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A Microwave Radiometric Method to Obtain the Average Path Profile of Atmospheric Temperature and Humidity Structure Parameters and its Application to Optical Propagation System Assessment

Free-Space Laser Communication and Atmospheric Propagation XXVII
Session 1: Atmospheric Propagation

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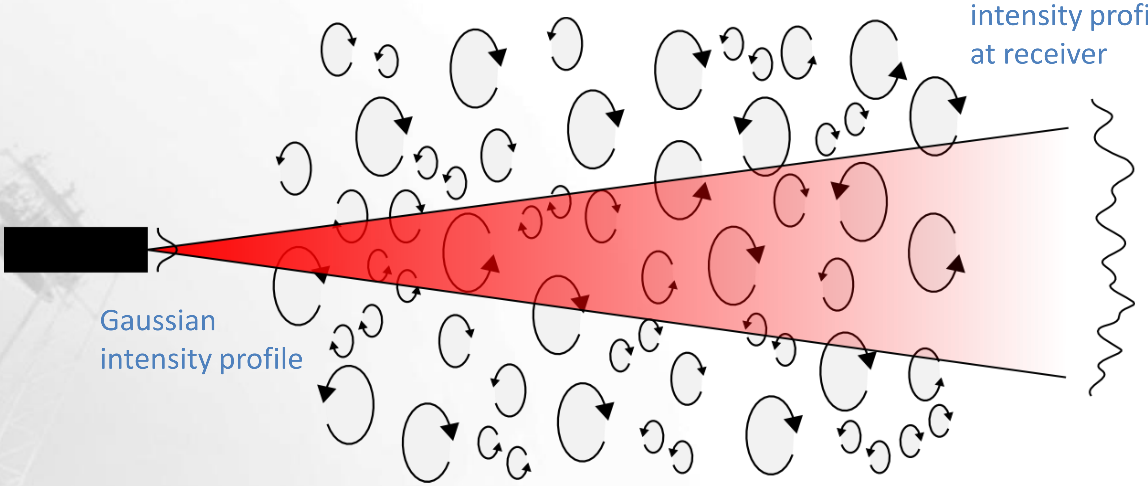
February 8, 2015 • *SPIE Photonics West* • *San Francisco, CA*

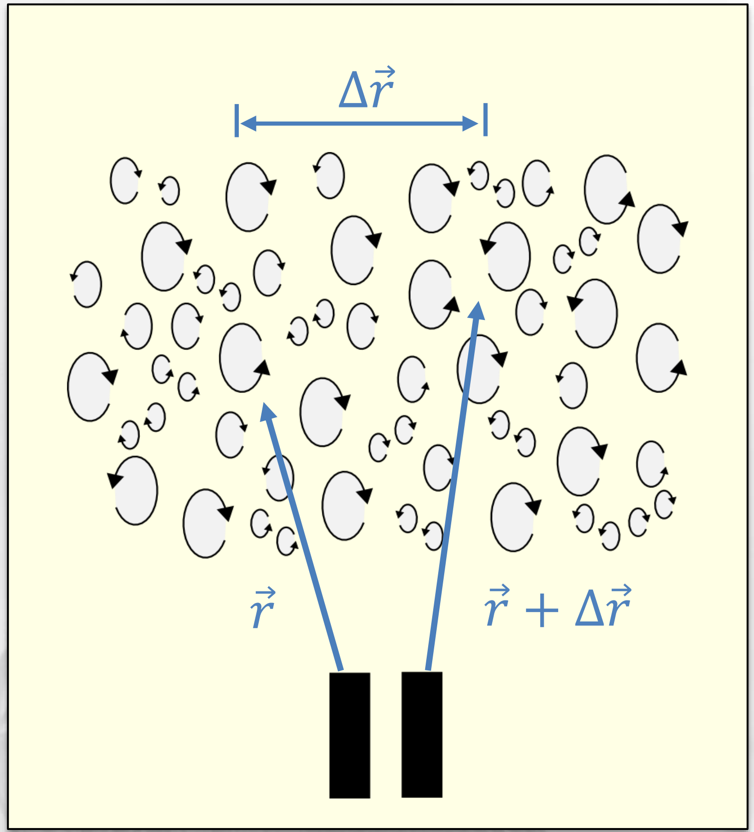
Turbulence Effects on Optical Beams

- Scintillation
- Beam broadening
- Spatial coherence
- Angle of arrival
- Temporal pulse stretching



Temporal and spatial intensity fluctuations at the receiving aperture results in power surges and fades

Turbulence Characterization



Requires simultaneous measurements at $\vec{r}$ and $\vec{r}+\Delta\vec{r}$

- Temperature (or humidity) structure function

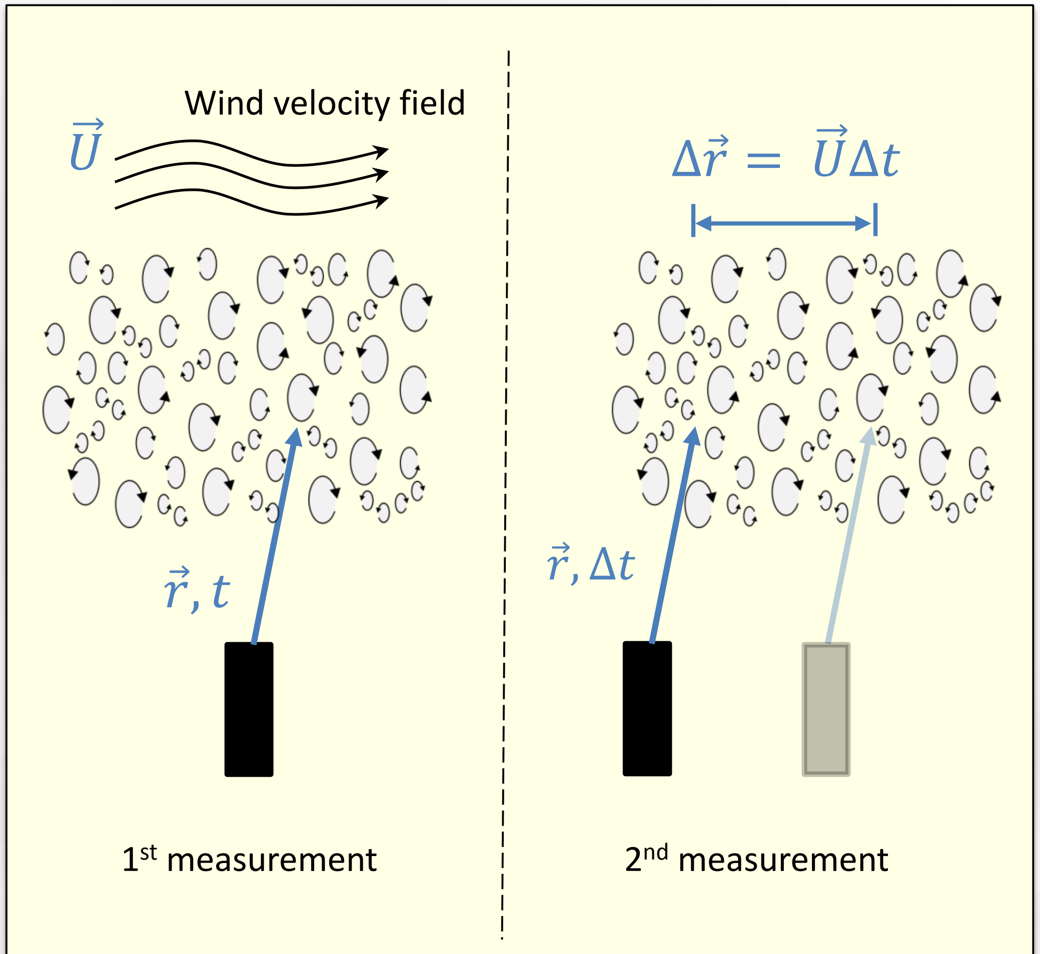
$$\boxed{D_T(\vec{r},\vec{r}+\Delta\vec{r}) = \left\langle \left[T(\vec{r}) - T(\vec{r}+\Delta\vec{r})\right]^2 \right\rangle}$$

- Contains the spatial statistics of the temperature field
- Within a range of certain $\Delta\vec{r}$, the well-known Kolmogorov "2/3" law holds

$$\boxed{D_T(\vec{r},\vec{r}+\Delta\vec{r}) = C_T^2 \left|\Delta\vec{r}\right|^{2/3}}$$

Temperature structure parameter

Single Radiometer Turbulence Characterization



- Temperature (or humidity) measurements are taken at a fixed location
- Wind velocity field shifts the turbulent air mass by a distance $\Delta\vec{r} = \vec{U}\Delta t$ between measurements
- For a particular altitude, the structure function is now

$$\boxed{D_T(t, t + U\Delta t) = \left\langle \left[T(t) - T(t + U\Delta t) \right]^2 \right\rangle}$$

Due to the radiometer integration time Δt, Kolmogorov-Obukhov turbulence theory and the Taylor frozen flow hypothesis must be modified

Energy Transfer Spectra

- From the Boussinesq approximation (eddy viscosity model) of the Navier-Stokes equations, it is possible to obtain equations involving the Fourier spectra of the turbulent energy, wind velocity and temperature fluctuations*

Energy spectrum related to Reynolds stresses

Buoyancy

Kinematic viscosity

Turbulent energy fluctuations spectrum

Energy transfer spectra due to wind velocity

$$\begin{cases} F(k) - \phi_{13}(k)\frac{d\bar{U}}{dz} + \beta\phi_{3T}(k) - 2\nu k^2\phi(k) = 0 \\ F_{TT}(k) - \phi_{3T}(k)\frac{d\bar{T}}{dz} - 2\nu_T k^2\phi_{TT}(k) = 0 \end{cases}$$

Energy transfer spectra due to temperature fluctuations

Energy spectrum related to heat flux

Thermal diffusivity

Temperature fluctuations spectrum

*Tchen, C.M., "On the Spectrum of Energy in Turbulent Shear Flow," *J. Res. Natl. Bureau of Standards* **50** (1), 51-62 (1953)

Modified Turbulence Spectrum

- Case 1: Near the boundary surface
  - Significant stratification and shear

$$\phi_{TT}(k) = Ak^{-1}$$

$$A = 2^{1/2}\left|\frac{d\bar{U}}{dz}\right|\left|\frac{d\bar{T}}{dz}\right|^{-2}\gamma^{-1}b^{-2}N^{2}\epsilon^{-1}$$

TKE dissipation

Heisenberg constant ≈ 1

Thermal diffusivity/ kinematic viscosity

Thermal dissipation

- Case 2: Free atmosphere
  - No stratification or shear

$$\phi_{TT}(k) = Bk^{-5/3}$$

$$B = \left(\frac{2}{3}\right) 4^{1/3}\gamma^{-2/3}b^{-1}N\epsilon^{-1/3}$$

- General:
  - Asymptotically reduces to either Case 1 or Case 2

$$V_{TT}(k) = \frac{AB}{B|k| + A|k|^{5/3}}$$

Relating to the Structure Functions

$$D_T(\Delta t) = 2\int_{-\infty}^{\infty}\left(1-\left\langle e^{-ik(\bar{U}+v)\Delta t}\right\rangle\right)V_{TT}(k)dk$$

Average wind velocity

Wind velocity fluctuations

The connection of the temporal statistics of the temperature $T(t)$ to the spatial spectrum $V_{TT}(k)$ is through the Fourier-Stieltjes transform

Evaluation of the integral is analytical in terms of Meijer G functions, however two useful series expansions can be obtained for the asymptotic cases

$$D_T(\Delta t) \approx \underbrace{C_T^2(\bar{U}\Delta t)^{2/3}}_{\text{Kolmogorov “2/3” law}}\left[1-0.11\frac{\langle v^2\rangle}{\bar{U}^2}\right]$$

$\bar{U}\Delta t \ll 1$

Crossover frequency
$\boldsymbol{k_C = (B/A)^{3/2}}$

$\bar{U}\Delta t \gg 1$

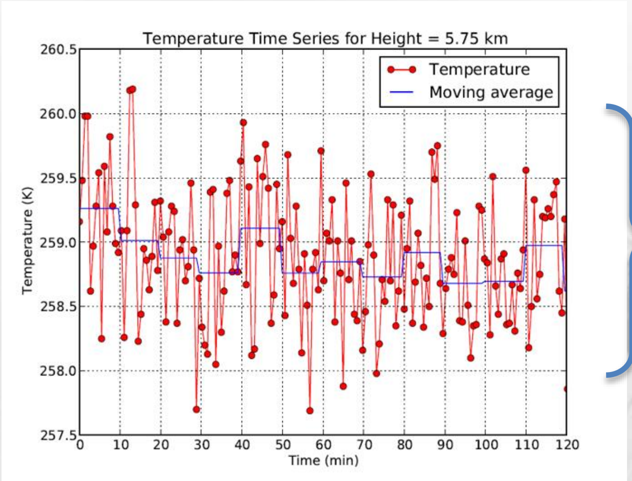
$$D_T(\Delta t) \approx \frac{C_T^2}{4}k_C^{-2/3}\left[0.57722+\log(k_C)+\log(\bar{U}\Delta t)-\frac{1}{2}\frac{\langle v^2\rangle}{\bar{U}^2}\right]$$

Instrumentation

- NASA TDRSS ground terminal site located at White Sands, NM
- Radiometrics MP-3000A
- 35 calibrated channels
  - 300 MHz bandwidth/channel
  - 21 K-band (22 to 30 GHz)
  - 14 V-band (51 to 59 GHz)
- 1.1 second integration time per channel
- Total $\Delta t \approx 40$ second sample period
- Temperature resolution ≈ 0.1 K

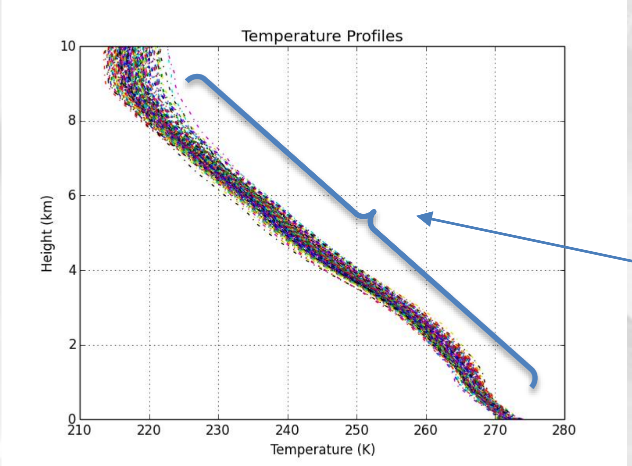
Temperature Data Analysis

Example fluctuations over a 2-hour time period



Fluctuation standard deviation, σ ≈ 0.5 K – 3 K

Average profiles for each 10 min window

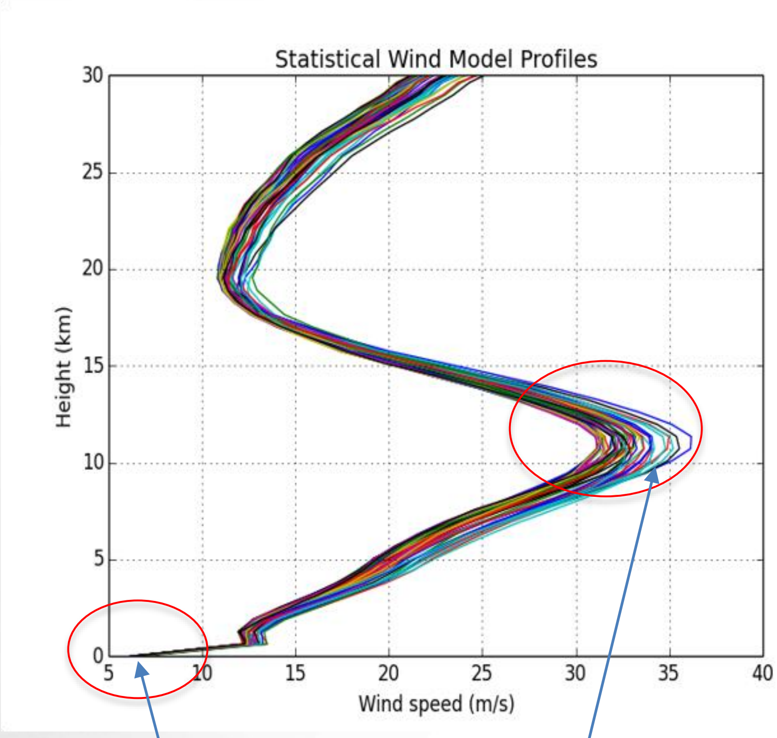


Average lapse rate of ≈ -5.474 K/km

- Measurements taken in January, 2013
- Dataset comprised of about 2100 profiles taken over a 24-hour period
- Each temperature time series divided into 10-minute moving average windows

Statistical Wind Model

- Vertical profiles of horizontal wind speed
- SPARC Data Center High-resolution radiosonde measurements at Santa Theresa, NM
- Statistics derived from 2376 wind profiles
- Principal component analysis (PCA) used for data reduction and retention of key features of the wind behavior

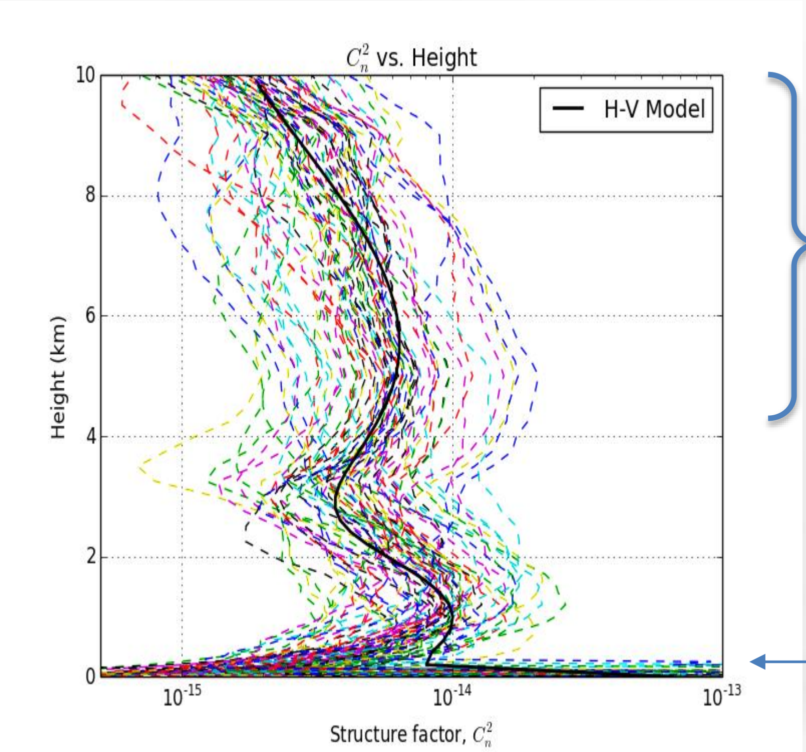


Ground wind speeds typically between 5 and 8 m/s

Max tropopause wind speeds typically between 30 and 40 m/s

Results for C_n^2

Refractive index structure parameter profiles



$$HV(h) = Ae^{-\frac{h}{H_A}} + Be^{-\frac{h}{H_B}} + Ch^{10}e^{-\frac{h}{H_C}} + De^{-\frac{(h-H_D)^2}{2d^2}}$$

General Hufnagel-Valley model

Free atmospheric estimates of C_n^2 about 10x – 100x larger than expected

Ground estimates of $C_n^2 \approx 10^{-13}$ m$^{-2/3}$

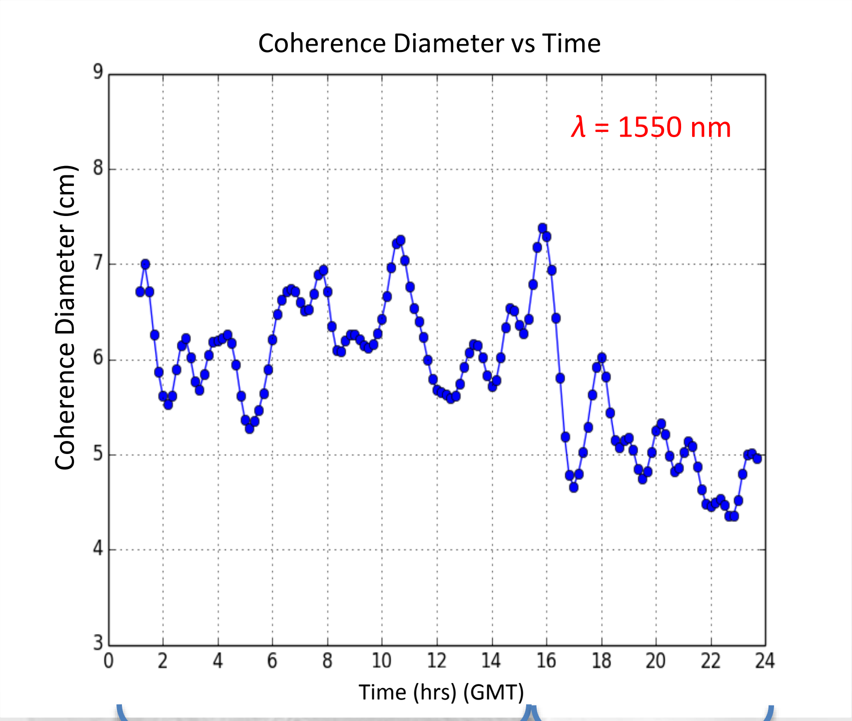
- At optical wavelengths the refractive index structure parameter is a function of C_T^2 only

$$C_n^2 = \left(10^{-6} \times \frac{77.689\langle P \rangle}{\langle T \rangle^2}\right)^2 C_T^2$$

Average prevailing pressure (hPa)

Specific atmospheric conditions during data compilation were not available, thus a nominal value of $k_c = 15\ \mathrm{m}^{-1}$ was assumed

Results – Coherence Diameter



Night time variation between 5.2 cm – 7.5 cm

Day time variation between 4.3 cm – 6 cm

- Coherence diameter, also known as the Fried parameter

$$r_0 = \left[0.423k^2 \int_0^H C_n^2(h)dh)\right]^{-3/5}$$

- Determines resolution limitations of telescopes

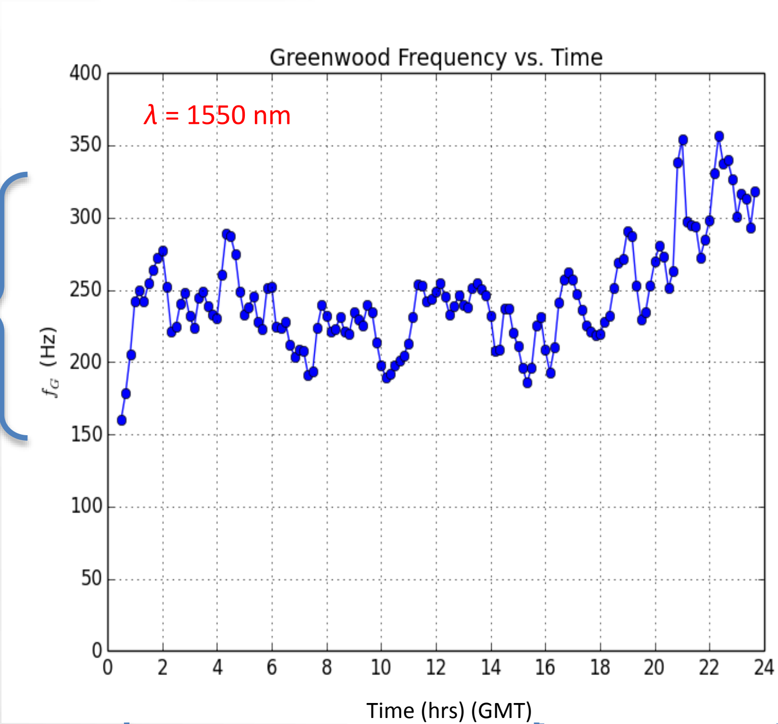
$D > r_0$ ⇒	atmosphere limited
$D < r_0$ ⇒	diffraction limited

- Also determines the spacing of actuators in adaptive optical systems

Results - Greenwood Frequency

- The Greenwood frequency specifies the response characteristic required of an atmospheric adaptive optics system to mitigate the refractive index perturbations

$$f_G = 0.255 \left[k^2 \int_0^H C_n^2(h) \left(\bar{U}(h)\right)^{5/3} dh \right]^{3/5}$$

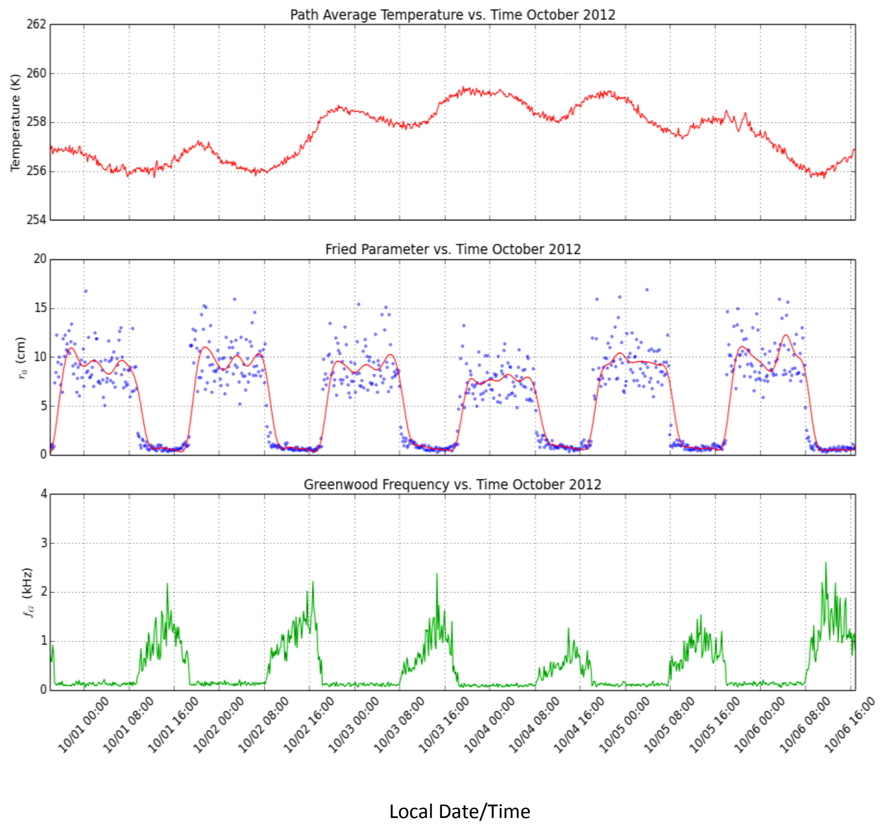


These values are about a factor of 3 larger than expected for the experimental site in January

Night time variation between 150 – 290 Hz

Day time variation between 200 – 350 Hz

Results – Five Days in October 2012



- Diurnal variations are easily resolved
- Verification that the resolution requirements of the radiometer are sufficient for this method

Coherence diameter varies between about 1 cm (day) and 10 – 18 cm (night)

Greenwood frequency varies between ≈ 2.5 kHz (day) and 200 Hz (night)

Summary

- Atmospheric remote sensing method using a single microwave profiling radiometer to obtain temperature and humidity turbulence structure parameters
- Augmented Kolmogorov turbulence theory to account for boundary effects in a general stratified atmosphere
- Test case shows promising results; however Greenwood frequencies and coherence diameters are over/under estimated
- A more rigorous turbulence spectrum derivation is required
- Ground-based measurements of the gradient Richardson numbers are required for better estimation of the crossover frequency k_c
- Concurrent radiosonde measurements of structure parameters along with the radiometer is needed for appropriate comparison