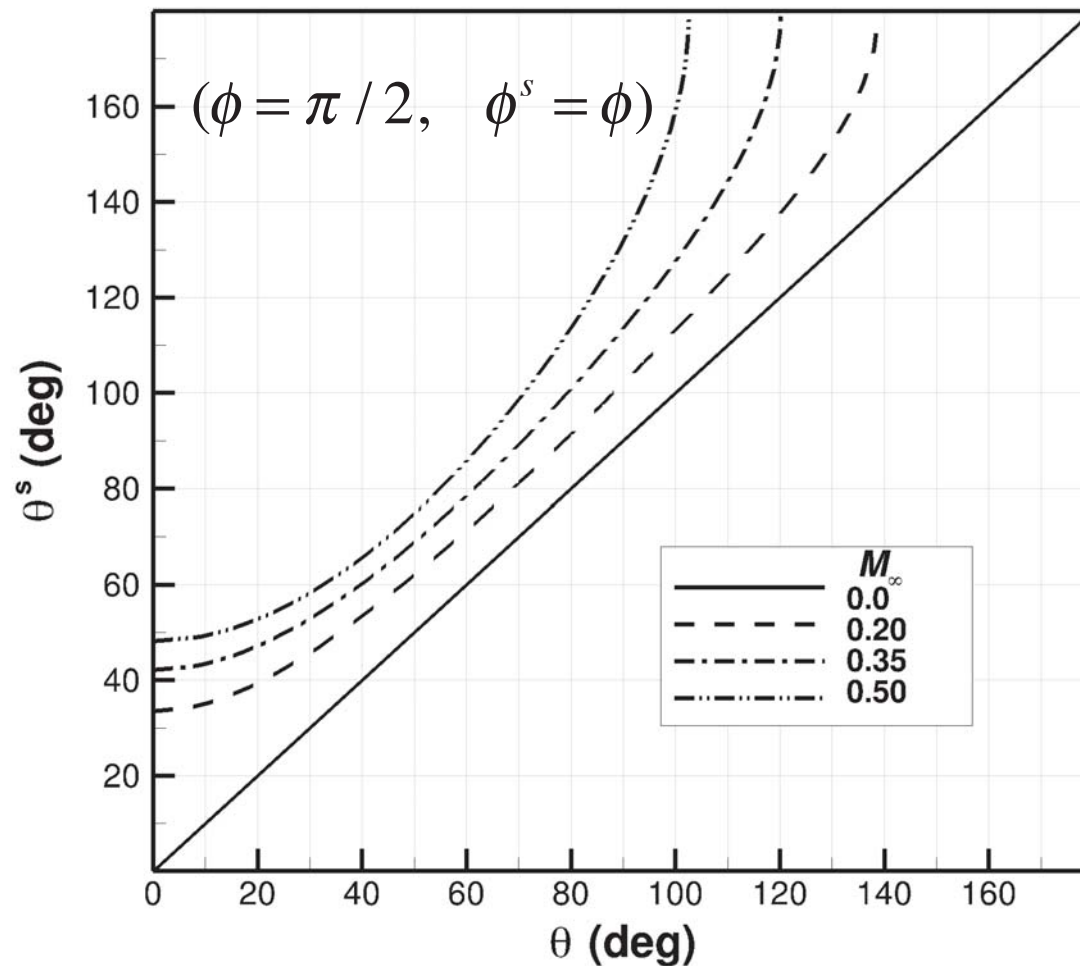
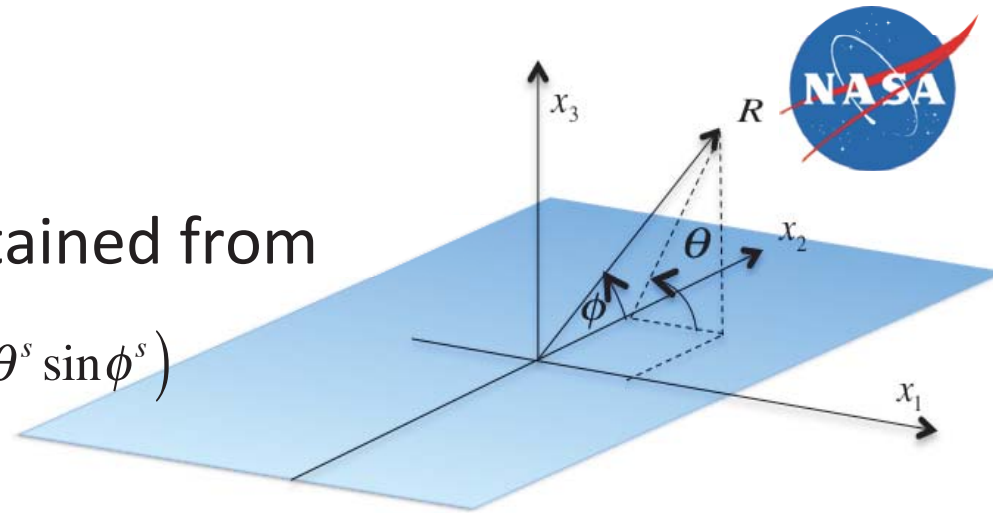
$$\mathbf{G}(\vec{x}, \vec{y}; \omega) \sim -i \frac{e^{i\Theta(\vec{k}_t^s, \vec{x}, \omega)}}{(2\pi)^3 R} \frac{\sin \theta^s \sin^2 \phi^s}{c_\infty^2 c(y_3)} \frac{b_2(\vec{k}_t^s, \omega) V_1(\vec{k}_t^s, y_3, \omega)}{W_o(\vec{k}_t^s, \omega, \vec{Z})} \frac{\Im}{\left(1 - \frac{U(y_3)}{c_\infty} \sin \phi^s \cos \theta^s\right)^2}$$

Stationary-Phase Point

- Stationary point angles are obtained from

$$\tan \theta = -S(\theta^s, \phi^s, M_\infty) / (M_\infty + (1 - M_\infty^2) \cos \theta^s \sin \phi^s)$$

$$\sin \theta \tan \phi = -S(\theta^s, \phi^s, M_\infty) / \cos \phi^s$$

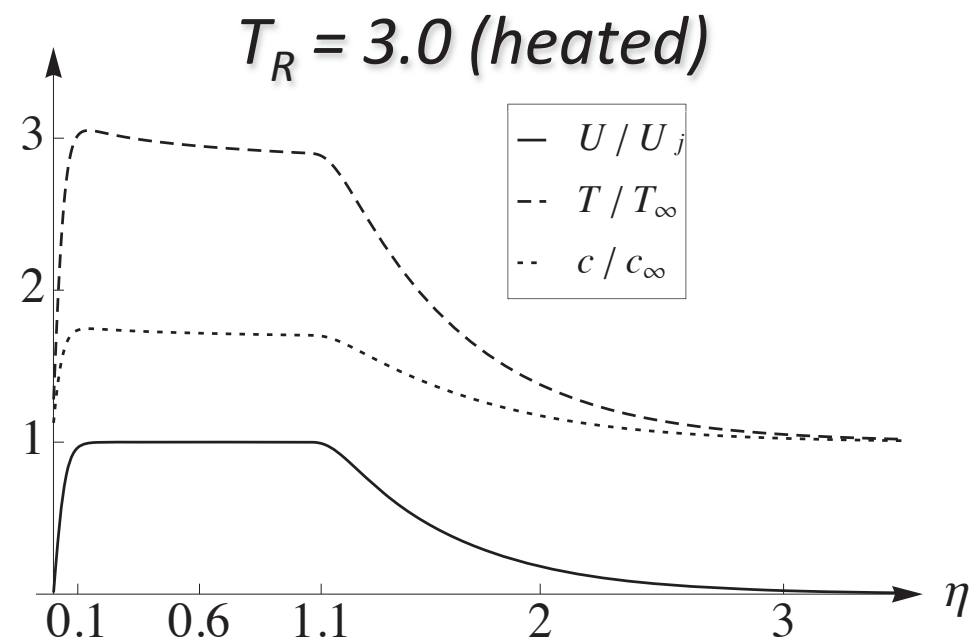
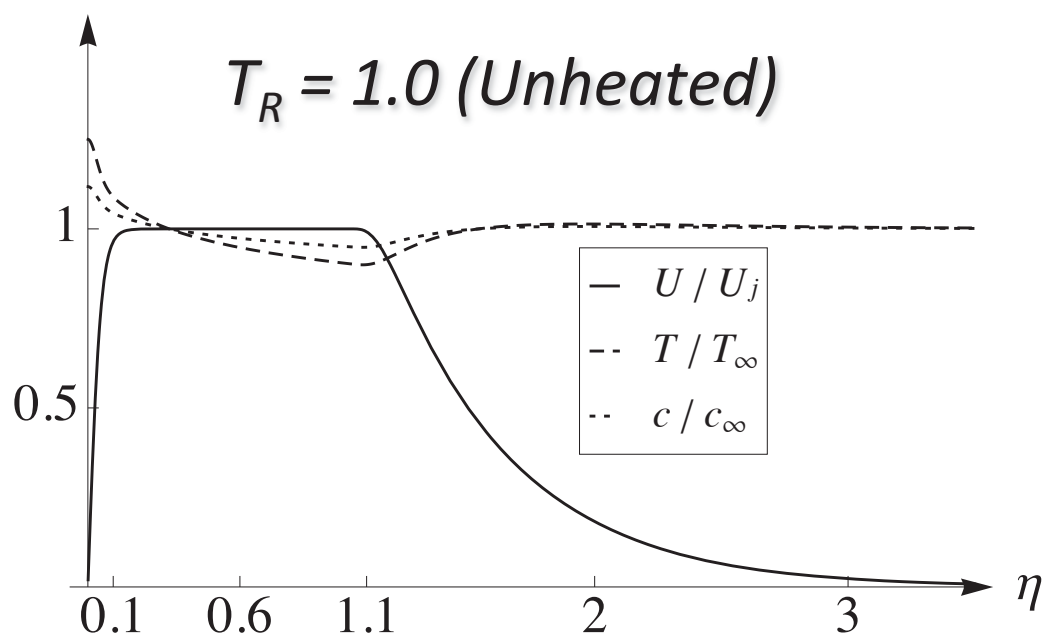
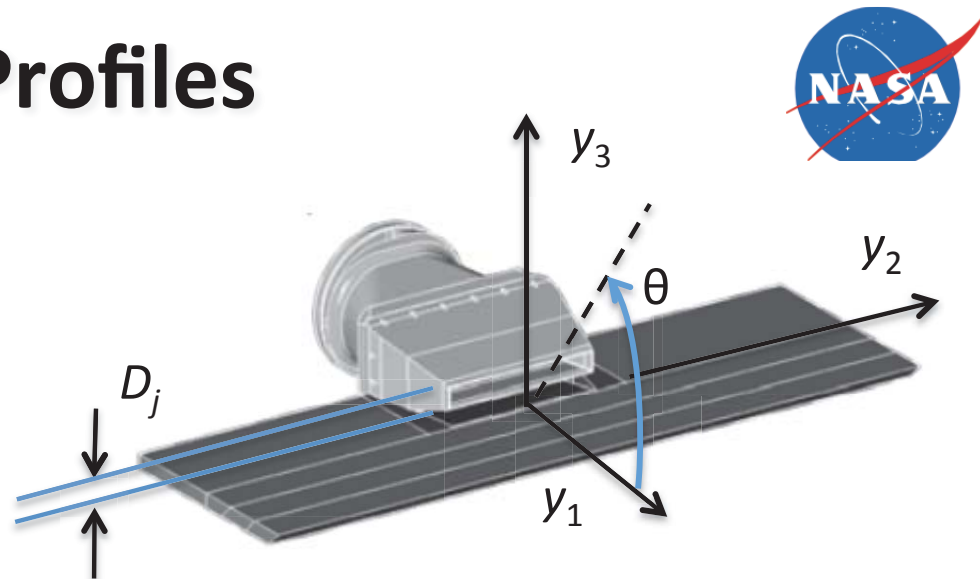


Temperature & Velocity Profiles

$$(M_\infty = 0, U_j / c_\infty = 0.90)$$

- Analytical profiles are selected for mean velocity & static temperature

$$\eta = y_3 / D_j$$

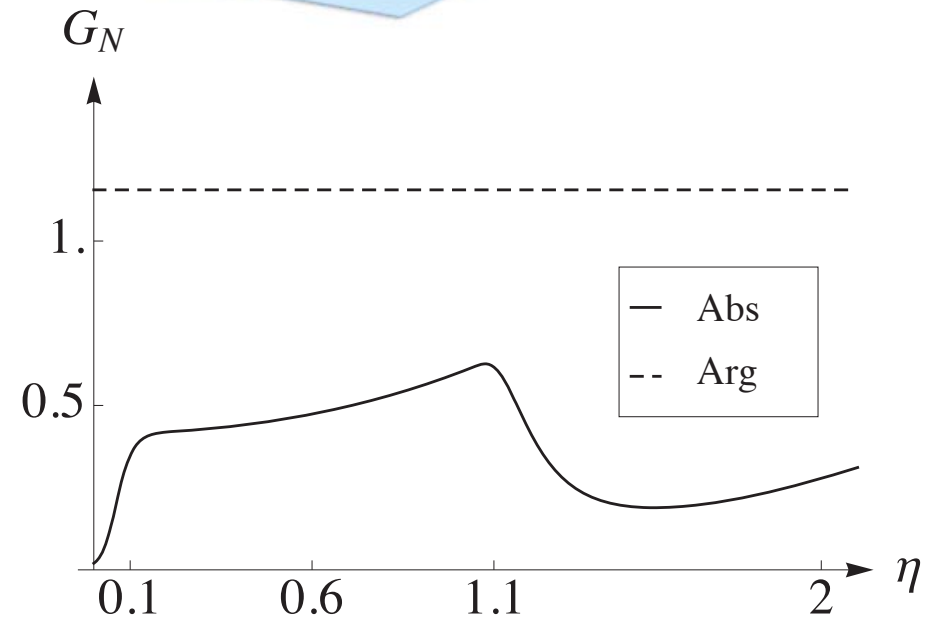
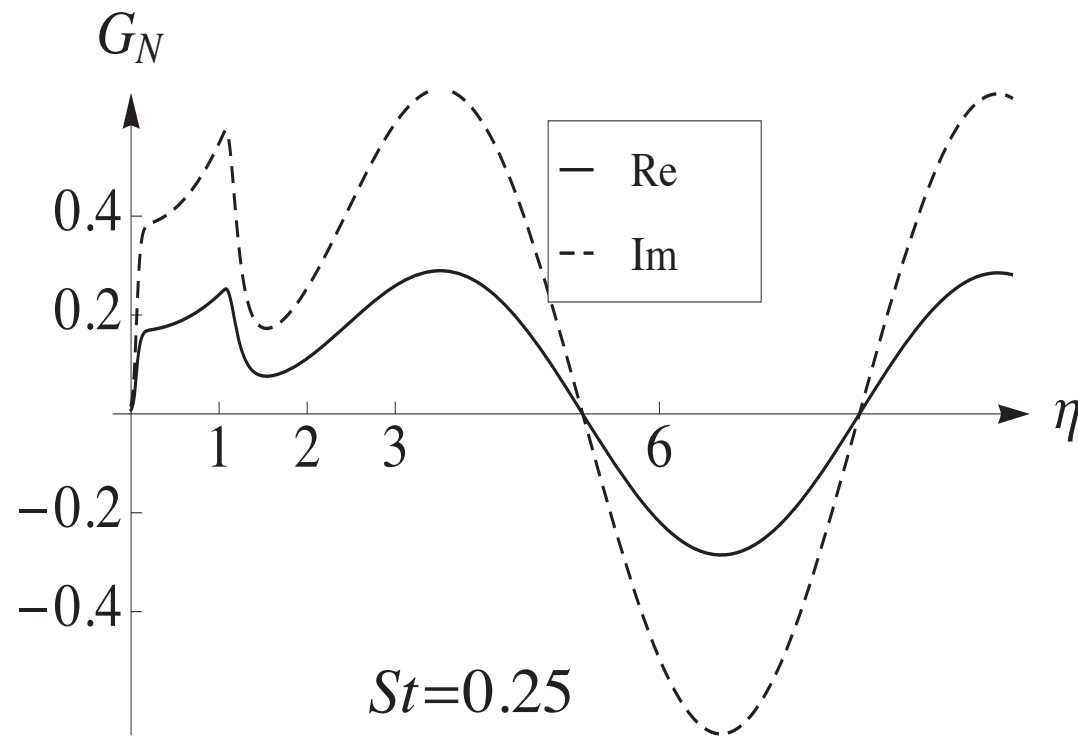
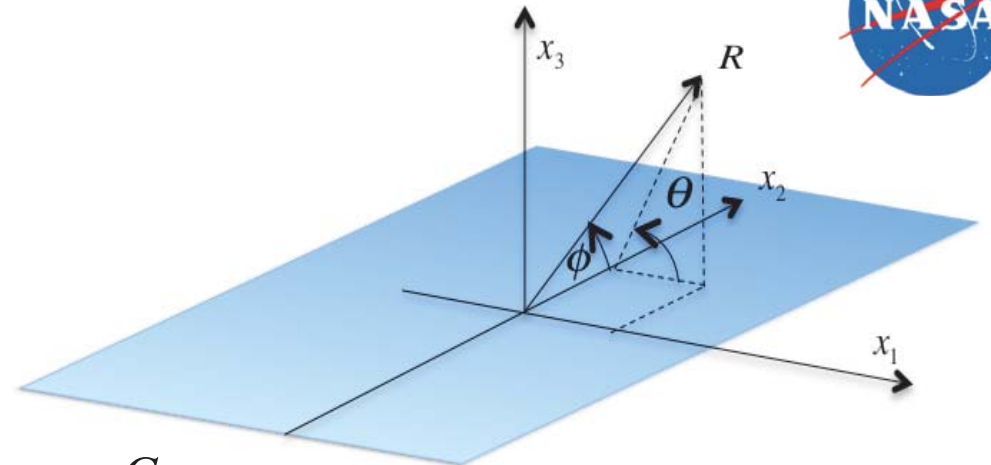


Numerical Results

$(\phi = \pi/2, \theta = \pi/4, U_j / c_\infty = 0.90, T_R = 3.0)$

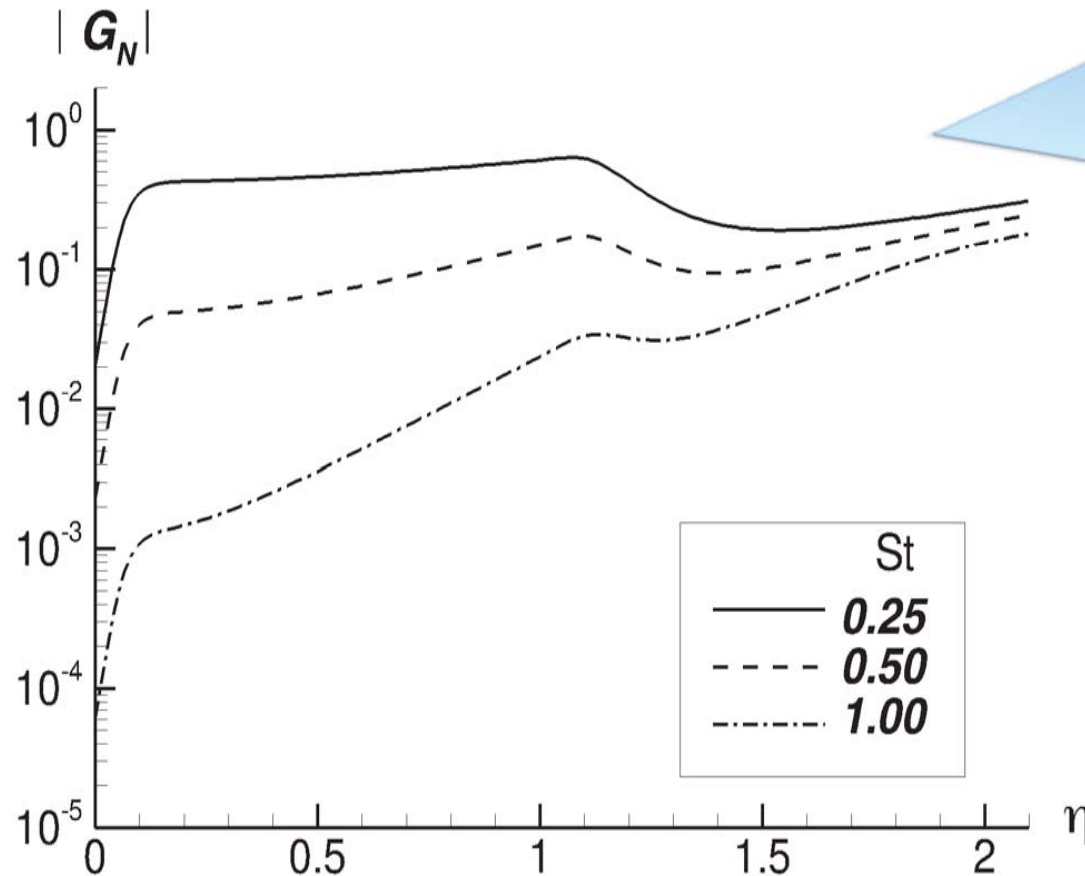
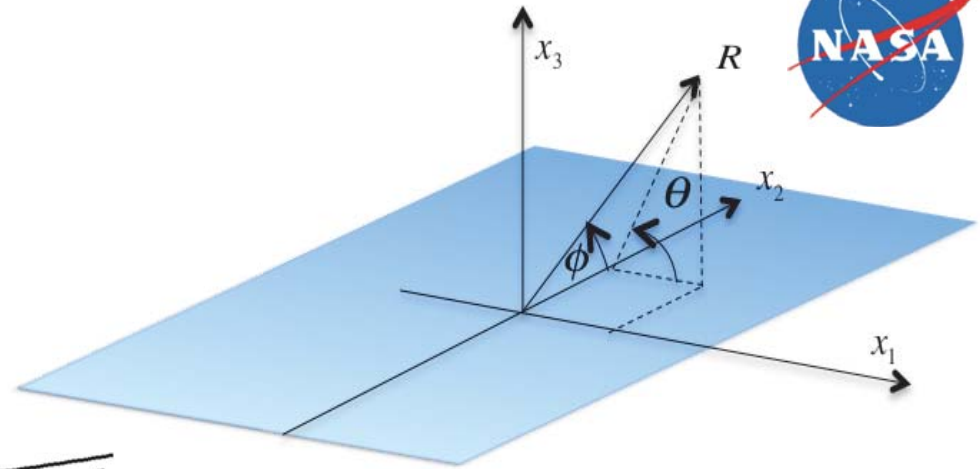
- Normalized GF $G_N \equiv \frac{\pi c_\infty^3 \mathbf{G}}{G_{FS}}$

$$St \equiv \frac{\omega D_j}{2\pi U_j}$$



Effect of Frequency

($\phi = \pi/2, \theta = \pi/4, U_j / c_\infty = 0.90, T_R = 3.0$)

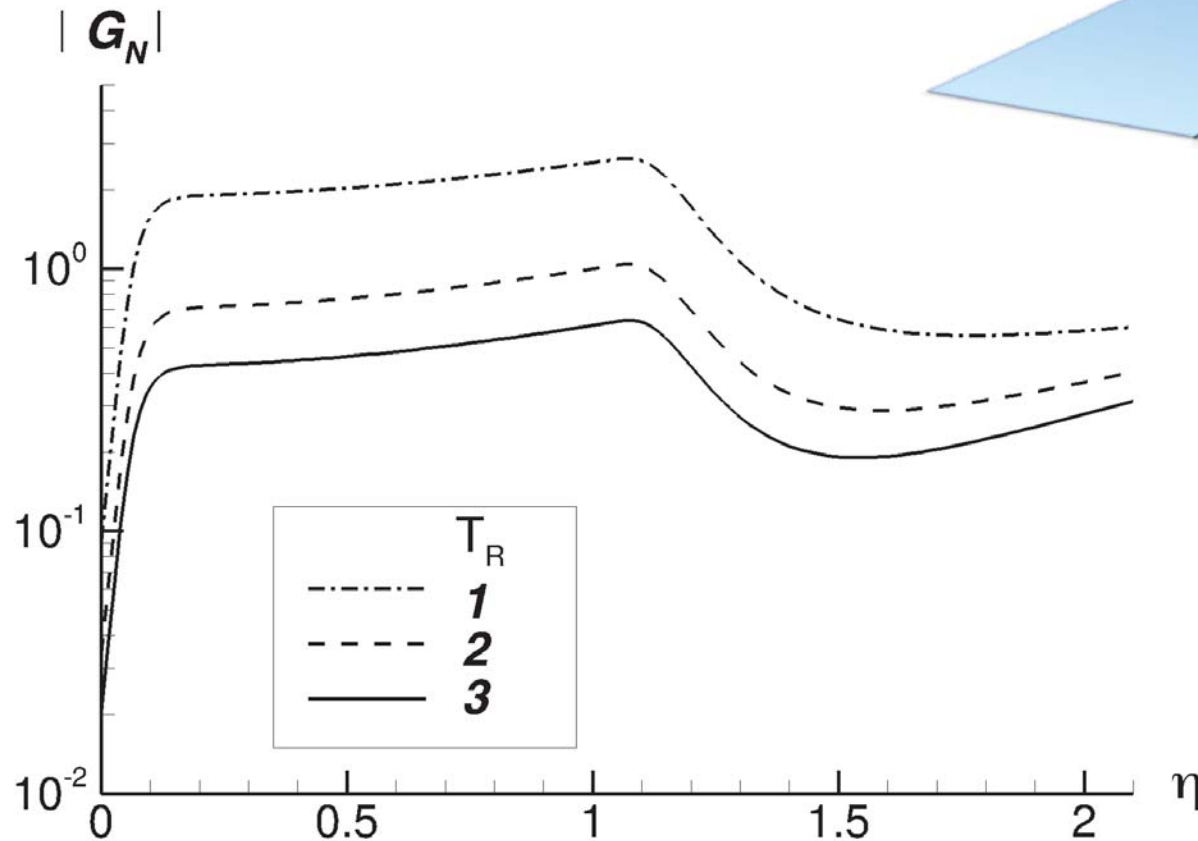
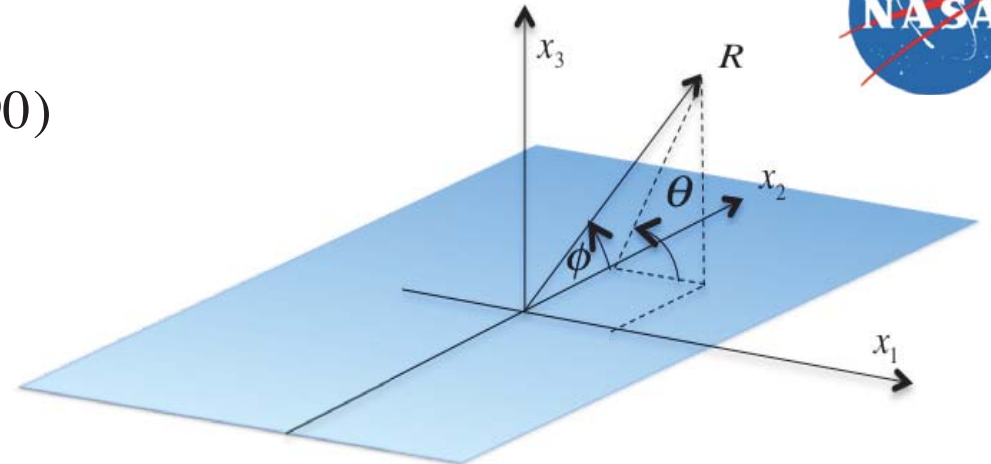


$$St \equiv \frac{\omega D_j}{2\pi U_j}$$

- GF amplitude decreases with increasing frequency
- In a uniform mean flow $G_N \sim 1/\omega$

Effect of Temperature

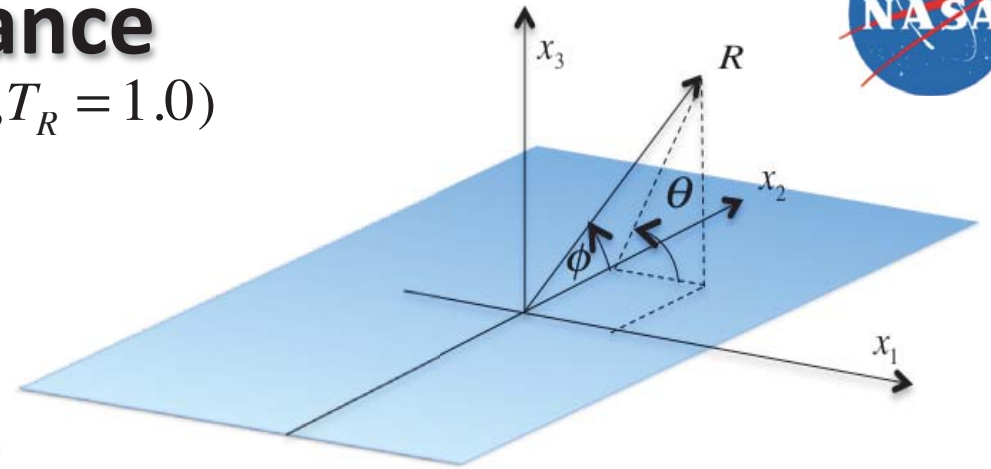
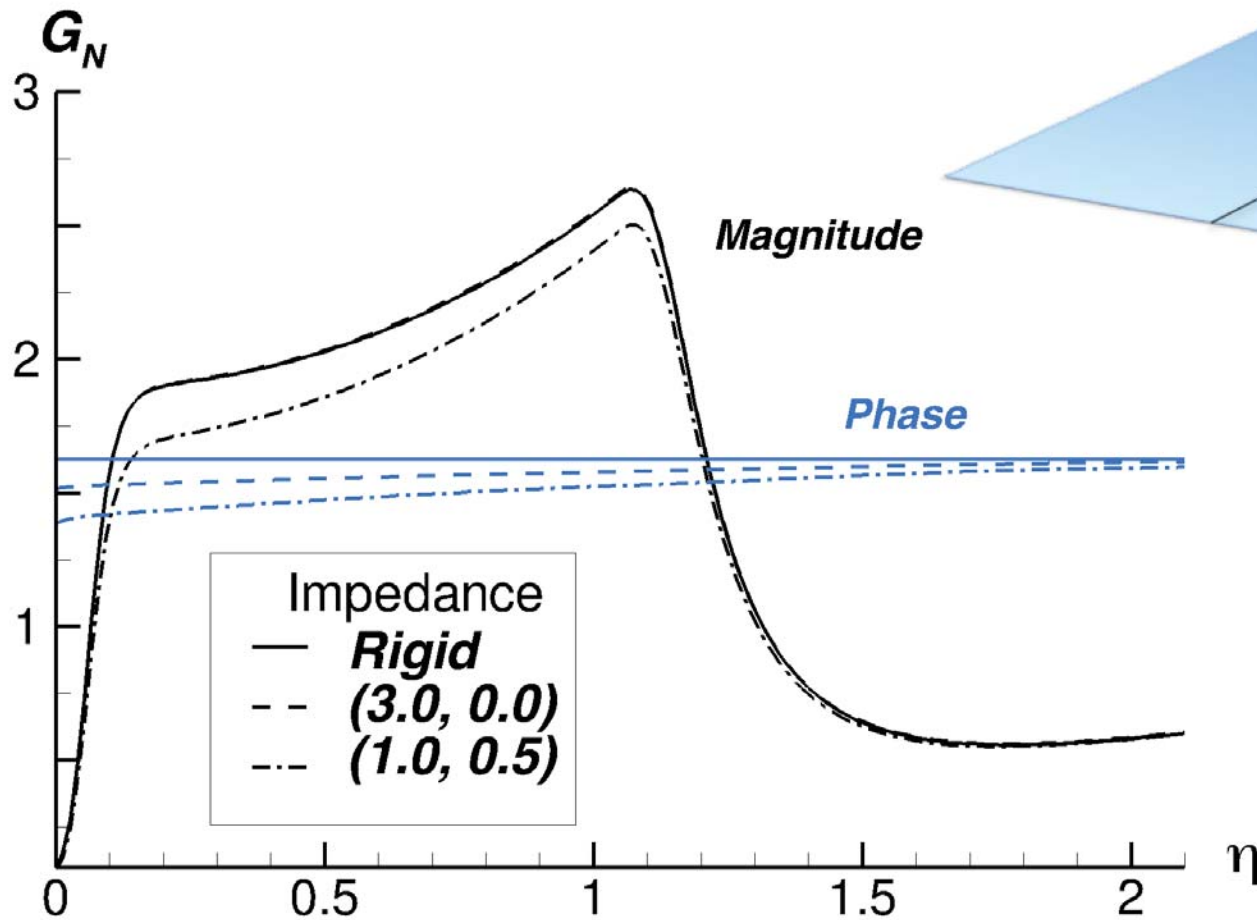
$(\phi = \pi/2, \theta = \pi/4, St_o = 0.25, U_j / c_\infty = 0.90)$



- GF amplitude decreases with increasing temperature

Effect of Surface Impedance

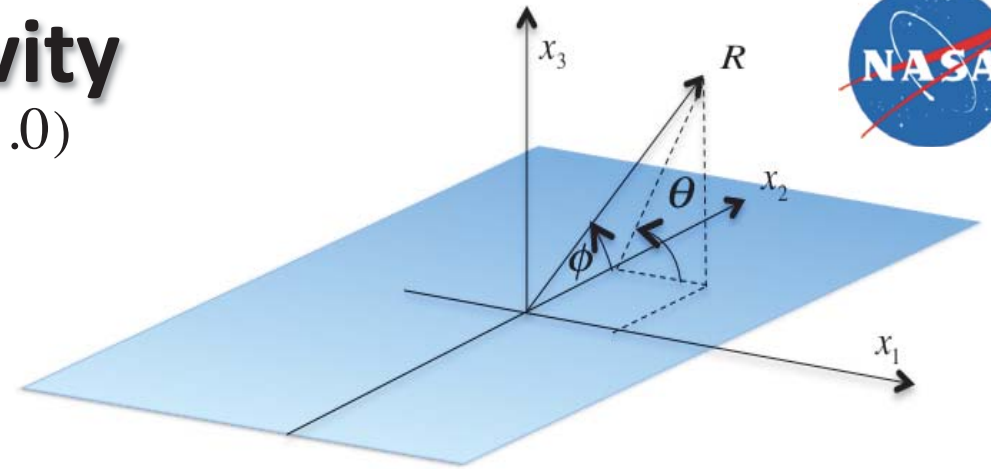
($\phi = \pi/2, \theta = \pi/4, St_o = 0.25, U_j / c_\infty = 0.90, T_R = 1.0$)



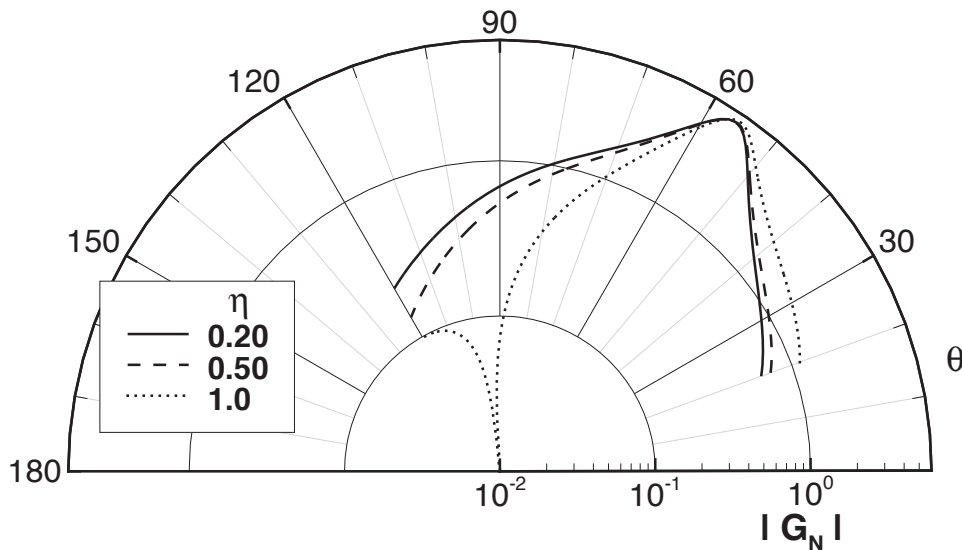
- Phase factor depends on source location for non-rigid surface

Flight Effect on Directivity

($\phi = \pi/2, St_o = 0.25, U_j / c_\infty = 0.90, T_R = 1.0$)

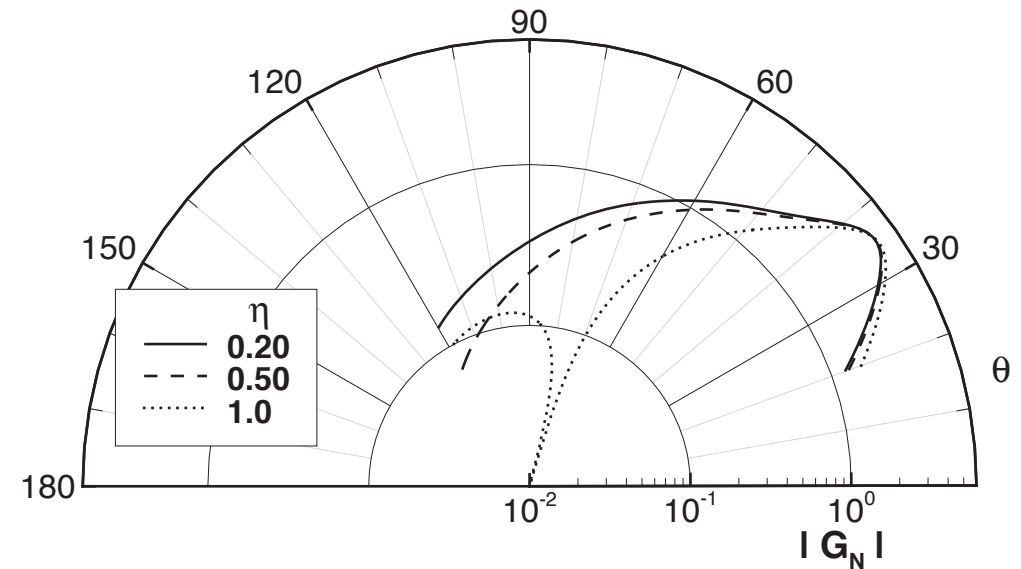


$M_\infty = 0.0$



$St = 0.25, U_j = 0.90, TR = 10, Ma = 0.0, Z = \text{Rigid}, \phi = \pi/2$

$M_\infty = 0.35$



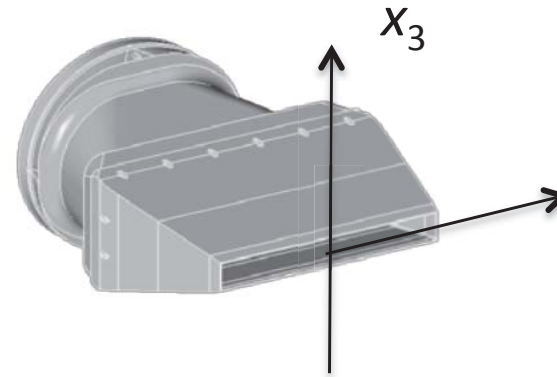
$St = 0.25, U_j = 0.90, TR = 1.0, Ma = 0.35, Z = \text{Rigid}, \phi = \pi/2$

- GF peaks at smaller down-stream angles as flight Mach number is increased

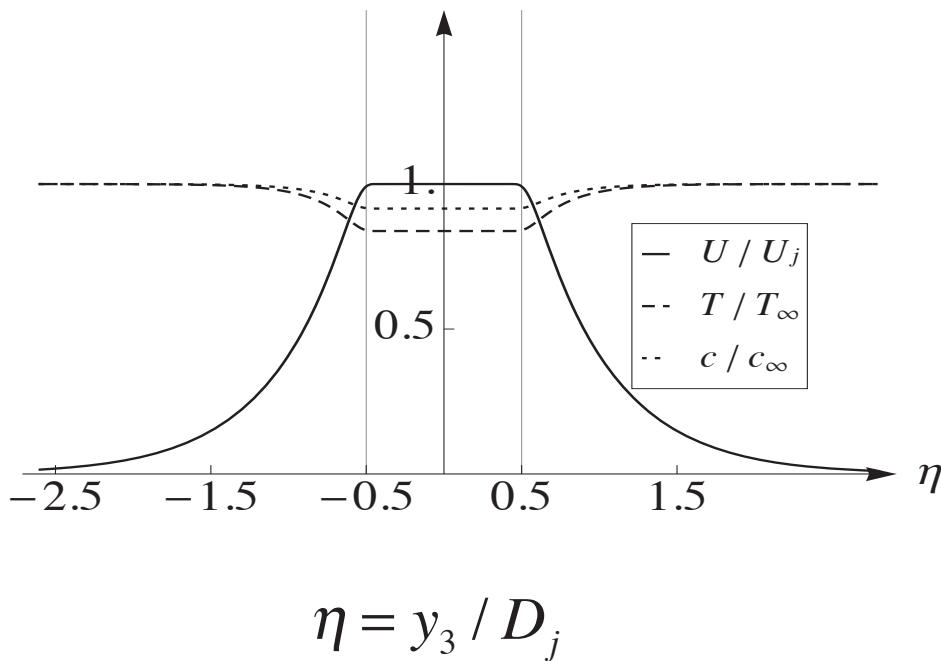
Isolated Rectangular Jet

$$V_j(x_3) = 1, \quad x_3 \rightarrow -\infty, \quad j = 1, 2$$

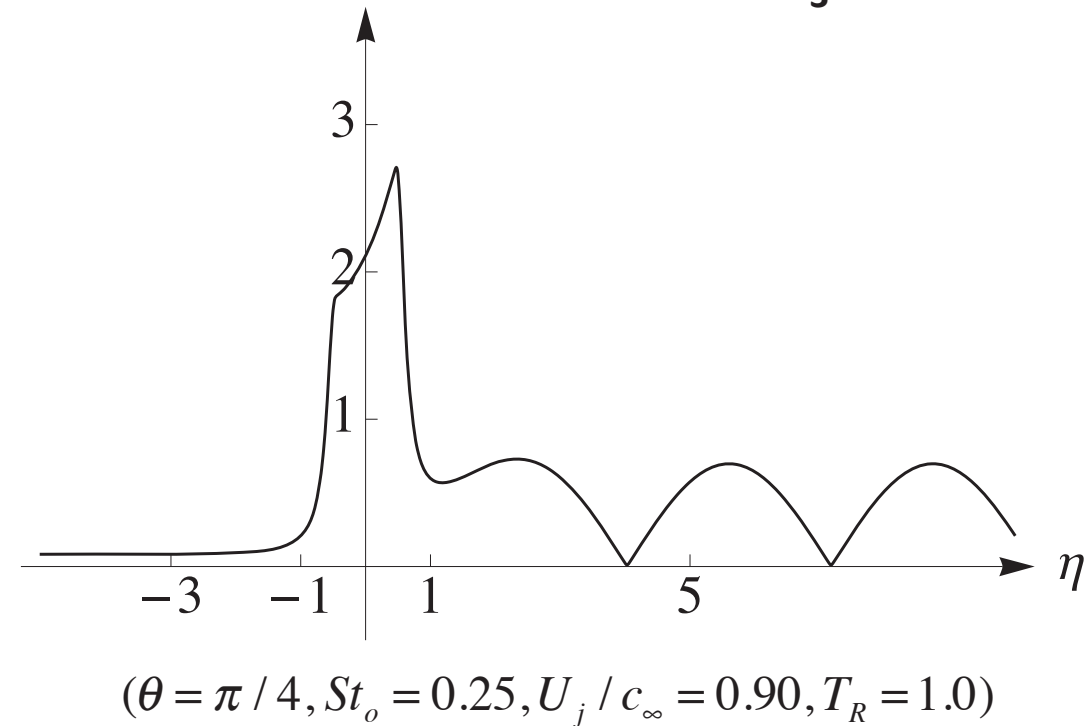
$$\frac{\partial V_j}{\partial x_3} - (-1)^j i \chi_\infty V_j = 0, \quad x_3 \rightarrow \begin{cases} -\infty, & j = 1 \\ +\infty, & j = 2 \end{cases}$$



Jet Profile – Unheated



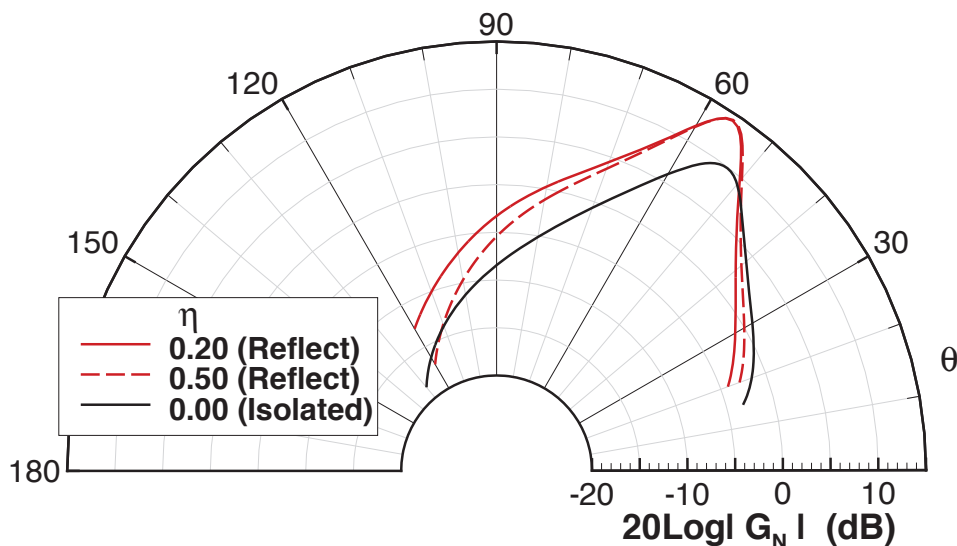
$|G_N|$ Observer at $x_3 \rightarrow +\infty$



Isolated vs. Reflected Jet

($M_a=0.90$, $T_R=1.0$)

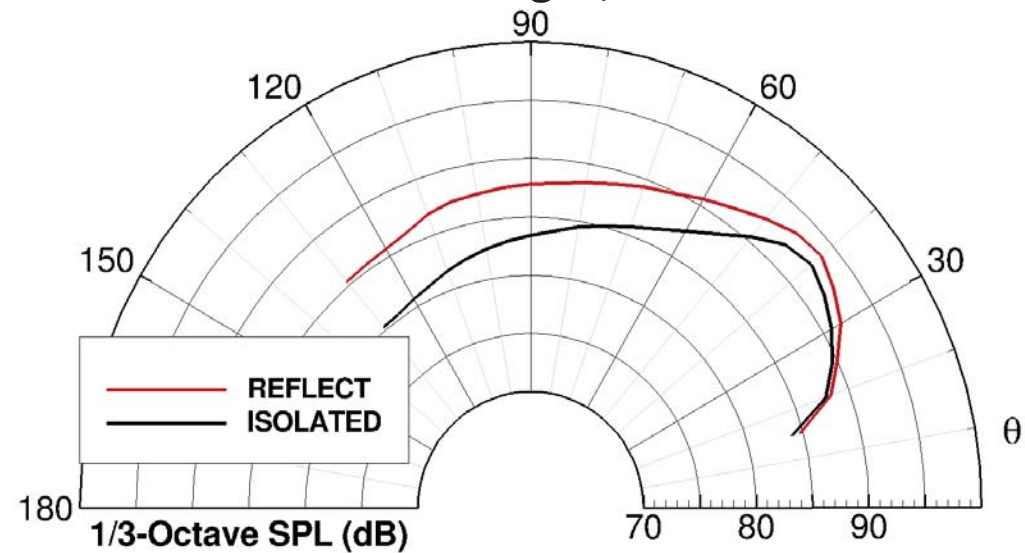
GF Directivity (St = 0.25)



$St_b = 0.25$, $U_j = 0.90$, $TR = 1.0$, $Ma = 0.0$, $\phi = \pi/2$

Measured SPL (St = 0.28, AR=8)

James Bridges, 2014



$X_{TE} / D_e = 5.6$, $D_e = 2.14''$

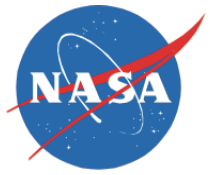
- A reflecting surface enhances the GF (5-6 dB) relative to an isolated jet (at polar angles larger than peak directivity angle)

Summary



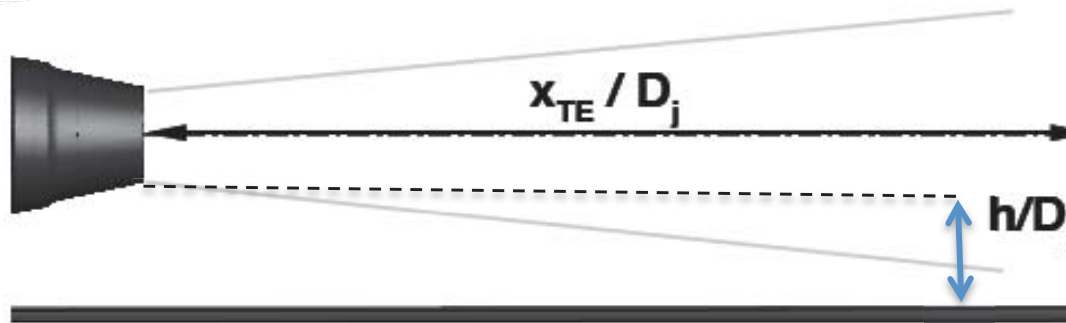
Within the region of nonzero sources:

- GF magnitude decreases with increasing frequency
- GF magnitude decreases with increasing temperature
- At a fixed polar angle GF magnitude varies with source location, generally increases at smaller downstream angle
- Phase factor varies with source location for non-rigid surface impedance
- GF peaks at smaller down-stream angles as flight Mach number is increased
- Presence of a reflecting surface enhances the GF magnitude (5-6 dB) relative to an isolated jet at polar angles larger than peak directivity angle.

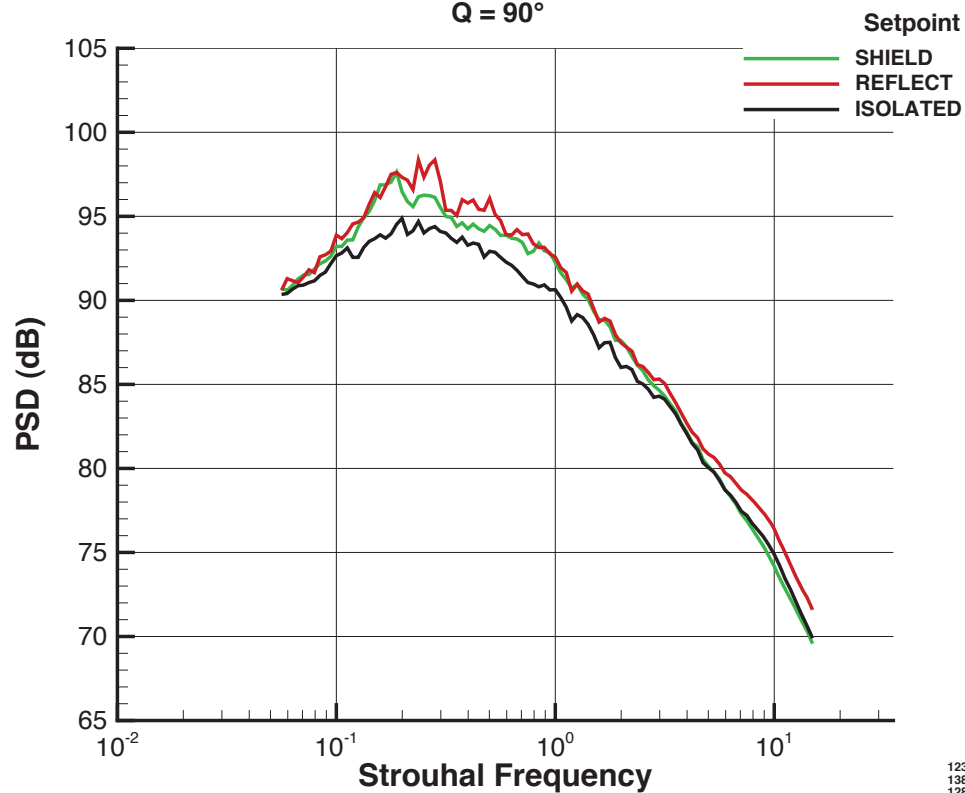


QUESTIONS ?

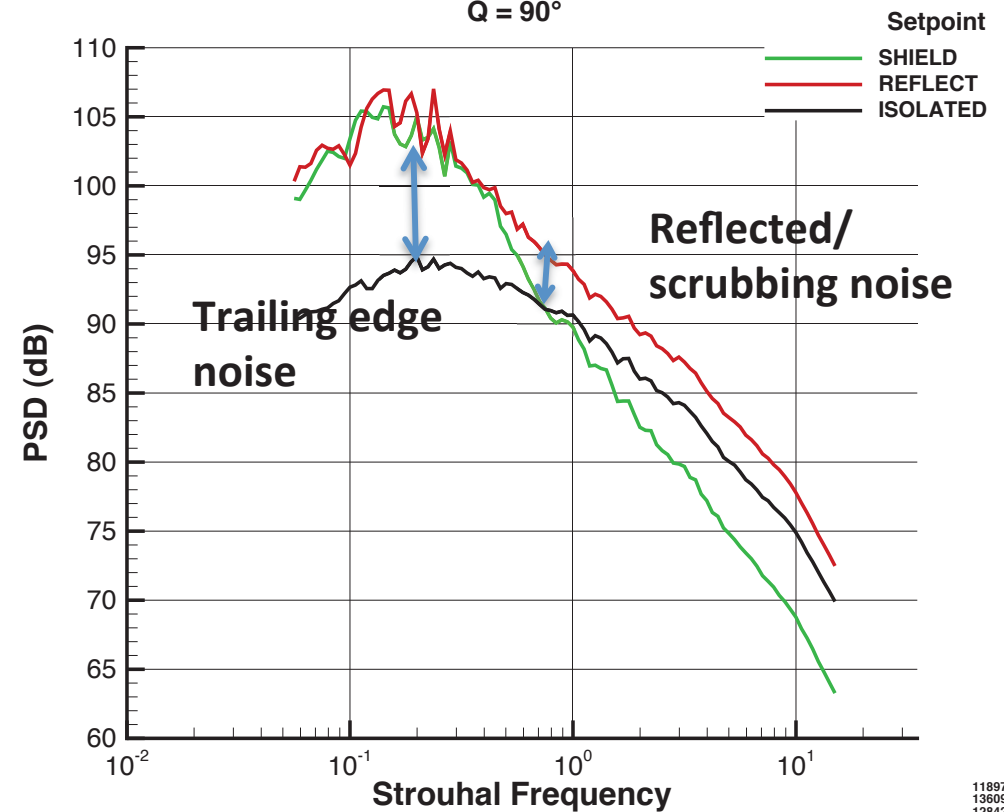
Measurements (JSIT Tests*)



SMC000, SP 7, $x_{TE} = 4$, $h = 0$
 $Q = 90^\circ$



SMC000, SP 7, $x_{TE} = 16$, $h = 0$
 $Q = 90^\circ$



*Cliff Brown (GRC), ASME paper GT2012

*Gary Podboy (GRC), ASME paper GT2012



Mean Flow – Analytical Profiles

- Axial Velocity $\eta = y_3 / D_j$

$$\frac{U(\eta)}{U_j} = \begin{cases} \tanh\left(\frac{D_j \eta}{d_1}\right), & \eta < 1.05 \\ \frac{1}{2}\left(1 + \frac{U_\infty}{U_j}\right) + \frac{1}{2}\left(1 - \frac{U_\infty}{U_j}\right) \tanh \frac{1}{d_2} \left(\frac{1/2}{\eta - 1} - \frac{\eta - 1}{1/2} \right), & \eta \geq 1.05 \end{cases}$$

- Temperature

$$T = T_1 + T_2$$

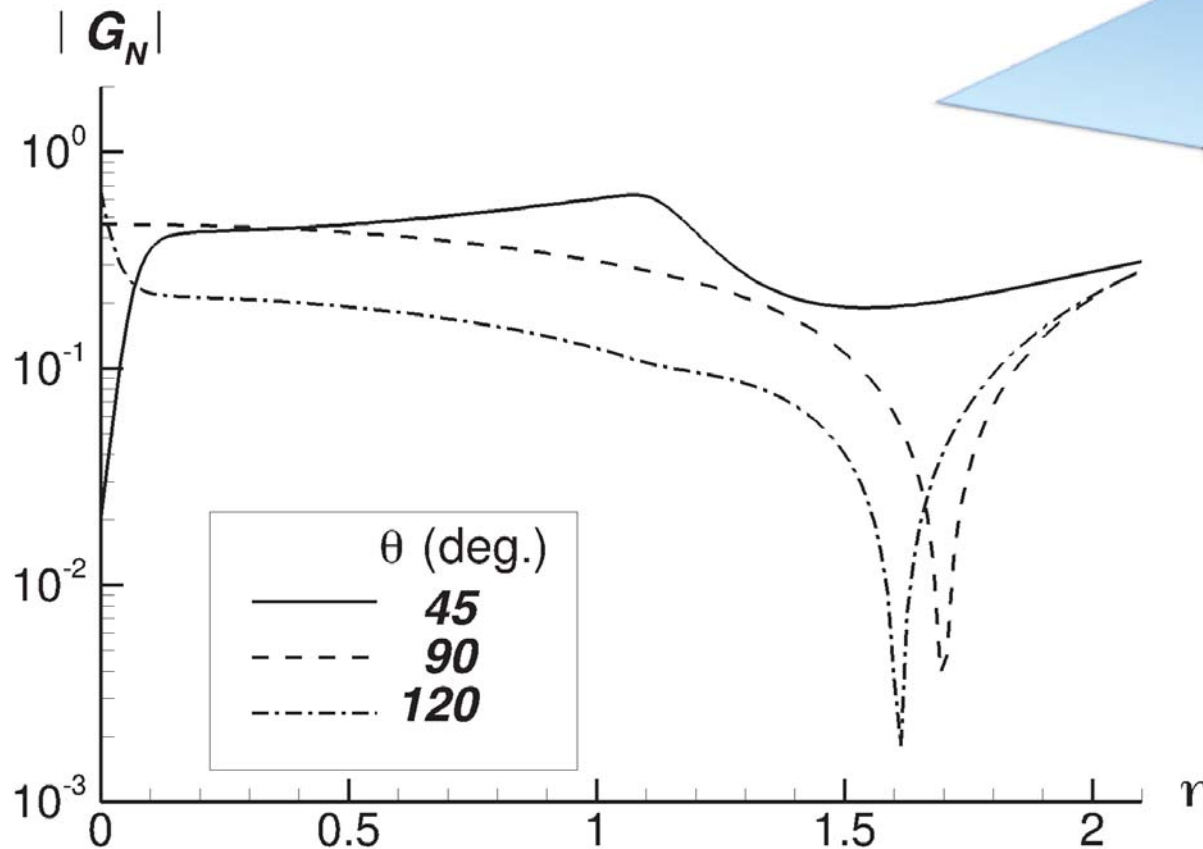
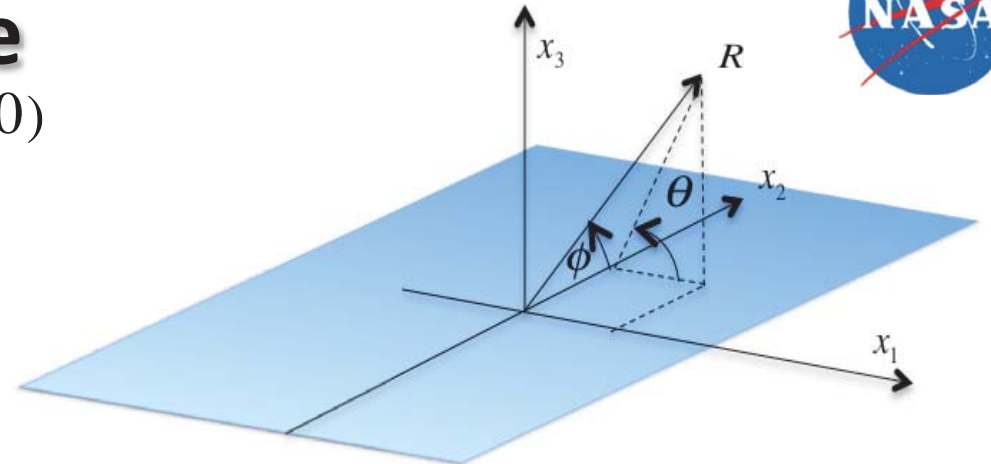
$$\frac{T_1(y_3)}{T_\infty} = 1 + (T_R - 1) \frac{U(y_3)}{U_j} - \frac{\gamma - 1}{2} \left(\frac{U(y_3)}{c_\infty} \right)^2 \quad \text{Crocco-Busemann Law}$$

$$\frac{T_2(y_3)}{T_\infty} = \frac{1}{d_3} \left(\frac{1}{2} + \frac{1}{2} \tanh \frac{1}{d_4} \left(\frac{1}{D_j \eta} - D_j \eta \right) \right) \quad \text{Frictional heat near the wall}$$

$$D_j = 2'' \quad \delta_o / D_j = 1.32 d_1 \quad (d_1, d_2, d_3, d_4) = (0.10, 2, 4, 3)$$

Effect of Observer Angle

($\phi = \pi/2, St_o = 0.25, U_j / c_\infty = 0.90, T_R = 3.0$)



- GF amplitude varies with source location, generally increases at smaller downstream angles