Recalibration of the Shear Stress Transport Model to Improve Calculation of Shock Separated Flows

Nicholas J. Georgiadis and Dennis A. Yoder
NASA Glenn Research Center

6th Annual Shock Wave/Boundary Layer Interaction (SWBLI)
Technical Interchange Meeting
April 2013

Material presented here is also in AIAA–2013–0685

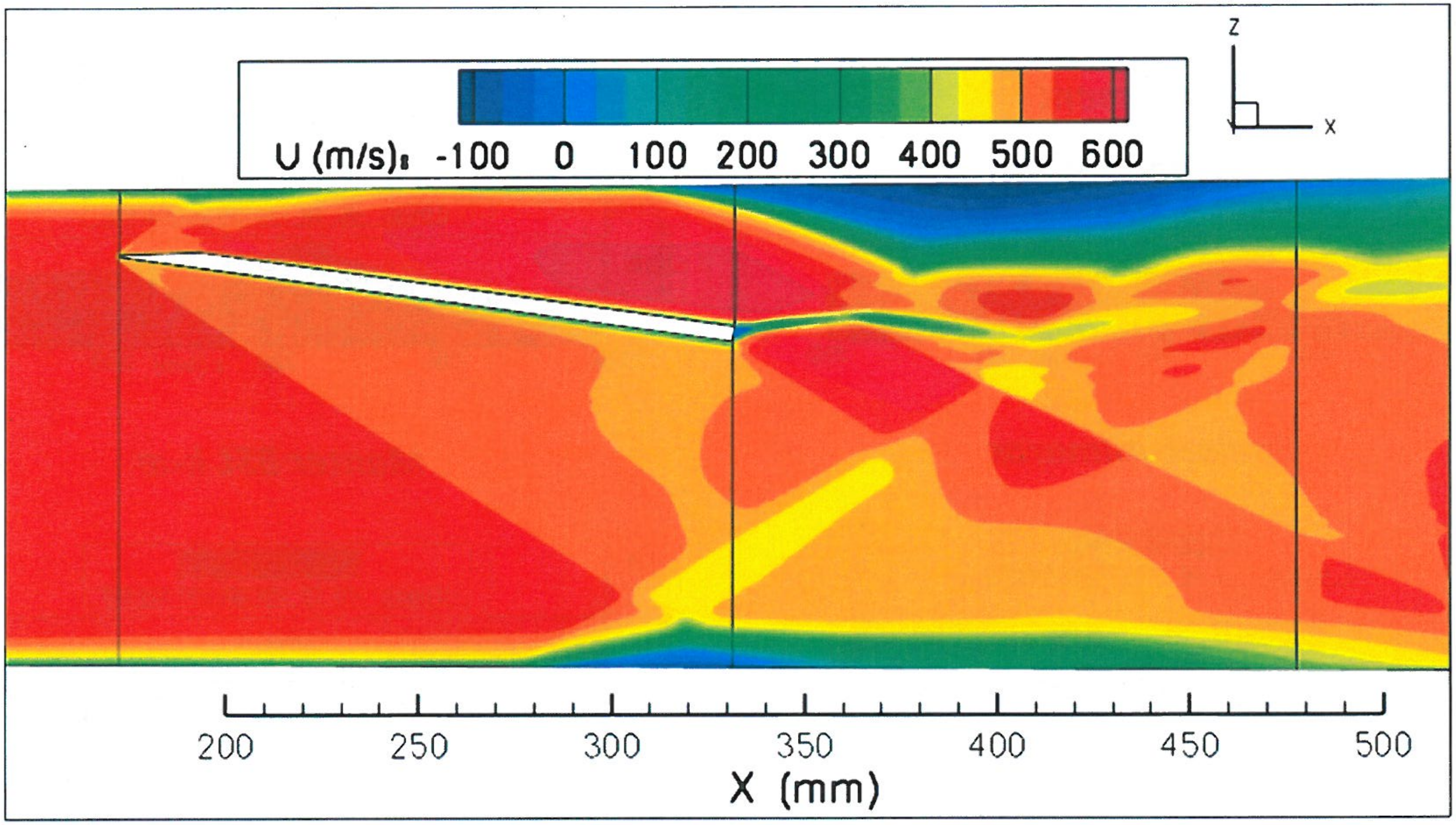
Motivation

- Shock-Wave / Turbulent Boundary Layer Interactions (SWTBLIs) are pervasive to all supersonic vehicles – external aerodynamics, inlets, isolators, etc.
- Accurate prediction of SWTBLIs with CFD remains elusive.
- Results from the 2010 AIAA SWTBLI workshop indicated deficiencies in RANS turbulence models for shock-separated flows.
- Some promise using LES/DNS, but these methods are not yet ready for engineering applications.

UFAST SWBLI Test Case

2010 AIAA SWTBLI Workshop

- Mach 2.25 flow approaching SWTBLI region.
- Several RANS and LES (including hybrid RANS-LES) solutions submitted; most widely used RANS turbulence models were SST and SA.

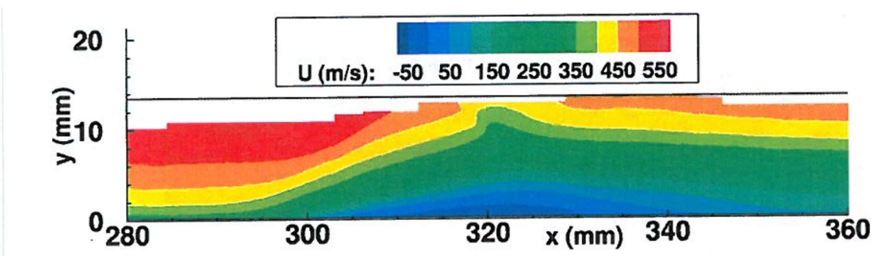


Motivation

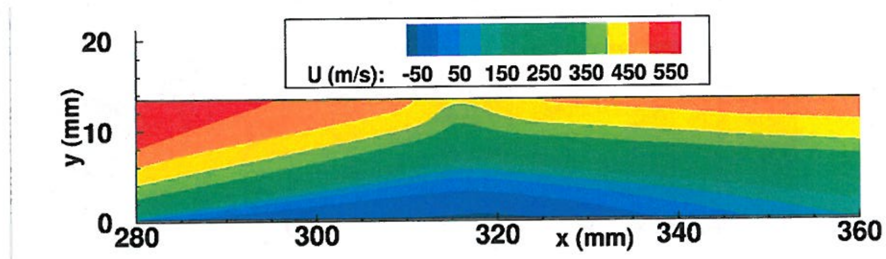
- 2010 AIAA SWTBLI workshop results: Most popular turbulence models utilized were Menter SST and Spalart-Allmaras.
- SST and BSL produce results on either side of experimental data for this case (UFAST) and most other SWTBLI cases we have examined.

UFAST Mach 2.25 SWTBLI Test Case:

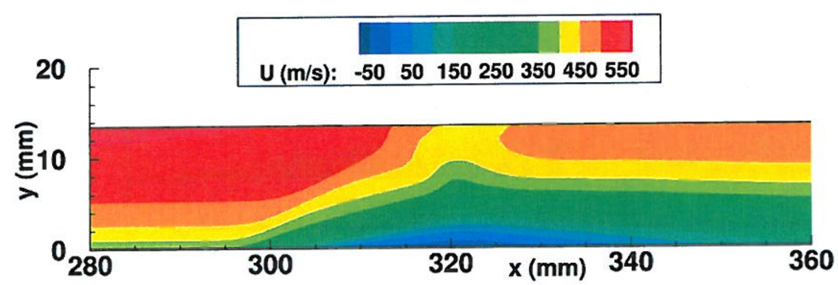
Experiment



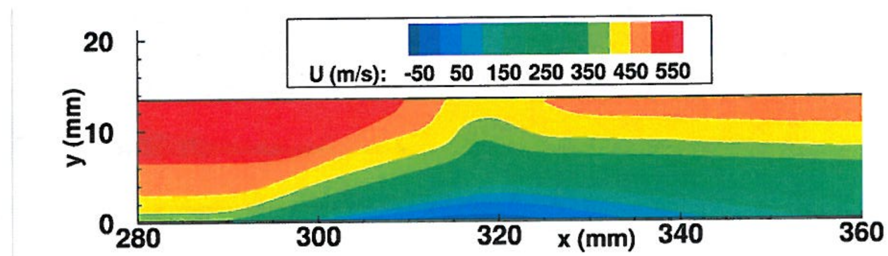
Menter SST k-ω



Spalart-Allmaras



Menter BSL k-ω



Presentation Outline

- Motivation
- Comparison of BSL and SST Models
- Investigation of Shear Stress Limiter
- Incompressible / Low-Speed Test Cases*:
  1. Flat Plate – zero pressure gradient boundary layer.
  2. Driver Axisymmetric Diffuser – adverse pressure gradient with separation.
  3. Backward Facing Step
- SWTBLI Cases:
  1. UFAST – Mach 2.25
  2. Schulein – Mach 5
  3. HIFiRE Scramjet Flow Paths – Mach 5.8 and 8.0 flight conditions
- Conclusions

** Grids and boundary conditions for incompressible test cases are taken from the Turbulence Model Benchmarking Working Group (TMBWG) website: turbmodels.larc.nasa.gov.*

Menter SST and BSL Models

- Baseline (BSL) model combined:
  1. Wilcox 1988 k-ω model – inner model.
  2. Jones Launder k-ε model – transformed to k-ω equation form – outer model.
  - F_1 function transitions from inner model to outer model at approximately 2/3 boundary layer thickness.
- Shear Stress Transport (SST) model is an extension of BSL model:
  1. Diffusion coefficient, σ_k, changed from 0.5 to 0.85 for inner model.
  2. Limiter placed on turbulent shear stress to not exceed 0.31 x turbulent kinetic energy (TKE). The motivation is to account for "transport of shear stress."

- The limiter originates from the observation of Bradshaw and others that -u'v' does not exceed 30% x TKE
- "Townsend structure parameter" = a_1 = -u'v' / k
- SST model setting a_1= 0.31 works well for low speed cases including mild adverse pressure gradient flows....optimal for higher speed?

Menter Two-Equation k-ω Baseline (BSL) Model

$$\frac{D(\rho k)}{Dt} = \tau_{ij}\frac{\partial u_i}{\partial x_j} - \beta^* \rho\omega k + \frac{\partial}{\partial x_j}\left[(\mu + \sigma_k \mu_t)\frac{\partial k}{\partial x_j}\right]$$

$$\frac{D(\rho\omega)}{Dt} = \frac{\gamma}{\upsilon_T}\tau_{ij}\frac{\partial u_i}{\partial x_j} - \beta\rho\omega^2 + \frac{\partial}{\partial x_j}\left[(\mu + \sigma_\omega \mu_t)\frac{\partial \omega}{\partial x_j}\right] + (1 - F_1)2\rho\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}$$

$$\boxed{\mu_t = \frac{\rho k}{\omega}}$$

$$CD_{kw} = \max(2\rho\sigma_{\omega 2}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}; 1\times 10^{-20})$$

$$F_1 = \tanh\left[(\arg_1)^4\right]$$

$$\arg_1 = \min\left[\max\left(\frac{k^{1/2}}{\beta^* \omega y}; \frac{500\upsilon}{\omega y^2}\right); \frac{4\rho\sigma_{\omega 2} k}{CD_{k\omega} y^2}\right]$$

$$\gamma = \frac{\beta}{\beta^*} - \frac{\sigma_\omega \kappa^2}{\sqrt{\beta^*}}$$

Constants: $\boldsymbol{\sigma_{k1} = 0.5}$, $\sigma_{\omega 1} = 0.5$, $\beta_1 = 0.075$
$\sigma_{k2} = 1.00$, $\sigma_{\omega 1} = 0.856$, $\beta_1 = 0.0828$
$a_1 = 0.31$, $\kappa = 0.41$, $\beta^* = 0.09$

Menter Two-Equation k-ω "Shear Stress Transport" (SST) Model

$$\frac{D(\rho k)}{Dt} = \tau_{ij}\frac{\partial u_i}{\partial x_j} - \beta^* \rho \omega k + \frac{\partial}{\partial x_j}\left[(\mu + \sigma_k \mu_t)\frac{\partial k}{\partial x_j}\right]$$

$$\frac{D(\rho\omega)}{Dt} = \frac{\gamma}{\upsilon_T}\tau_{ij}\frac{\partial u_i}{\partial x_j} - \beta\rho\omega^2 + \frac{\partial}{\partial x_j}\left[(\mu + \sigma_\omega \mu_t)\frac{\partial \omega}{\partial x_j}\right] + (1-F_1)2\rho\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}$$

$$\mu_t = \min\left(\frac{\rho k}{\omega}; \frac{a_1 \rho k}{\Omega F_2}\right)$$

$$CD_{kw} = \max(2\rho\sigma_{\omega 2}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}; 1\times 10^{-20})$$

$$F_1 = \tanh\left[(\arg_1)^4\right] \qquad F_2 = \tanh\left[(\arg_2)^2\right]$$

$$\arg_1 = \min\left[\max\left(\frac{k^{1/2}}{\beta^*\omega y}; \frac{500\upsilon}{\omega y^2}\right); \frac{4\rho\sigma_{\omega 2}k}{CD_{k\omega}y^2}\right] \qquad \arg_2 = \min\left(2\frac{k^{1/2}}{\beta^*\omega y}; \frac{500\upsilon}{\omega y^2}\right)$$

$$\gamma = \frac{\beta}{\beta^*} - \frac{\sigma_\omega \kappa^2}{\sqrt{\beta^*}}$$

Constants: $\sigma_{k1} = 0.85$, $\sigma_{\omega 1} = 0.5$, $\beta_1 = 0.075$
$\sigma_{k2} = 1.00$, $\sigma_{\omega 1} = 0.856$, $\beta_1 = 0.0828$
$a_1 = 0.31$, $\kappa = 0.41$, $\beta^* = 0.09$

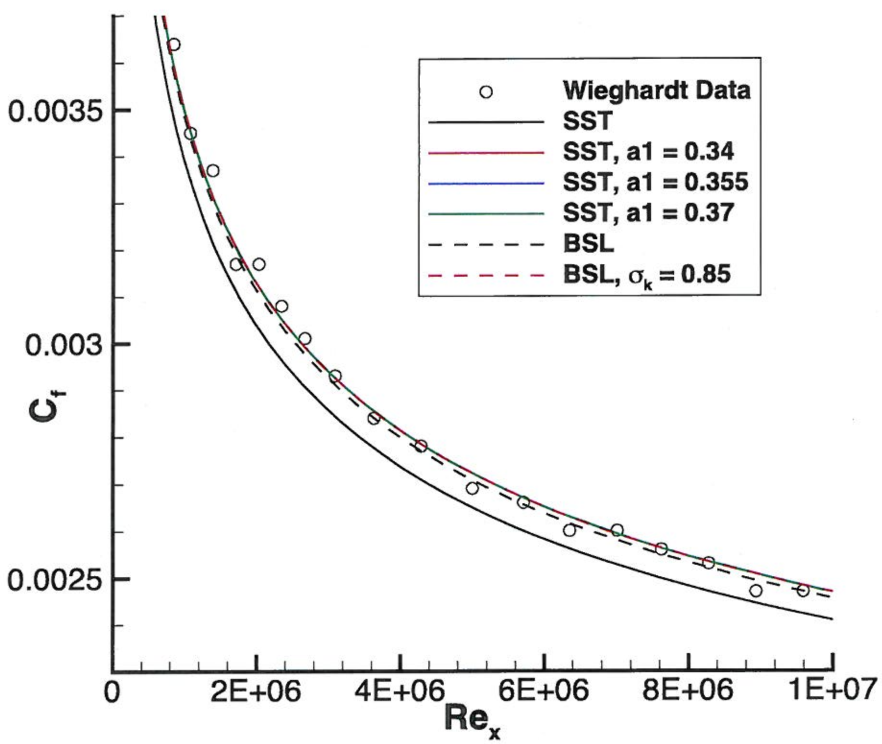
Investigation of Shear Stress Limiter

- The original SST model has become one of the most widely RANS used turbulence models.
- Shear stress limiter sets $a_1 = 0.31 = -u'v'/k$, using observations of Bradshaw, Townsend, and others -- for zero pressure gradient and mild adverse gradient flows.
- This limiter is only active in inner ¾ of boundary layer – via F_2 function.
- Experimental data (UFAST, Smits et al, others) shows $-u'v'/k$ exceeds 0.31 in SWTBLI flows.
- Others (Wilcox, Tan and Jin, Edwards) have investigated values for a_1 greater than 0.31.
- **This work:**
  1. **Investigate a range of a_1 from 0.31 to 0.40. (larger values for a_1 are very similar to BSL)**
  2. **Examine details of experimental turbulent measurements alongside computations to determine appropriate value(s) for a_1 in SWTBLI flows.**

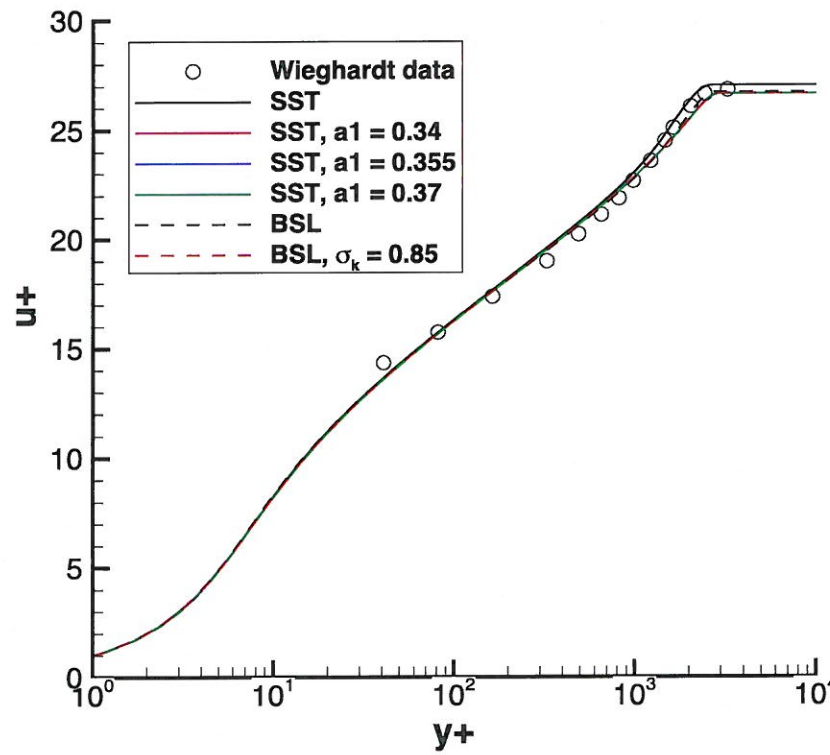
Mach 0.2 Boundary Layer – Zero Pressure Gradient

- Grid, BC's from TMBWG website.

Wall Skin Friction:

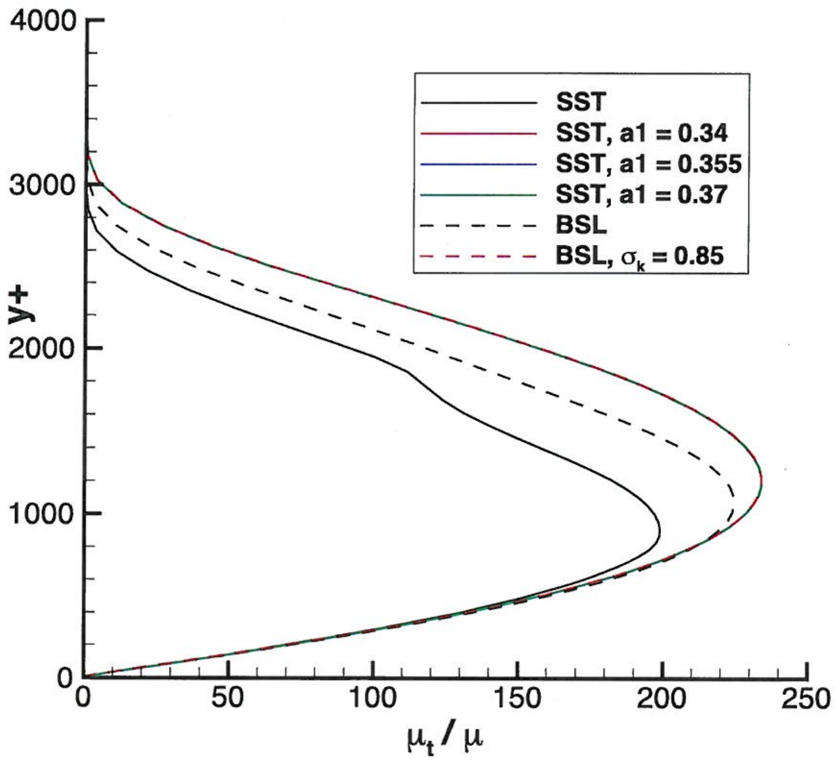


Mean velocity (Re_x = 4.3e6):

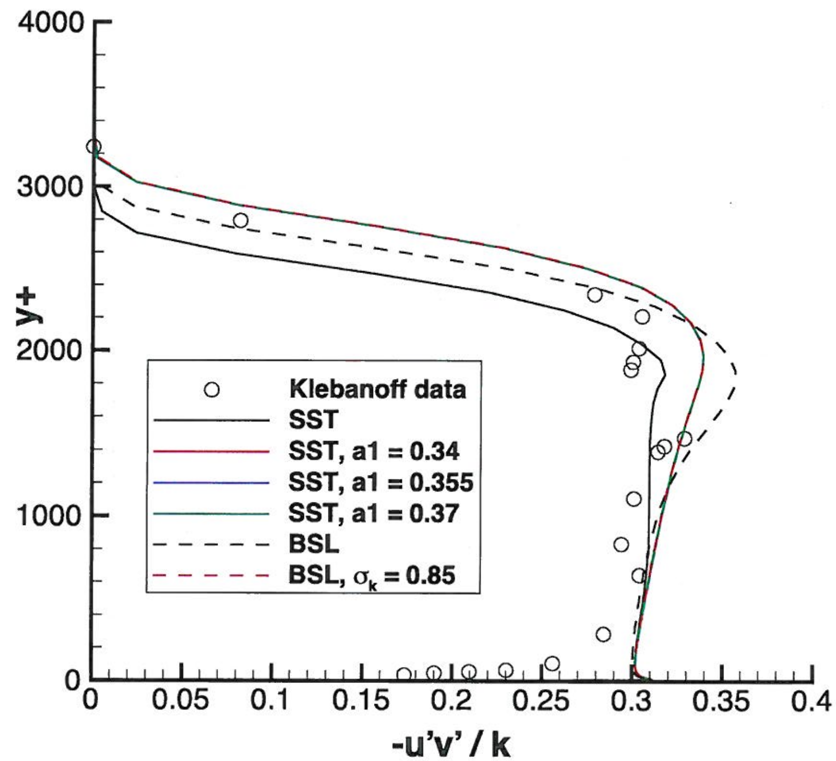


Mach 0.2 Boundary Layer – Zero Pressure Gradient

Eddy Viscosity (Re_x = 4.2e6):



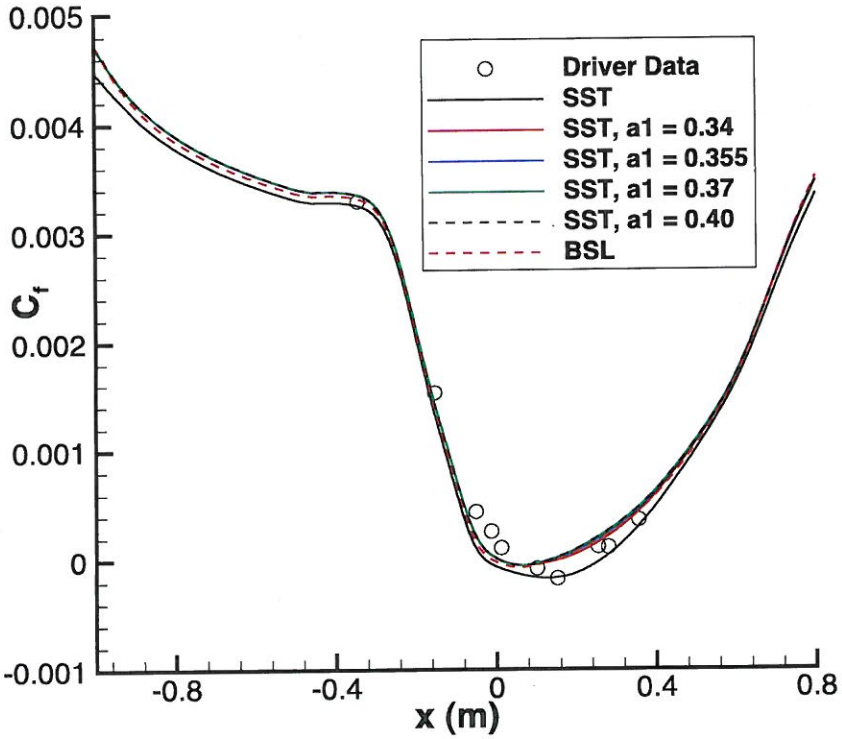
Shear Stress (Re_x = 4.2e6):



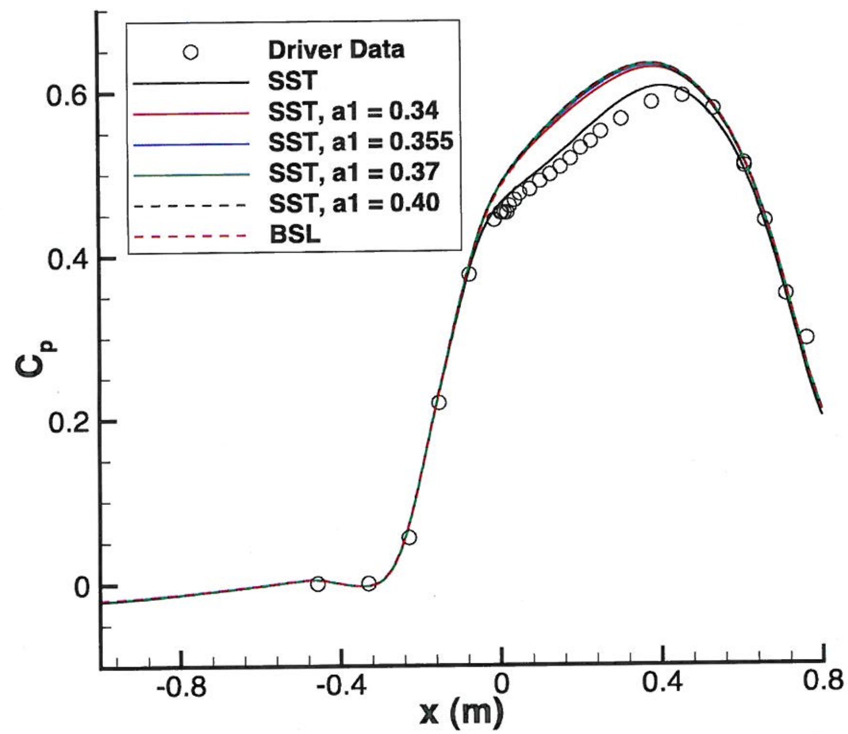
Driver Axisymmetric Diffuser

- Grid, BC's from TMBWG website.
- Original flow case was axisymmetric diffusing geometry within rectangular wind tunnel. Menter (*AIAA J. 1994*) defined an axisymmetric streamline to simplify calculations.

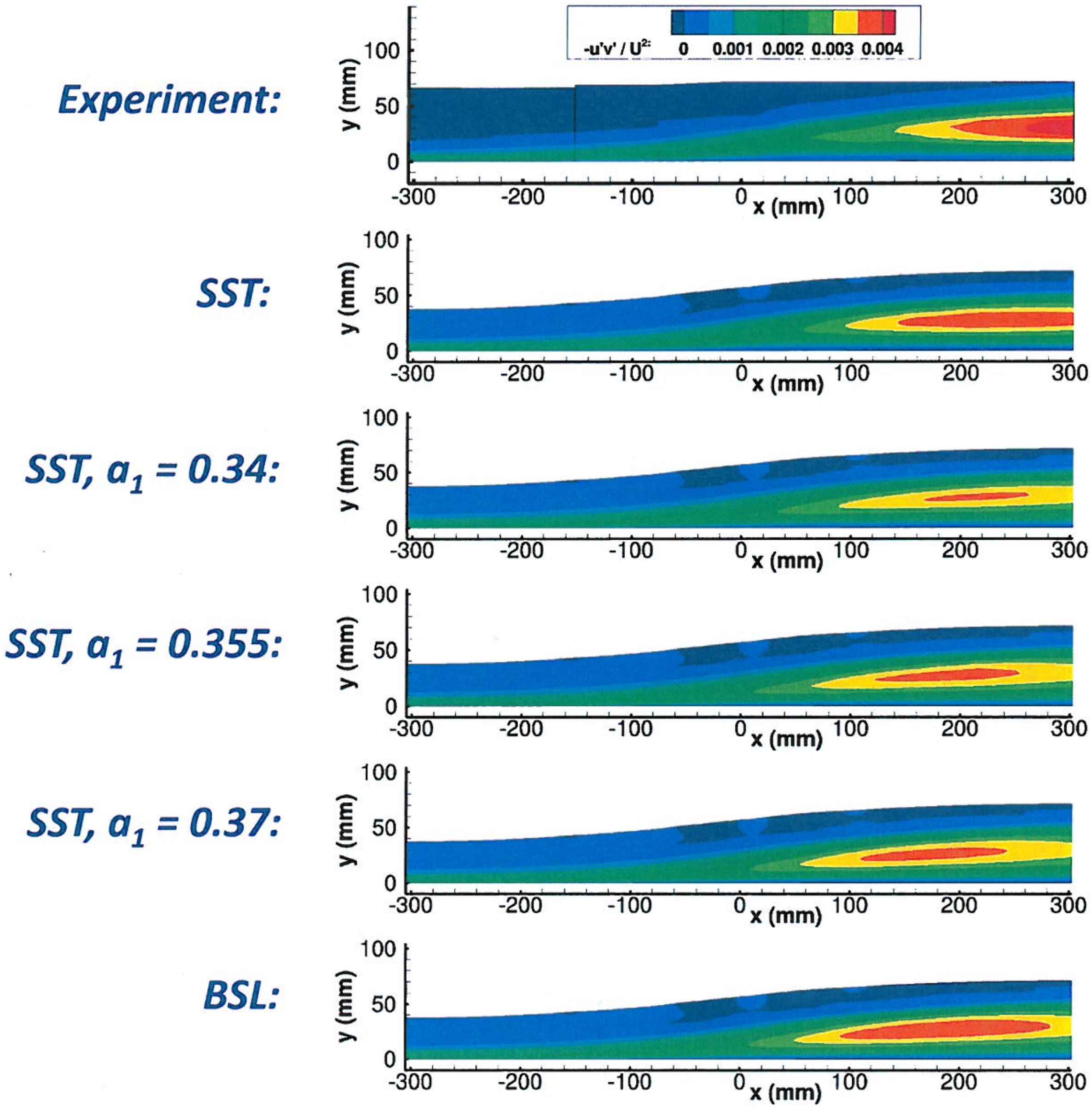
Wall Skin Friction:



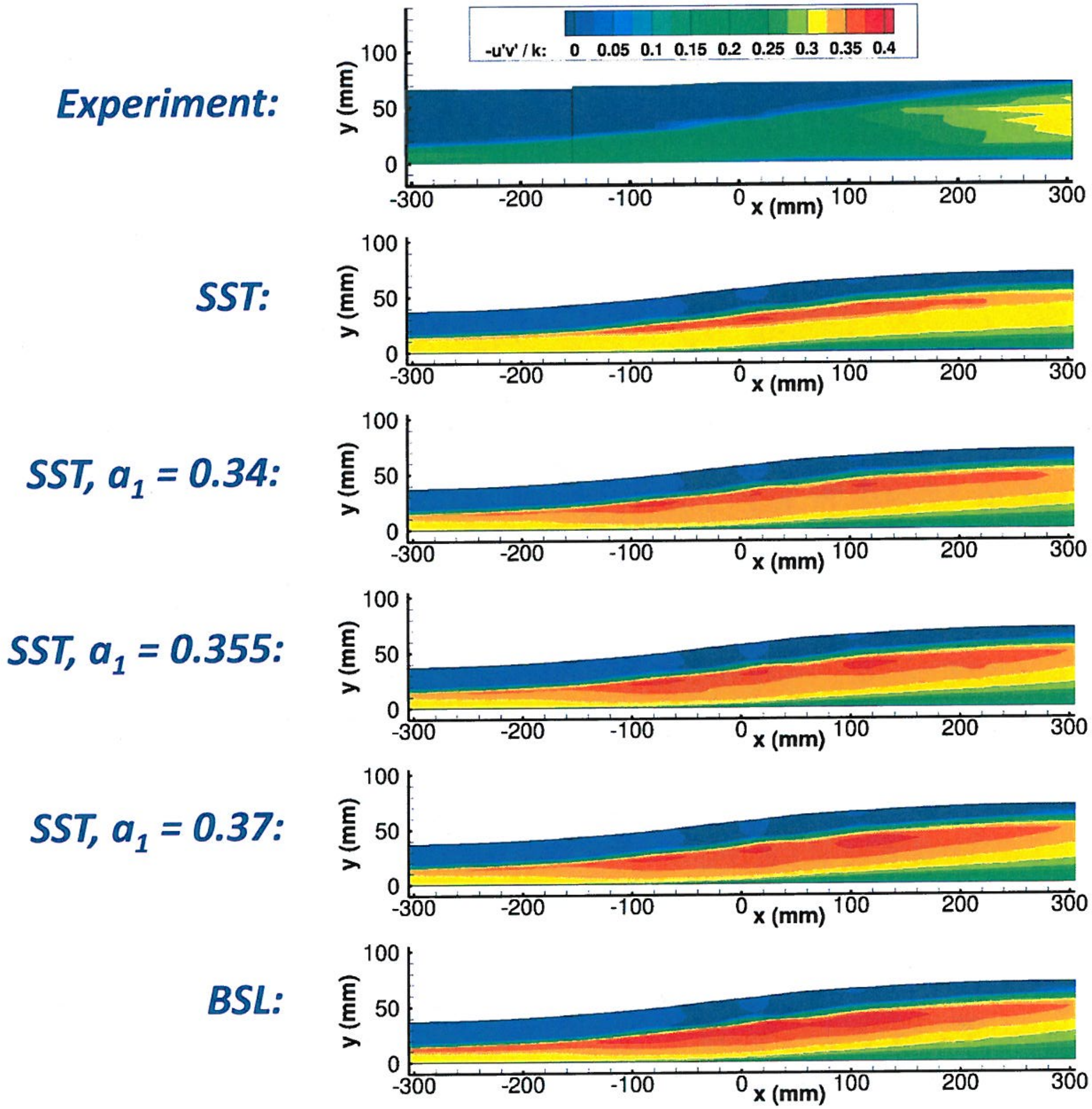
Wall Static Pressure:



Driver Diffuser – Turbulent Shear Stress



Driver Diffuser – Structure Parameter = -u'v'/k



Driver Diffuser – Turbulent Kinetic Energy

Experiment:

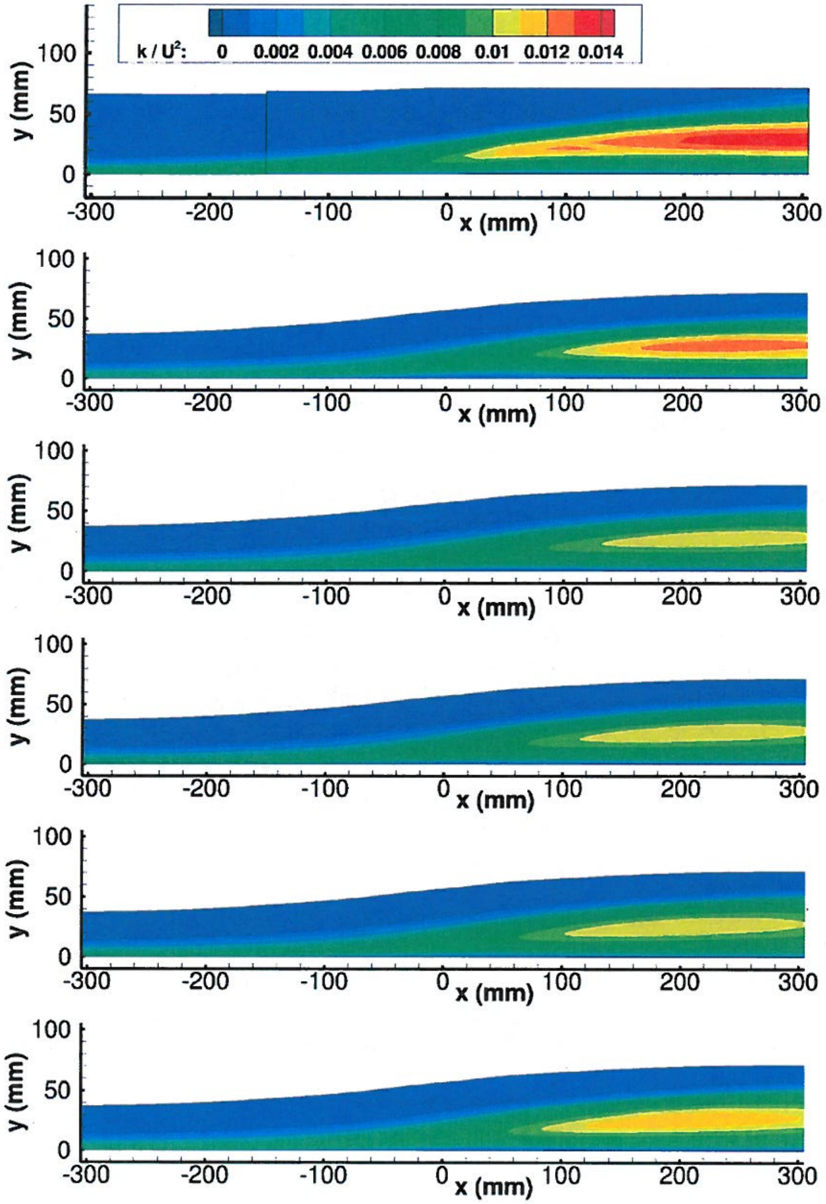
SST:

SST, a_1 = 0.34:

SST, a_1 = 0.355:

SST, a_1 = 0.37:

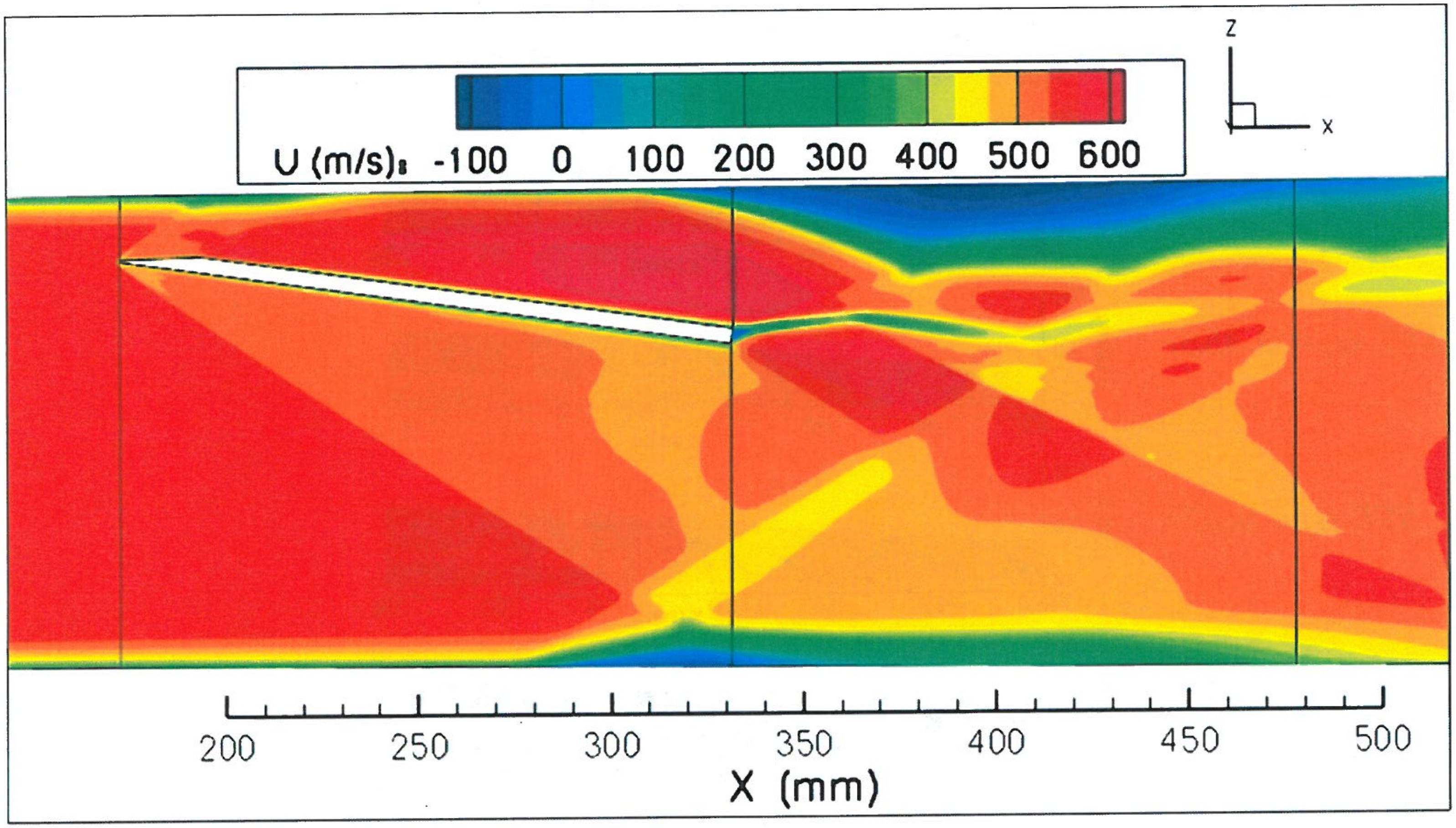
BSL:



UFAST SWBLI Test Case

2010 AIAA SWTBLI Workshop

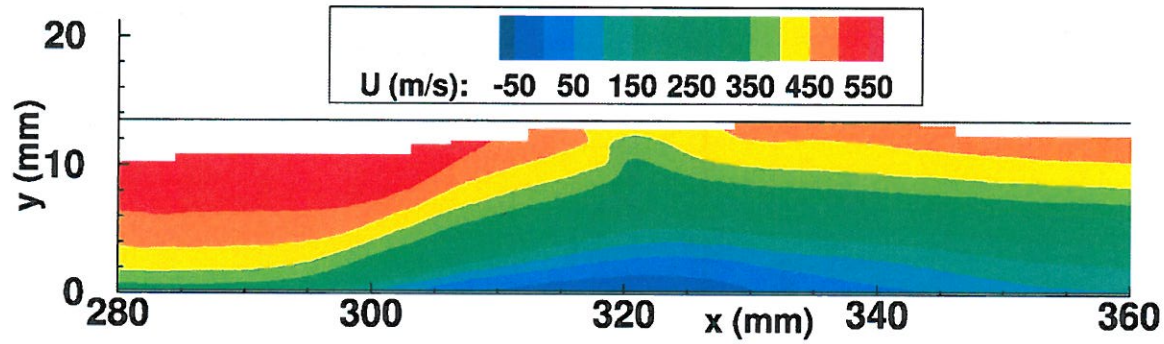
- Mach 2.25 flow approaching SWTBLI region.
- Several RANS and LES (including hybrid RANS-LES) solutions submitted; most widely used RANS turbulence models were SST and SA.



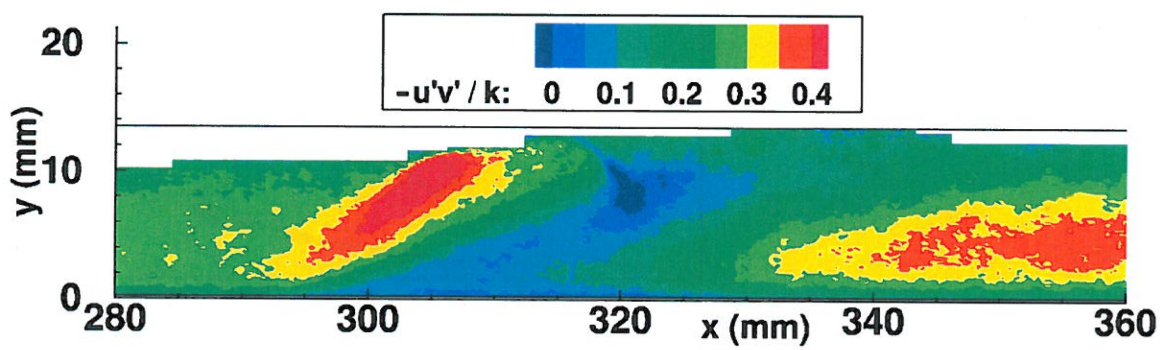
Structure Parameter – UFAST Experimental Data

- Turbulent shear stress exceeds 0.35 x TKE at beginning of interaction region and in region where boundary layer reattaches.

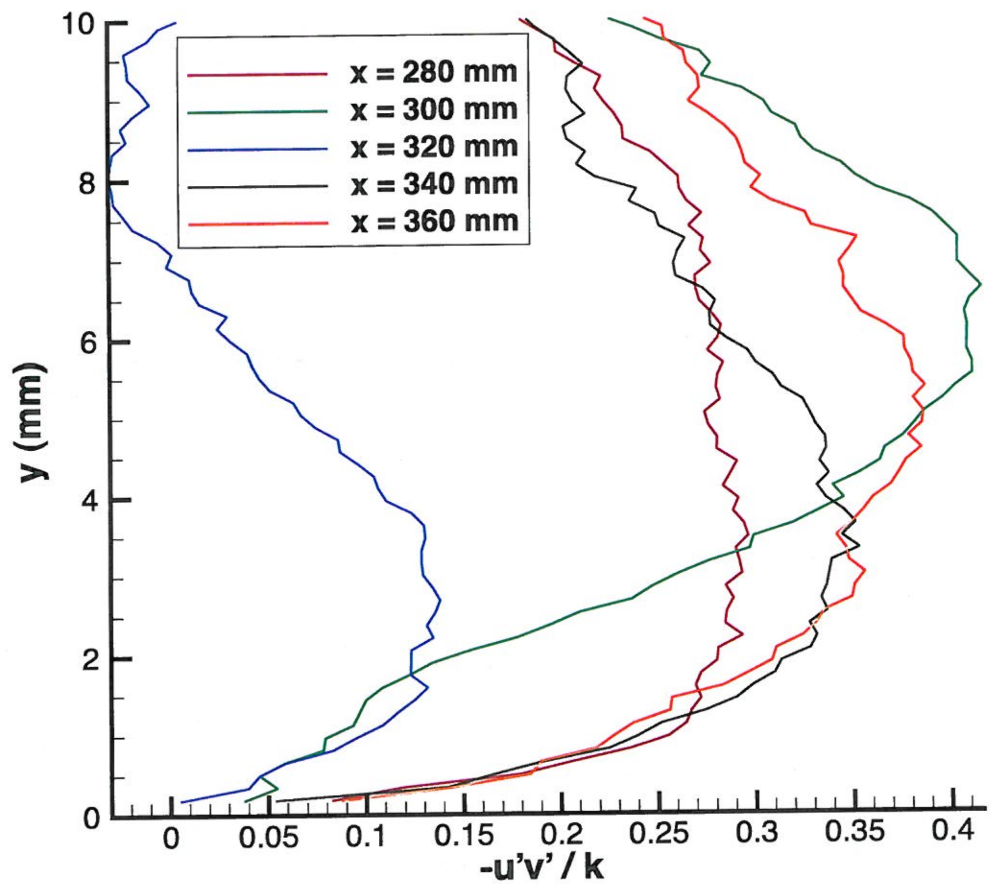
Mean velocity:



Structure parameter:

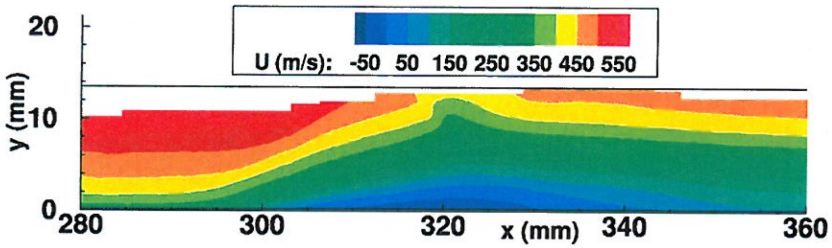


Structure Parameter – UFAST Experimental Data

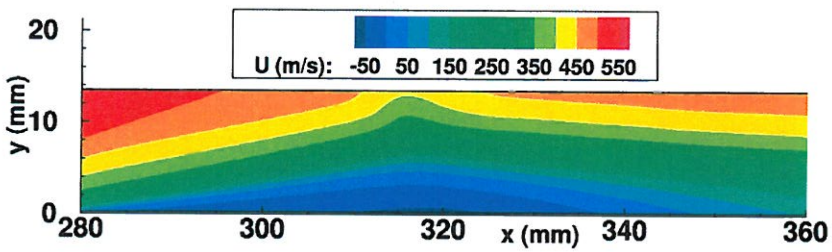


UFAST Velocity Contours

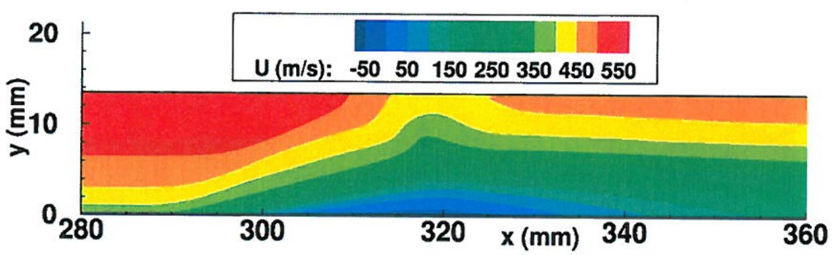
Experiment



Menter SST k-ω

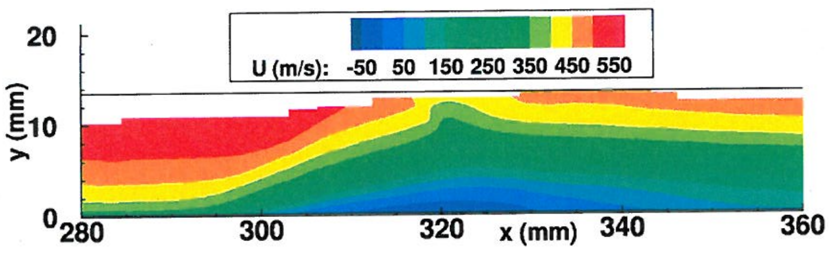


Menter BSL k-ω

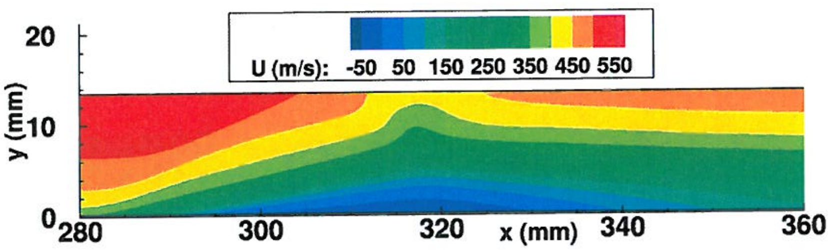


UFAST Velocity Contours

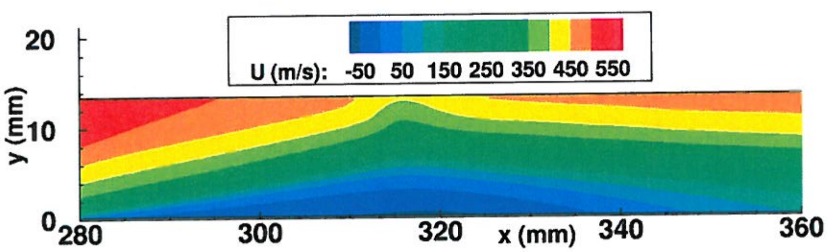
Experiment



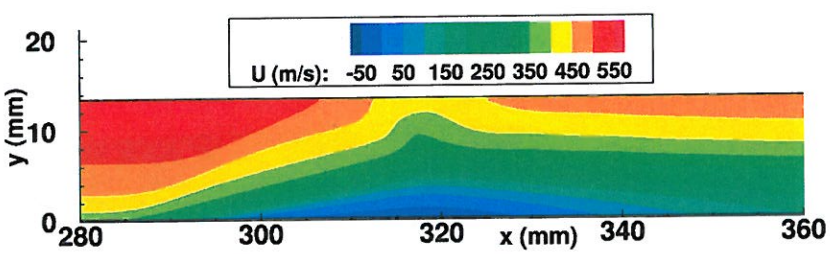
Menter SST k-ω, a_1 = 0.34



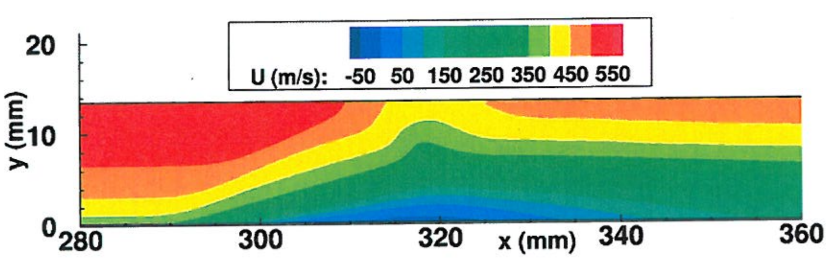
Menter SST k-ω



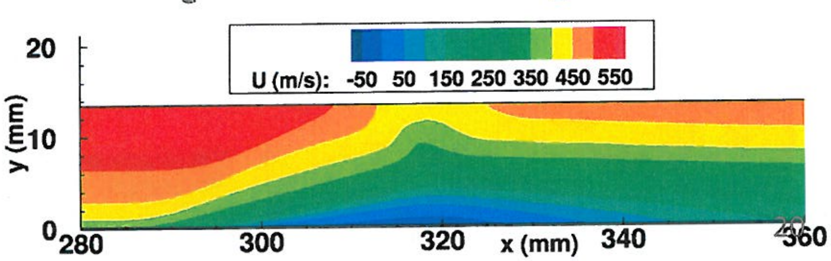
Menter SST k-ω, a_1 = 0.355



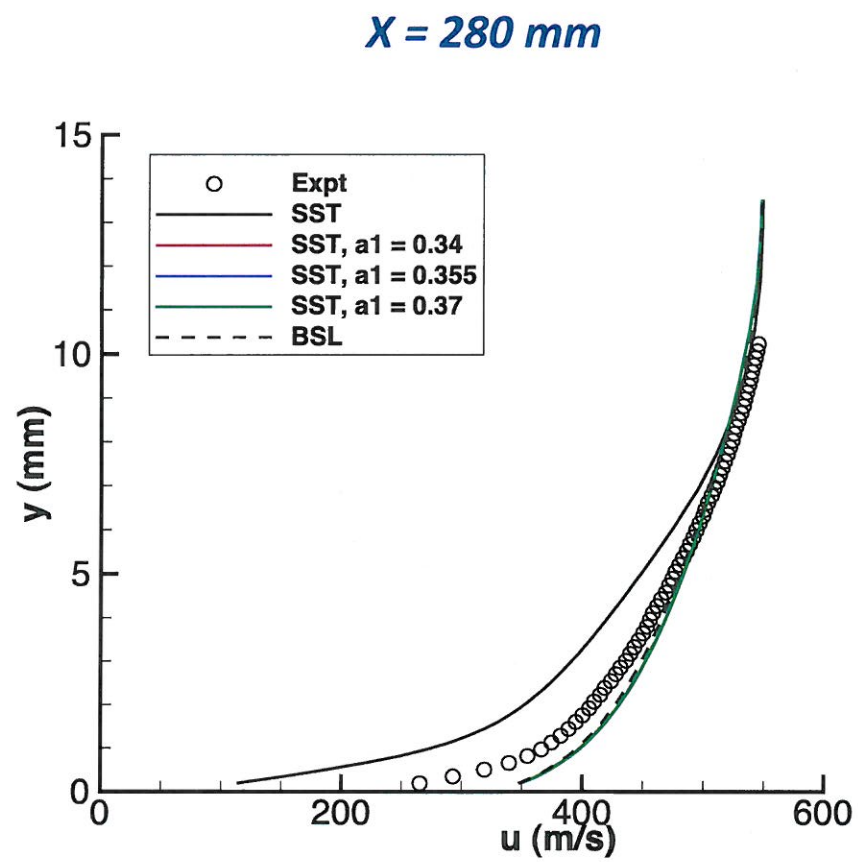
Menter BSL k-ω

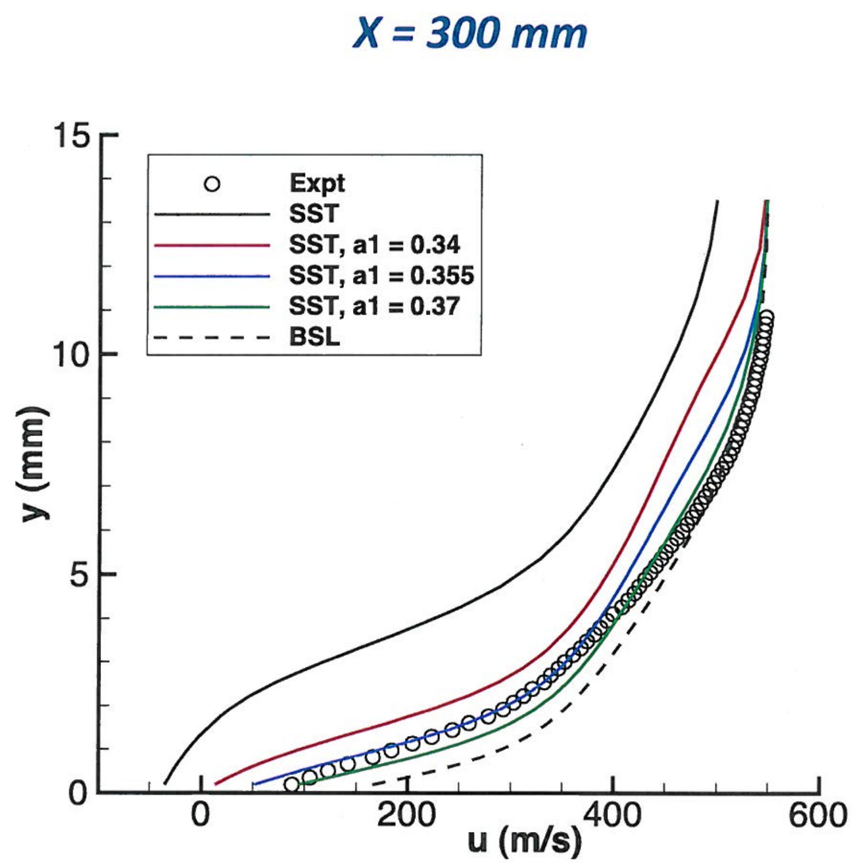


Menter SST k-ω, a_1 = 0.37



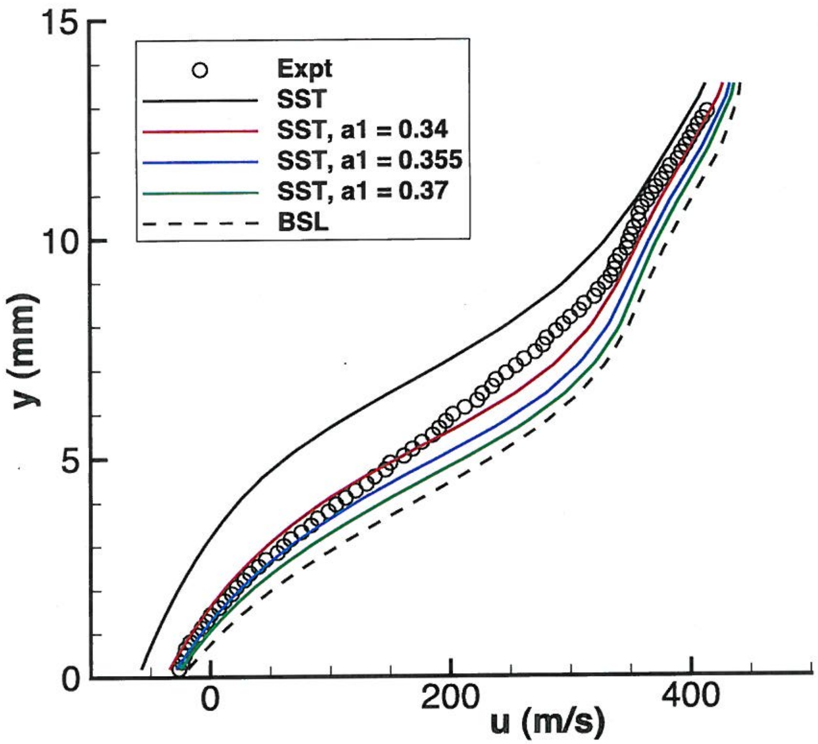
UFAST Velocity Profiles



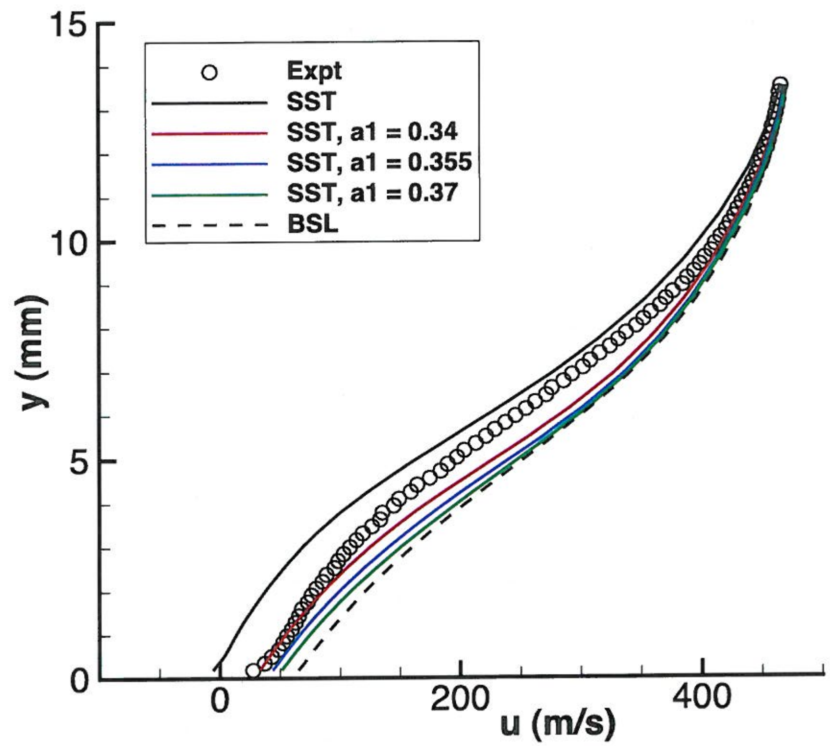


UFAST Velocity Profiles

X = 320 mm

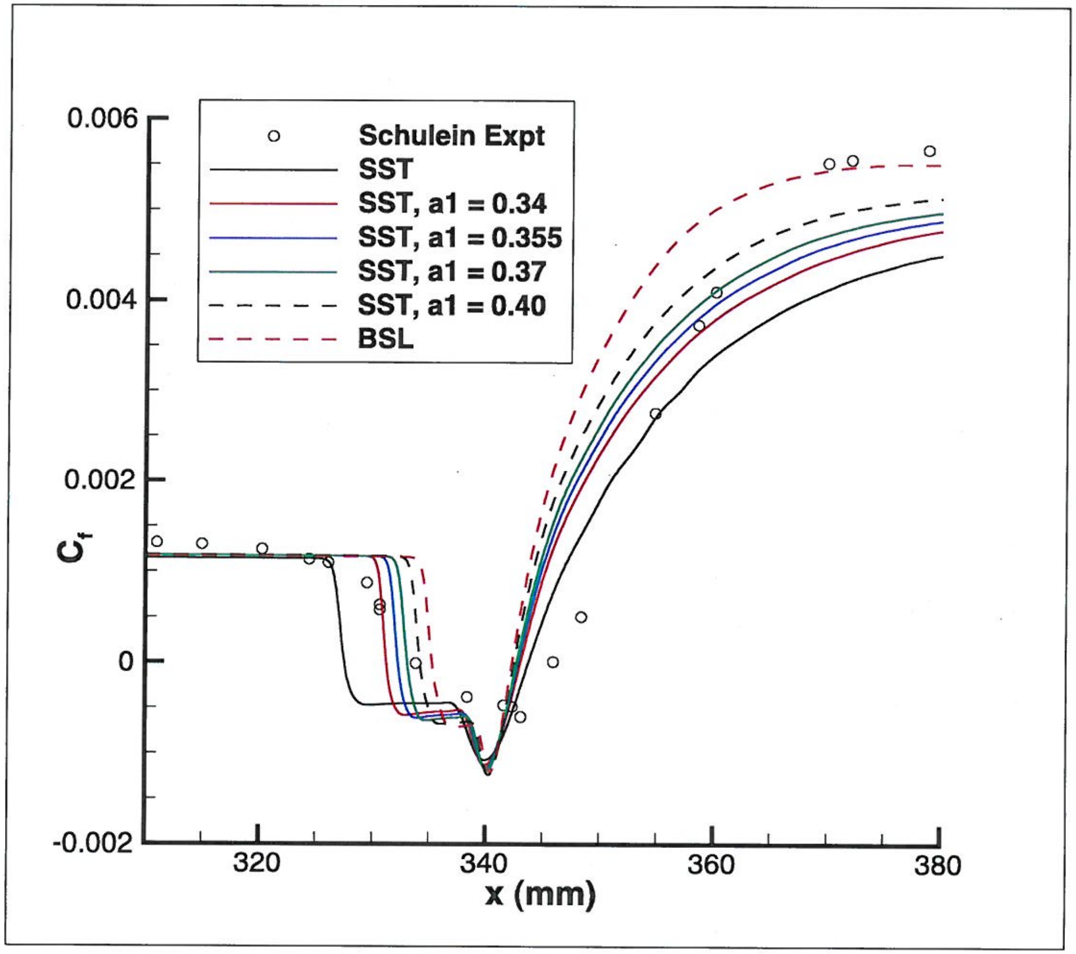


X = 340 mm

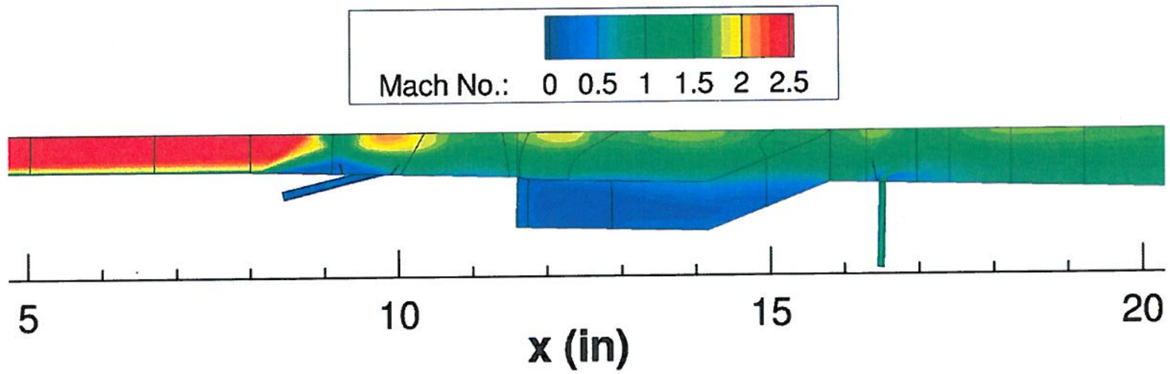


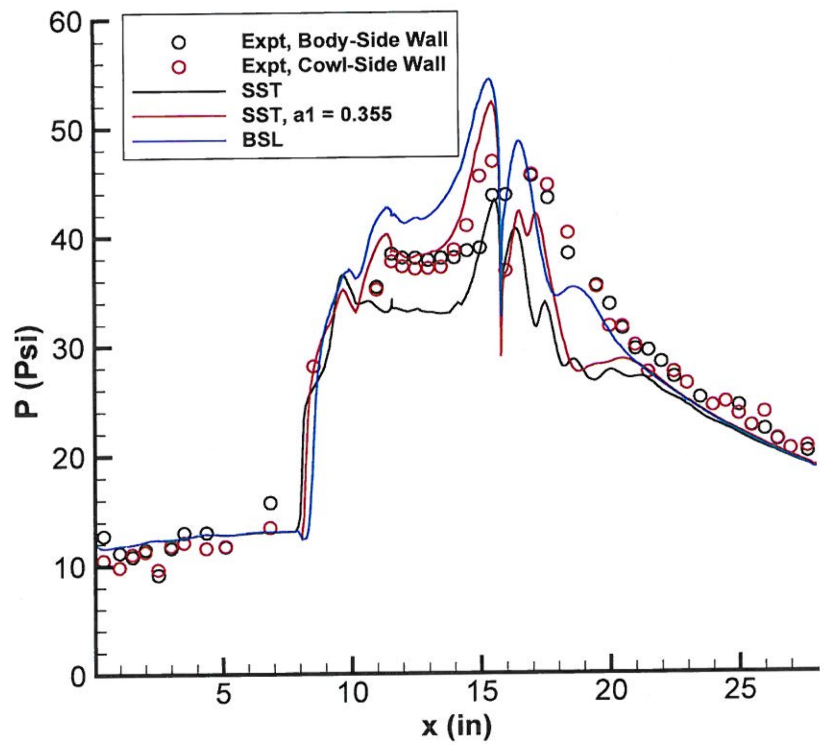
Mach 5 SWTBLI - Schulein

- Only wall shear stress data available for this case.

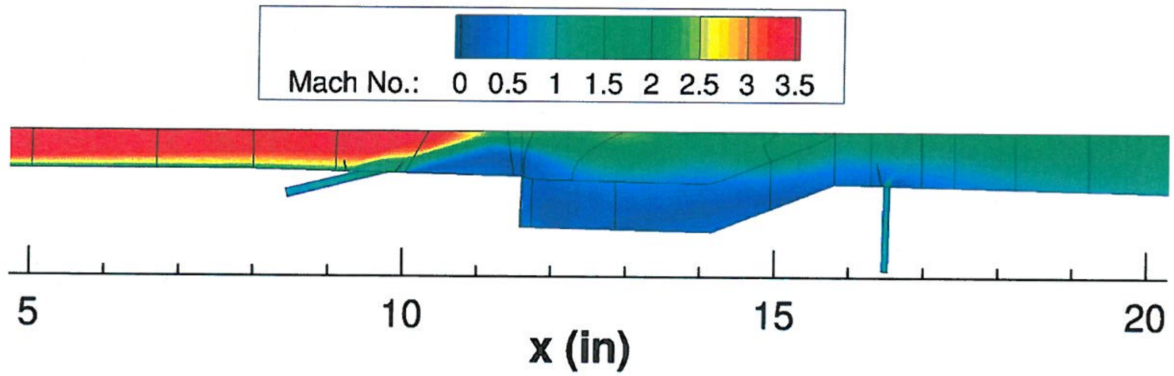


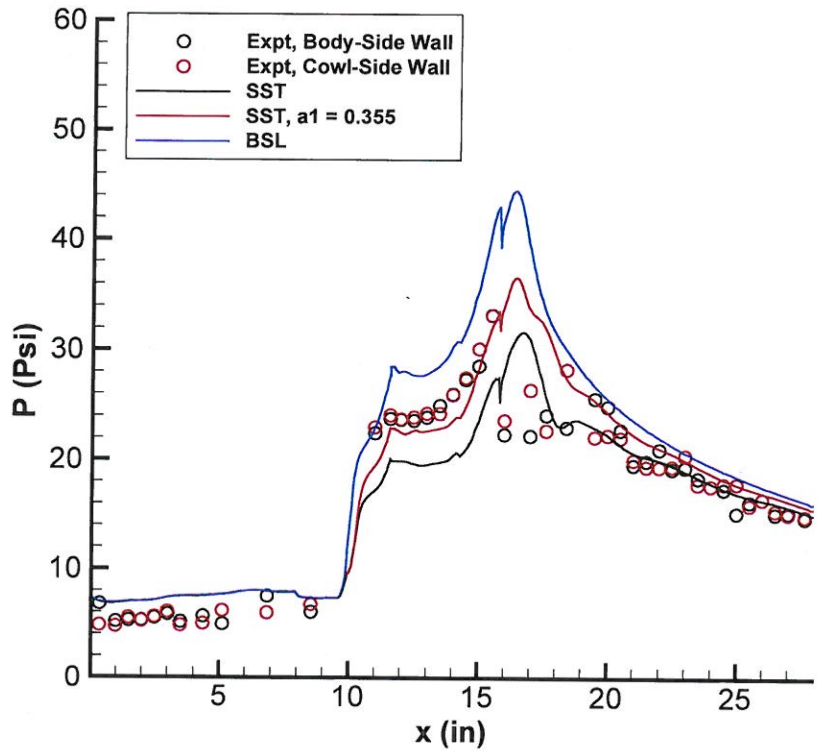
Hypersonic International Flight Research (HIFiRE) Direct-Connect Rig (HDCR) – Mach 5.8 Flight Case





Hypersonic International Flight Research (HIFiRE) Direct-Connect Rig (HDCR) – Mach 8.0 Flight Case





Conclusions

- BSL and SST models provide solutions on either side of experimental data for Shock-Wave / Turbulent Boundary Layer Interactions (SWTBLIs).
- This work investigated alternative values for the shear stress limiter constant, a_1, which is set to 0.31 in the original SST model.
- Incompressible models were investigated: For the Driver axisymmetric diffuser problem, SST provides best C_p results; but perhaps fortuitous – considering turbulence measurements.
- For SWTBLI problems, increasing a_1 results in less limiting of turbulent shear stress.....smaller separations.
- Experimental data indicates values for $a_1 = -u'v' / k$ larger than 0.31 are warranted – in agreement with computational results.
- $a_1 = 0.355$ is recommended value for SWTBLI problems.