Green's Functions for Prediction of Noise From Non-Axisymmetric Jets

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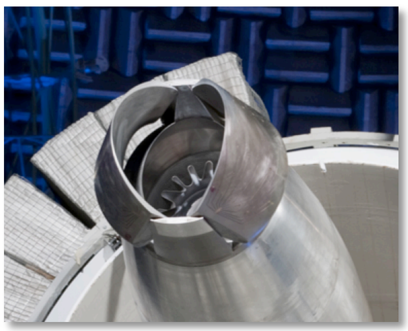
Presentation Outline

I. Jet Noise Prediction Needs
II. Acoustic Analogy Approach
III. Green's Functions
    i. Reduced-order models
        1) Conformal Mapping
            a. Application: Rectangular Jets
            b. Application: Twin Round Jets
        2) Orthogonal Function Expansion
            a. Applications
    ii. Numerical Methods
IV. Summary

Jet Noise Prediction Needs

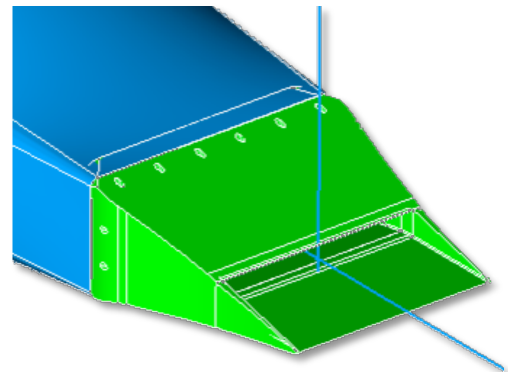
Next generation aircraft will involve complex exhaust system geometries

Non-circular exits



Multiple Streams

Nearby Solid Surfaces



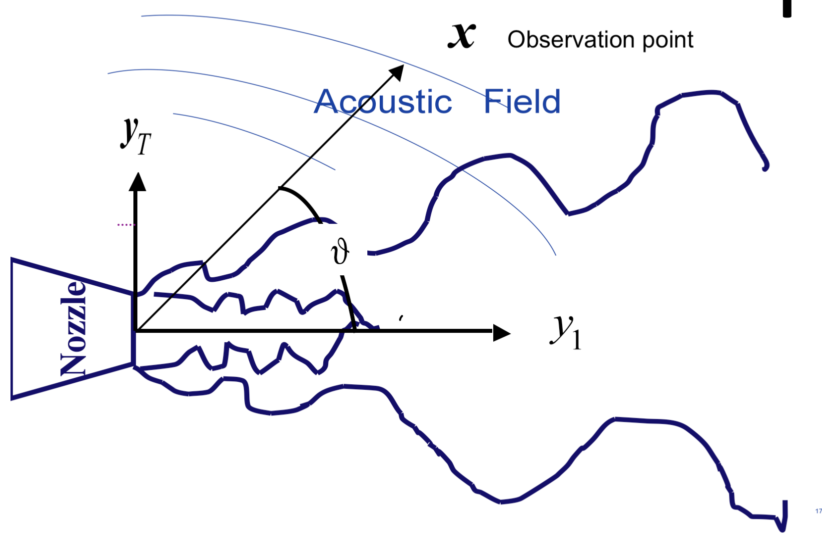
In order to make noise predictions for Next GEN exhaust systems, need to extend physics-based methods (JeNo) to be able to handle:

- Non-axisymmetric mean flows
- Interactions with solid surfaces

Acoustic Analogy Approach

- Many versions:
  - Lighthill (1952), Lilley (1972), … .
- Current work based on Goldstein (2003) formulation.
- Rearrange the Navier-Stokes equations to obtain:
  - A linear wave operator which governs sound propagation (mean flow interaction effects) through a specified base flow.
  - Nonlinear source terms.
- Model source terms.
- Compute propagation effects.
  - Solve linear inhomogeneous equations using a Green's function.

Acoustic Analogy Approach
Formula for the Acoustic Spectrum



Acoustic Analogy Approach for Prediction of Far-Field Acoustic Spectrum

$$I_\omega(\boldsymbol{x}) = 2\pi \int_V \int_{-\infty}^{\infty} \int_V \Gamma_{\lambda j}\left(\boldsymbol{x} \mid \boldsymbol{y};\omega\right) \Gamma^*_{\kappa l}\left(\boldsymbol{x} \mid \boldsymbol{y}+\boldsymbol{\eta};\omega\right) e^{-i\omega\tau} R_{\lambda j \kappa l}\left(\boldsymbol{y},\boldsymbol{\eta},\tau\right) d\boldsymbol{\eta}\, d\tau\, d\boldsymbol{y}$$

$\Gamma_{\lambda j}$ Propagator Function (Green's Function) -- Computed

$$R_{\lambda j \kappa l} = \varepsilon_{\lambda j,\sigma m} R_{\sigma m \gamma n} \varepsilon_{\kappa l,\gamma n} \quad ; \quad \varepsilon_{\lambda j,\sigma m} \equiv \delta_{\lambda \sigma}\delta_{jm} - \frac{\gamma - 1}{2}\delta_{\lambda j}\delta_{\sigma m}$$

$R_{\sigma m \gamma n}$ Reynolds Stress Auto-Covariance Tensor -- Modeled

Acoustic Analogy Approach
Adjoint Vector Green's Function

$$\frac{\tilde{D}g^a_{i4}}{D\tau} - g^a_{j4}\frac{\partial \tilde{v}_j}{\partial y_i} + \widetilde{c^2}\frac{\partial g^a_{44}}{\partial y_i} + \frac{\gamma - 1}{\bar{\rho}}\frac{\partial \tilde{\theta}_{ij}}{\partial y_j} g^a_{44} + \frac{\partial g^a_{54}}{\partial y_i} = 0$$

$$\frac{\tilde{D}g^a_{44}}{D\tau} + \frac{\partial g^a_{i4}}{\partial y_i} - (\gamma - 1) g^a_{44}\frac{\partial \tilde{v}_j}{\partial y_j} = -\delta(\boldsymbol{x} - \boldsymbol{y})\delta(t - \tau)$$

$$\frac{\tilde{D}g^a_{54}}{D\tau} + \frac{1}{\bar{\rho}}\frac{\partial \tilde{\theta}_{ij}}{\partial y_j} g^a_{i4} = 0$$

•Three-dimensional, time-dependent system of linear evolution equations for adjoint Green's function.

$$\text{Propagator} \Rightarrow \Gamma_{\lambda j} \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega(t-\tau)}\left[\frac{\partial g^a_{\lambda 4}(\boldsymbol{y},\tau|\boldsymbol{x},t)}{\partial y_j} - (\gamma - 1)\frac{\partial \tilde{v}_\lambda}{\partial y_j} g^a_{44}(\boldsymbol{y},\tau|\boldsymbol{x},t)\right] d(t-\tau)$$

•For design and concept-evaluation purposes, desire at most 'overnight' run times.

•Green's function computation most time-consuming part of prediction. ⇒ Need for reduced-order models.

Acoustic Analogy Approach
Weakly Non-Parallel Mean Flow

- Major simplification if mean flow is assumed weakly non-parallel.
  - Locally parallel approximation is leading-order solution.
    - Away from singularity in propagator at supersonic speeds and polar angles near the jet axis.
- Single equation for reduced Green's function:

$$\hat{G}_o\left(\boldsymbol{y}_T \mid \boldsymbol{x}_T; k, \omega\right) = i\left(\omega - Uk\right)\hat{g}_{44}^{a}$$

$$\frac{\partial}{\partial y_j}\frac{\widetilde{c^2}}{\left(\omega - kU\right)^2}\frac{\partial \hat{G}_o}{\partial y_j} + \left[1 - \frac{k^2\widetilde{c^2}}{\left(\omega - kU\right)^2}\right]\hat{G}_o = \frac{\delta\left(\boldsymbol{x}_T - \boldsymbol{y}_T\right)}{\left(2\pi\right)^2} \quad j = 2,3$$

- For observer in far field: $k = \frac{\omega}{c_\infty}\cos\vartheta$

Green's Functions
Reduced-Order Models
Conformal Mapping

- Assume (approximate) levels surfaces of mean velocity and temperature to coincide, $U = U(u)$; $T = T(u)$, and be concentric.

- For certain shapes, can choose u , and the corresponding orthogonal curves, $v = \text{constant}$, such that

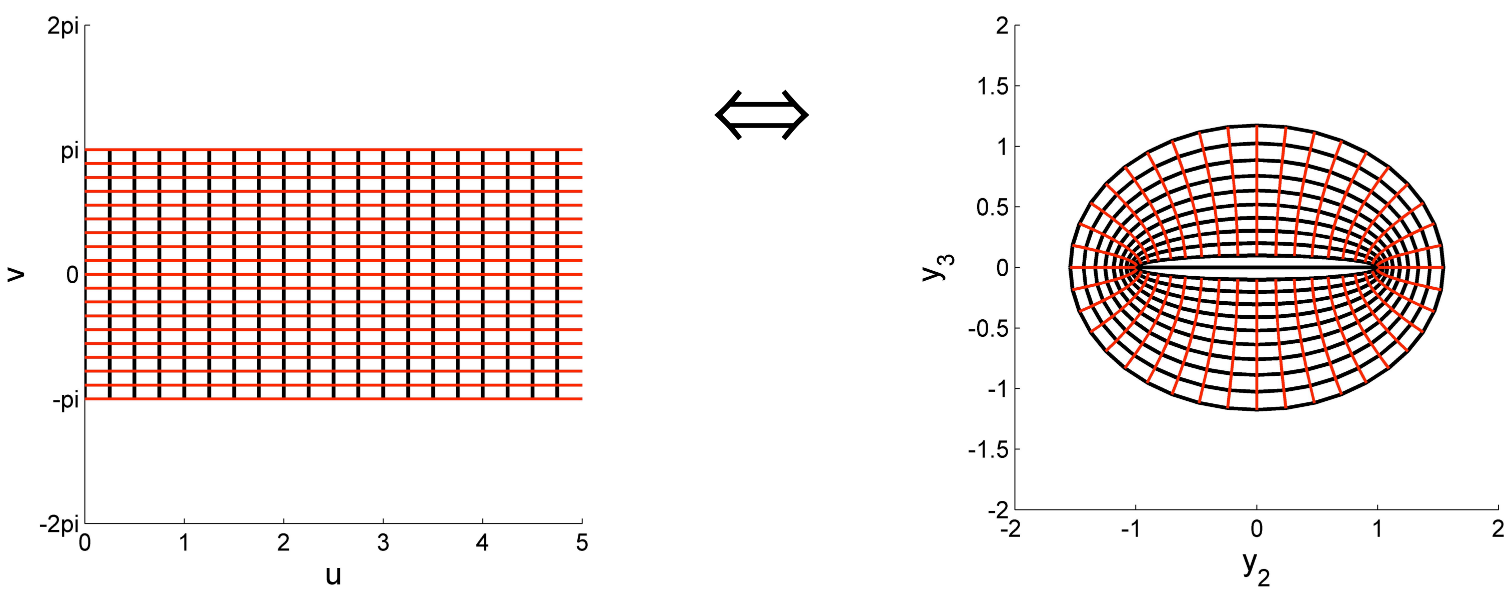
$$W\left(y_2 + iy_3\right) = u\left(y_2, y_3\right) + iv\left(y_2, y_3\right)$$

  maps the plane $y_2 + iy_3$ into a strip in the $\left(u + iv\right)$ plane where the Green's function:
  - Is periodic in v
  - Satisfies the appropriate far-field boundary conditions as $u \to \infty$
- Express Green's function in terms of a Fourier series.
- Reduce partial differential equation to coupled system of ordinary differential equations for Fourier modes.

Application: Rectangular Jets

- Mean flow in cross-flow planes approximated by concentric ellipses.
- Conformal mapping to cylindrical elliptical coordinates.

$$y_2 + iy_3 = C\cosh\left(u + iv\right),\;\; C \text{ is a real constant}$$

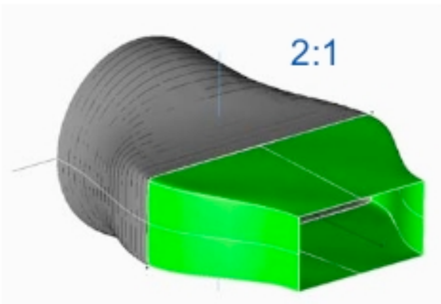


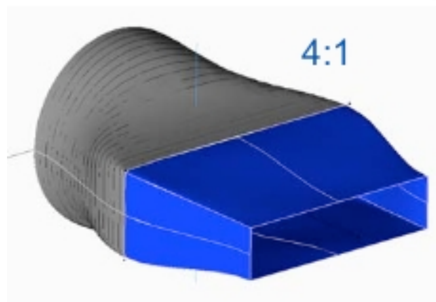
Application: Rectangular Jets Noise Prediction Scheme

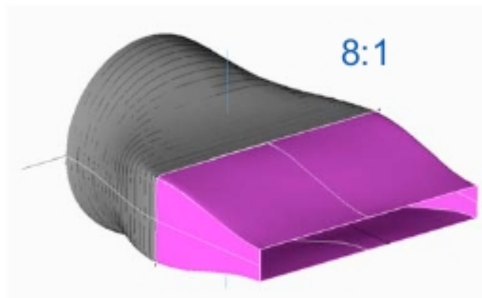
- Conformal mapping to elliptical coordinates for Green's function.
- Hybrid (space-time/frequency) source model of Leib and Goldstein (2011).
  - Scalings from low-speed flow surveys (from K. Zaman)
- WIND US RANS solutions as input (from F. Frate)
- Compare with acoustic data (from J. Bridges)

Application: Rectangular Jets
EXTENSIBLE RECTAGULAR NOZZLES

- Aspect ratios two, four and eight baseline rectangular nozzles.

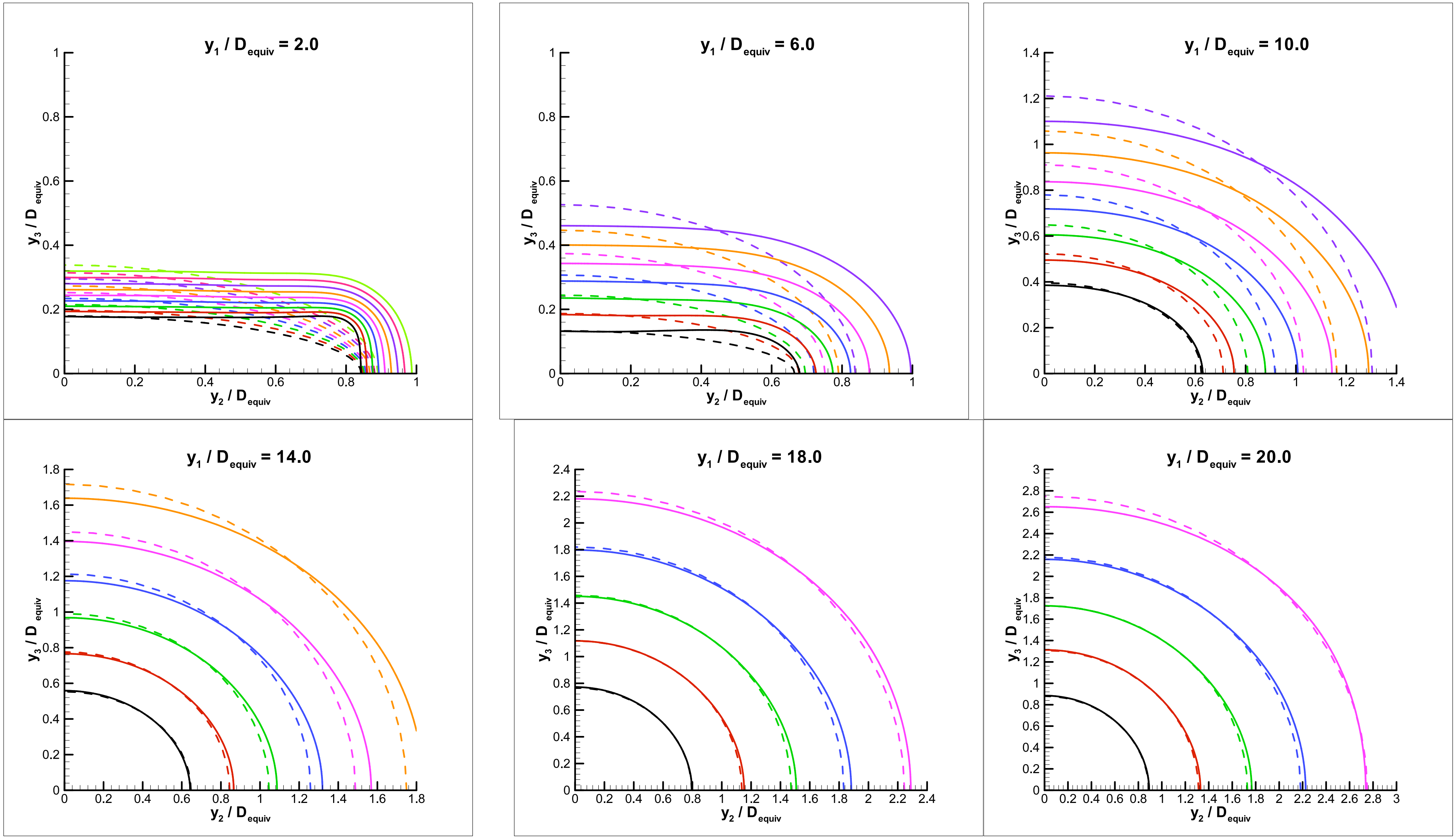






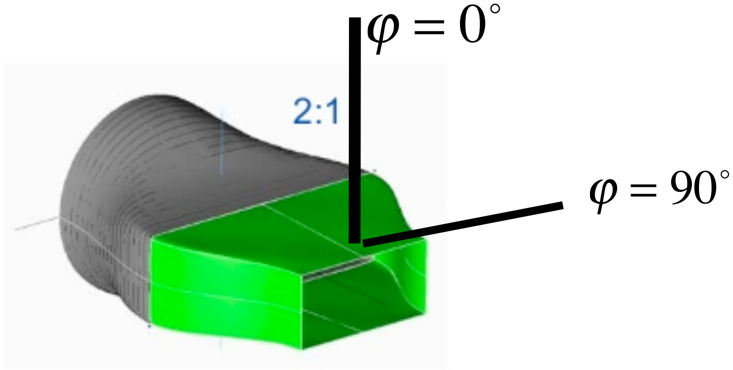
- Equivalent diameter = 2.13 inches.
- NPRs = 1.197,1.439,1.856
  - Acoustic Mach numbers at exit = 0.5, 0.7,0.9
  - Unheated (Cold)

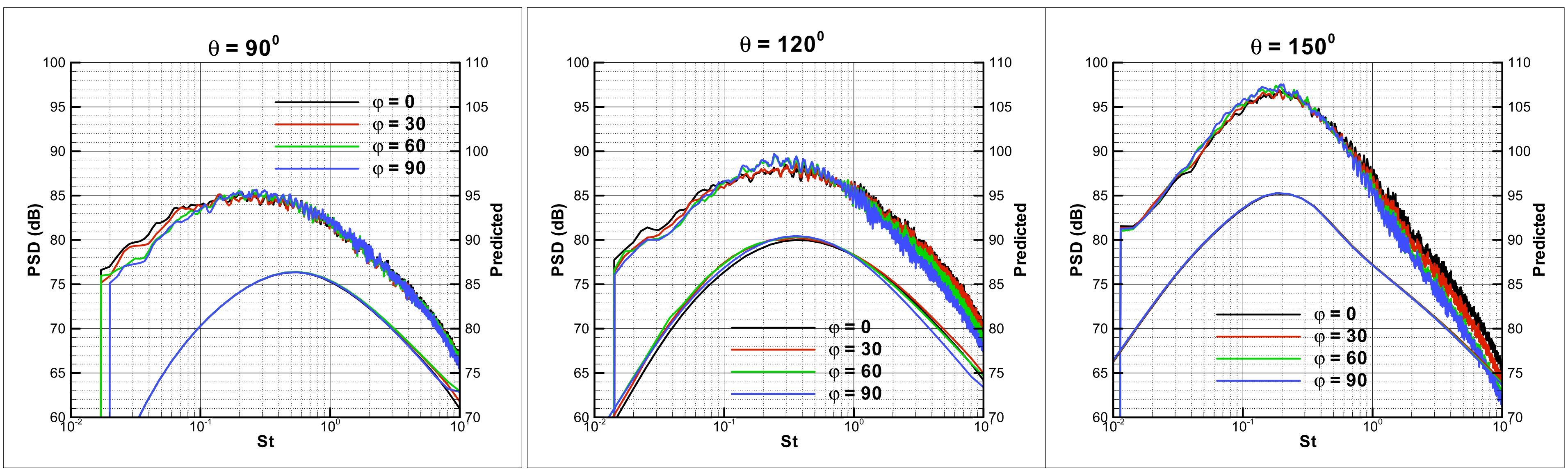
Application: Rectangular Jets
Mean Flow Model – 4:1 Aspect Ratio



Application: Rectangular Jets Noise Predictions

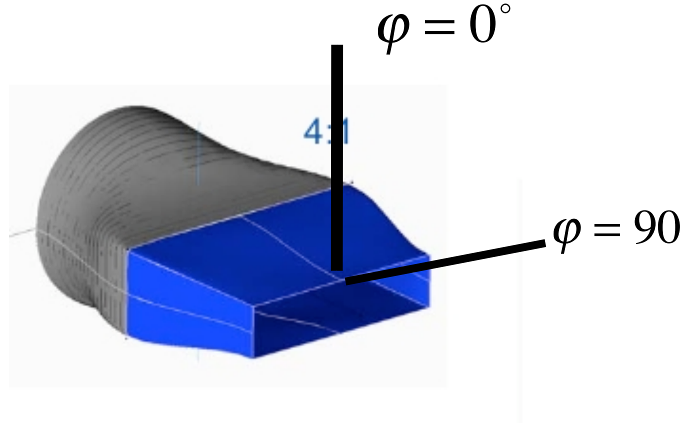
Aspect Ratio Two
Set Point 5:
NPR=1.439
MJ=0.7

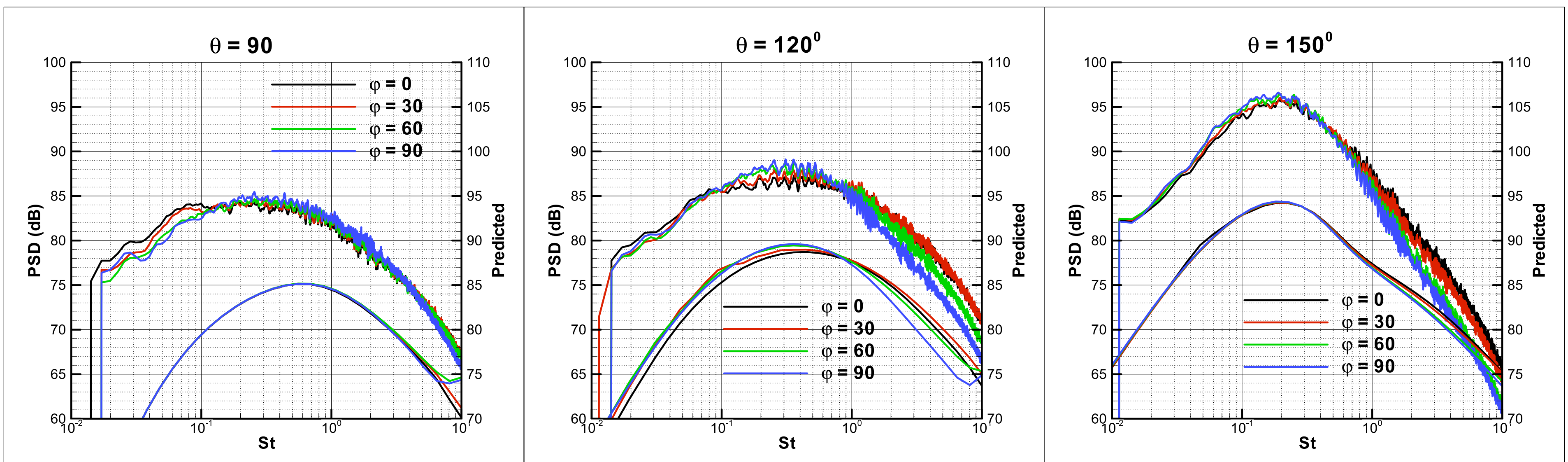




Application: Rectangular Jets Noise Predictions

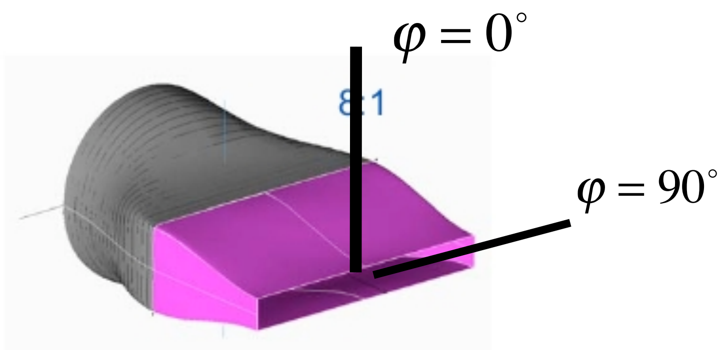
Aspect Ratio Four
Set Point 5:
NPR=1.439
MJ=0.7

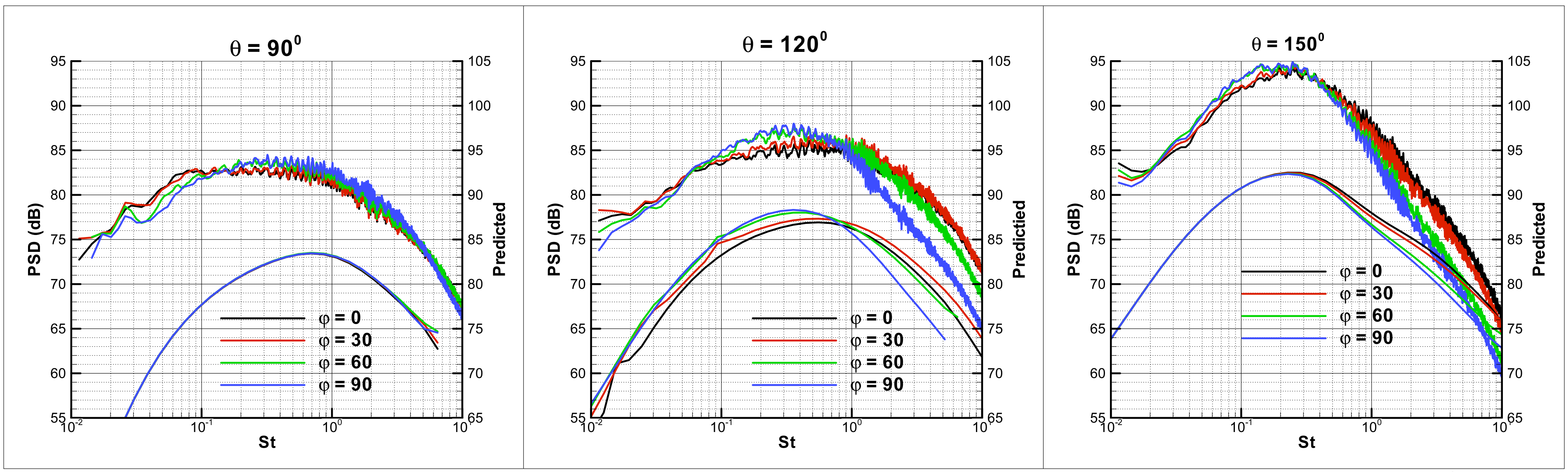




Application: Rectangular Jets Noise Predictions

Aspect Ratio Eight
Set Point 5:
NPR=1.439
MJ=0.7

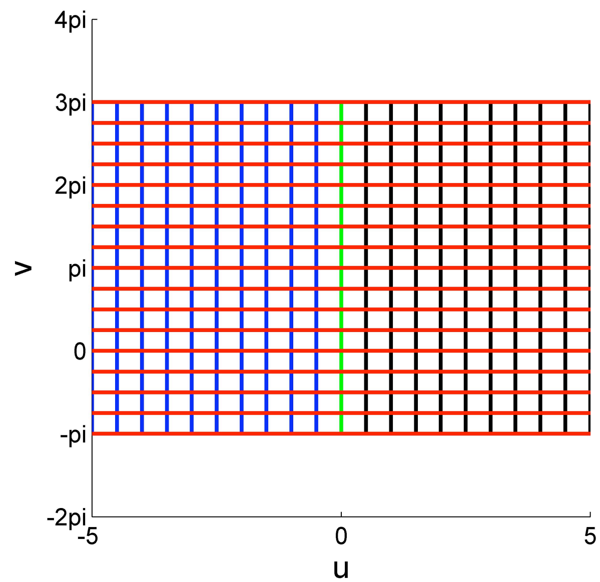




Application: Twin Round Jets

- Conformal mapping to Cassinian ovals

$$u + iv = \ln\left[\left(y_2 + iy_3\right)^2 - C^2\right],\ \ C \text{ is a real constant}$$



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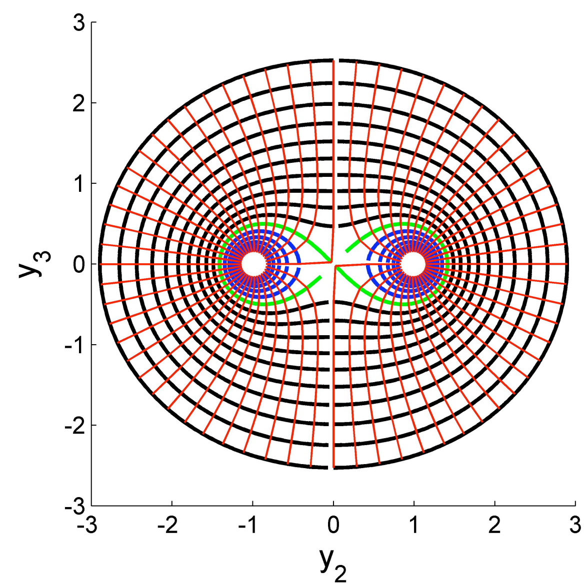


Application: Twin Round Jets
Test Case for Green's Function Solver

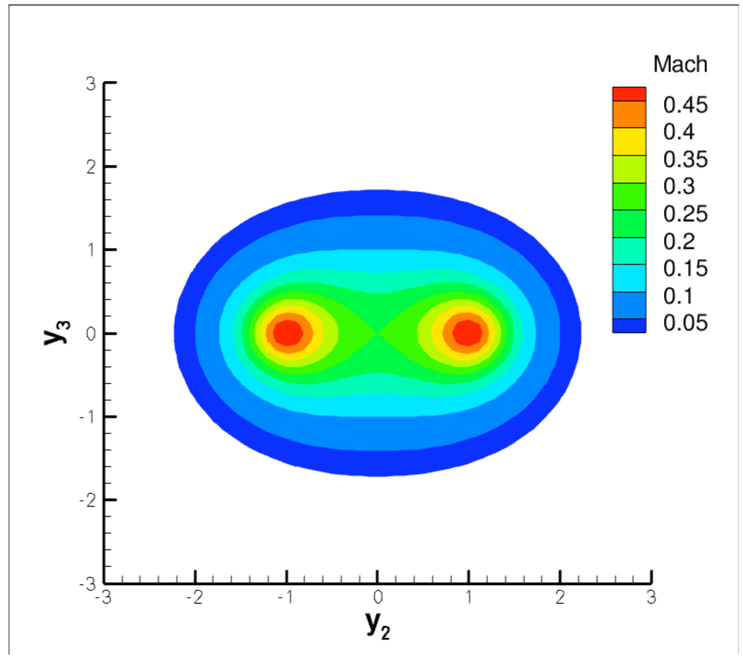
Analytic Function for Mean Flow

$$M(u) = \frac{M_0}{2}\left[1 - \tanh u\right]$$

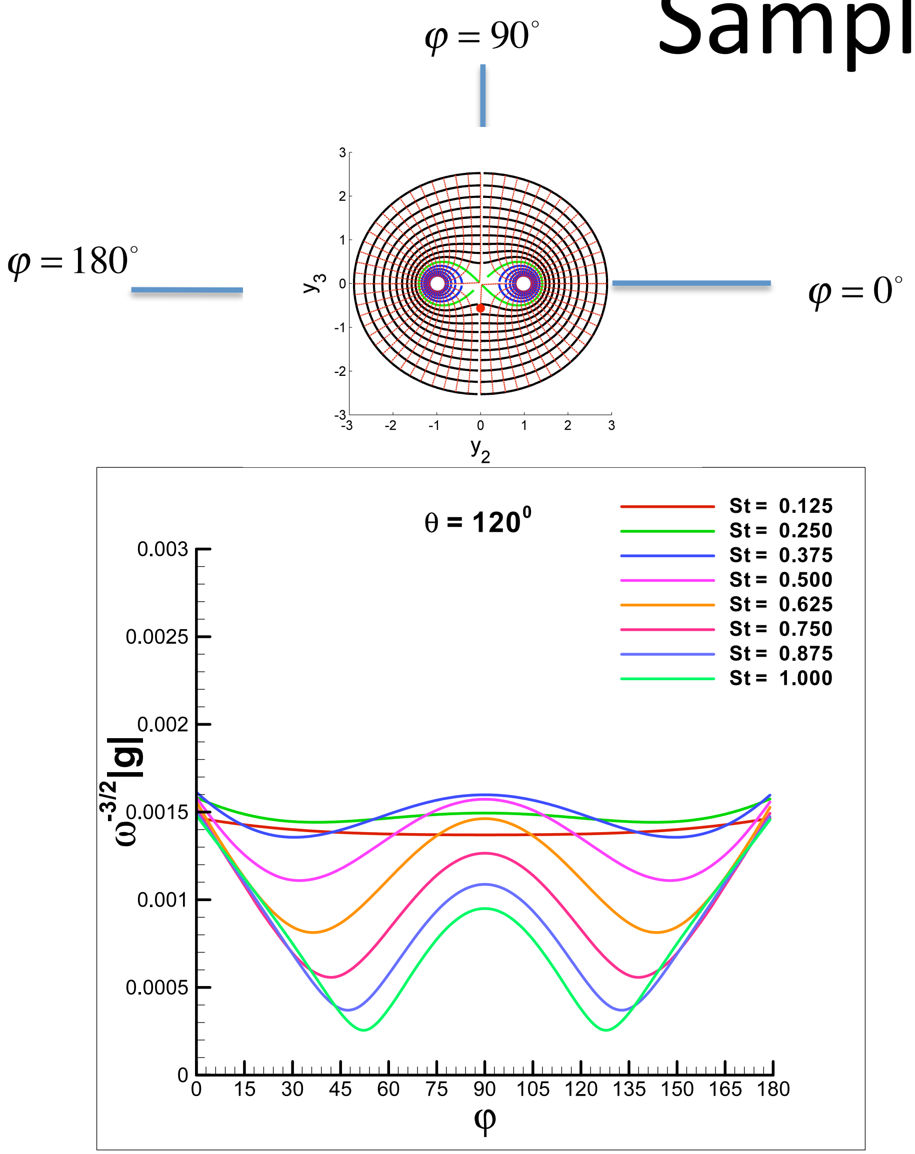
$$\widetilde{c^2}(u) \equiv c_\infty^2$$

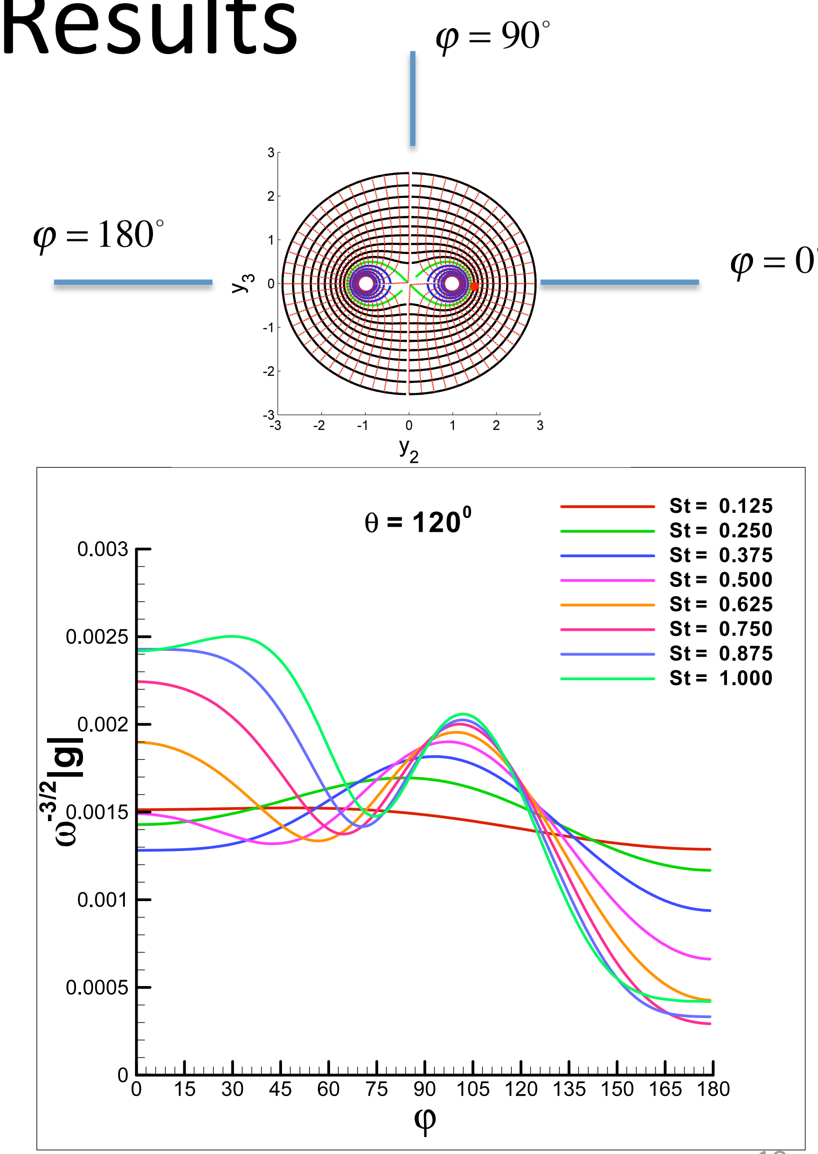
$$u = \ln\left|\left(y_2 + iy_3\right)^2 - C^2\right|$$

$$C = 1 \;\;,\;\; M_0 = 0.5$$

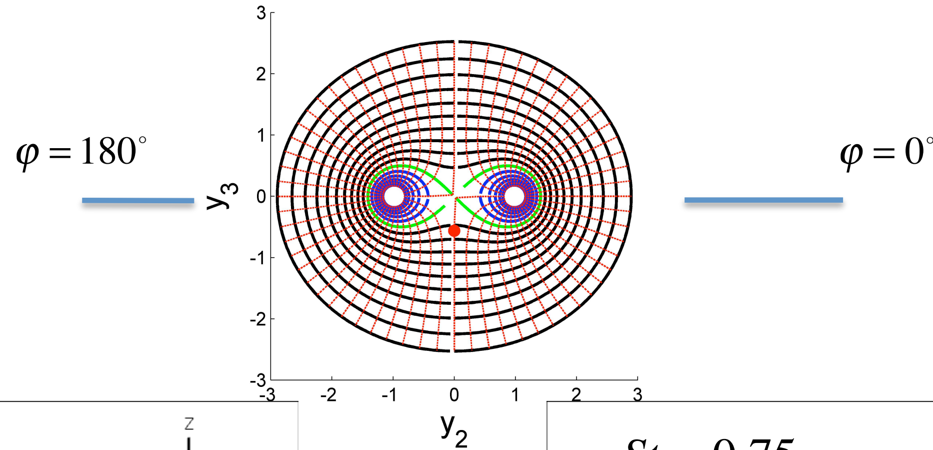


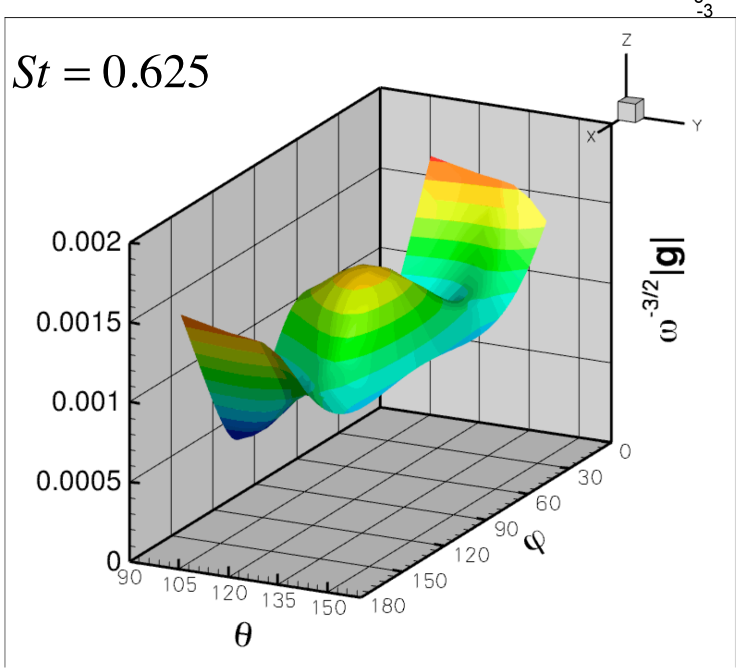
Application: Twin Round Jets
Sample Results

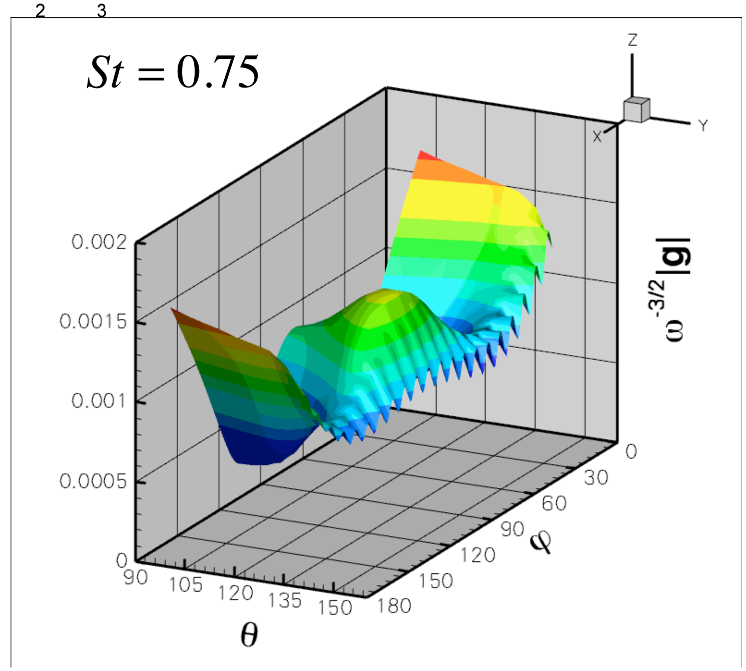




Application: Twin Round Jets
Green's Function Directivity







Green's Function Reduced-Order Models Orthogonal Function Expansion

- Represent mean flow by sum of orthogonal functions.

$$M(y_2, y_3) = \sum_{m=0}^{l} a_m(\rho)\Psi_m(\varphi) \qquad \rho = \rho(y_2, y_3) \quad ; \quad \varphi = \varphi(y_2, y_3)$$

  - For computational efficiency, a relatively small number of functions is desired.

- Expand Green's function in series of these orthogonal functions

$$\tilde{g}(y_2, y_3) = \sum_{m=0}^{\infty} g_m(\rho)\Psi_m(\varphi)$$

- Solve system of coupled ordinary differential equations for Green's function modes:
  - Direct solution of banded system.
  - Iterative solution (Mani).

Green's Function
Reduced-Order Models
Orthogonal Function Expansion

Example: Polar coordinates and Fourier series expansion

$$
\left.
\begin{aligned}
-i\left(\omega - Uk\right)\hat{g}^{a}_{14} + ik\widetilde{c^{2}}\,\hat{g}^{a}_{44} &= 0 \\
-i\left(\omega - Uk\right)\hat{g}^{a}_{r4} + \frac{\partial U}{\partial r}\hat{g}^{a}_{14} - \widetilde{c^{2}}\,\frac{\partial \hat{g}^{a}_{44}}{\partial r} &= 0, \\
-i\left(\omega - Uk\right)\hat{g}^{a}_{\varphi 4} + \frac{1}{r}\frac{\partial U}{\partial \varphi}\hat{g}^{a}_{14} - \widetilde{c^{2}}\,\frac{1}{r}\frac{\partial \hat{g}^{a}_{44}}{\partial \varphi} &= 0, \\
-i\left(\omega - Uk\right)\hat{g}^{a}_{44} - \left[\left(\frac{\partial \hat{g}^{a}_{r4}}{\partial r} + \frac{1}{r}\frac{\partial \hat{g}^{a}_{\varphi 4}}{\partial \varphi} + \frac{1}{r}\hat{g}_{r4}\right)\right] + ik\hat{g}^{a}_{14} &= \frac{1}{\left(2\pi\right)^{2}}\frac{\delta\left(r - r_0, \varphi - \varphi_0\right)}{r}
\end{aligned}
\right\}
$$

Solution of the form :

$$U\left(r,\varphi\right) = \sum_{n=-l}^{l} U_n\left(r\right)e^{in\varphi}$$

$$\hat{g}^{a}_{\sigma 4}\left(r,\varphi\right) = \sum_{n=-\infty}^{\infty} G^{(n)}_{\sigma 4}\left(r\right)e^{in\varphi}$$

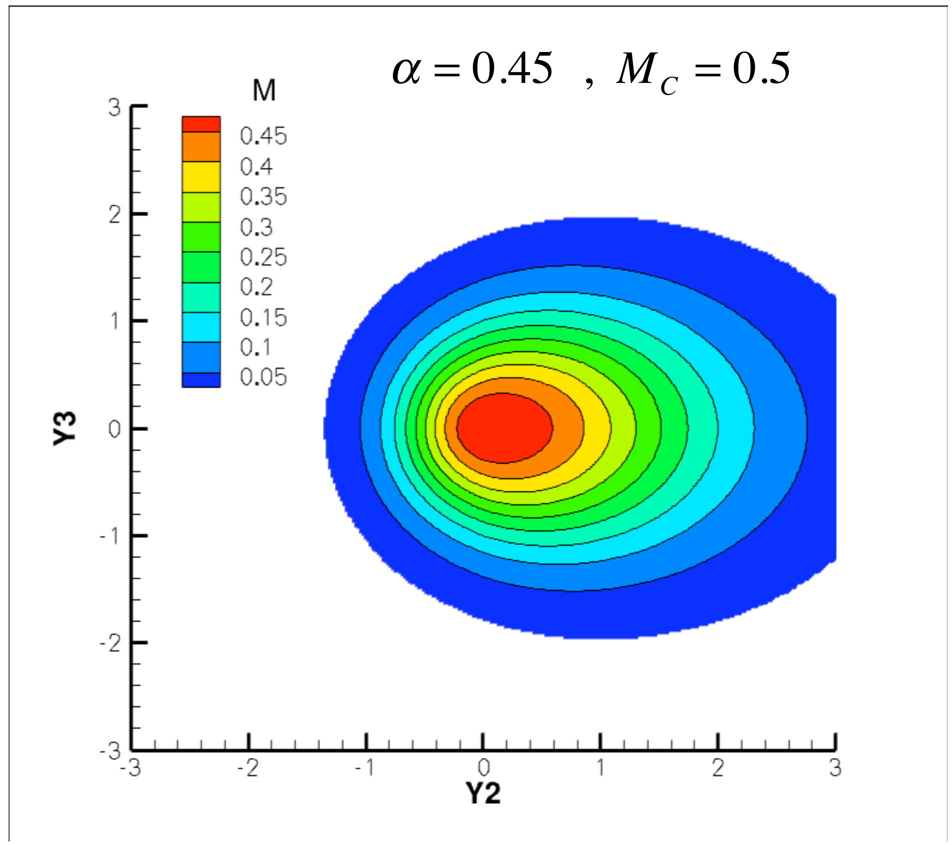
Application: Offset Stream

$$M(r,\varphi) = M_C e^{-(1+\alpha-2\alpha\cos\varphi)r^2}$$

Fourier Expansion of Mean Flow

$$M(r,\varphi) = \sum_{n=-\infty}^{\infty} M_n(r) e^{in\varphi}$$

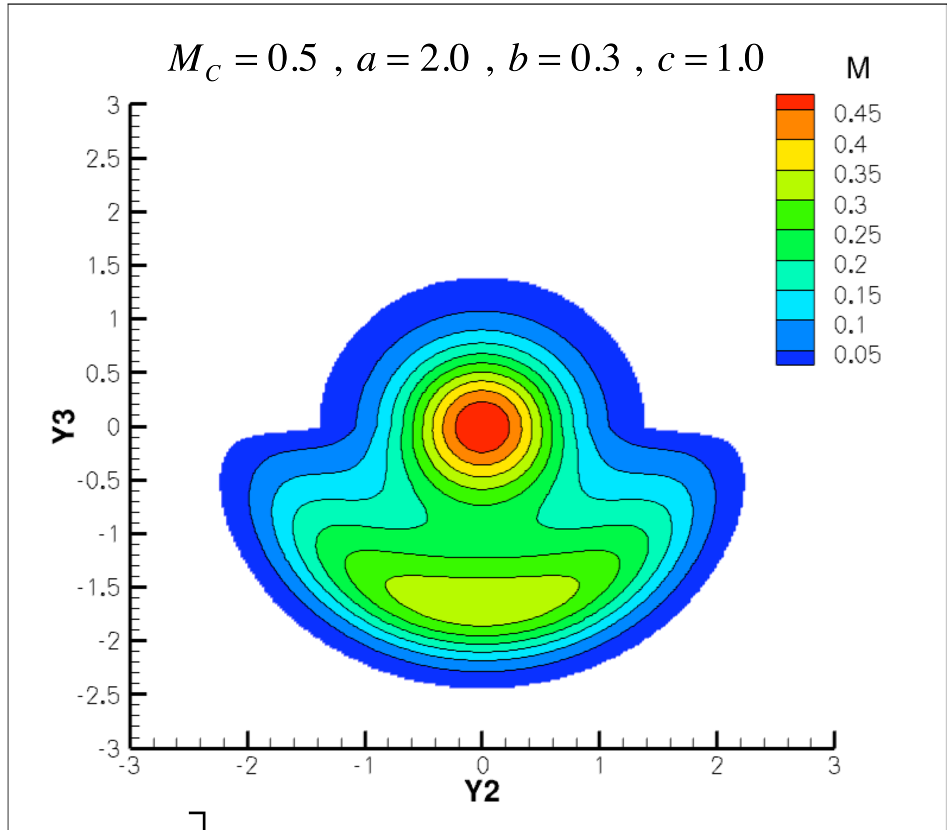
$$M_n = M_C e^{-(1+\alpha)r^2} I_n\left(2\alpha r^2\right)$$



Application: Fluid Shield

$$M(r,\varphi) = M_C\left[e^{-ar^2} + br^2 e^{-c(r-1)^4} g(\varphi)\right]$$

$$g(\varphi) = \begin{cases} 0 \quad , & 0 \le \varphi < \pi \\ -\sin\varphi \quad , & \pi \le \varphi < 2\pi \end{cases}$$



Fourier Expansion of Mean Flow

$$M(r,\varphi) = \sum_{n=-\infty}^{\infty} M_n(r) e^{in\varphi}$$

$$M_n = M_C\left[e^{-ar^2}\delta_{n,0} + br^2 e^{-c(r-1)^4}\left(\frac{1}{2\pi(1-n^2)}\right)\left[(-1)^n + 1\right]\right] \ , \ n \ne \pm 1$$

$$M_{\pm 1} = \pm M_C \frac{i}{4} br^2 e^{-c(r-1)^4}$$

Green's Function
Numerical Methods

- Develop code for numerical solution of the acoustic analogy equations Green's function.
  - Collaboration with John Goodrich, GRC
- Use for:
  - Validation of reduced-order models.
  - High-resolution calculations for cases of special interest.
  - Study effects of non-parallel mean flow.

Green's Function Numerical Methods

- Time-domain, finite-difference method.
- Computational domain surrounded by damping layers.
- High-order boundary conditions (related to Giles and Thompson) on outer boundary of damping layers.

- Status:
  - Current code is written for the Linearized Euler Equations.
  - Have validated a two-dimensional version of code with analytical solution for a uniform flow .
- Plans:
  - Extend to three dimensions.
  - Extend to non-uniform mean flow.
  - Adapt for solution of acoustic analogy equations.

Summary

- To support development of noise-reduction concepts:
  - Develop reduced-order models for Green's function of acoustic analogy equations.
    - Rectangular Jets
    - Twin Round Jets
    - Offset Jets
    - FLADE
  - Integrate with source model for noise prediction.
    - Rectangular Jets
  - Develop code for numerical solution of Green's function of acoustic analogy equations.