

# Wing Shape Sensing from Measured Strain

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**Patent Pending: Patent App No. 14/482784**



# Overview

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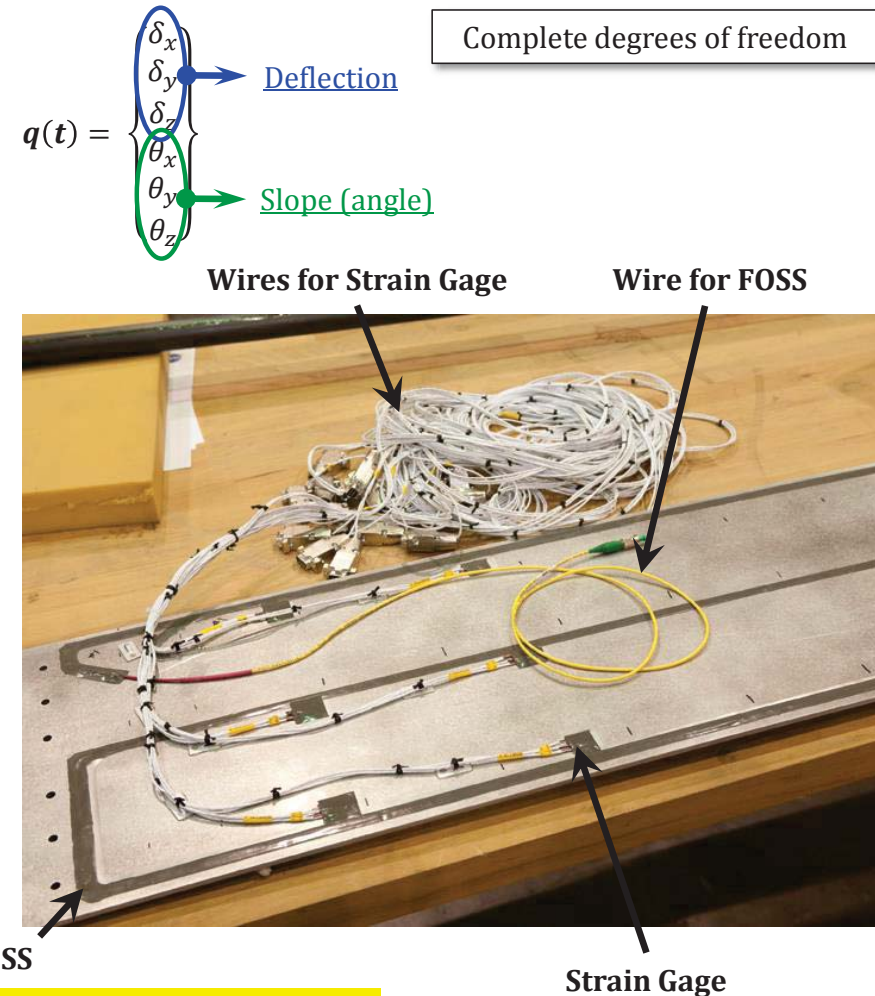
- ☐ What the technology does
- ☐ Previous technologies
- ☐ Technical features of new technology
- ☐ Computational Validation
  - ❖ Uniform 1g load
  - ❖ Wing tip torsion load
  - ❖ Aerodynamic load under  $1^\circ$  angle of attack at Mach 0.715
- ☐ Experimental Testing
  - ❖ Leading-edge load
  - ❖ Uniform load
- ☐ Conclusions



# What the technology does

## Problem Statement

- ❑ Improving fuel efficiency for an aircraft
  - ❖ Reducing **weight** or **drag**
    - Similar effect on fuel savings
  - ❖ Multidisciplinary design optimization (design phase) or active control (during flight)
- ❑ Real-time measurement of deflection, slope, and loads in flight are a valuable tool.
- ❑ Wing deflection and slope (complete degrees of freedom) are essential quantities for load computations during flight.
  - ❖ Loads can be computed from the following governing equations of motion.
 
$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{G}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{F}_a(\mathbf{Mach}, \mathbf{q}(t))$$
    - Internal Loads: using finite element structure model
      - ✓  $\mathbf{M}\ddot{\mathbf{q}}(t)$ ,  $\mathbf{G}\dot{\mathbf{q}}(t)$ ,  $\mathbf{K}\mathbf{q}(t)$  : Inertia, damping, and elastic loads
    - External Load: using unsteady aerodynamic model
      - ✓  $\mathbf{F}_a$  : Aerodynamic load
- ❑ Traditionally, strain over the wing are measured using strain gages.
  - ❖ Cabling would create **weight and space limitation** problems.
  - ❖ A **new innovation** is needed. **Fiber optic strain sensor (FOSS)** is an ideal choice for **aerospace** applications.



Wing deflection & slope will be computed from measured strain.



# Previous technologies

## Beam theory; Sectional bending moment and shear loads

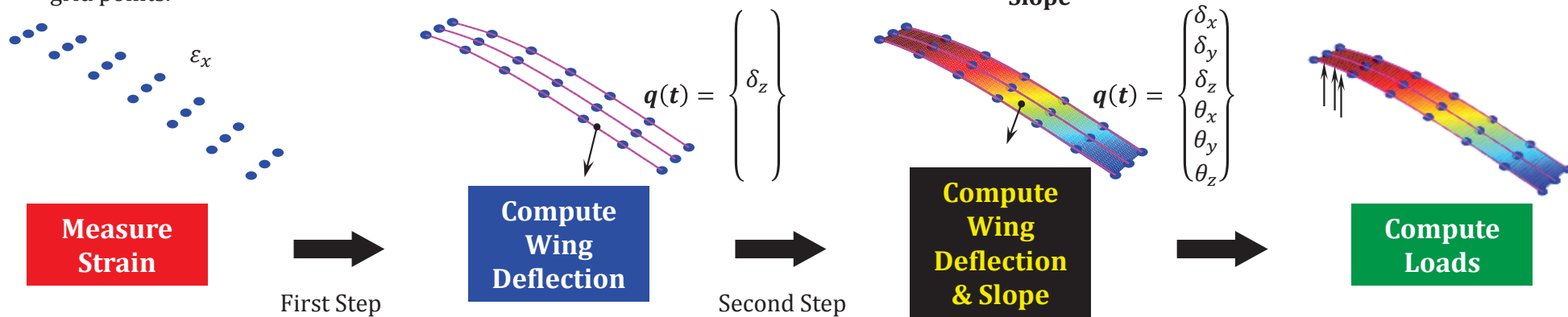
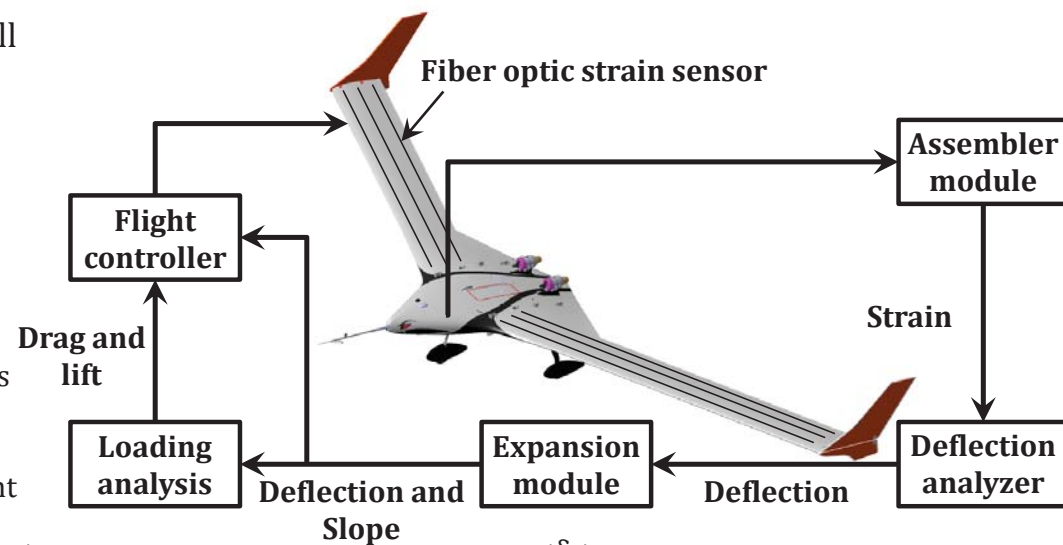
- ❑ Liu, T., Barrows, D. A., Burner, A. W., and Rhew, R. D., “Determining Aerodynamic Loads Based on Optical Deformation Measurements,” AIAA 2001-0560
  - ❖ **NASA LRC**; Application is limited for “**beam**”.
- ❑ Shkarayev, S., Krashantisa, R., and **Tessler**, A., “An Inverse Interpolation Method Utilizing In-Flight Strain Measurements for Determining Loads and Structural Response of Aerospace Vehicles,” Proceedings of Third International Workshop on Structural Health Monitoring, 2001
  - ❖ **University of Arizona** and **NASA LRC**; **using an inverse interpolation formulation**.
- ❑ Kang, L.-H., Kim, D.-K., and Han, J.-H., “Estimation of Dynamic Structural Displacements using fiber Bragg grating strain sensors,” 2007
  - ❖ **KAIST**; displacement-strain-transformation (DST) matrix. Use **strain mode shape**. Application was based on **beam structure**.
- ❑ Igawa, H. et al., “Measurement of Distributed Strain and Load Identification Using 1500 mm Gauge Length FBG and Optical Frequency Domain Reflectometry,” 20th International Conference on Optical Fibre Sensors, 2009
  - ❖ **JAXA**; **using inverse analysis**. “**Beam**” **application only**.
- ❑ Ko, W. and Richards, L., “Method for real-time structure shape-sensing,” US Patent #7520176B1, April 21, 2009
  - ❖ **NASA AFRC**; **closed-form equations** (based on **beam theory**)
- ❑ Richards, L. and Ko, W., “Process for using surface strain measurements to obtain operational loads for complex structures,” US Patent #7715994, May 11, 2010
  - ❖ **NASA AFRC**; “**sectional**” bending moment and shear force along the “**beam**”.
- ❑ Moore, J.P., “Method and Apparatus for Shape and End Position Determination using an Optical Fiber,” U.S. Patent No. 7813599, issued October 12, 2010
  - ❖ **NASA LRC**; **curve-fitting**
- ❑ Park, Y.-L. et al., “Real-Time Estimation of Three-Dimensional Needle Shape and Deflection for MRI-Guided Interventions,” *IEEE/ASME Transactions on Mechatronics*, Vol. 15, No. 6, 2010, pp. 906-915
  - ❖ **Harvard University**, **Stanford University**, and **Howard Hughes Medical Institute**; Uses **beam theory**.
- ❑ Carpenter, T.J. and Albertani, R., “Aerodynamic Load Estimation: Pressure Distribution from Virtual Strain Sensors for a **Pliant Membrane** Wing,” AIAA 2013-1917
  - ❖ **Oregon State University**; Aerodynamic loads are estimated from measured strain using virtual strain sensor technique.

Previous technologies are applied to a beam structure.

# Technical features of new technology

## Proposed solutions:

- ❑ The new method for obtaining the deflection over a flexible full 3D aircraft structure is based on the following two steps.
  - ❖ First Step: Compute wing deflection along fibers using measure strain data
    - Wing deflection will be computed along the fiber optic sensor line.
    - Strains at selected locations will be “fitted”.
    - These fitted strain will be integrated twice to have deflection information. (Relative deflection w.r.t. the reference point)
    - This is a finite element model independent method.
  - ❖ Second Step: Compute wing slope and deflection of entire structures
    - Slope computation will be based on a finite element model dependent technique.
    - Wing deflection and slope will be computed at all the finite element grid points.



A new two-step theory is investigated for predicting the deflection and slope of an entire structure using strain measurements at discrete locations.



# Technical features of new technology (continued)

## First Step

- ❖ Use piecewise least-squares method to minimize noise in the measured strain data (strain/offset)
- ❖ Obtain cubic spline (Akima spline) function using re-generated strain data points:

$$\frac{d^2 \delta}{ds^2} = -\epsilon(s)/c(s)$$

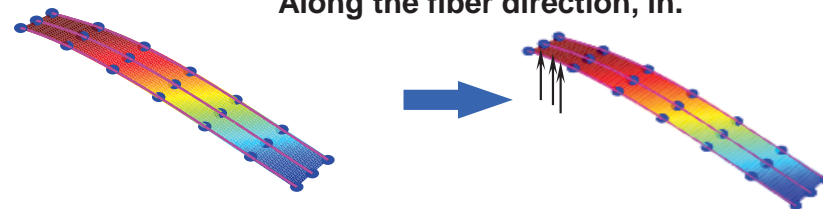
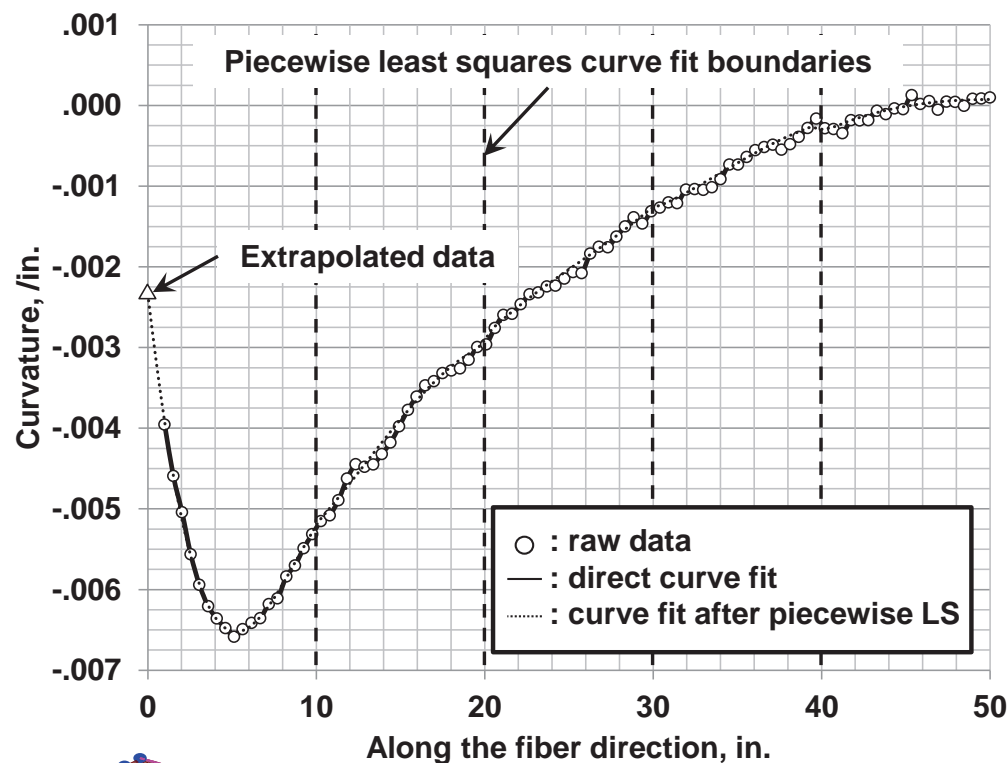
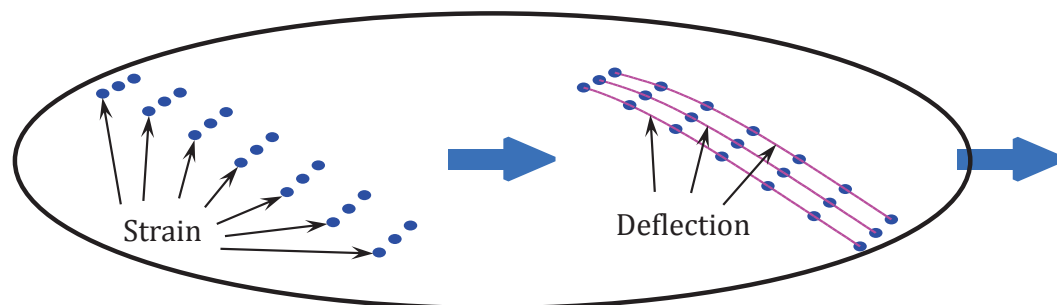
- ❖ Integrate fitted spline function to get slope data:

$$\frac{d\delta}{ds} = \theta(s)$$

- ❖ Obtain cubic spline (Akima spline) function using computed slope data

- ❖ Integrate fitted spline function to get deflection data:

$$\delta(s)$$



A measured strain is fitted using a piecewise least-squares curve fitting method together with the cubic spline technique.



# Technical features of new technology (continued)

## □ Second Step: Based on General Transformation

❖ For all model reduction/expansion techniques, there is a relationship between the **master (measured or tested)** degrees of freedom and the **slave (deleted or omitted)** degrees of freedom which can be written in general terms as

➤  $\{q\} = \begin{Bmatrix} q_M \\ q_S \end{Bmatrix} = [T]\{\tilde{q}_M\}$ :  $\{q\}$ = general displacement vector

✓ Where, an eigen-matrix is defined as  $\begin{Bmatrix} q_M \\ q_S \end{Bmatrix} = \begin{bmatrix} \Phi_M \\ \Phi_S \end{bmatrix} \{\eta\}$ ;  $\{\eta\}$ = orthogonal displacement vector

➤ Transformation matrix  $[T]$  can be one of the followings:

✓ Guyan (or static) condensation, dynamic condensation, improved reduced system (IRS), or system equivalent reduction expansion process (SEREP)

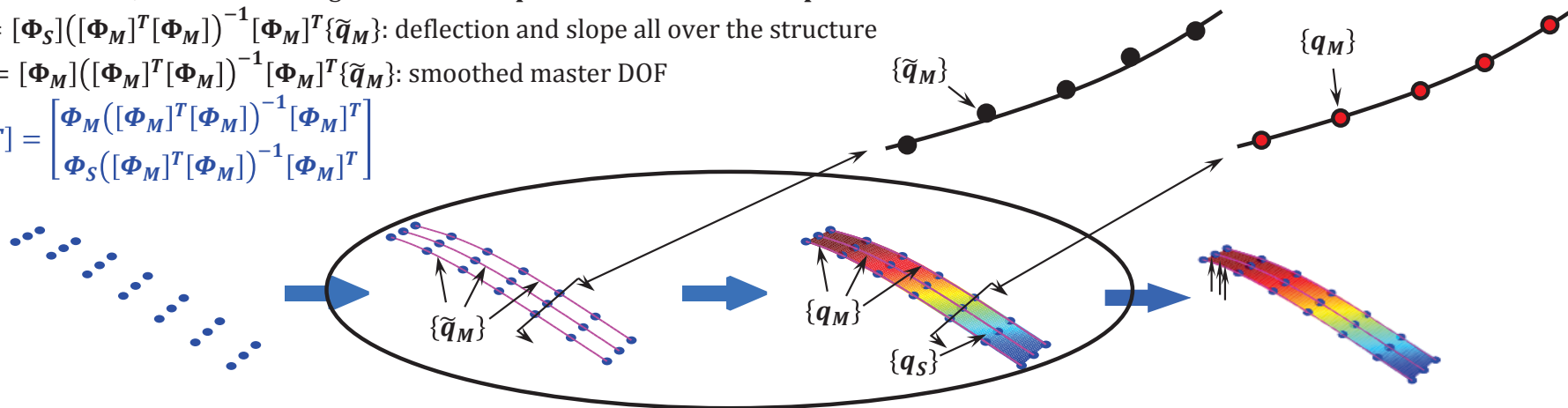
## □ Expansion of displacement using SEREP: kinds of least-squares method; most accurate reduction-expansion technique

❖  $\{\tilde{q}_M\}$ : master DOF; **deflection** along the fiber “**computed from the first step**”

❖  $\{q_S\} = [\Phi_S]([\Phi_M]^T[\Phi_M])^{-1}[\Phi_M]^T\{\tilde{q}_M\}$ : deflection and slope all over the structure

❖  $\{q_M\} = [\Phi_M]([\Phi_M]^T[\Phi_M])^{-1}[\Phi_M]^T\{\tilde{q}_M\}$ : smoothed master DOF

➤  $[T] = \begin{bmatrix} \Phi_M([\Phi_M]^T[\Phi_M])^{-1}[\Phi_M]^T \\ \Phi_S([\Phi_M]^T[\Phi_M])^{-1}[\Phi_M]^T \end{bmatrix}$



Computed deflection along the fibers are combined with a finite element model of the structure in order to interpolate and extrapolate the deflection and slope of the entire structure through the use of the System Equivalent Reduction and Expansion Process.



# Computational Validation

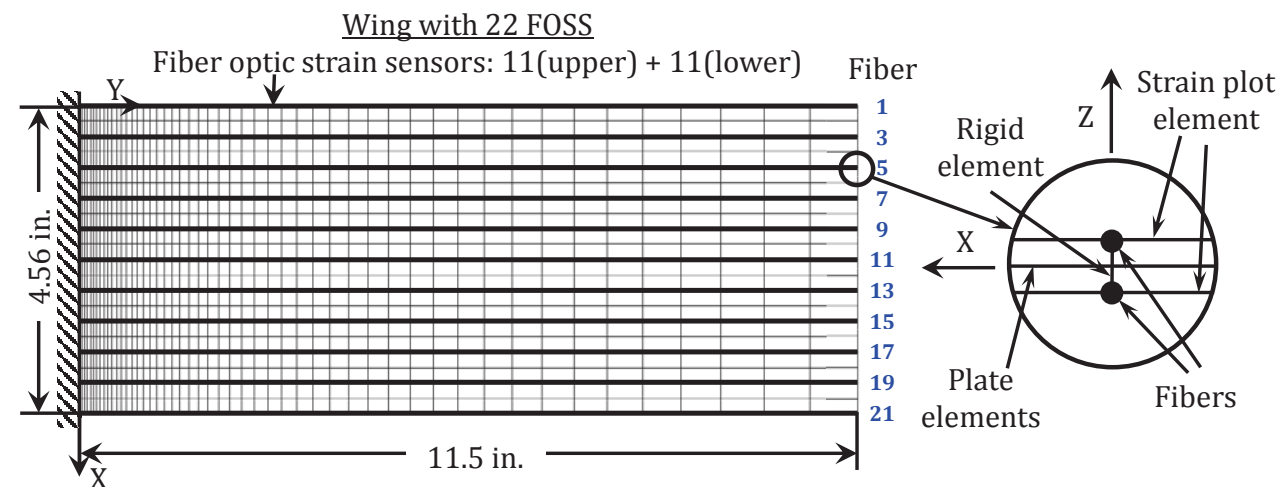
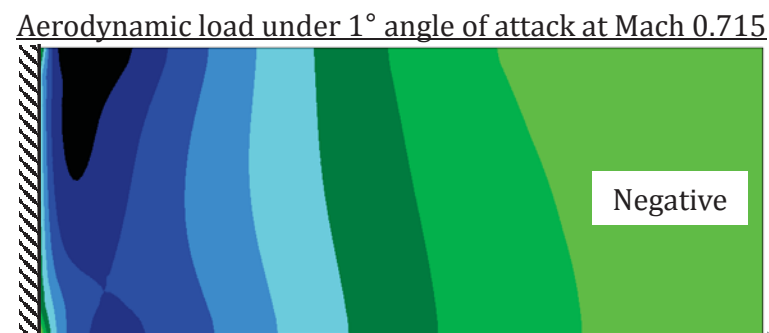
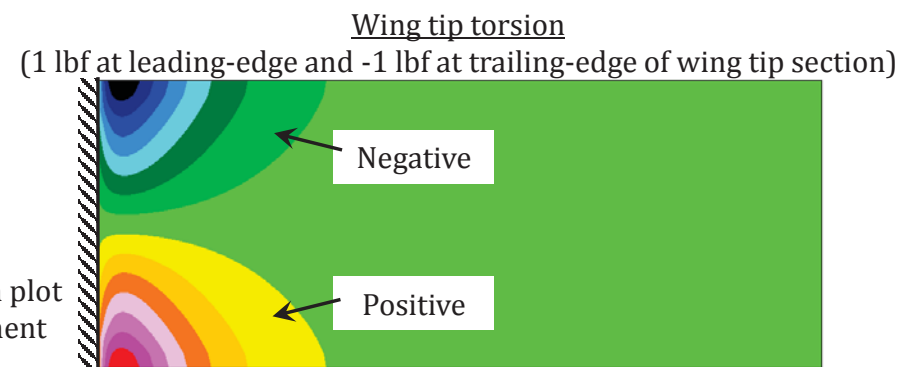
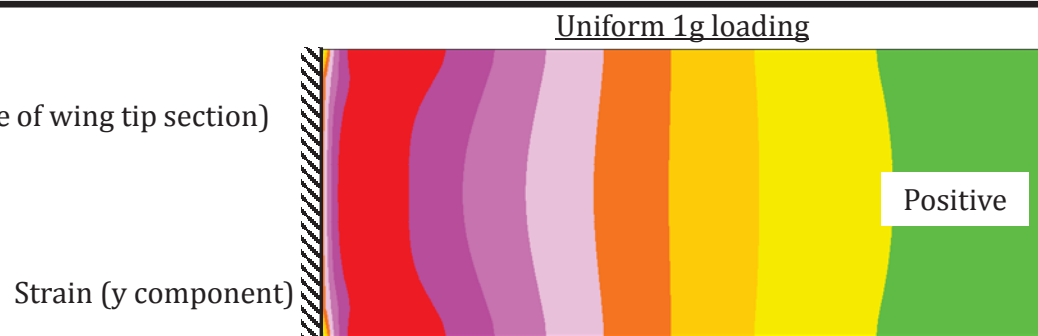


**Cantilevered rectangular wing model**



# Cantilevered Rectangular Wing Model

- ❑ Wind tunnel test wing (thickness = 0.065 in.)
  - ❖ Uniform 1g load
  - ❖ Wing tip torsion (1 lbf at leading-edge and -1 lbf at trailing-edge of wing tip section)
  - ❖ Aerodynamic load under 1° angle of attack at Mach 0.715
- ❑ MSC/NASTRAN
  - ❖ Compute strain
  - ❖ Compute deflection (**target**)
- ❑ ZAERO
  - ❖ Compute aerodynamic load
- ❑ Two-step approach
  - ❖ Compute deflection from computed strain
  - ❖ Compare computed deflection with respect to target value



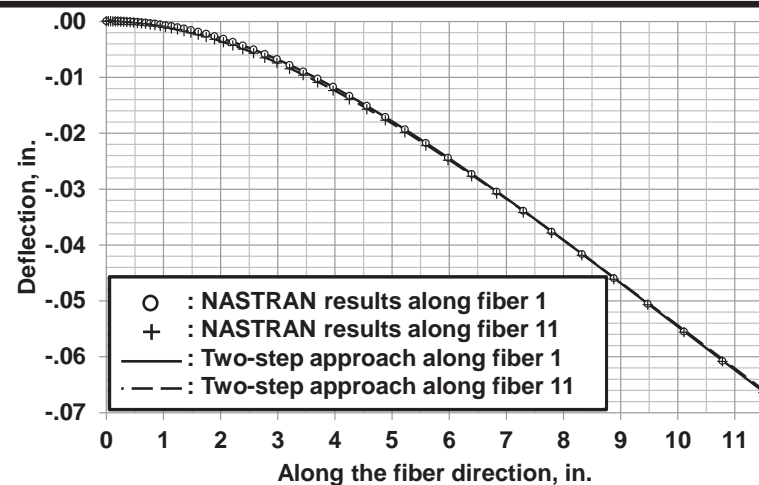
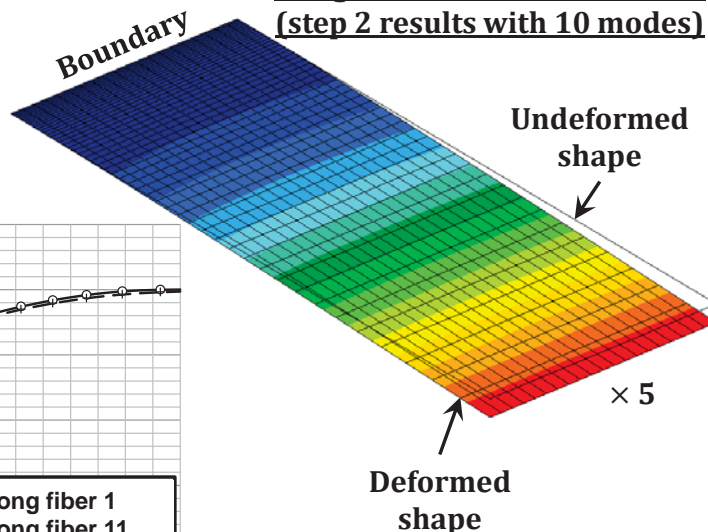


# Cantilevered Rectangular Wing Model: Uniform 1g

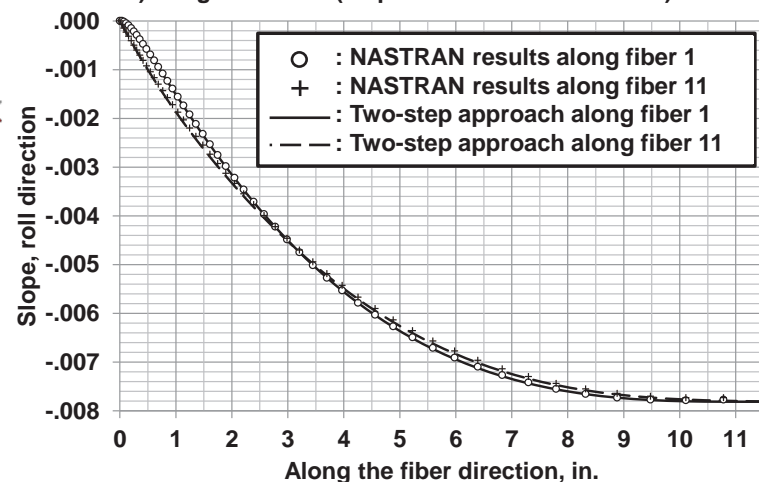
□ Uniform 1g load



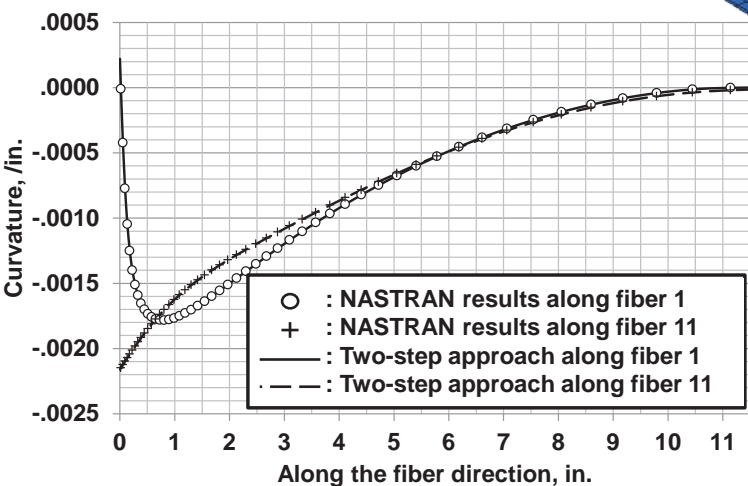
Wing deflection over FE model  
(step 2 results with 10 modes)



b) Wing deflection (step 2 results with 10 modes)



d) Wing span-wise slope (step 2 results with 10 modes)

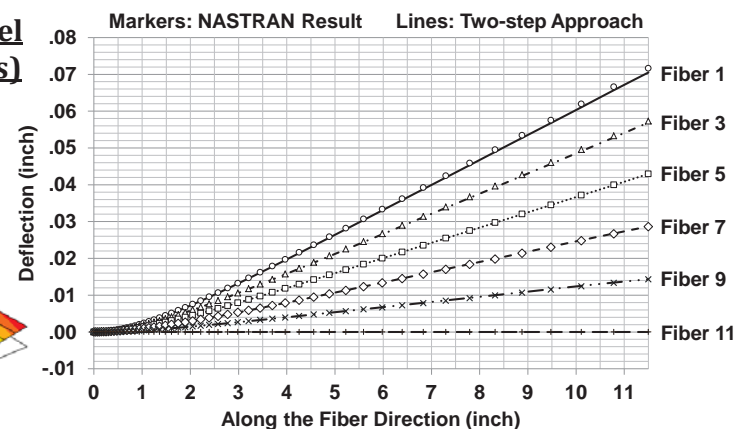
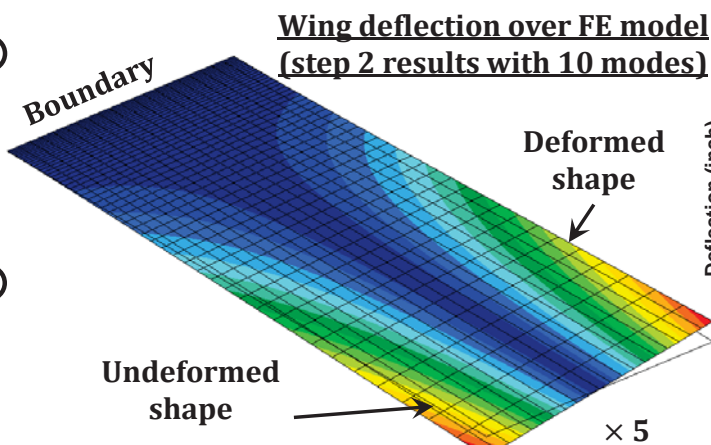
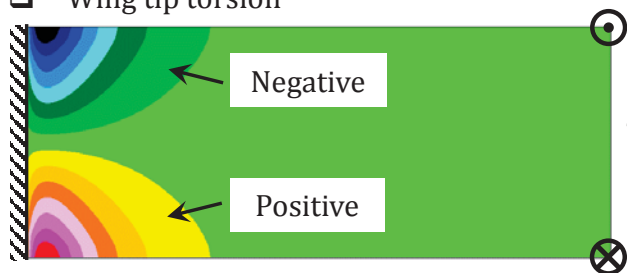


a) Curvature distribution (step 1 results)

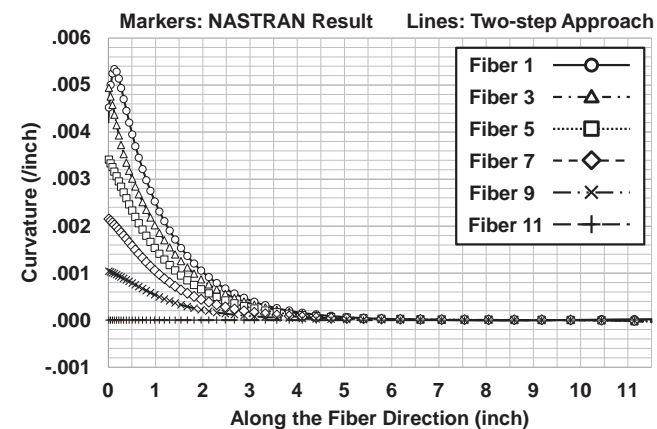


# Cantilevered Rectangular Wing Model: Wing tip torsion

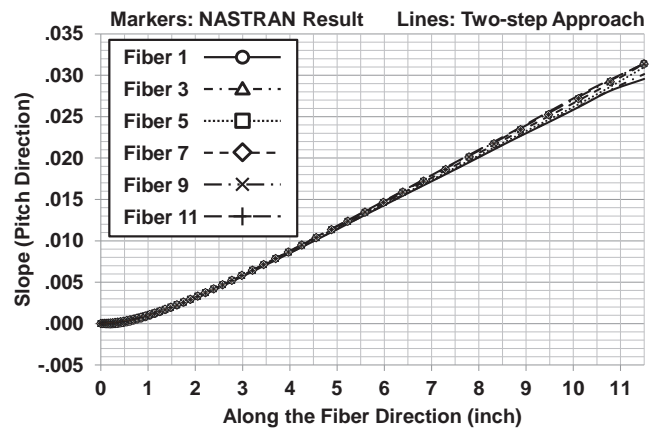
□ Wing tip torsion



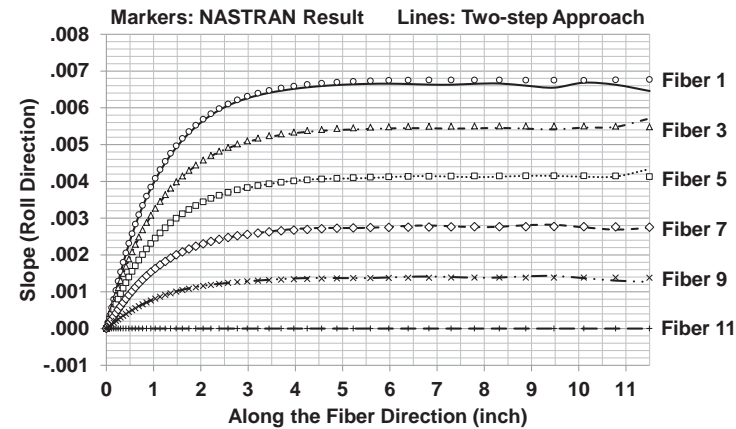
(d) Step 2 results with 50 Modes



(a) Strain distribution (step 1 results)



(d) Step 2 results with 50 Modes

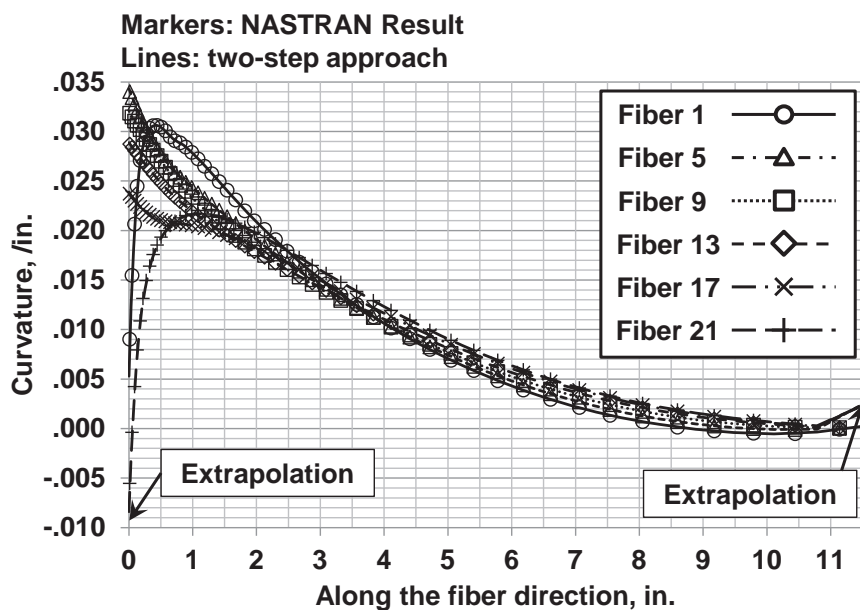
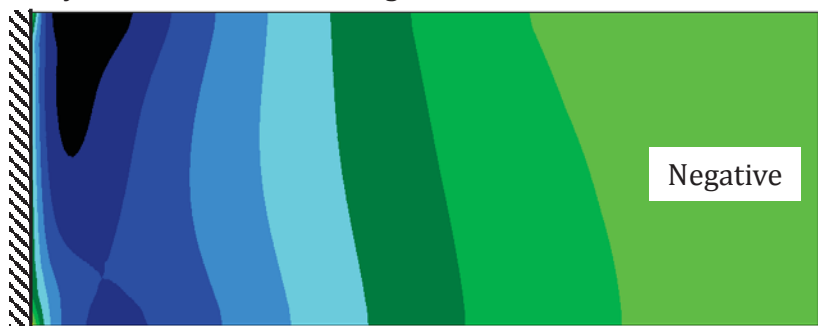


(d) Step 2 results with 50 Modes

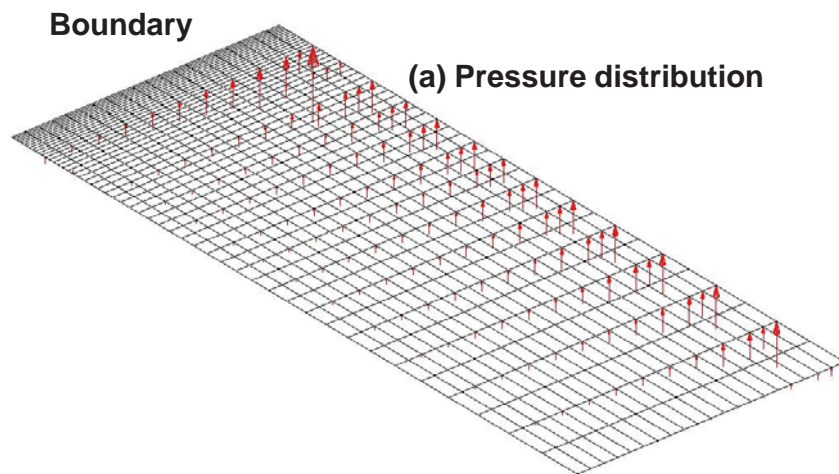
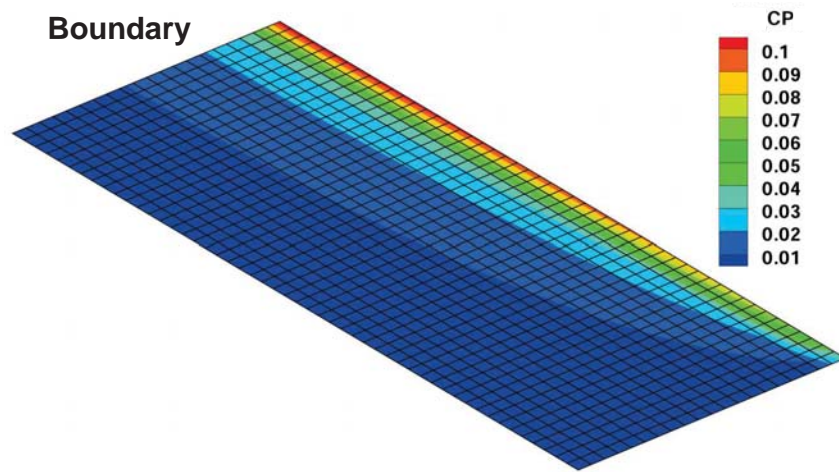


# Cantilevered Rectangular Wing Model: Aerodynamic load

□ Aerodynamic load under  $1^\circ$  angle of attack at Mach 0.715



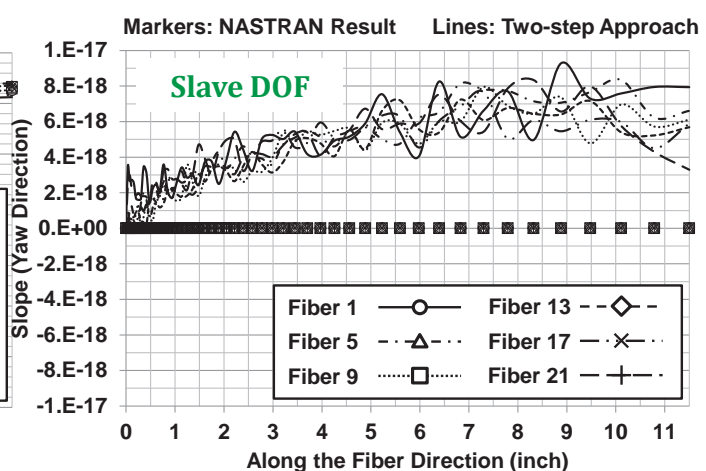
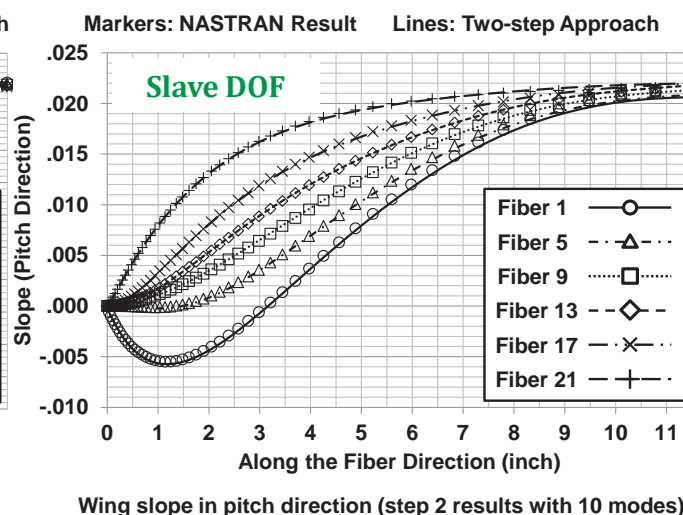
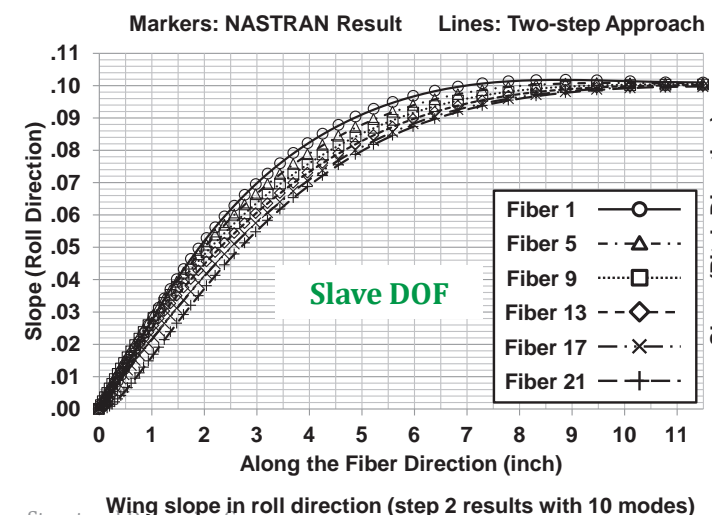
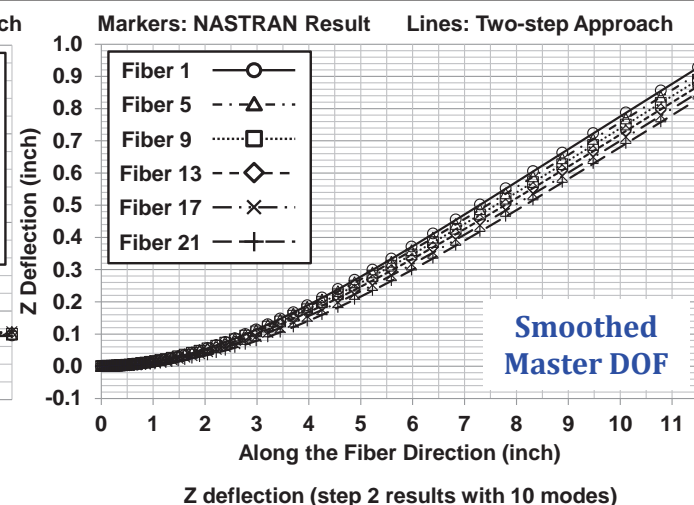
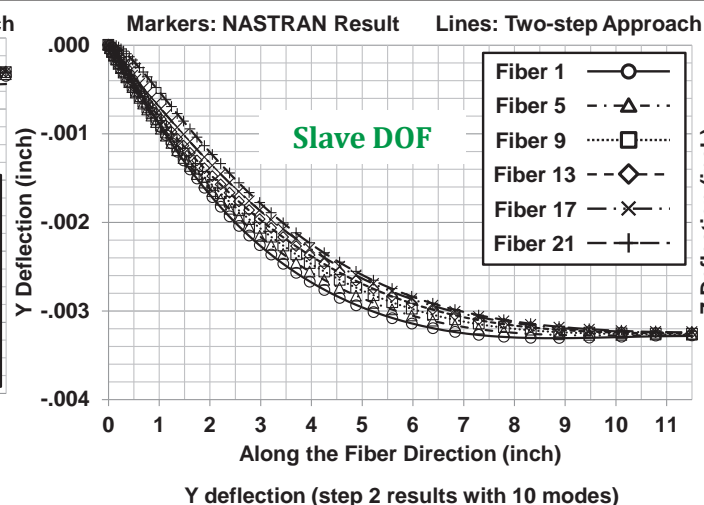
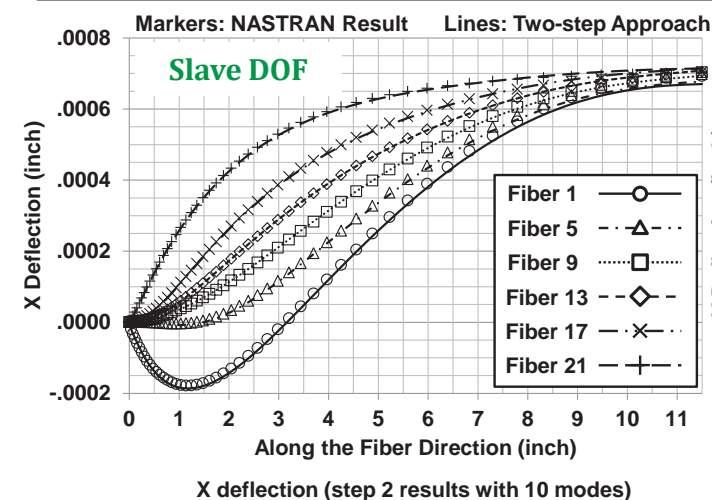
a) Curvature distribution (step 1 results)



(b) Splined load vectors



# Cantilevered Rectangular Wing Model: Aerodynamic load (continued)







## Cantilevered Rectangular Wing Model: Aerodynamic load (continued)

☐ Aerodynamic load under  $1^\circ$  angle of attack at Mach 0.715 (continued)

☐ Wing tip deflections

### Deflection in X, Y, & Z direction

Fiber number	Target (inch)	Computed X deflection (inch)		Relative error (%)	
		Step 1	Step 2 with 10 modes	Step 1	Step 2 with 10 modes
1	0.0006927	N/A	0.0006703	N/A	-3.2
5	0.0006975	N/A	0.0006776	N/A	-2.8
9	0.0007029	N/A	0.0006936	N/A	-1.3
13	0.0007047	N/A	0.0007081	N/A	0.48
17	0.0007035	N/A	0.0007144	N/A	1.6
21	0.0007023	N/A	0.0007143	N/A	1.7
Fiber number	Target (inch)	Computed Y deflection (inch)		Relative error (%)	
		Step 1	Step 2 with 10 modes	Step 1	Step 2 with 10 modes
1	-0.003280	N/A	-0.003282	N/A	0.08
5	-0.003272	N/A	-0.003278	N/A	0.18
9	-0.003260	N/A	-0.003268	N/A	0.26
13	-0.003248	N/A	-0.003257	N/A	0.28
17	-0.003239	N/A	-0.003246	N/A	0.23
21	-0.003235	N/A	-0.003242	N/A	0.20
Fiber number	Target (inch)	Computed Z deflection (inch)		Relative error (%)	
		Step 1	Step 2 with 10 modes	Step 1	Step 2 with 10 modes
1	0.9280	0.9269	0.9275	-0.12	-0.06
5	0.9085	0.9091	0.9087	0.07	0.02
9	0.8889	0.8894	0.8895	0.06	0.08
13	0.8691	0.8696	0.8699	0.05	0.09
17	0.8493	0.8497	0.8498	0.05	0.06
21	0.8296	0.8300	0.8297	0.04	0.01

$\{q_s\}$ : slave DOF

Input to Step 2

Structural Dynamics Group

$\{\tilde{q}_M\}$ : master DOF  $\{q_M\}$ : smoothed master DOF

Boundary

Deformed shape

Air

Undeformed shape

**Wing deflection over FE model  
(step 2 results with 10 modes)**

Smoothing effect



# Cantilevered Rectangular Wing Model: Aerodynamic load (continued)

☐ Aerodynamic load under 1° angle of attack at Mach 0.715 (continued)

☐ Wing tip slopes

**Slope in roll, pitch, & yaw direction**

$\{q_s\}$ : slave DOF

Not usable for Step 2 computations

Y deflection

Fiber number	Target	Computed roll slope		Relative error (%)	
		Step 1*	Step 2 with 10 modes	Step 1	Step 2 with 10 modes
1	0.10090	0.1010	0.10100	0.12	0.08
5	0.10070	0.1016	0.10090	0.96	0.18
9	0.10030	0.1012	0.10060	0.94	0.26
13	0.09993	0.1009	0.10020	0.93	0.28
17	0.09966	0.1006	0.09989	0.92	0.23
21	0.09954	0.1004	0.09974	0.88	0.20
Fiber number	Target	Computed pitch slope		Relative error (%)	
		Step 1	Step 2 with 10 modes	Step 1	Step 2 with 10 modes
1	0.02131	N/A	0.02063	N/A	-3.2
5	0.02146	N/A	0.02085	N/A	-2.9
9	0.02163	N/A	0.02134	N/A	-1.3
13	0.02168	N/A	0.02179	N/A	0.5
17	0.02165	N/A	0.02198	N/A	1.5
21	0.02161	N/A	0.02198	N/A	1.7
Fiber number	Target	Computed yaw slope		Absolute error	
		Step 1	Step 2 with 10 modes	Step 1	Step 2 with 10 modes
1	2.2e-31	N/A	7.9e-18	N/A	0.0000
5	1.9e-31	N/A	6.6e-18	N/A	0.0000
9	1.7e-31	N/A	6.1e-18	N/A	0.0000
13	1.7e-31	N/A	5.7e-18	N/A	0.0000
17	1.4e-31	N/A	5.9e-18	N/A	0.0000
21	1.2e-31	N/A	3.3e-18	N/A	0.0000

Relative error (%)	
Step 1	Step 2 with 10 modes
N/A	0.08
N/A	0.18
N/A	0.26
N/A	0.28
N/A	0.23
N/A	0.20
Relative error (%)	
Step 1	Step 2 with 10 modes
N/A	-3.2
N/A	-2.8
N/A	-1.3
N/A	0.48
N/A	1.6
N/A	1.7

X deflection

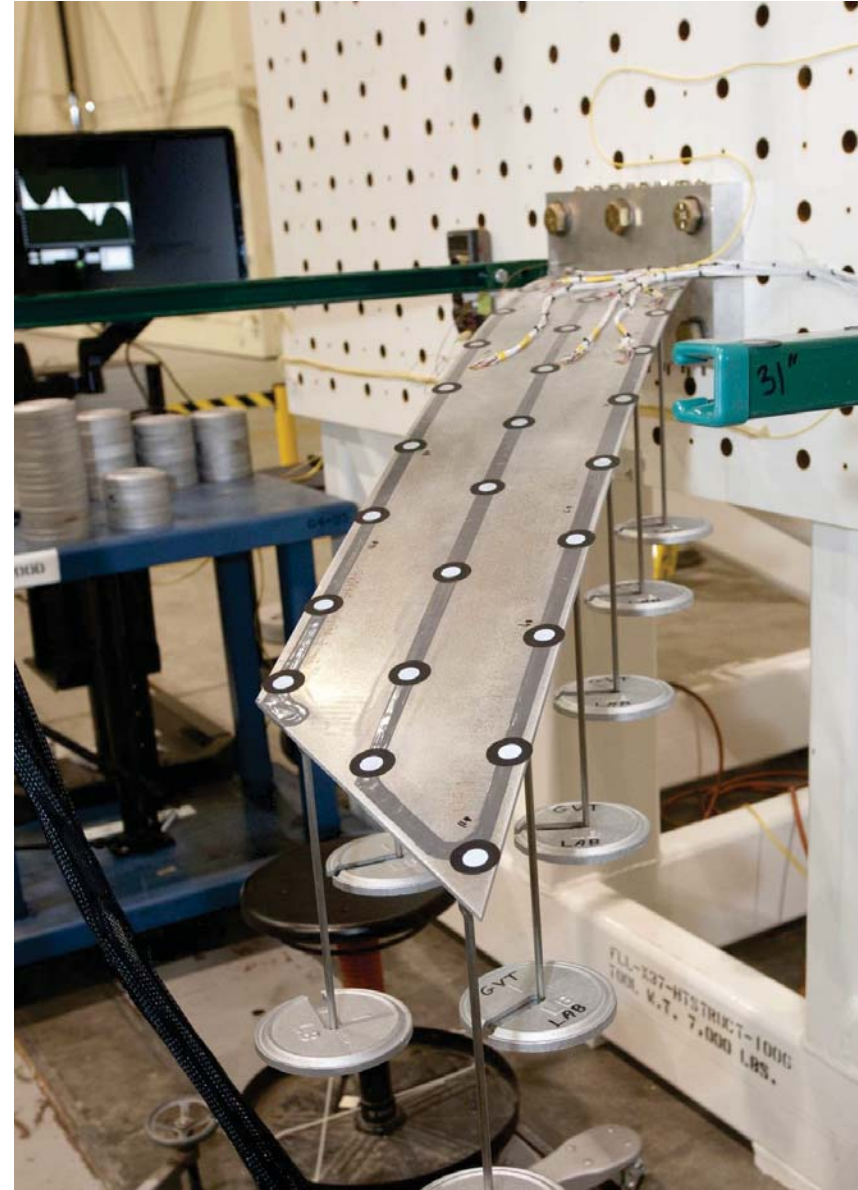
\*: Roll slope without effect of X and Y deflections (These slopes are not used during step 2 computation.)



# Experimental Testing



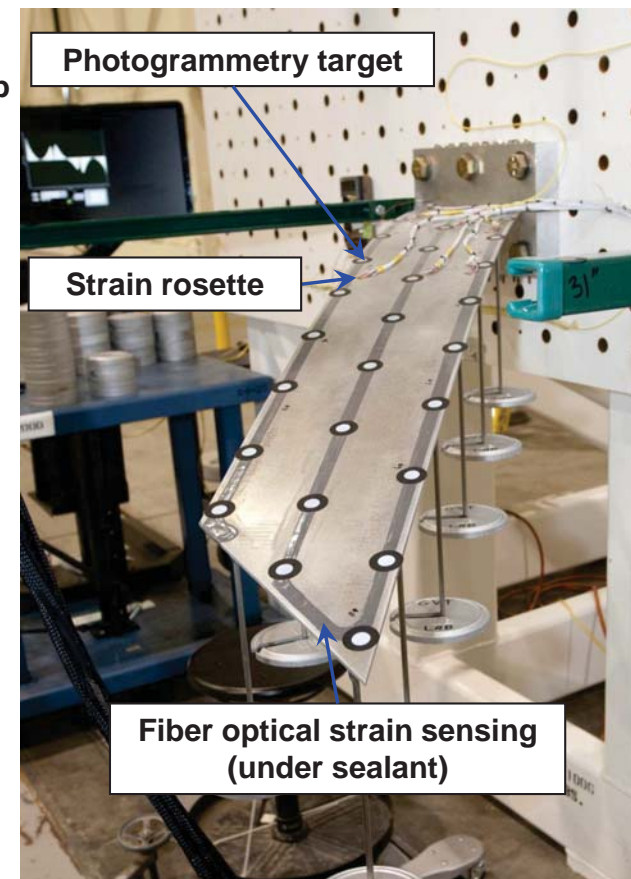
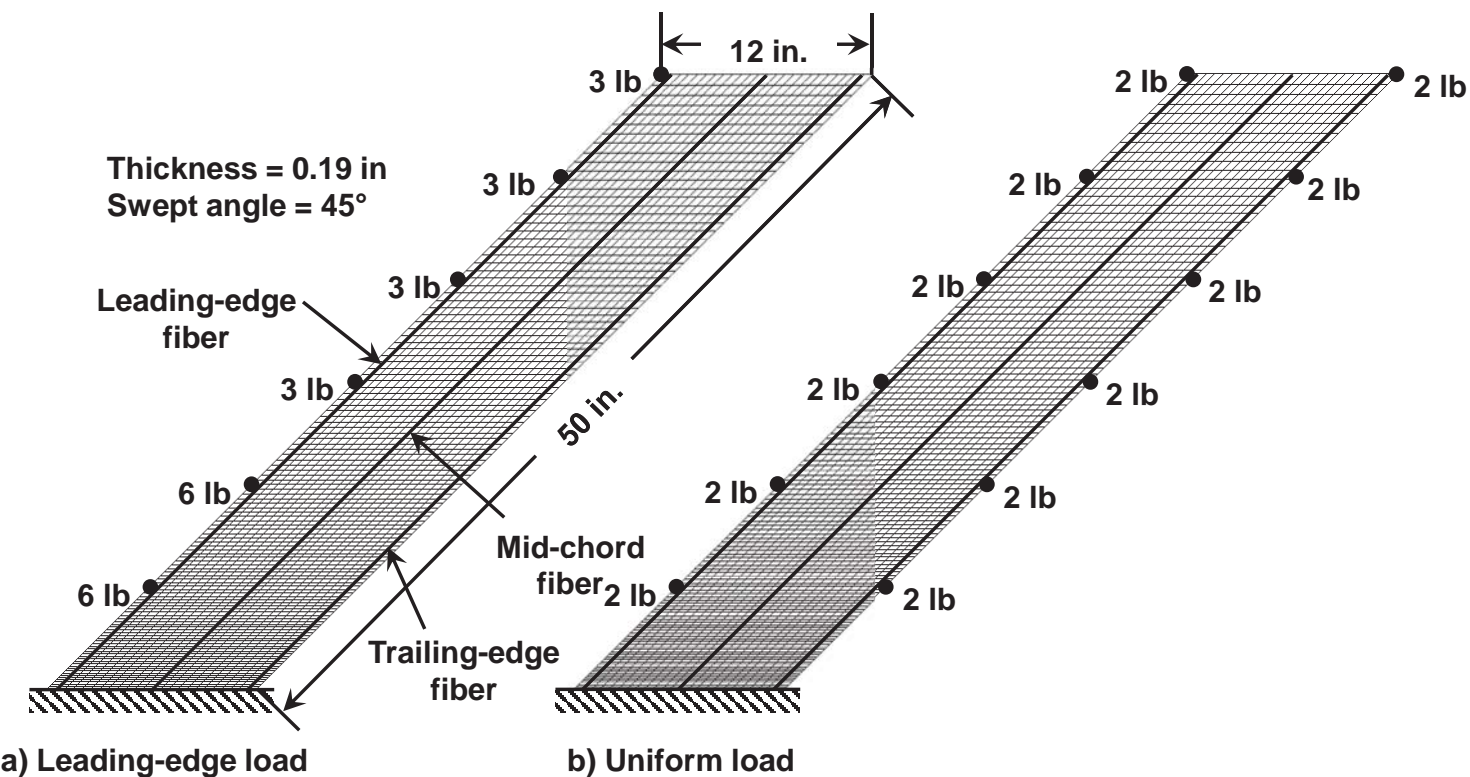
Swept test plate





# Swept Test Plate

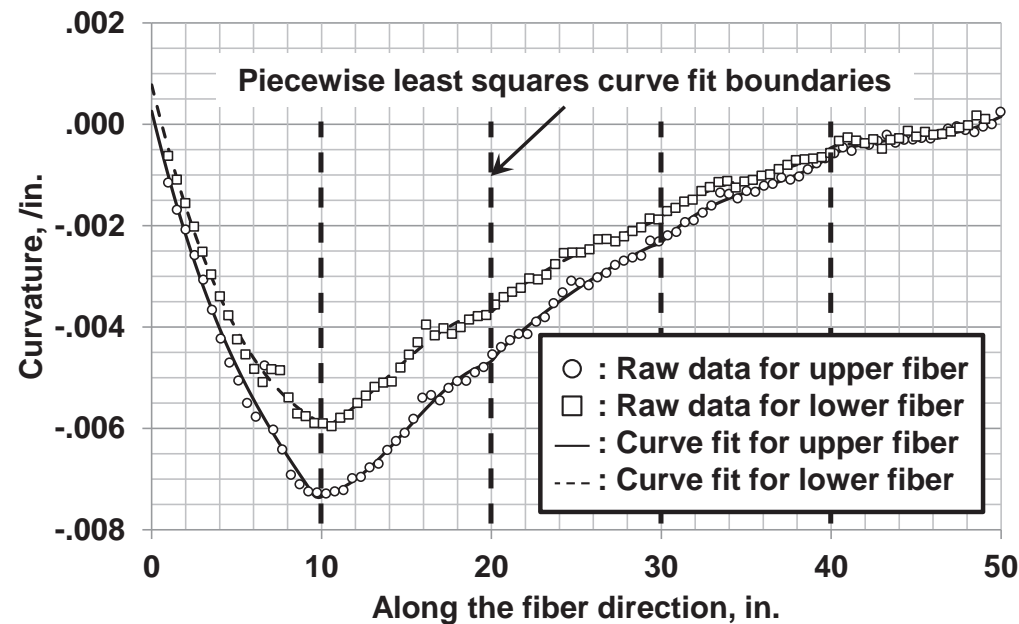
☐ Tested at NASA AFRC



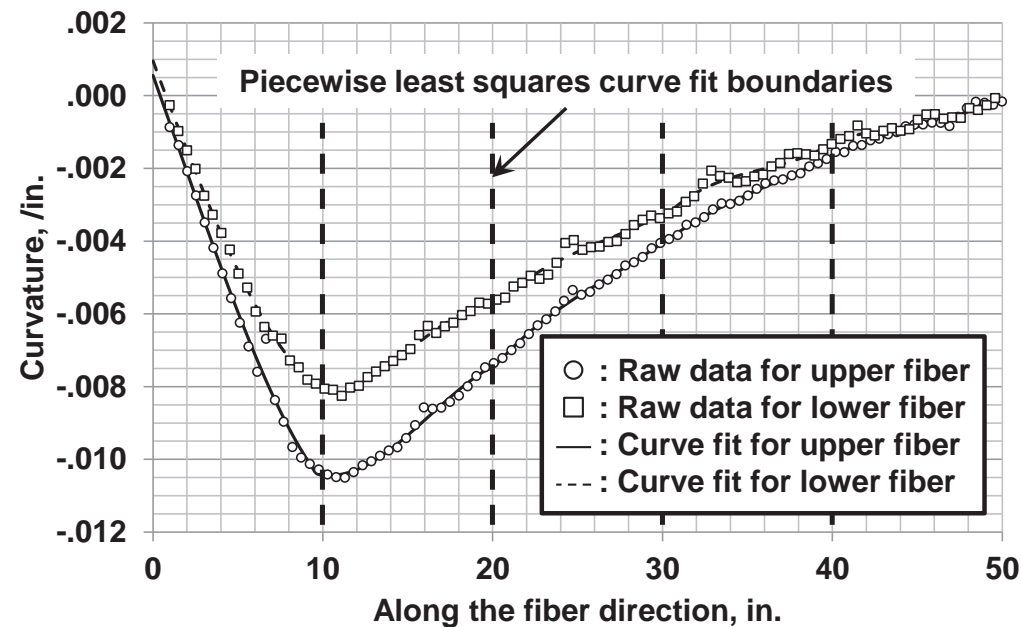


## Swept Test Plate (continued)

- ❑ Averaging the curvatures calculated by using each fiber individually eliminates the effect of the axial load.
- ❑ This computation is performed after curve-fitting each set of data individually to minimize noise.



(a) Leading-edge load

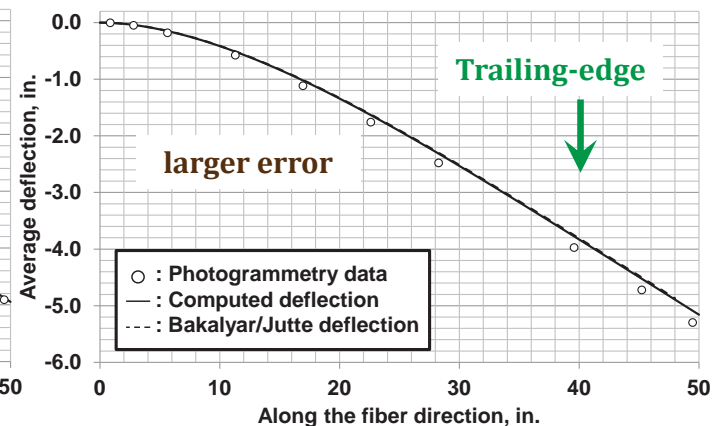
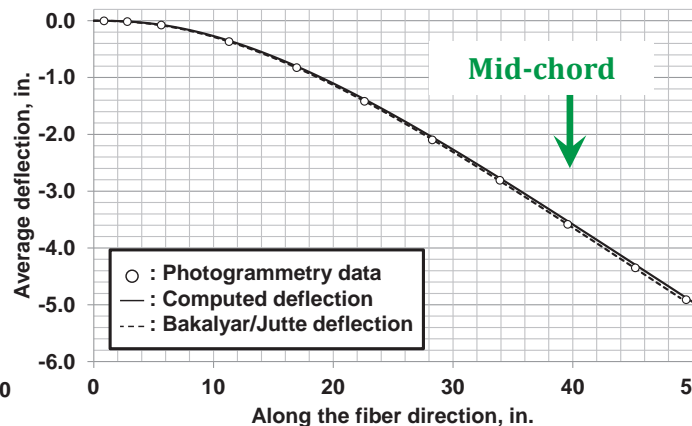
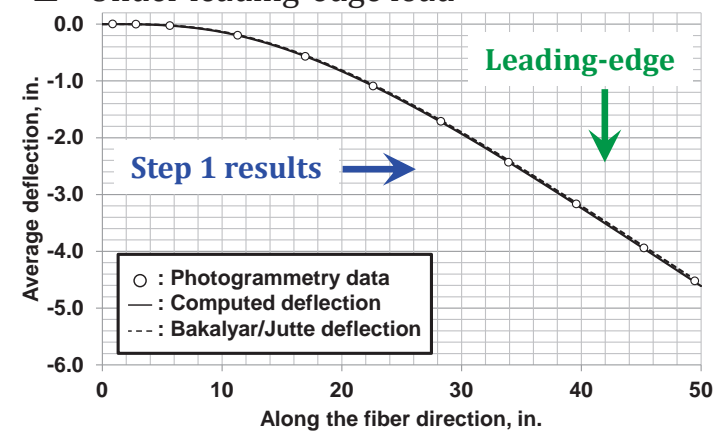


(b) Uniform load



# Swept Test Plate (continued)

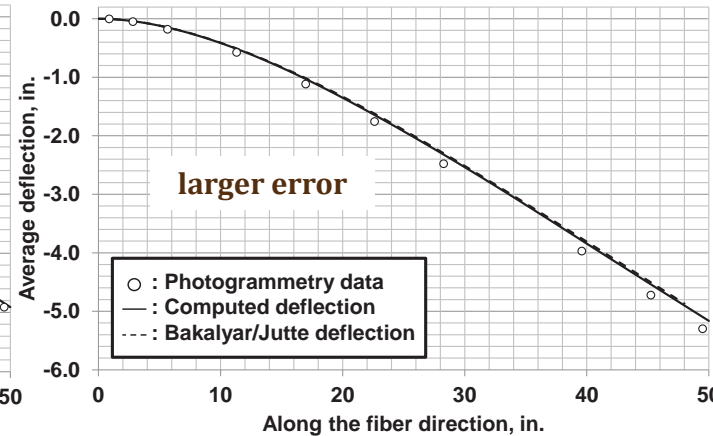
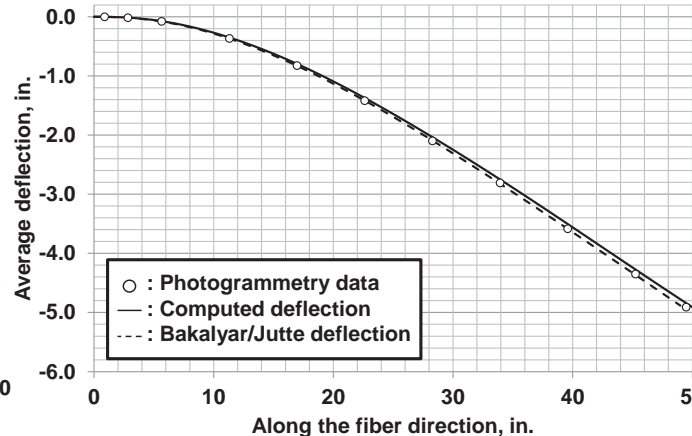
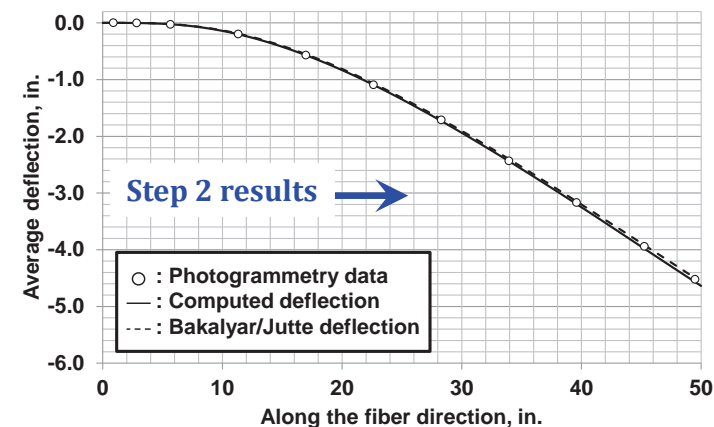
Under leading-edge load



a) Leading-edge fiber (step 1 result)

c) Mid-chord fiber (step 1 result)

e) Trailing-edge fiber (step 1 result)



b) Leading-edge fiber (step 2 result with 10 modes)

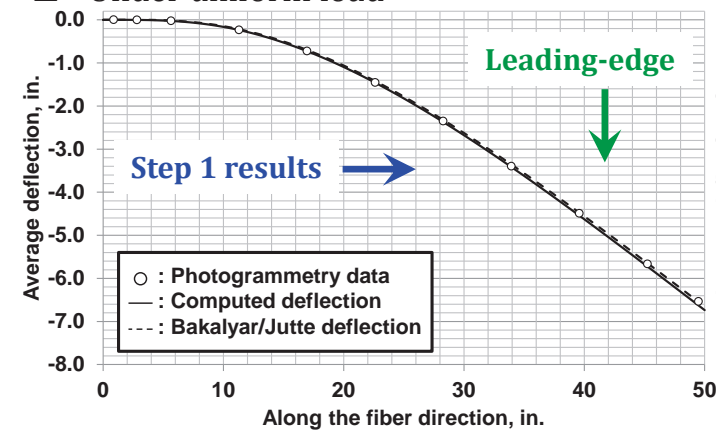
d) Mid-chord fiber (step 2 result with 10 modes)

f) Trailing-edge fiber (step 2 result with 10 modes)

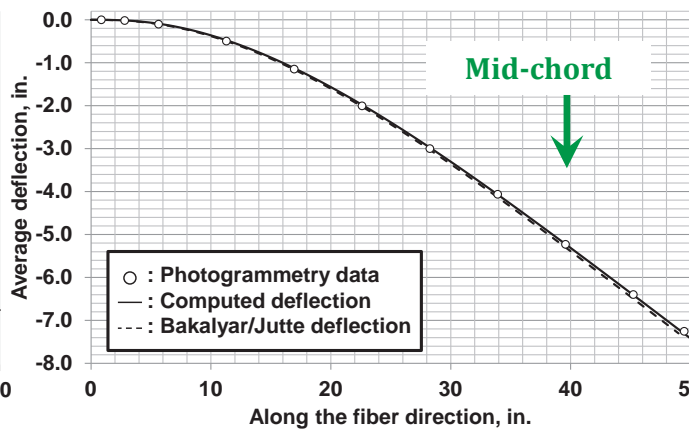


# Swept Test Plate (continued)

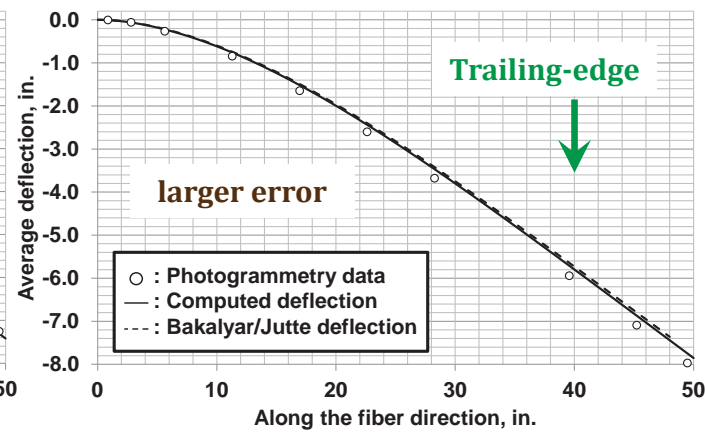
□ Under uniform load



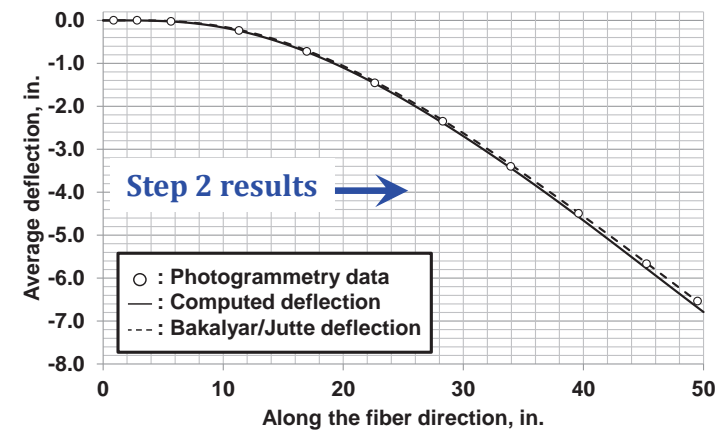
a) Leading-edge fiber (step 1 result)



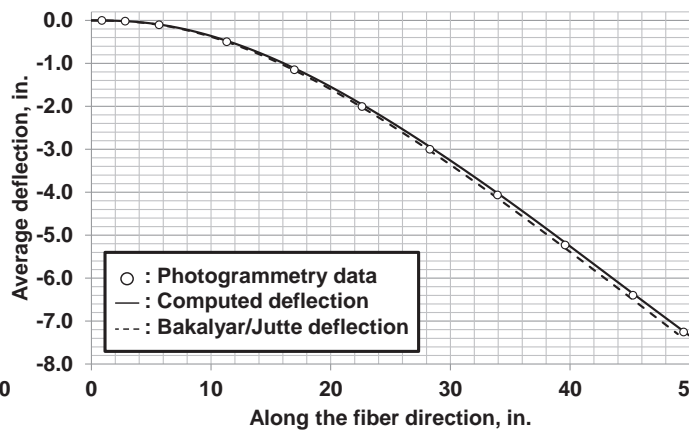
c) Mid-chord fiber (step 1 result)



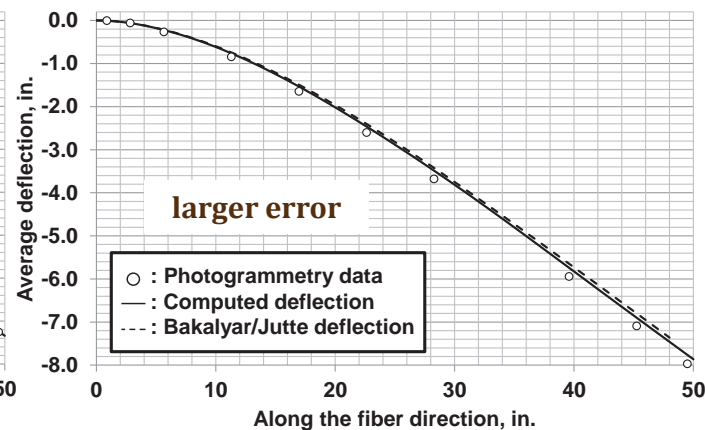
e) Trailing-edge fiber (step 1 result)



b) Leading-edge fiber (step 2 result with 10 modes)



d) Mid-chord fiber (step 2 result with 10 modes)



f) Trailing-edge fiber (step 2 result with 10 modes)





# Swept Test Plate (continued)

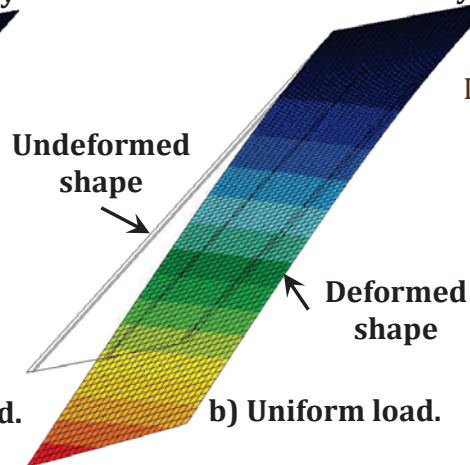
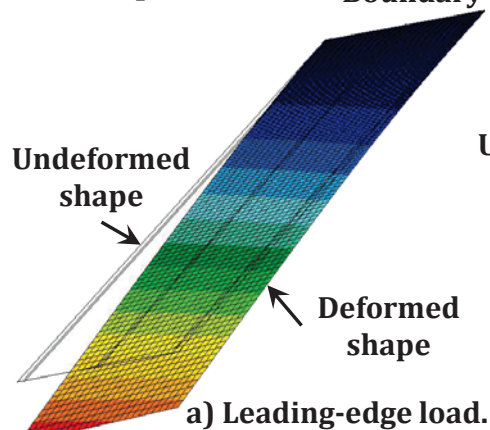
- Deformed wing shape (step 2 results with 10 modes)

	Measured (inch)	Computed (inch)			Relative error (%)		
		Bakalyar and Jutte*	Step 1	Step 2 with 10 modes	Bakalyar and Jutte	Step 1	Step 2 with 10 modes
Leading-edge load							
Leading edge fiber	-4.525	-4.500	-4.542	-4.569	-0.55	0.38	0.97
Middle fiber	-4.912	-4.952	-4.880	-4.843	0.81	-0.65	-1.40
Trailing edge fiber	-5.300	-5.067	-5.091	-5.097	-4.40	-3.90	-3.80
Uniform load							
Leading edge fiber	-6.541	-6.546	-6.630	-6.684	0.08	1.40	2.20
Middle fiber	-7.256	-7.408	-7.313	-7.238	2.10	0.79	-0.25
Trailing edge fiber	-7.971	-7.667	-7.750	-7.763	-3.80	-2.80	-2.60

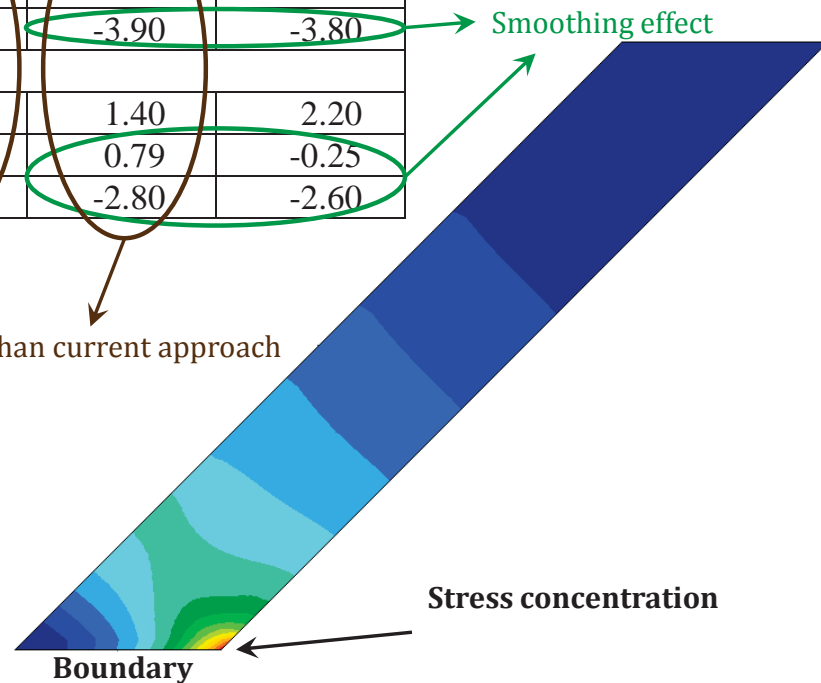
\*: extrapolated result

Boundary

Boundary



In general larger error than current approach





# Conclusions

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- ❑ The **two-step approach** for computing all the degrees of freedom in a structural FE model from measured strain along the FOSS is successfully applied to a cantilevered rectangular wing model and a test plate.
  
- ❖ The first experiment investigates the accuracy of the theory by applying it to a cantilevered rectangular wing model analyzed using the MSC/NASTRAN and ZAERO codes.
  - 1g uniform load case
  - Wing tip torsion load
  - Aerodynamic loading
  - ✓ **All six computed DOFs have excellent matching with target values.**
  
- ❖ The second experiment applies the theory to experimental data collected from a test plate fabricated and tested at the NASA AFRC.
  - **The deflections calculated from the experimental model are extremely accurate.**



# Questions ?

