The Multiscale Generalized Method of Cells and its Utility in Predicting the Deformation and Failure of Woven CMCs

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Outline

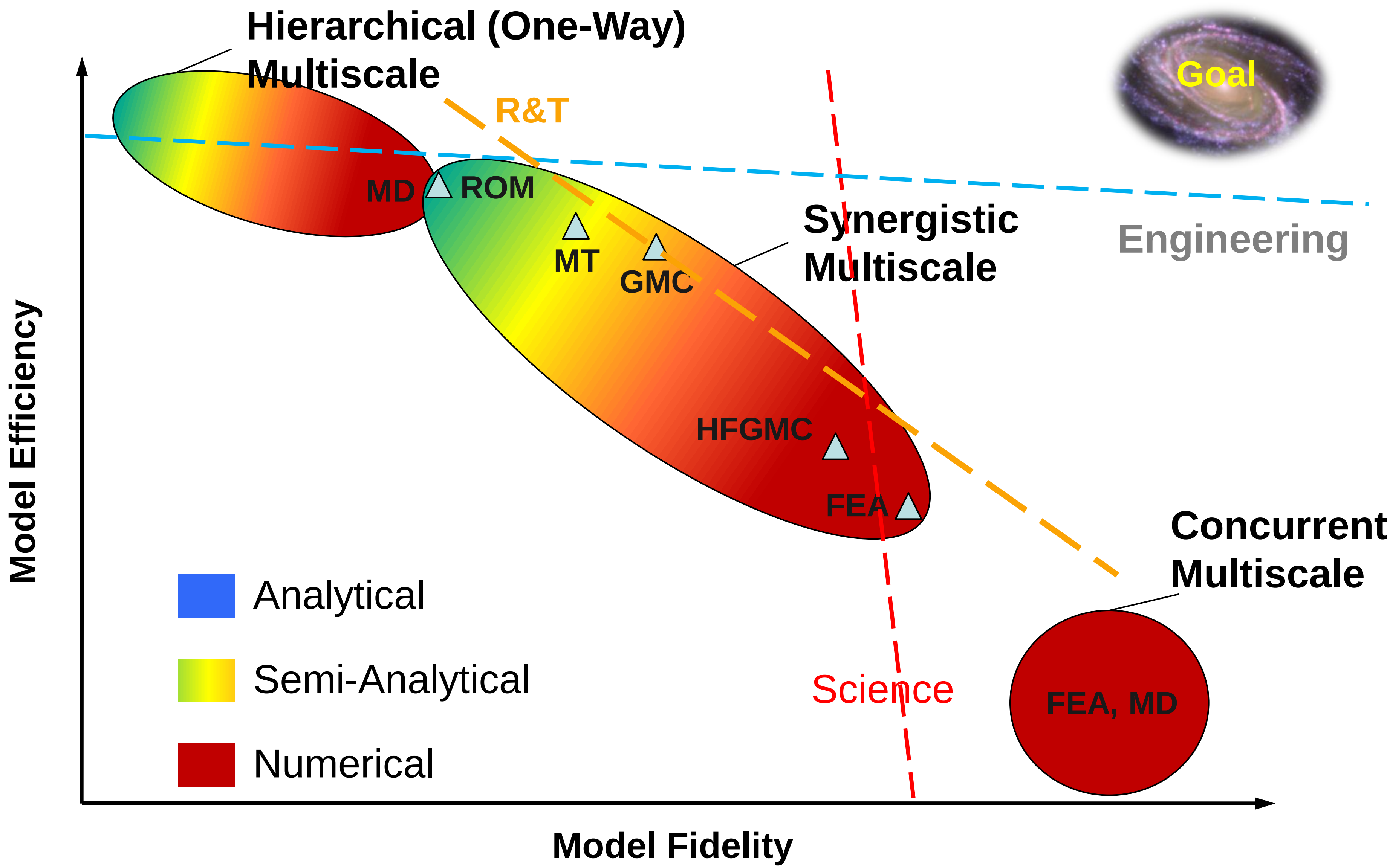
- **I**ntegrated **m**ultiscale **M**icromechanics **A**nalysis **C**ode (**ImMAC**)
- Multiscale Generalized Method of Cells (MSGMC)
- Modeling of Woven Fabrics (Plain & 5HS)
- Results
  - Tensile (Deterministic, Stochastic)
    - ✓ Load and Unloading
  - Creep
- Concluding Remarks

Presentation Objective:

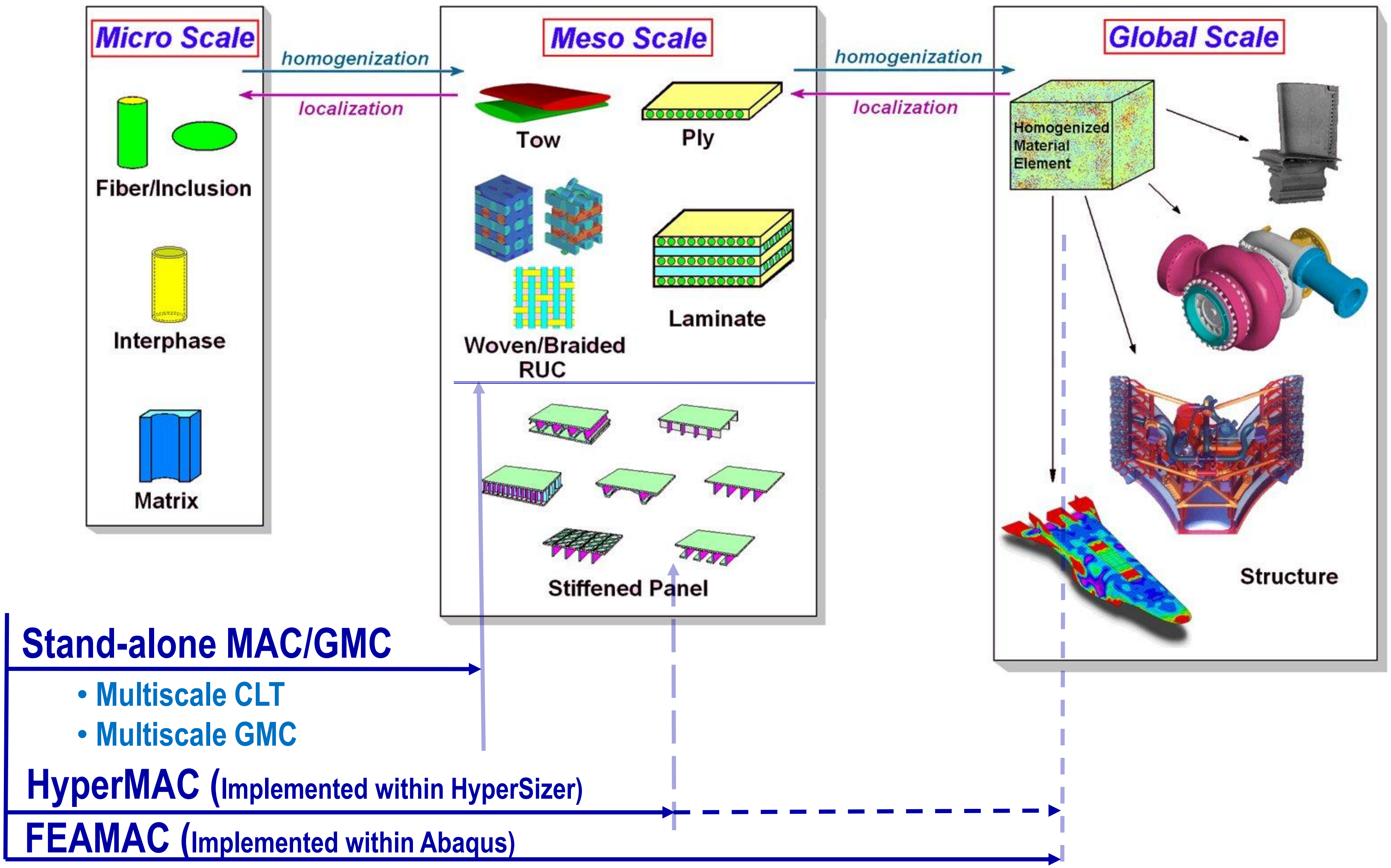
Apply a **synergistic** multiscale modelling technique to woven composites to determine underlying reasons for nonlinear response

- Understand influence (i.e., primary, secondary, etc.) of architectural parameters (e.g., fiber/void volume fraction, weave geometry, tow geometry, void geometry)at multiple length scales on the mechanical response of CMCs.
- Analyze the significance of effects and compare to material scatter

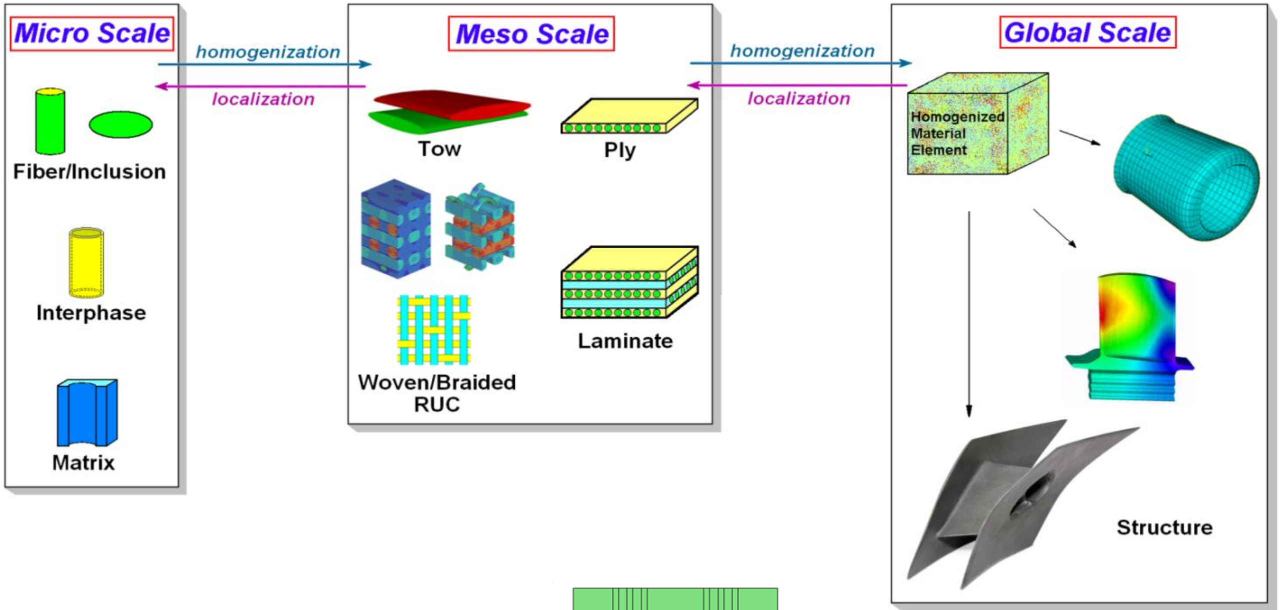
Goal is to Balance Efficiency vs Fidelity

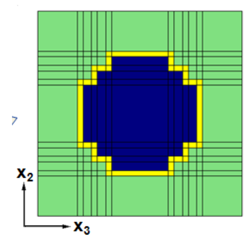


NASA's Integrated multiscale Micromechanics Analysis Code (ImMAC) Suite



MAC/GMC is Evolving Anisotropic Thermoelastic Inelastic and Damage Constitutive Model





Micro-level Field Equations (subcell)

$$\boldsymbol{\varepsilon}^{(\alpha\beta\gamma)} = \mathbf{A}^{(\alpha\beta\gamma)}\,\bar{\boldsymbol{\varepsilon}} + \mathbf{D}^{(\alpha\beta\gamma)}\left(\boldsymbol{\varepsilon}_s^I + \boldsymbol{\varepsilon}_s^T\right)$$

$$\boldsymbol{\sigma}^{(\alpha\beta\gamma)} = \mathbf{C}^{(\alpha\beta\gamma)}\left[\mathbf{A}^{(\alpha\beta\gamma)}\,\bar{\boldsymbol{\varepsilon}} + \mathbf{D}^{(\alpha\beta\gamma)}\left(\boldsymbol{\varepsilon}_s^I + \boldsymbol{\varepsilon}_s^T\right) - \left(\boldsymbol{\varepsilon}^{I(\alpha\beta\gamma)} + \boldsymbol{\varepsilon}^{T(\alpha\beta\gamma)}\right)\right]$$

Macro-level Constitutive Equations

$$\bar{\boldsymbol{\sigma}} = \mathbf{B}^*\left(\bar{\boldsymbol{\varepsilon}} - \bar{\boldsymbol{\varepsilon}}^I - \bar{\boldsymbol{\varepsilon}}^T\right)$$

$$\mathbf{B}^* = \frac{1}{dhl}\sum_{\alpha=1}^{N_\alpha}\sum_{\beta=1}^{N_\beta}\sum_{\gamma=1}^{N_\gamma} d_\alpha h_\beta l_\gamma \mathbf{C}^{(\alpha\beta\gamma)}\mathbf{A}^{(\alpha\beta\gamma)}$$

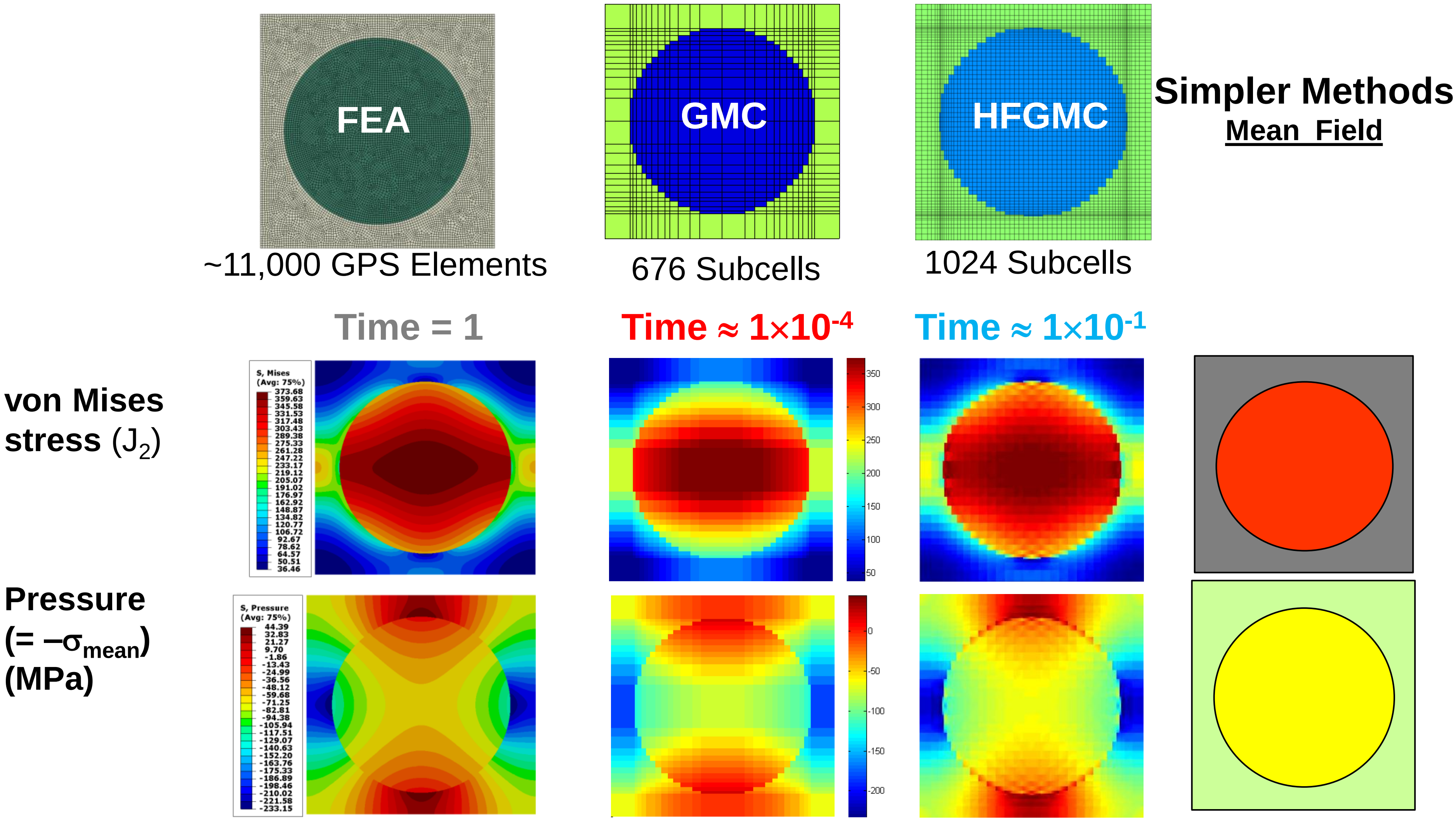
$$\bar{\boldsymbol{\varepsilon}}^I = \frac{-\left[\mathbf{B}^*\right]^{-1}}{dhl}\sum_{\alpha=1}^{N_\alpha}\sum_{\beta=1}^{N_\beta}\sum_{\gamma=1}^{N_\gamma} d_\alpha h_\beta l_\gamma \mathbf{C}^{(\alpha\beta\gamma)}\left(\mathbf{D}^{(\alpha\beta\gamma)}\boldsymbol{\varepsilon}_s^I - \boldsymbol{\varepsilon}^{I(\alpha\beta\gamma)}\right)$$

$$\bar{\boldsymbol{\varepsilon}}^T = \frac{-\left[\mathbf{B}^*\right]^{-1}}{dhl}\sum_{\alpha=1}^{N_\alpha}\sum_{\beta=1}^{N_\beta}\sum_{\gamma=1}^{N_\gamma} d_\alpha h_\beta l_\gamma \mathbf{C}^{(\alpha\beta\gamma)}\left(\mathbf{D}^{(\alpha\beta\gamma)}\boldsymbol{\varepsilon}_s^T - \boldsymbol{\varepsilon}^{T(\alpha\beta\gamma)}\right)$$

Fidelity vs. Efficiency in Composite Micromechanics

Comparison of Local Stress Invariants

Transverse Loading; 50% Glass/Epoxy



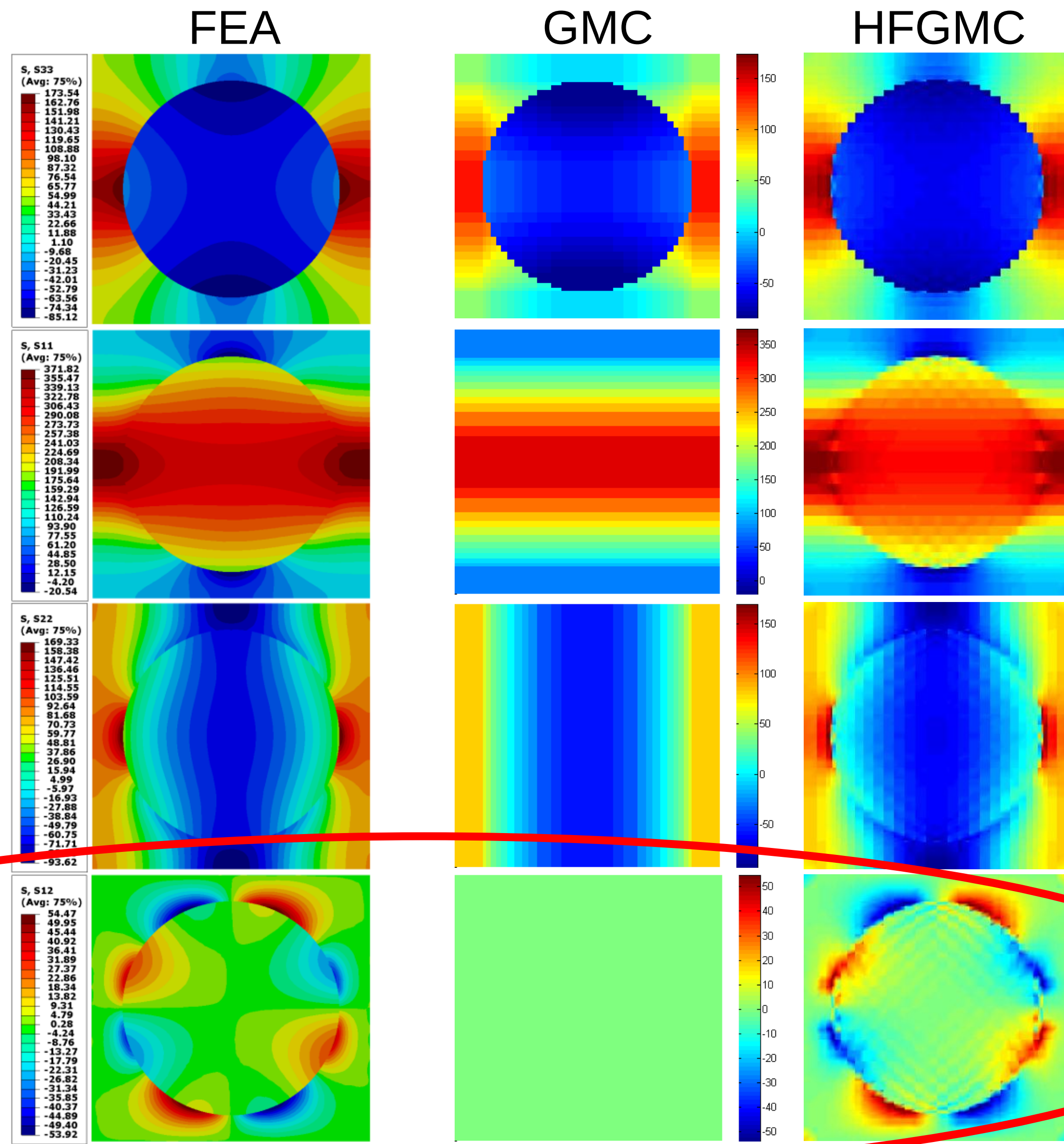
Individual Stress Components

Axial stress (MPa)

Transverse stress in loading direction (MPa)

Transverse stress (MPa) normal to loading direction

Transverse shear stress (MPa)



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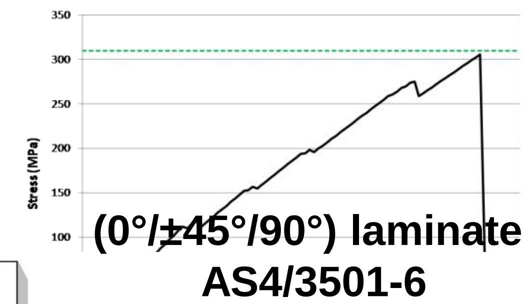
Failure Criterion for Strength and Durability

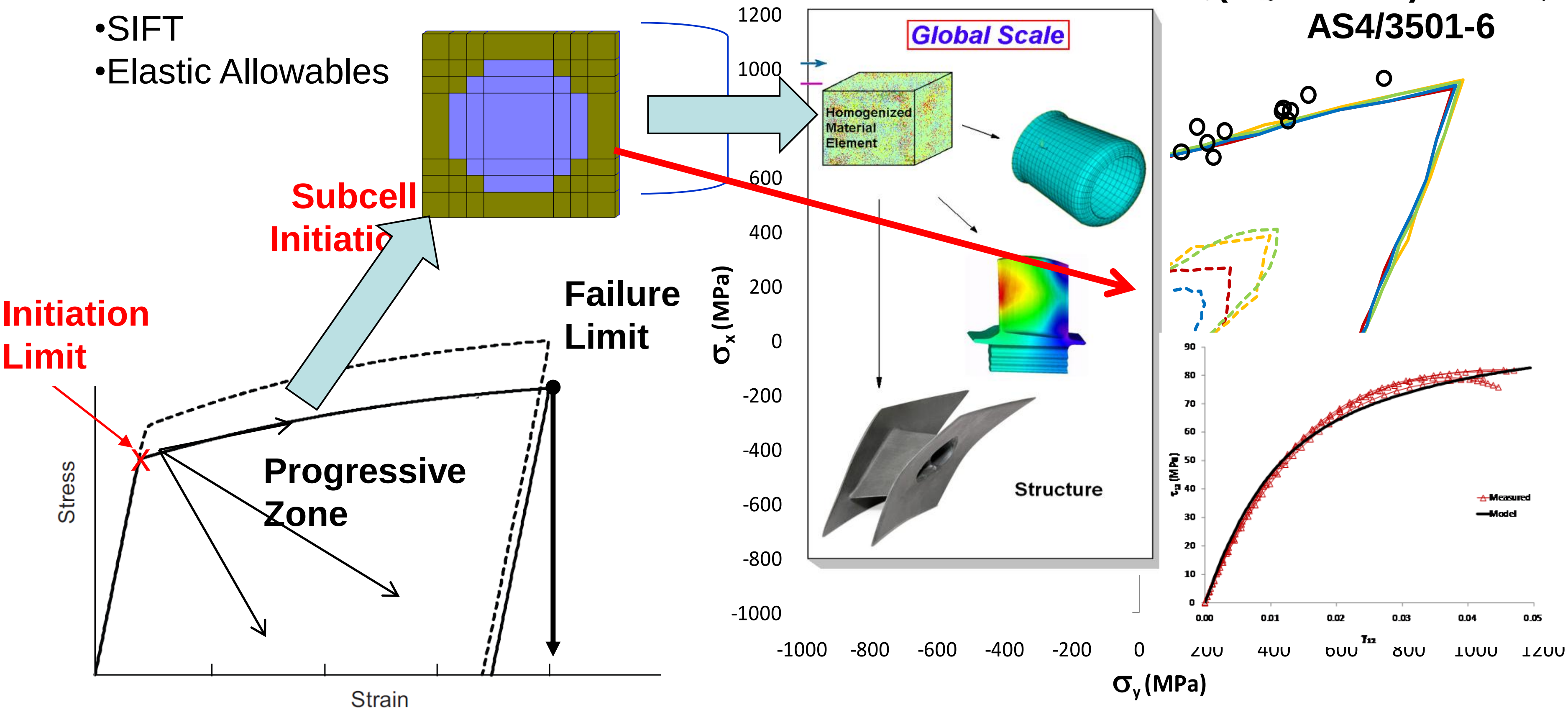
<u>Subcell Elimination Criterion</u>

- Max. Stress Theory
- Max. Strain Theory
- Tsai-Hill Theory
- Tsai-Wu
- SIFT
- Elastic Allowables

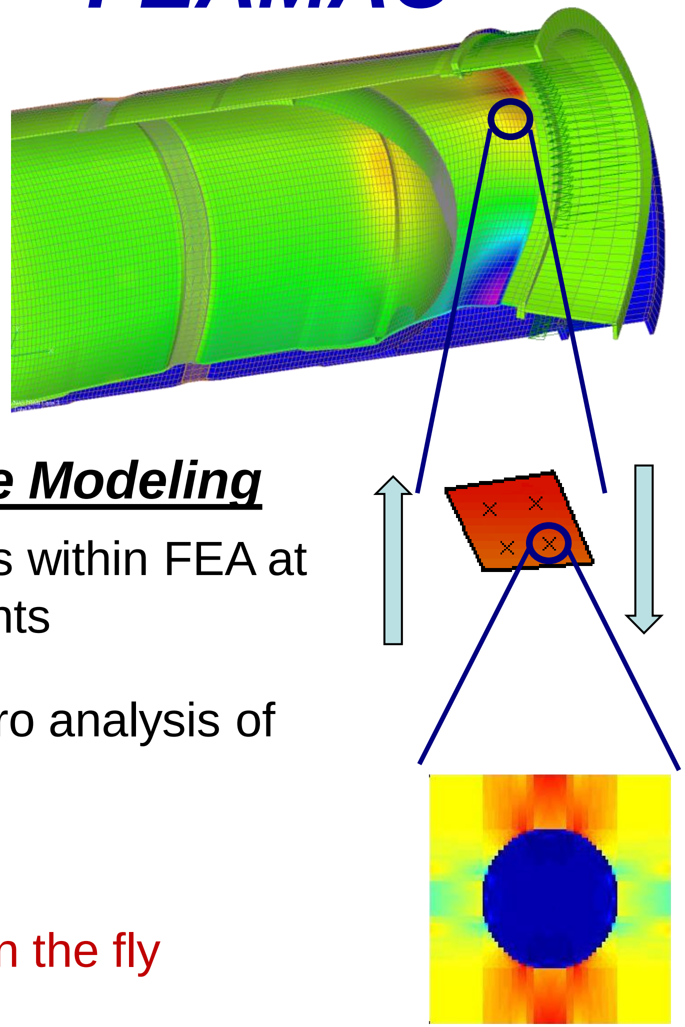
<u>Progressive Damage Criterion</u>

- *Scalar Damage (Triaxial)*
- *MMCDM*
- Smeared Crack Band





Integrated *Multiscale Analysis of Arbitrary Composite Structures with FEAMAC*



Structure-Scale FEA

Element/Integration Point

MAC/GMC micromechanics analysis

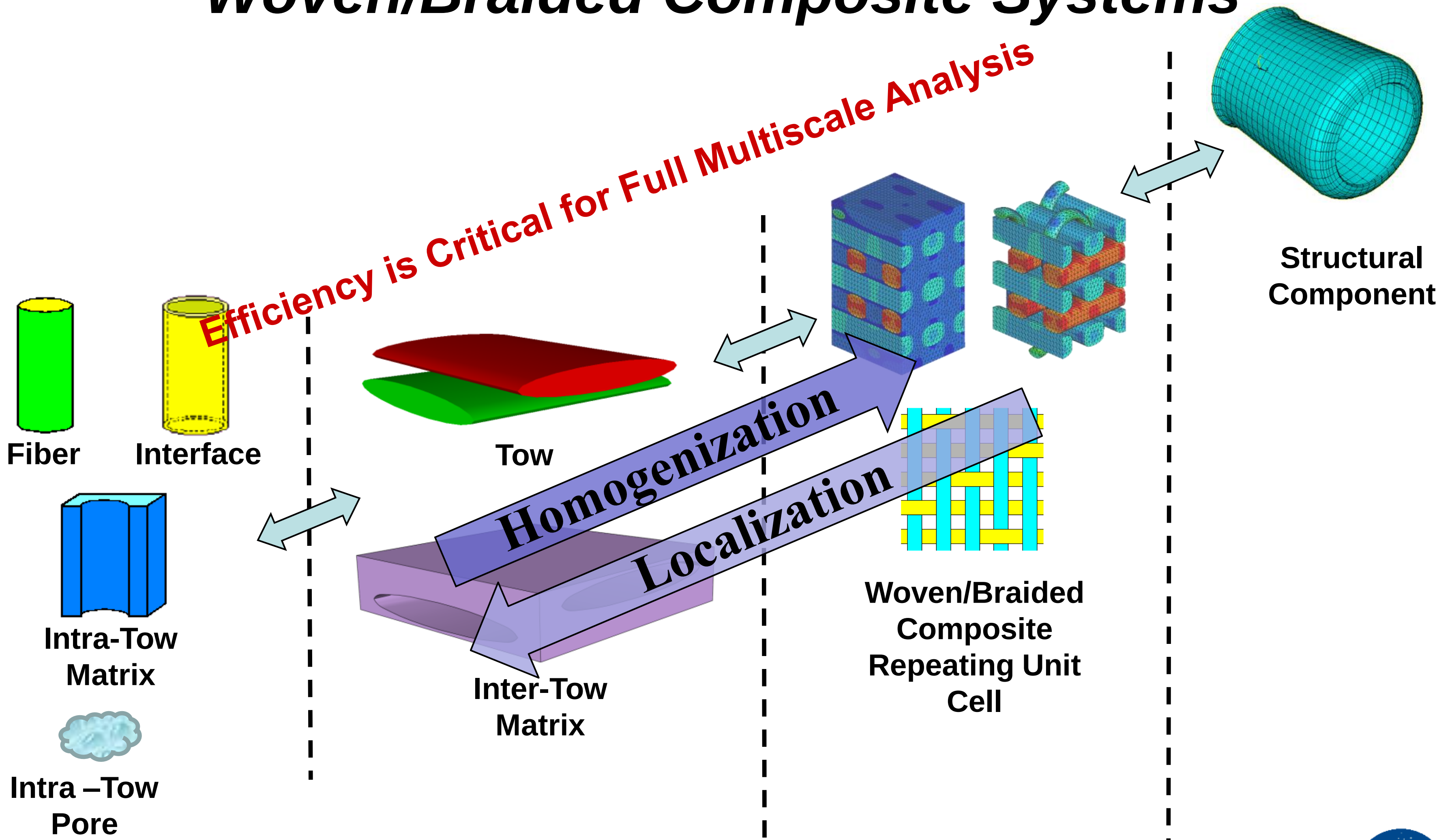
Synergistic Multiscale Modeling

- Embed micromechanics within FEA at element integration points
- New tool for micro/macro analysis of composite structures:

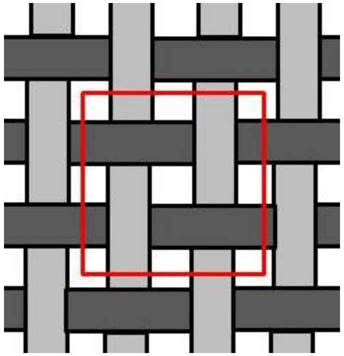
  FEAMAC

- Localize/homogenize on the fly

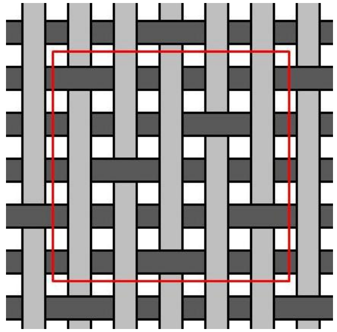
Utilize Novel Multiscale Generalized Method of Cells (MSGMC) For Concurrent Analysis of Woven/Braided Composite Systems



Problem Definition



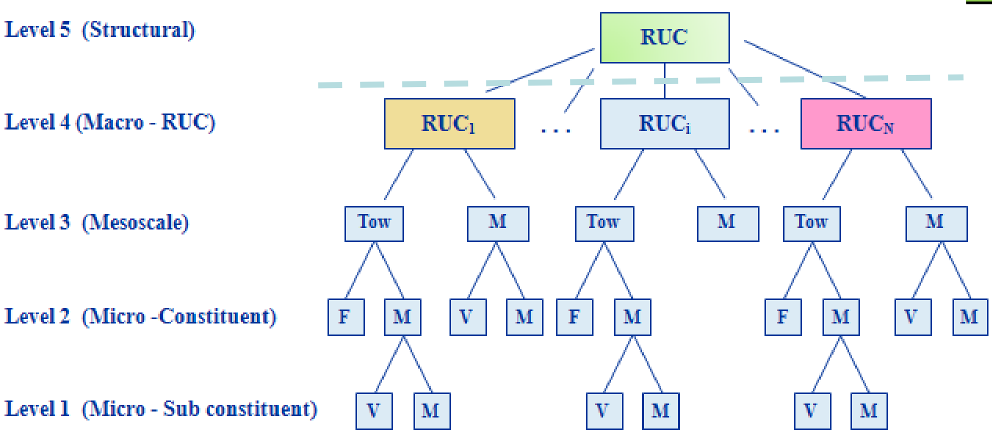
Plain



5 Harness Satin (5HS)

Weave Properties

Type	5HS
Overall Fiber Volume Fraction	36%
Tow Volume Fraction	78%
Tow Width	1.25 mm
Tow Spacing	0.34 mm
Thickness	2.5 mm
Matrix	CVI-SiC



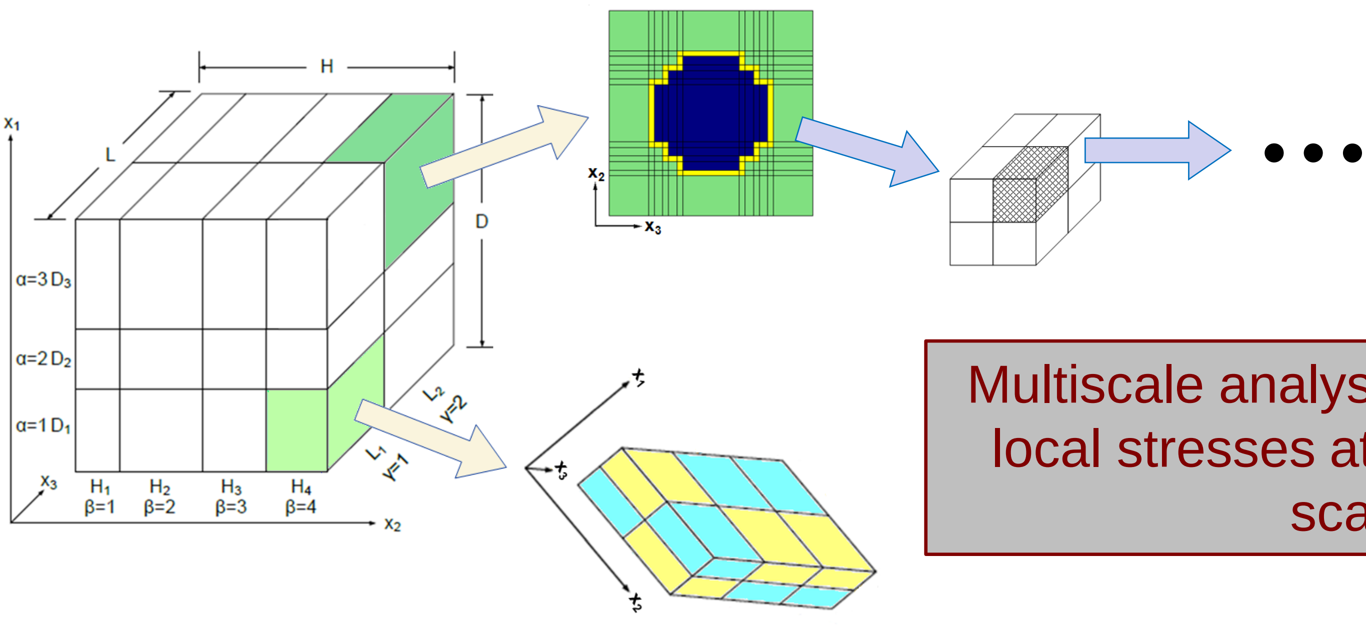
Tow Properties

Fiber Vol Fraction within Tow	46%
Tow Packing Structure	Square
Fiber	IBN-Sylramic
Matrix	CVI-SiC
Interface	BN

Current Multiscale Analysis Involves 4 Scales And 3 Homogenizations/Localizations

Multiscale Generalized Method of Cells(MSGMC) Overview

- Newly developed recursive GMC methodology
  - Each length scale in each subcell can call a separate GMC analysis
- Works for any arbitrary multiphase material
  - Elastic / Inelastic / Damage



Multiscale analysis can determine local stresses at different length scales

$$\sigma^{\{\alpha\beta\gamma\}\{\beta g\}} = C^{\{\alpha\beta\gamma\}\{\beta g\}} A^{\{\alpha\beta\gamma\}\{\beta g\}} A_{tt}^{\{\alpha\beta\gamma\}} A_{ip}^{\{\beta\gamma\}} \Delta\varepsilon$$

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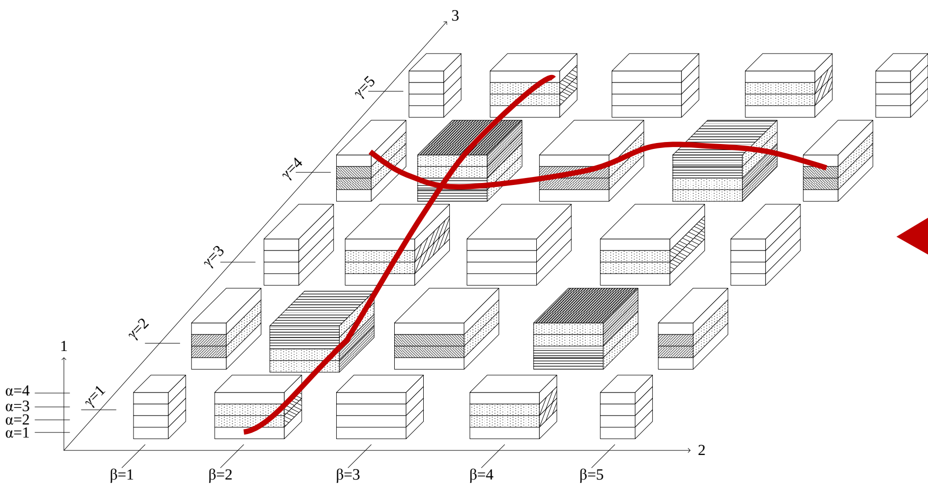
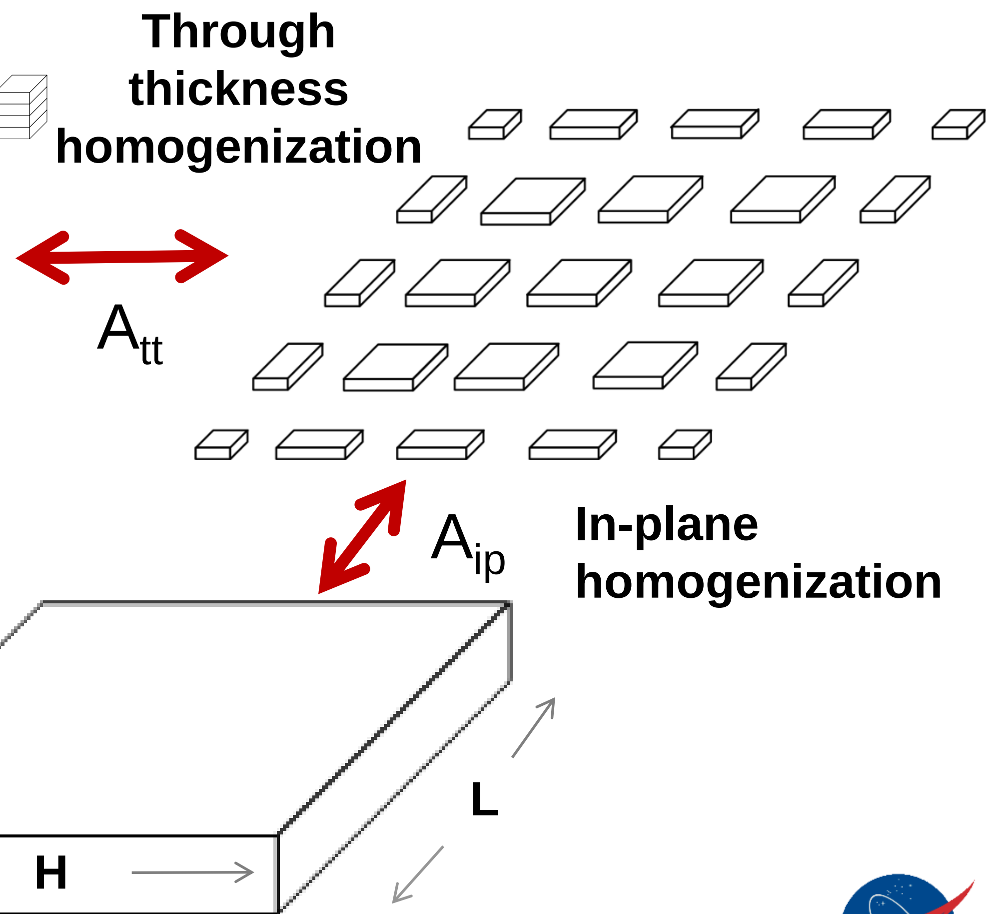
Macroscale (Weave) Two Step Homogenization

To compensate for lack of normal-shear coupling within GMC a two-step homogenization scheme is employed for woven composites.

(Bednarcyk & Arnold, *IJSS*, 41, 2003)

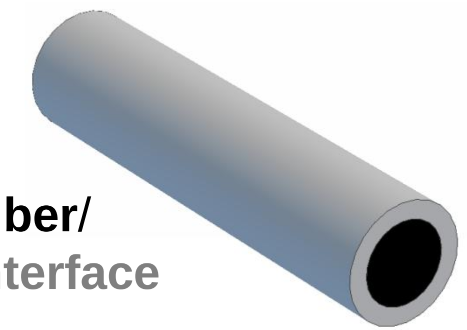


Plain Weave



Constituent Constitutive Model and Strain Localization

Microscale



Fiber/
Interface

Assume Fiber and Interface Linear Elastic Hashin Fiber Failure Criteria (1980)

$$f = \frac{\sigma_{11}^{\;2}}{\sigma_{axial}^{\;2}} + \frac{1}{\tau_{axial}^{\;2}}\left(\sigma_{13}^{2} + \sigma_{12}^{2}\right)$$

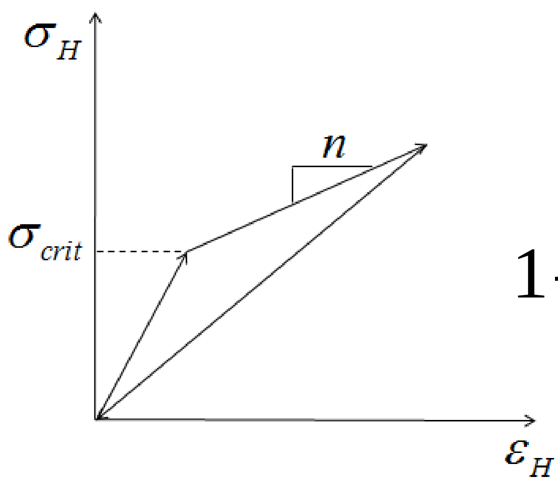
Assume Linear Elastic with a Scalar Damage constitutive relationship

$$\sigma^{\{\alpha\beta\gamma\}\{\beta\gamma\}} = \left(1-\phi^{\{\alpha\beta\gamma\}\{\beta\gamma\}}\right)C^{\{\alpha\beta\gamma\}\{\beta\gamma\}}\varepsilon^{\{\alpha\beta\gamma\}\{\beta\gamma\}}$$

Matrix damage driven by magnitude of triaxiality

$$\text{If } \sigma_H > \sigma_{critical}$$

$$f = 3\varepsilon_H nK - \sigma_H = 0$$



K° = initial bulk modulus

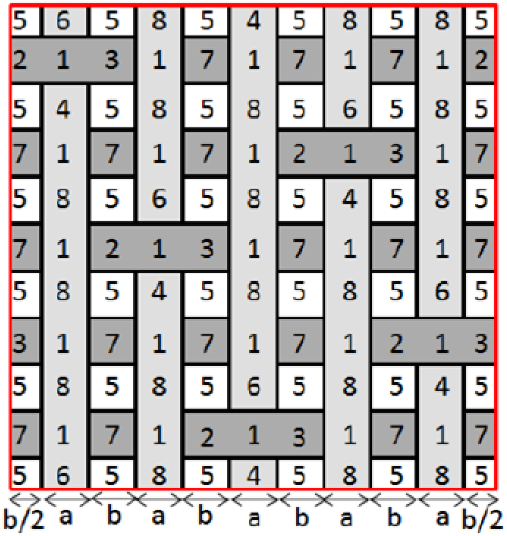
$$\sigma_H = I_1(\sigma) = \frac{\left(\sigma_{11}+\sigma_{22}+\sigma_{33}\right)}{3}$$

$$\varepsilon_H = I_1(\varepsilon) = \frac{\left(\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33}\right)}{3}$$

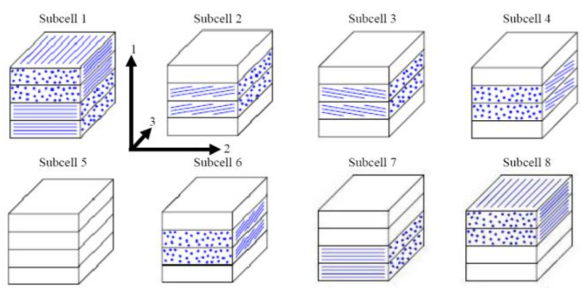
$$1-\phi^{i+1} = \lambda^{i+1} = \frac{n\Delta\varepsilon_H^{i+1} + \lambda^{i}\varepsilon_H^{i+1}}{\left(\Delta\varepsilon_H^{i+1} + \varepsilon_H^{i+1}\right)}$$

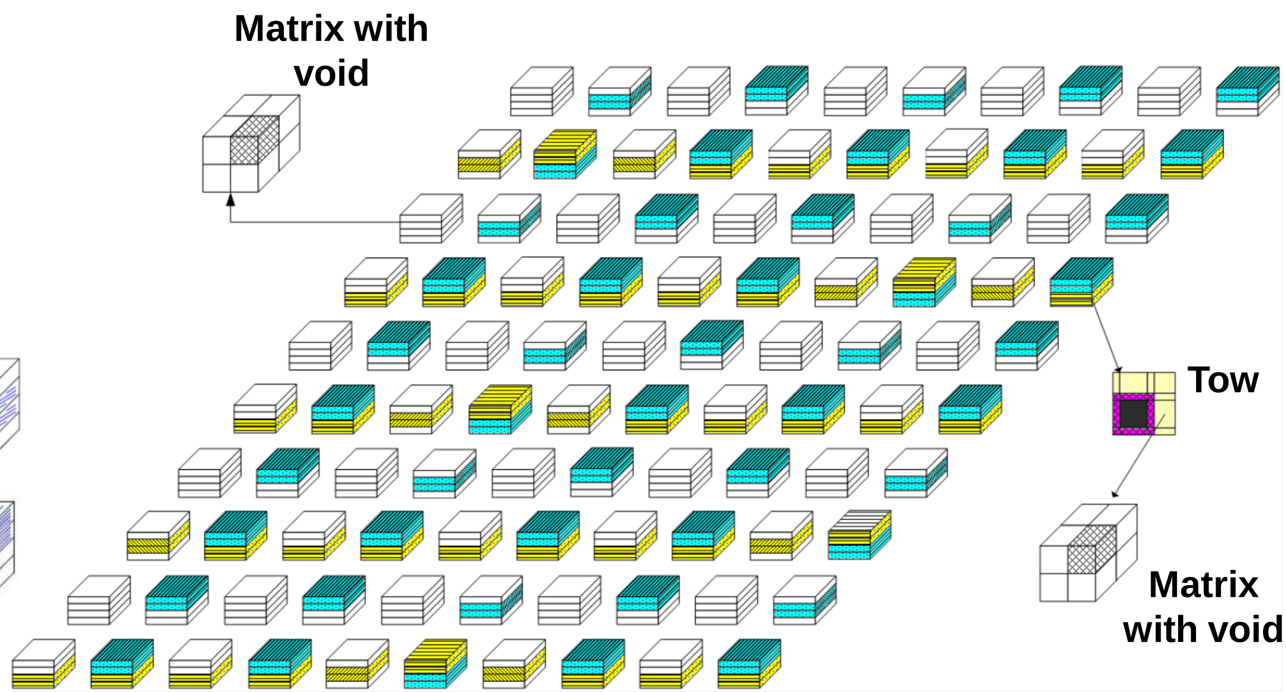
$$\left(\varepsilon^{i+1} = \varepsilon^{i} + \Delta\varepsilon^{i+1}\right)$$

Full Multiscale Modeling of 5HS Weave with Porosities



5HS and most other orthogonal weaves can be discretized into 8 unique subcell groups. Furthermore model tow and matrix with voids using lower scale RUCs

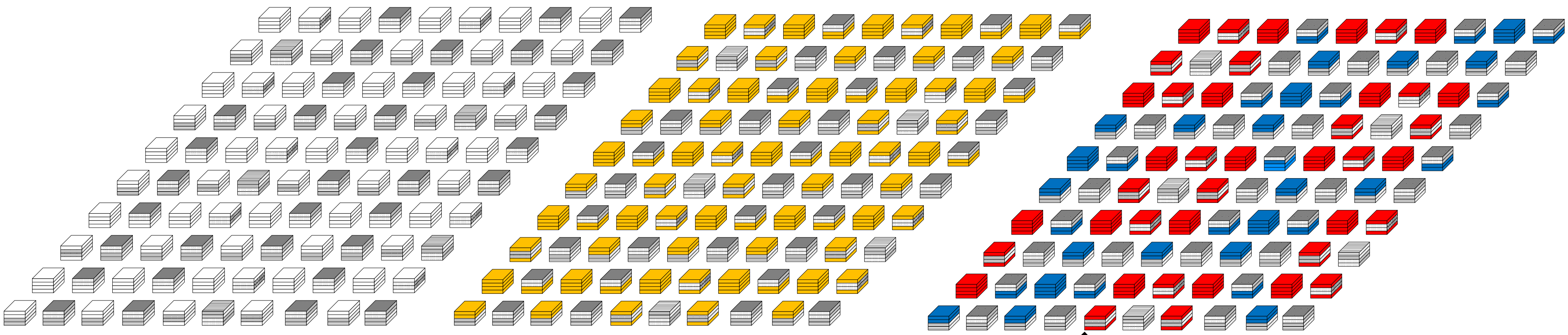




Three Void Modeling Schemes Considered

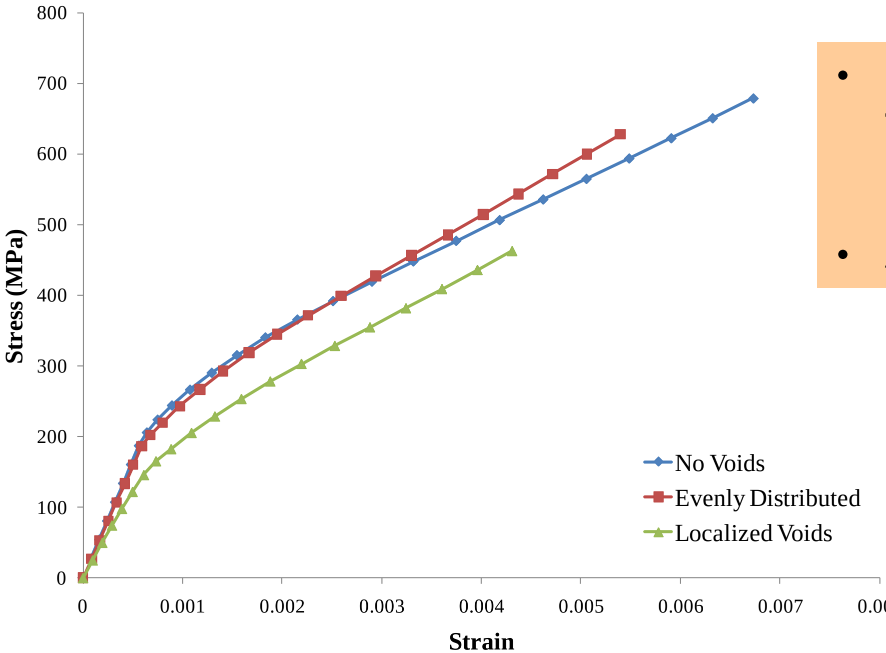
Voided Matrix Response Achieved via Separate GMC Analysis

No Voids   Evenly Distributed Voids   Localized Voids



Gold = 12.7% voids ; Red = 90% voids; Blue = 5% voids

Load direction



- **Localization of porosity significantly influences failure response**
  a) Knee – 33% delta
  b) Strain to Failure – 25-55% delta
- Assuming uniform distribution of voids similar to no voids

Fiber

Name	iBN-Sylramic
Modulus	400 GPa
Poisson's Ratio	0.2
Axial Strength	2.2 GPa
Shear Strength	900 MPa

Interface

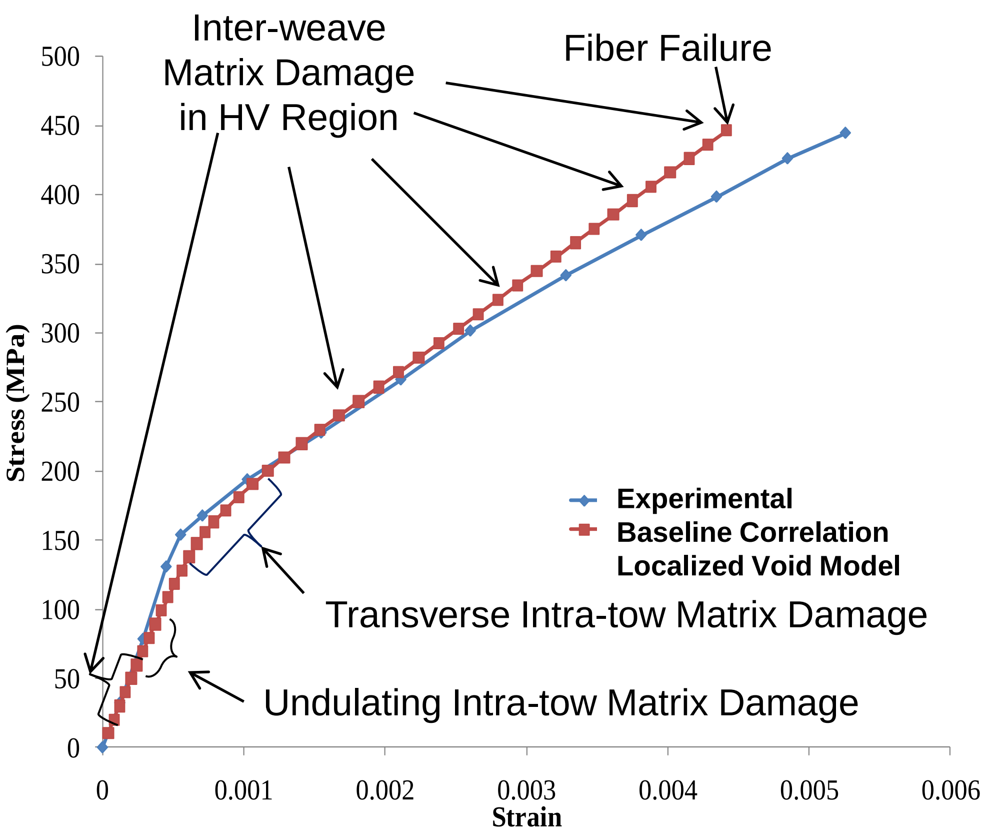
Name	Boron Nitride
Modulus	22 GPa
Poisson's Ratio	0.22

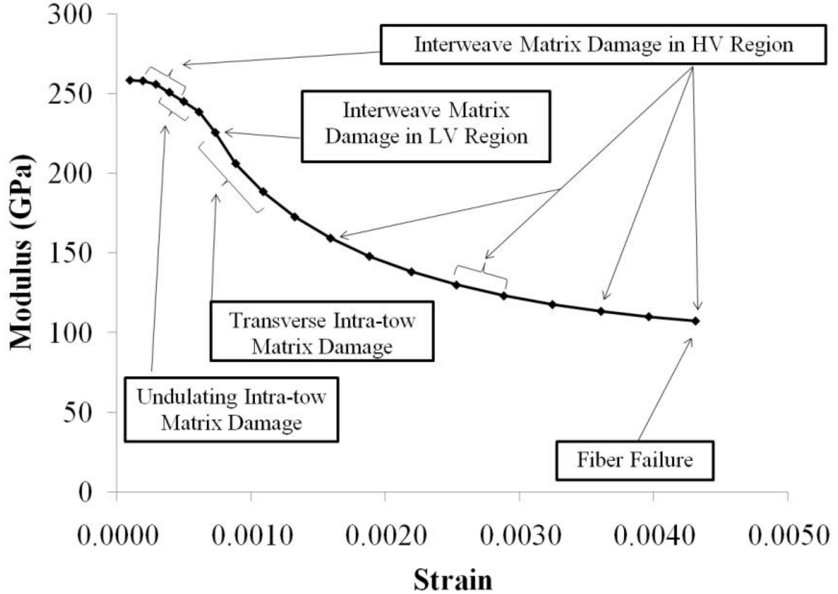
Matrix

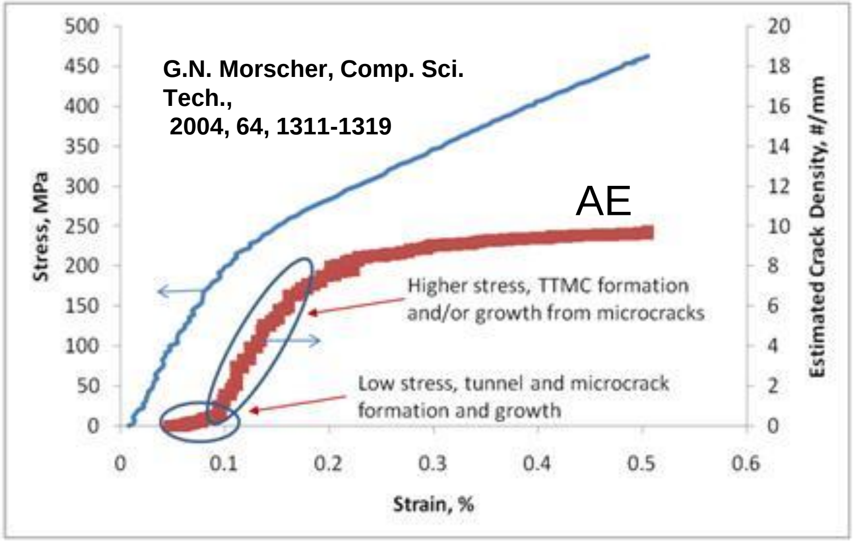
Name	CVI-SiC
Modulus	420 GPa
Poisson's Ratio	0.2
σ_{dam}	180 MPa
n	0.04

Simulation Identifies Local Damage Events / Mechanisms Explaining Nonlinearities in Macro Stress Strain Curve

Assuming 5HS RUC with Localized Porosities





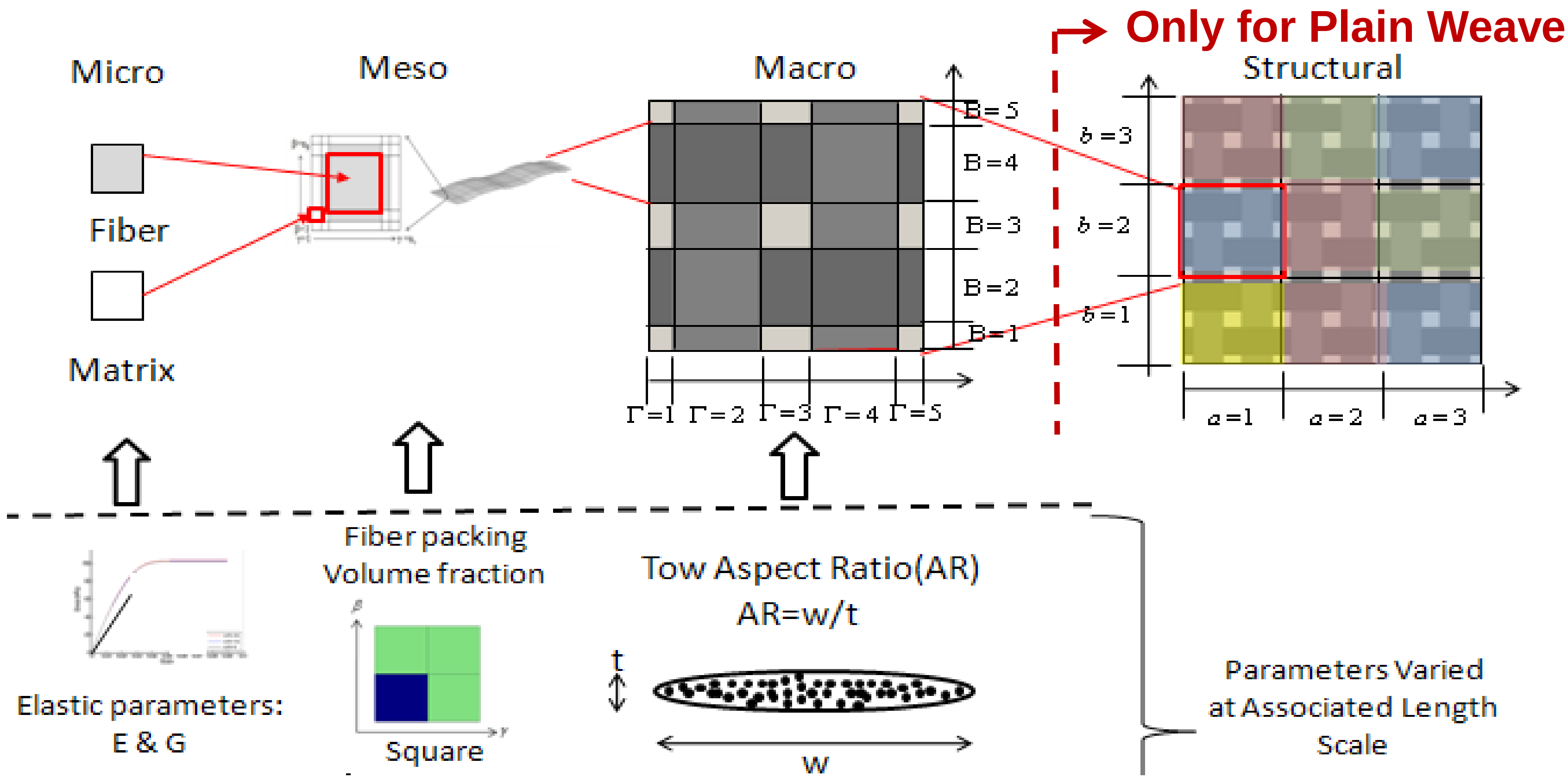


Fiber: Elastic , Hashin Fiber Failure Criteria (includes shear stress)

Interface: Elastic (very compliant 1/20th)

Matrix: Elastic, Hydrostatic-Driven Damage

Study Effects Of Micro, Meso, And Macro Parameters on Macroscale Response



Architectural Parameter	Relevant Length Scale	Values
Tow Fiber Volume Fraction	Meso	0.46,0.48,0.50
Tow Void Volume Fraction	Meso	0.01,0.05,0.07
Tow Aspect Ratio	Macro	8,10,12

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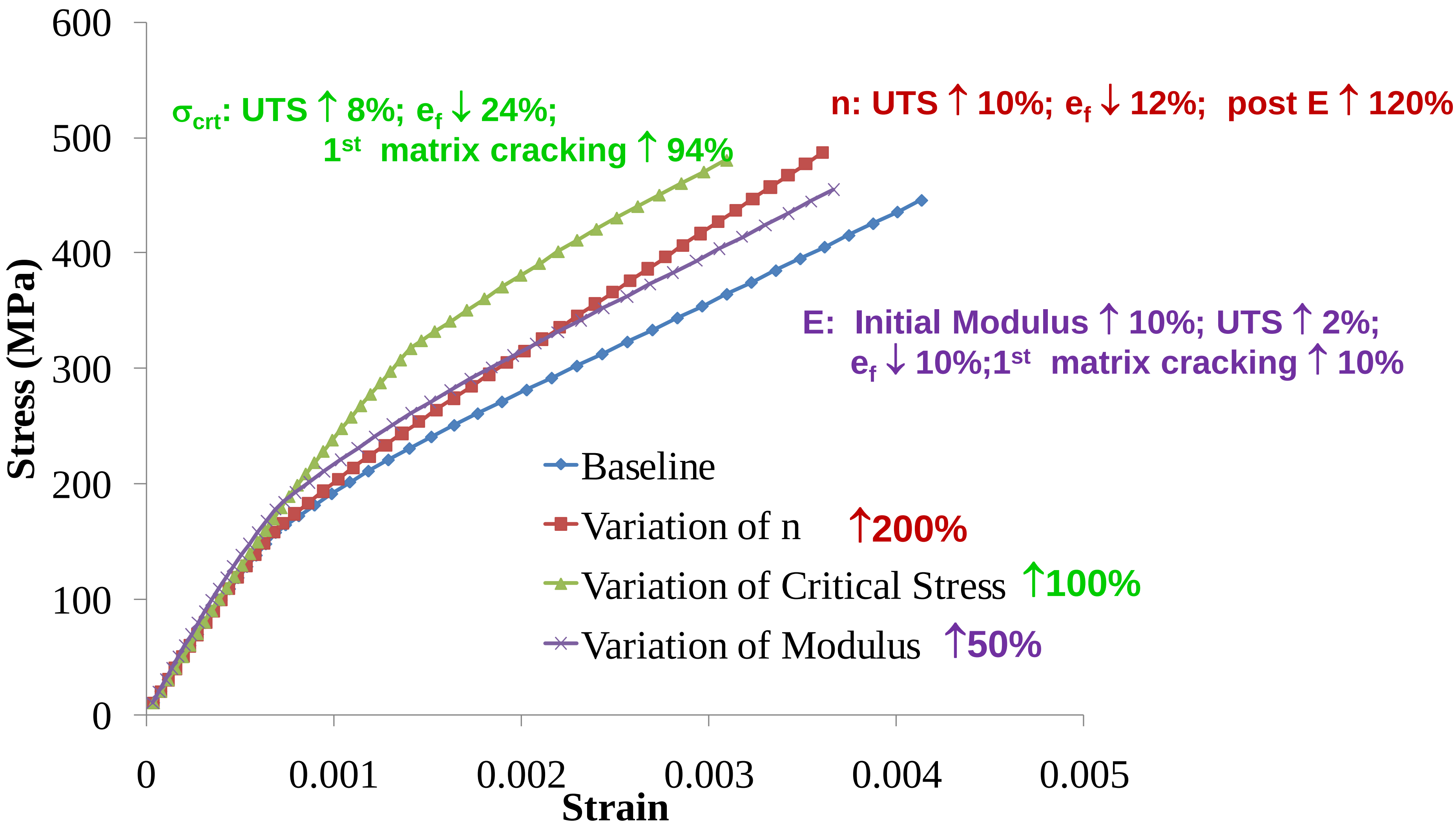
Influence of Varying Matrix Material Parameters on the Macroscale Response



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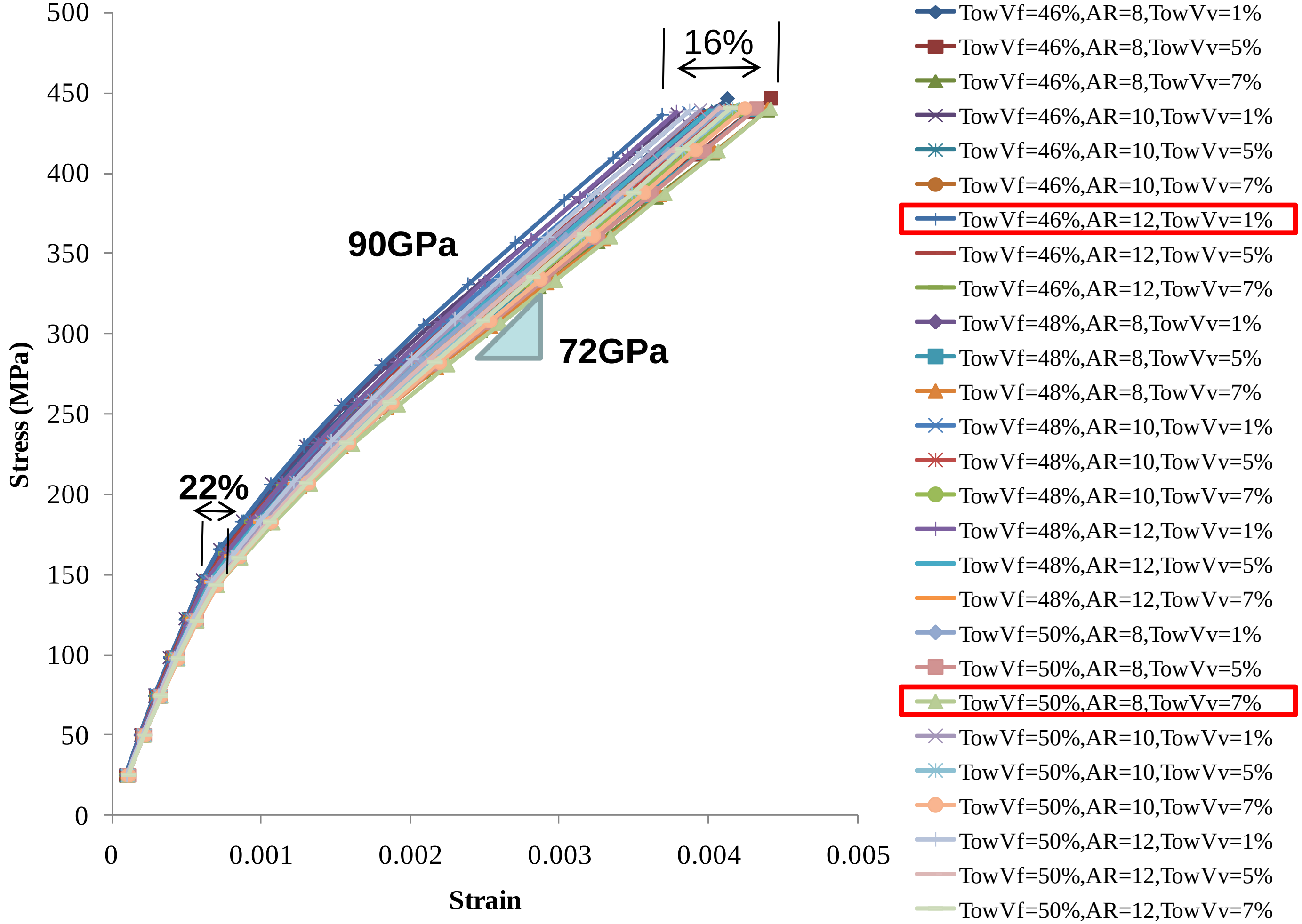
Depicts Entire Range Of Macro Response Curves Given the 27 Variations In Architectural Parameters

Utilized Localized Void Model

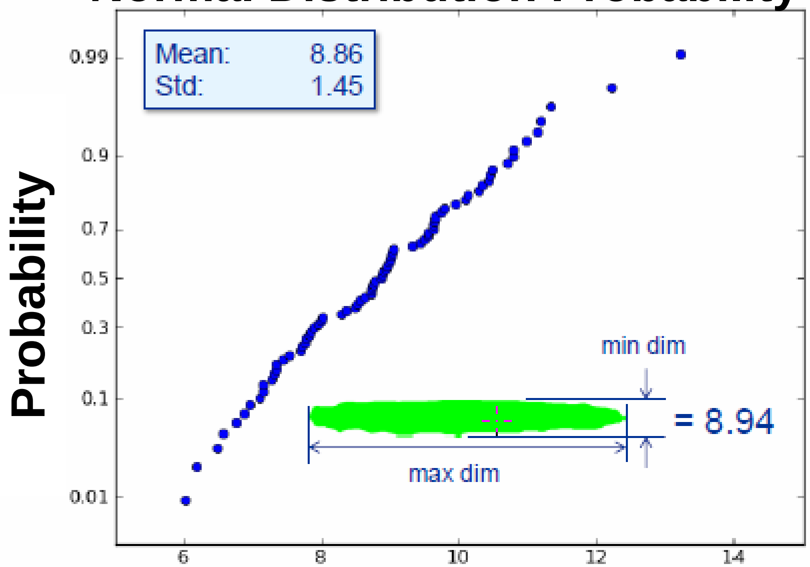
Architectural Variations clearly contribute to variation in measured material response.

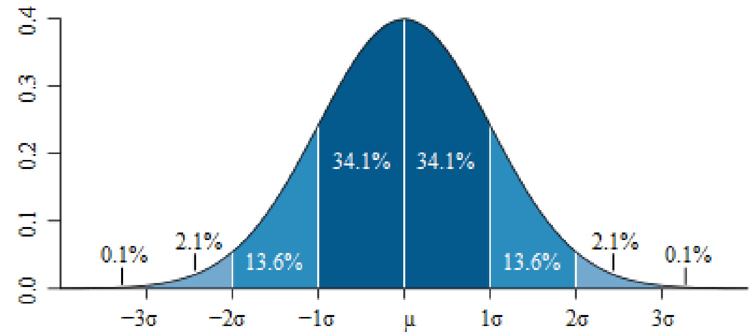
- **Initial Modulus ≈ 24%**
- **UTS ≈ 2%**
- **1st matrix cracking ≈ 16%**
- **Post matrix cracking Modulus ≈ 24%**
- **ε_f impacted ≈ 16%**



Assumed Normal Distributions for Architectural Parameters

Normal Distribution Probability Plot*



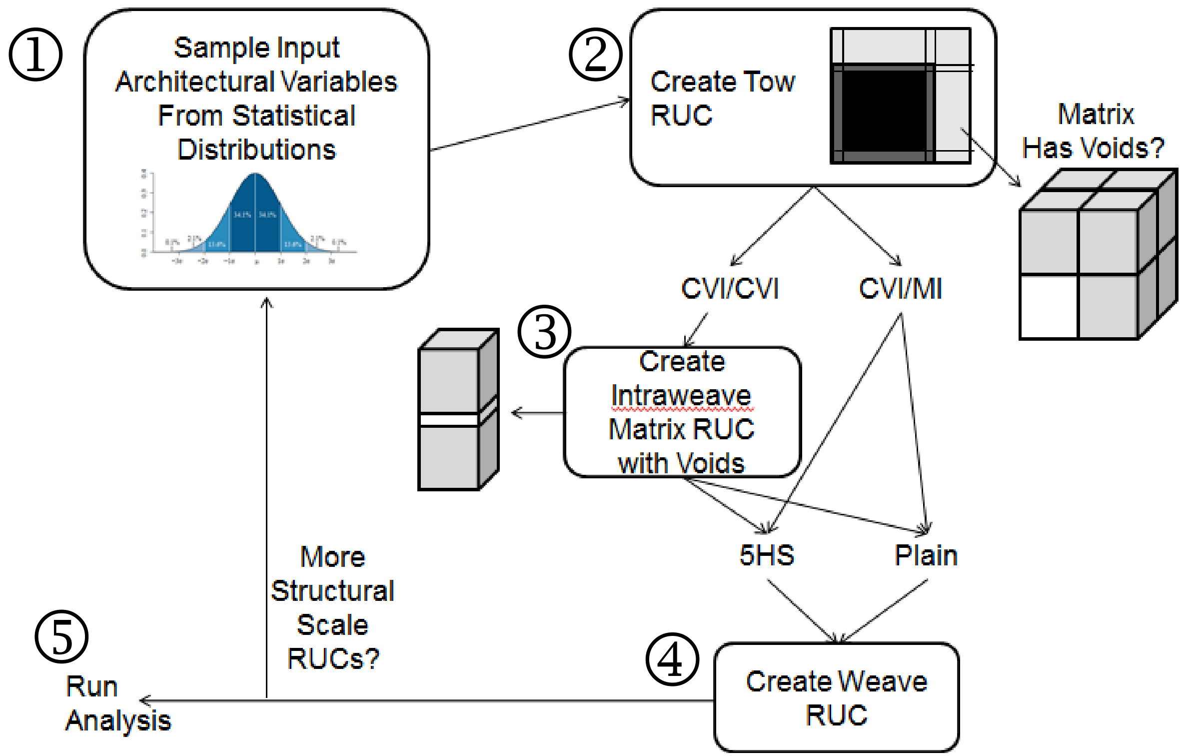


Parameter	Mean	Std. Dev.
Tow Fiber Volume Fraction	0.48	0.033
Tow Aspect Ratio	8	.533
Tow Void Volume Fraction	0.05	0.01
Localized Weave Void Volume Fraction	0.75	0.05

Note: Material Properties held fixed at Baseline Values; Void shape – sheet like

*Bonacuse, P., Subodh M., and Goldberg, R.; "CHARACTERIZATION OF THE AS MANUFACTURED VARIABILITY IN A CVI SIC/SIC WOVEN COMPOSITE, Proceedings of ASME Turbo Expo 2011, GT2011, June 6-10, 2011, Vancouver, Canada

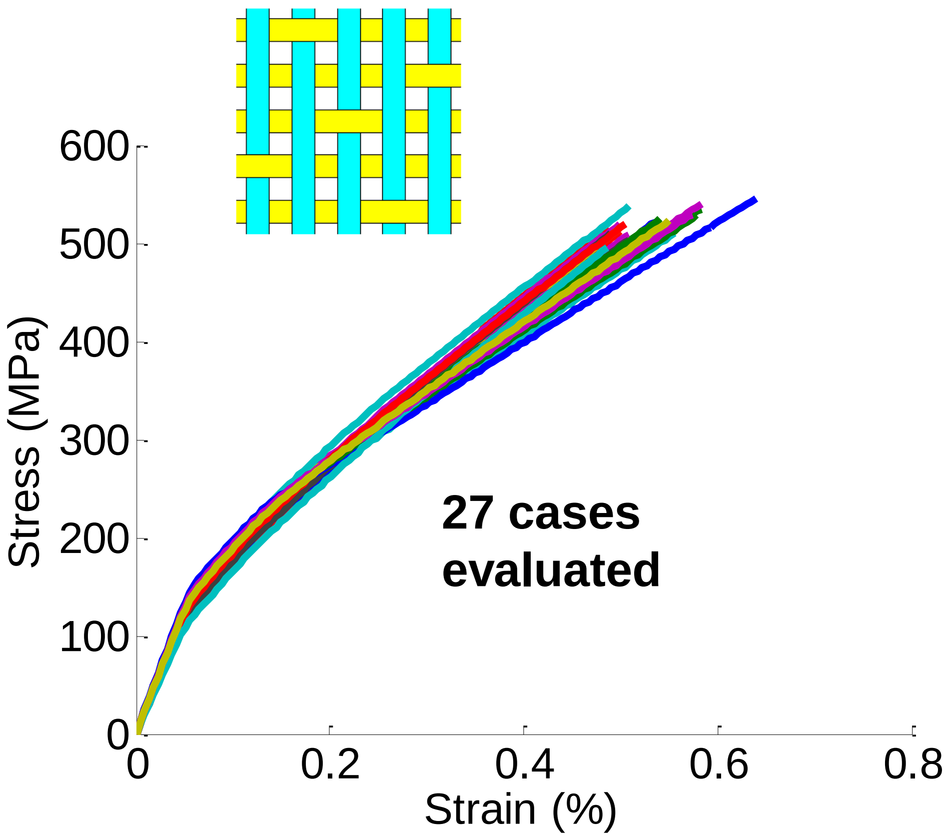
Procedure for Incorporating Stochastics Requires Significant Computation Resources



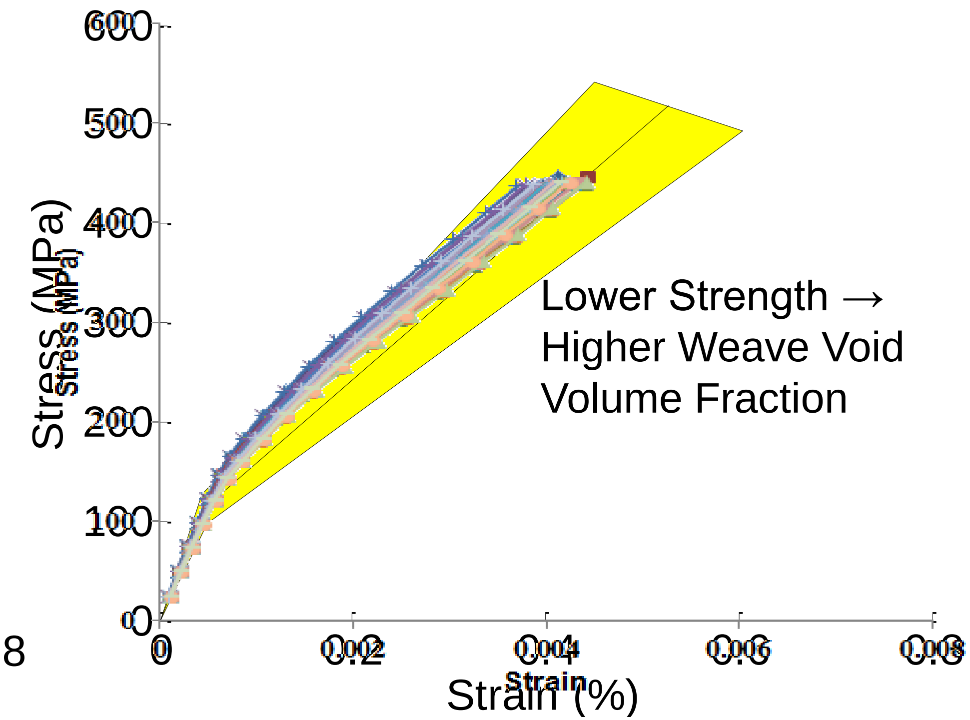
Weave Type	Time / Increment (sec)	Typical Increments	Total Time (sec)	No. of Subcells
5HS (1x1)	12	200	4000 (1.1hrs)	93,800
PW (1x1)	1.5	150	225	18,840
PW (6x6)	53	200	10600 (2.9 hrs)	678,276

Macro Stress-Strain Response Curves Given Stochastic Assumption of Architectural Parameters

Utilized Localized Void Model



Statistical Reconstruction 95% Confidence*



*determined from normality assumption using bilinear approximation

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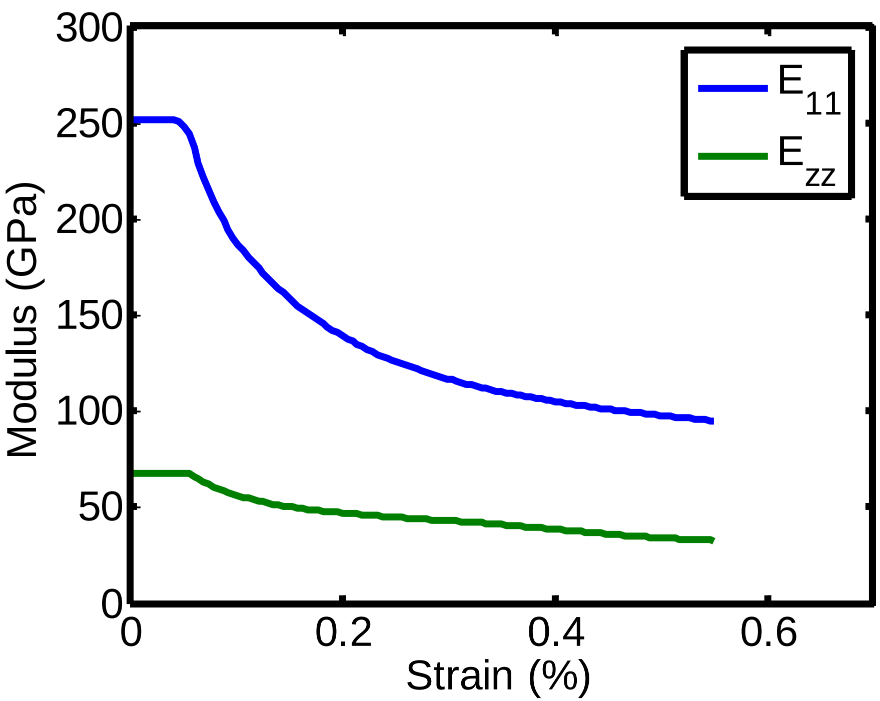
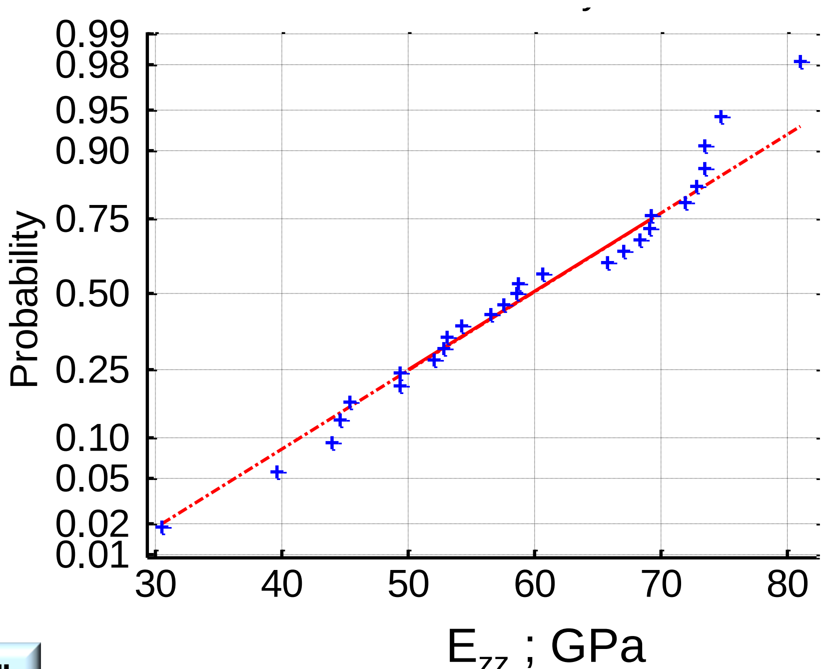
Secant Through Thickness Moduli (E_{zz}) Degrades With Loading As Does In-plane (E_{11})



Norm Probability Plot of E_{zz} → Has some skewness towards right…maybe log normal?



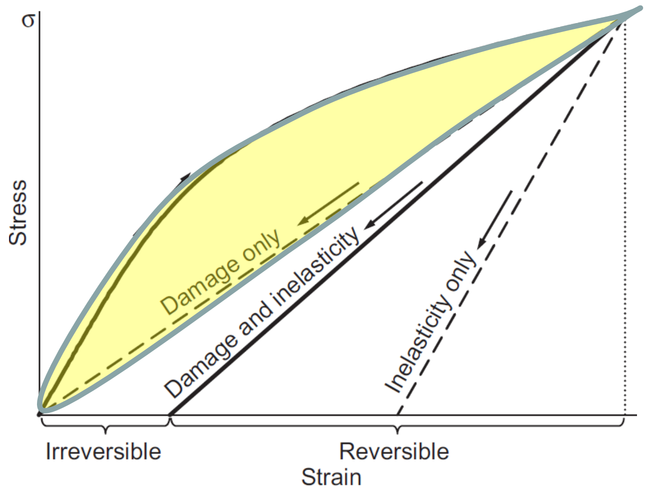
Note: In composites many material “properties” evolve with loading history !

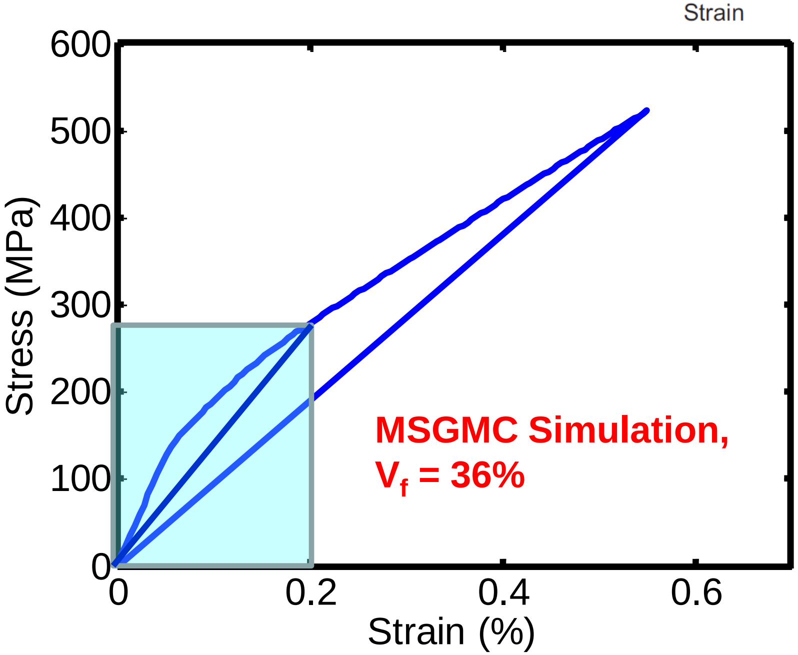
Normal in ≠ Normal out

E_{zz} = 59.02 ± 12.5 ; 68.2% confidence

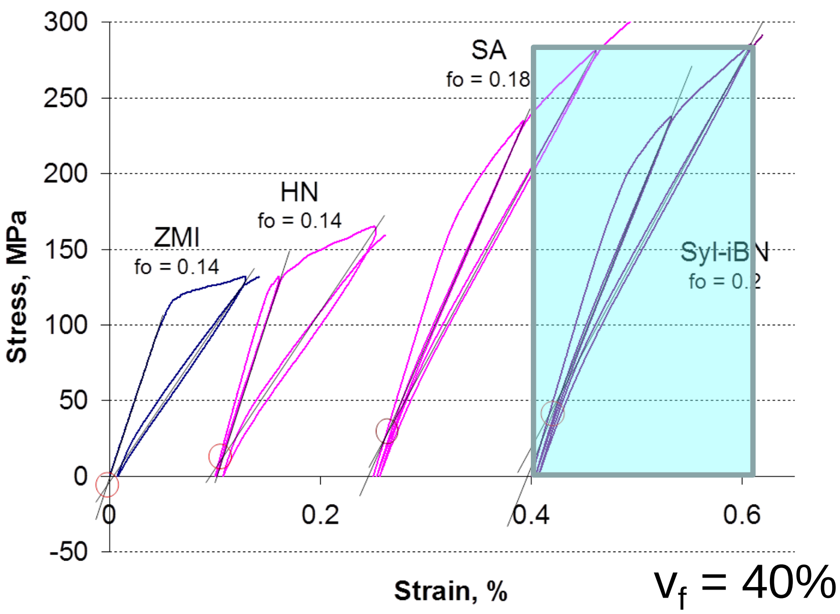
E_{zz} = 59.02 ± 25 ; 95.4% confidence

Loading Histories with Unloading Are Critical For Deducing Mechanisms Driving Nonlinear Response



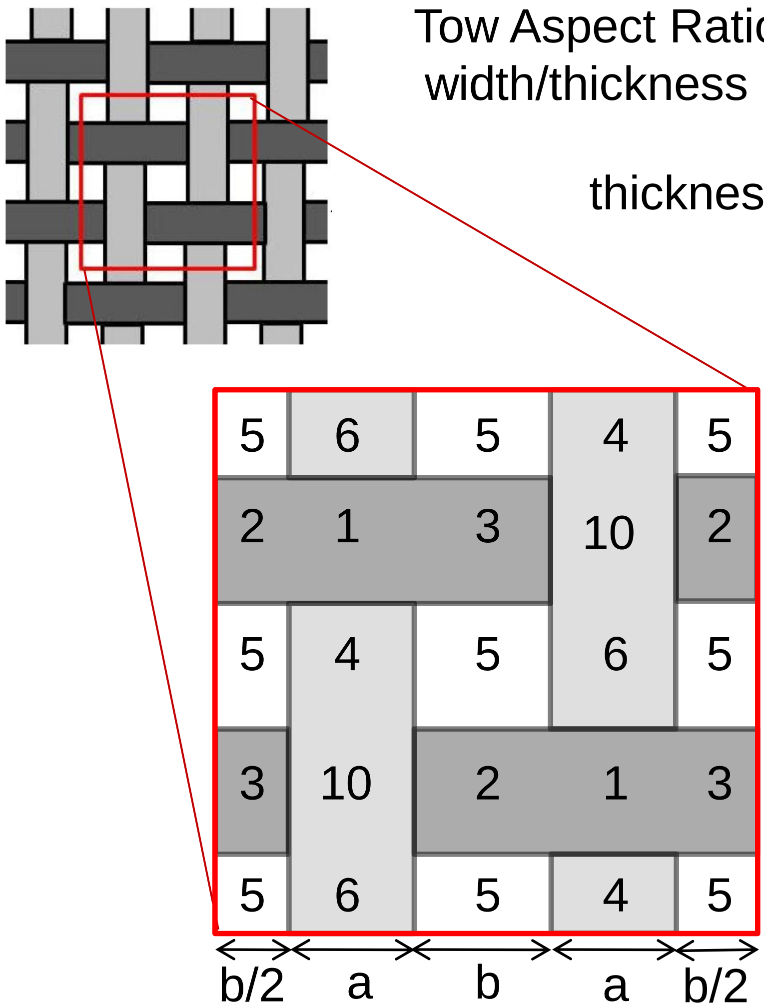


Morscher, G.: 2008
Experimental Unloading Response Returns to Zero – <u>indicating nonlinearity due to damage</u>



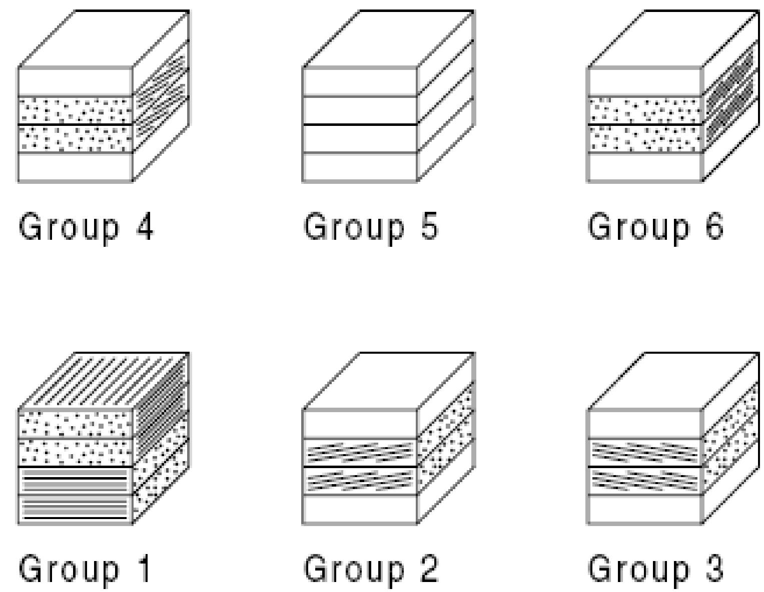
Examine Plain Weave Discretization to Study Architectural Parameters on Structural Scale

Tow Aspect Ratio= width/thickness



width

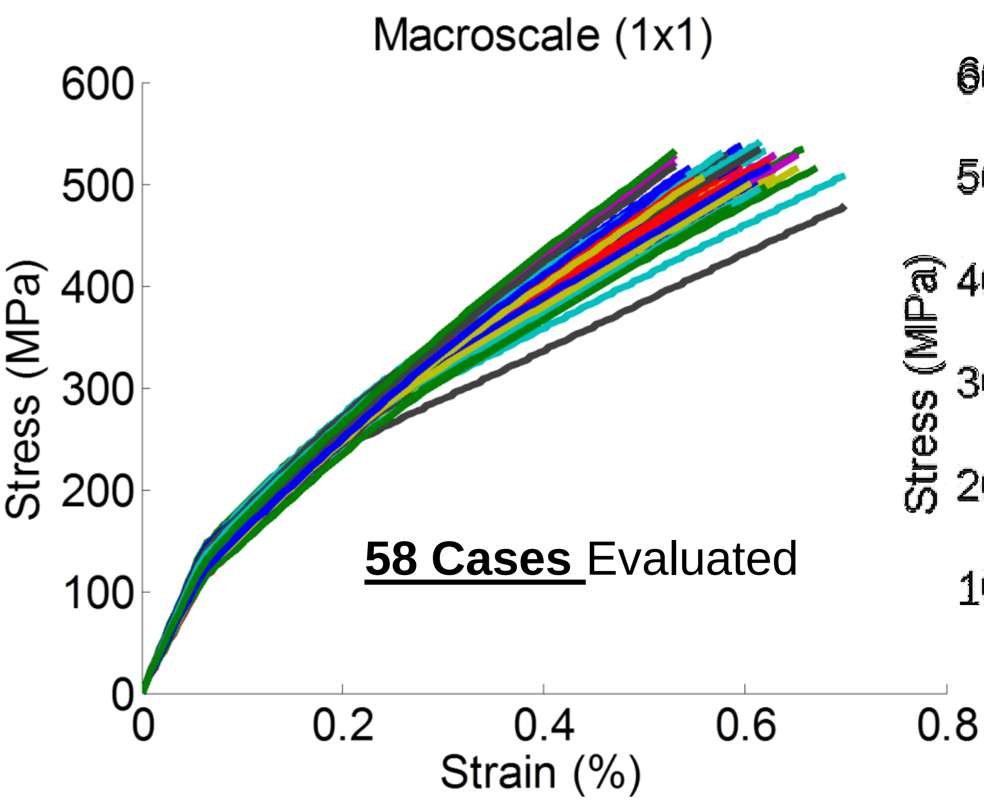
thickness



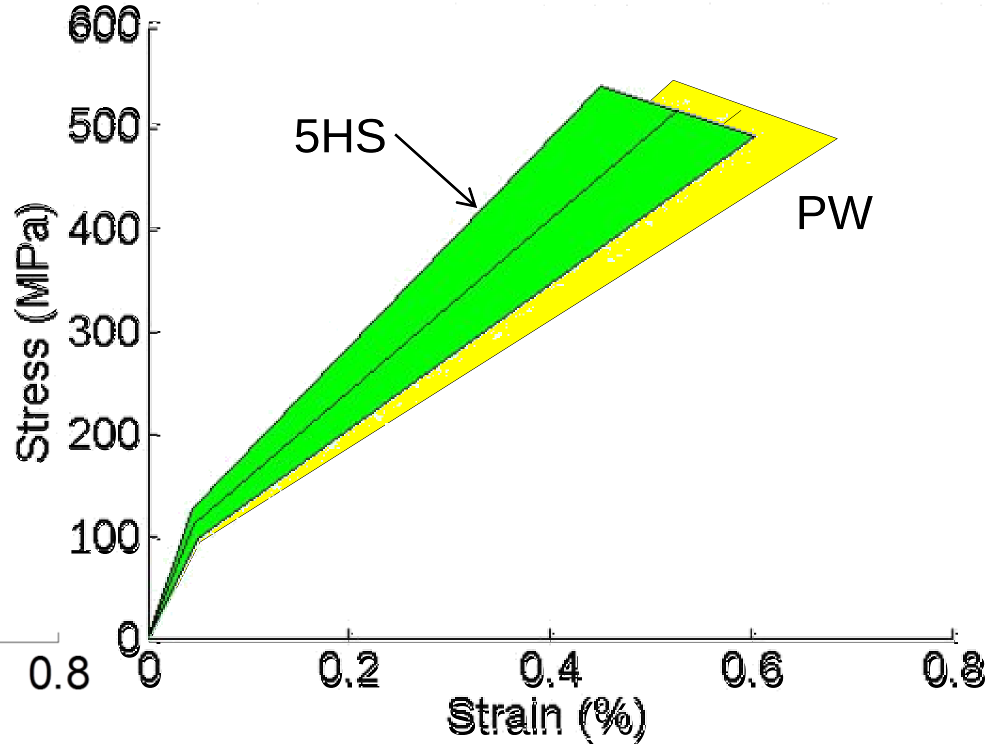
Subcell group properties determined from lower length scales

Macroscale – Plain Weave Discretization

Assumes Normal Distribution for all Architectural Parameters



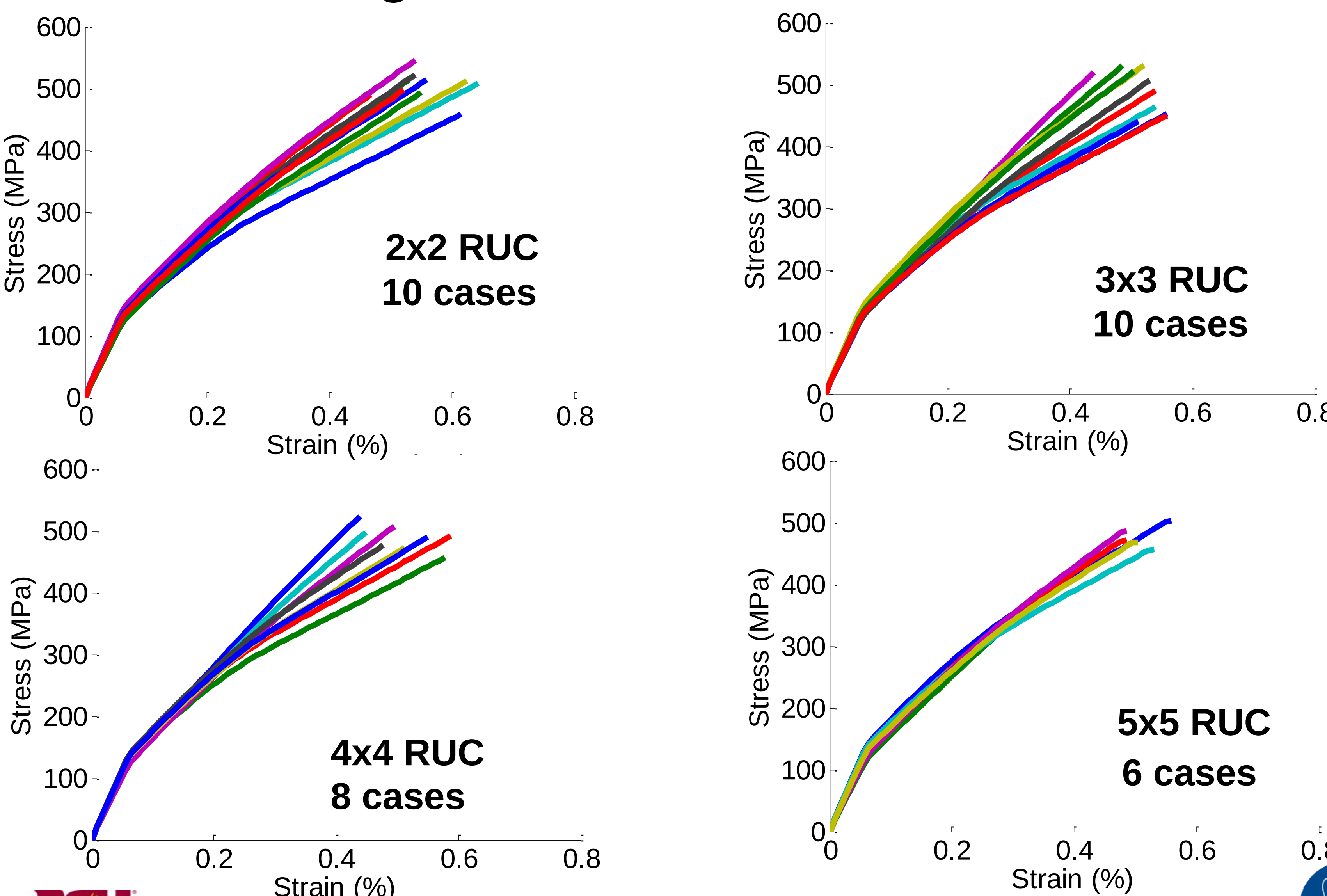
Statistical Reconstruction 95% Confidence*



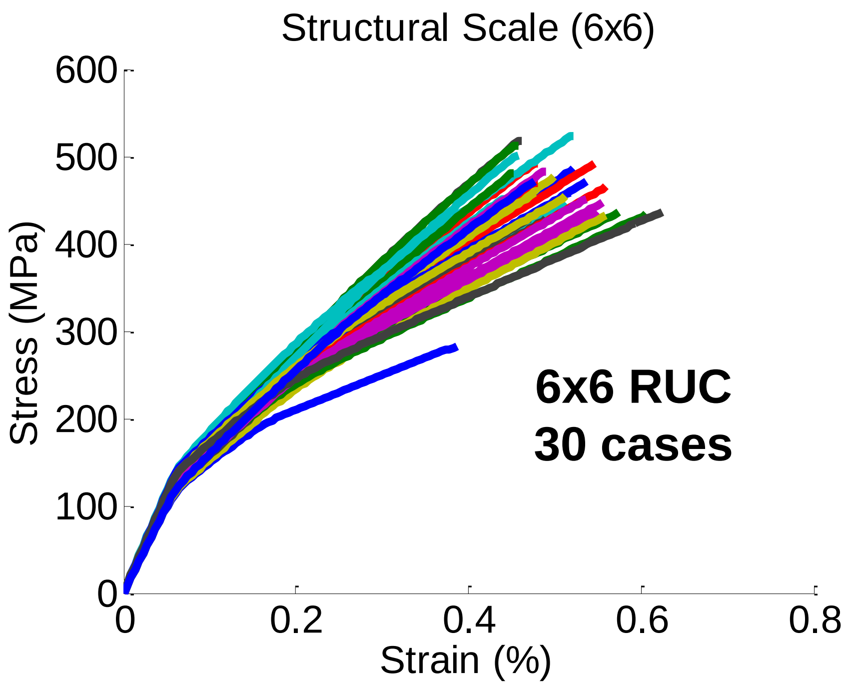
*determined from normality assumption using bilinear approximation

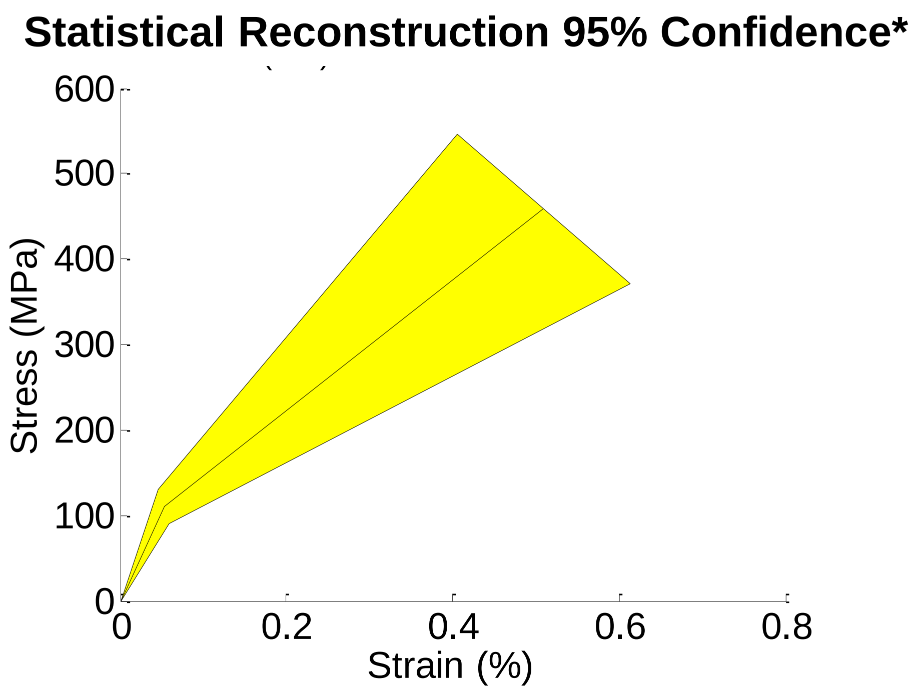
PW slightly less stiff and more nonlinear than 5HS

Sensitivity To Architectural Features Changes With Increasing Structural Scale: Plain Weave



Sensitivity To Architectural Features Changes With Increasing Structural Scale: Plain Weave



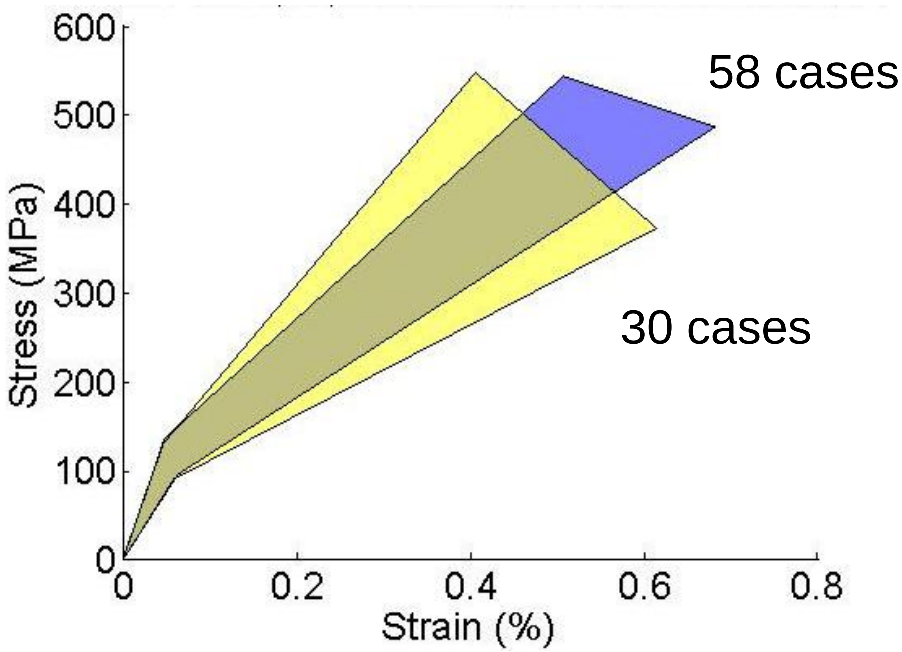


Property	E11 (GPa)	PLS	H (GPa)	σ_{UTS}
Mean	211	111	73.7	460
± σ	9.5	10	6.5	42.5
± 2σ	19	20	13	85

Arizona State University

Comparison of Reconstructed 95% (2σ) Confidence Plain Weave Stress-Strain Response

Blue = 1x1, Yellow = 6x6



- **Composite Stiffness, PLS (first matrix cracking), Secondary Modulus, statistically unaffected by increasing size of RUC**
- **Failure stress/strain is the only value that we can say with 95% confidence is influenced by architectural details**

Property	E11 (GPa) 1x1	E11(GPa) 6x6	PLS 1x1	PLS 6x6	H (GPa) 1x1	H(GPa) 6x6	σ_{UTS} 1x1	σ_{UTS} 6x6
Mean	209.5	211	116	111	74.5	73.7	512.5	460
± σ	13	9.5	10	10	6.5	6.5	15	42.5
± 2σ	26	19	20	20	13	13	30	85

Conclusion

1. ***Demonstrated that a synergistic analysis using the multiscale generalized method of cells (MSGMC) can accurately represent woven CMC tensile behavior (loading/unloading)***
    - ***4 level of scales analyzed***
    - ***Nonlinear behavior due to damage – demonstrated by unloading***
    - ***Critical invariant is I_1 (brittle) not J_2 (metals)***
    - ***Failure mechanisms capture via local continuum damage model***
2. ***Non-uniform distribution of voids/porosities must be incorporate within the RUC - accurate deformation and failure response***
3. **Variations in Weave Parameters (micro, meso, and macro) appear to contribute to variation in measured material <u>macrolevel</u> response.**
    a) ***Primary Variables appear to be***
        - ***Constituent material constants (micro)***
        - ***Spatial distribution of void locations (meso); shape is sheet like***
    b) ***Secondary Variables appear to be***
        i. ***Tow void content (meso)***
        ii. ***Tow Aspect Ratio (meso)***
        iii. ***Tow volume fraction (macro)***
4. ***Assuming Normal Probability Distributions → showed that only the ultimate failure stress/strain (statistically speaking) is influenced at the <u>structural level</u> by lower scale features .***

Future Work

1. *Examine the influence of these parameters on the time-dependent material response and corresponding life.*
2. *Incorporation of constituent property distribution in the analysis*
3. *Incorporate environmental degradation (due to oxidation / moisture)*
4. *Multivariate statistics and stochastic processes for coupled architectural/material parameters*
5. *Incorporate MSGMC into ImMAC 5.0*

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THANK YOU

QUESTIONS

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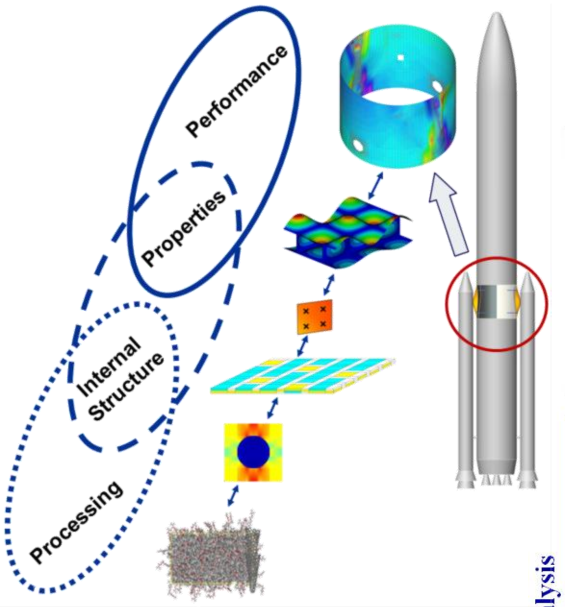
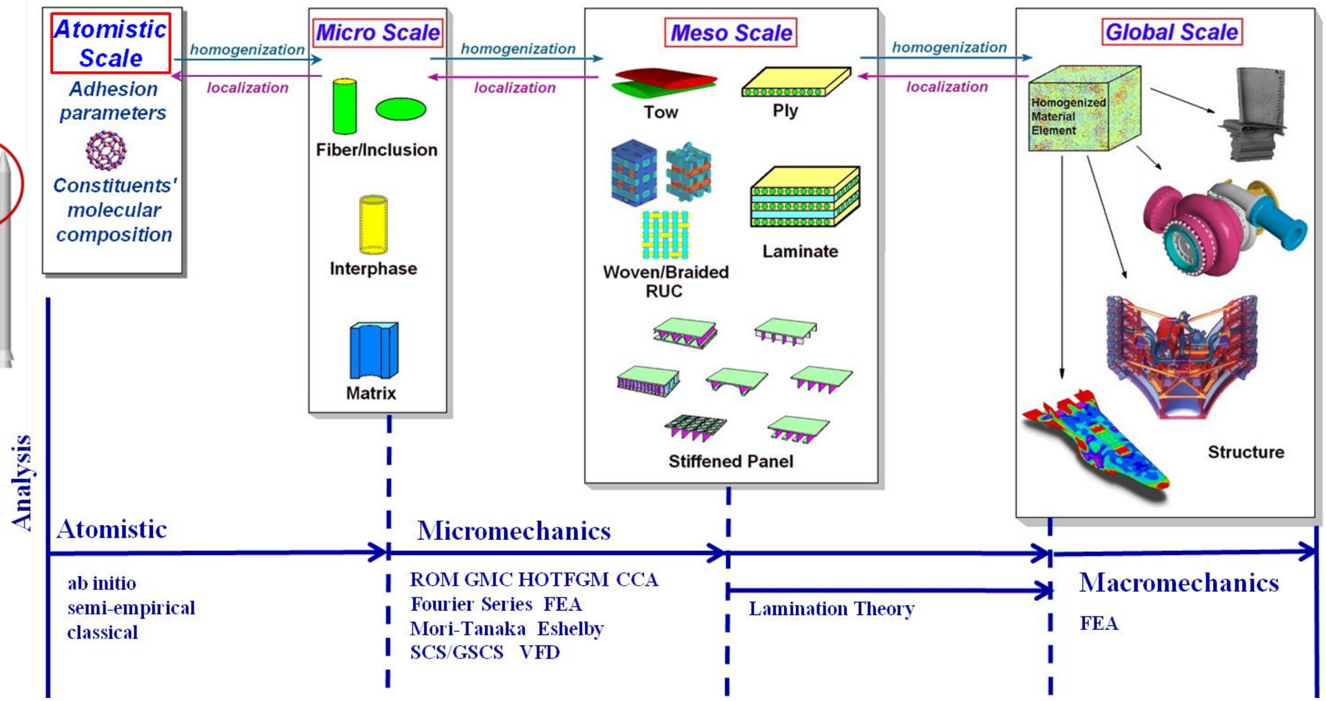


MACE

Multiscale Analysis Center of Excellence



COLLABORATORS

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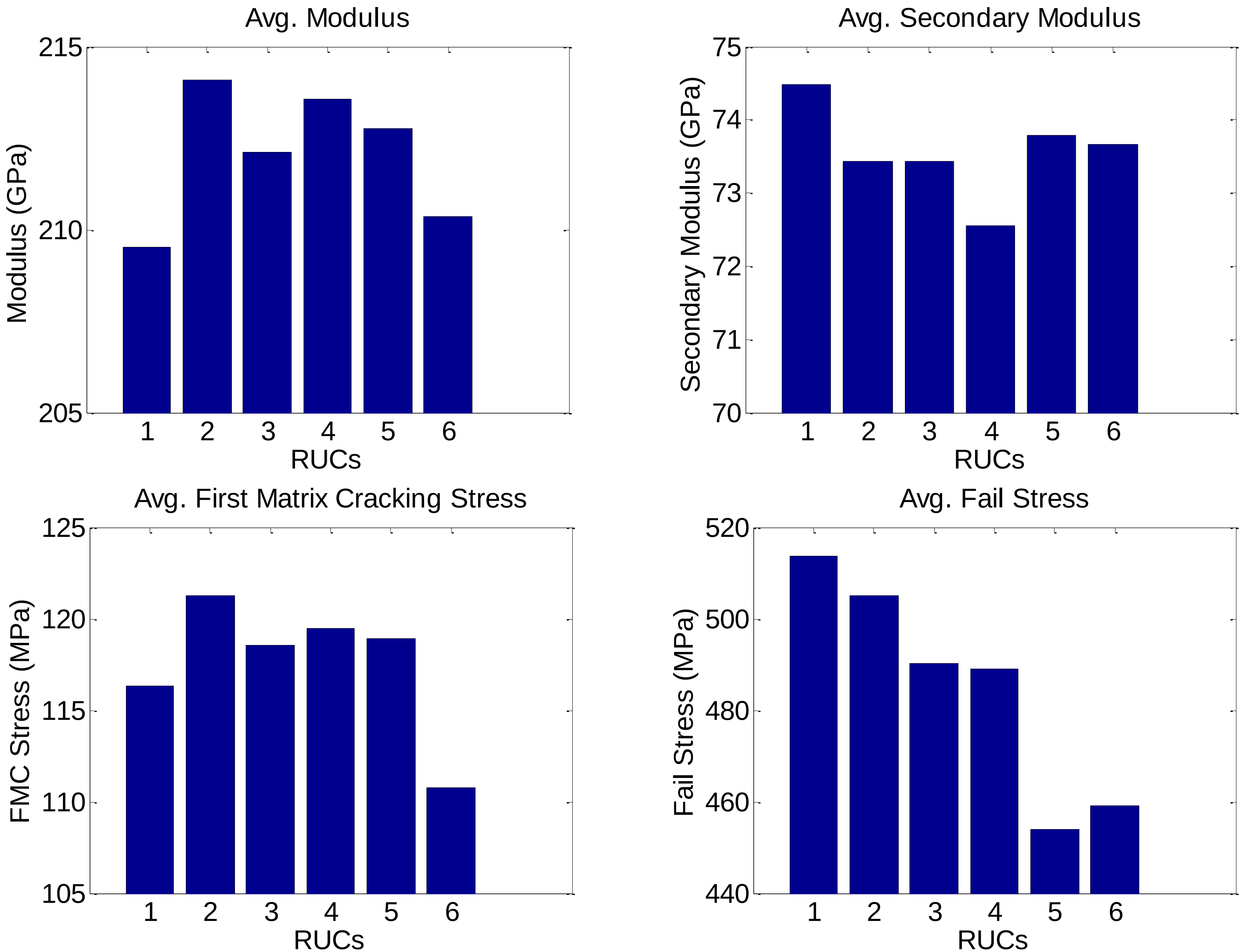
NASA Multiscale Analysis Center of Excellence (MACE) Established in 2010 at GRC

Average Values of Four Key Composite Response Attributes: E, PLS, H and σ_{UTS}



Average Values of Four Key Composite Response Attributes: E, PLS, H and σ_{UTS}

Remember 5x5 has lowest DoF