Rudder/Speed Brake Actuator Probabilistic Fatigue Life and Reliability Analysis - A Case Study

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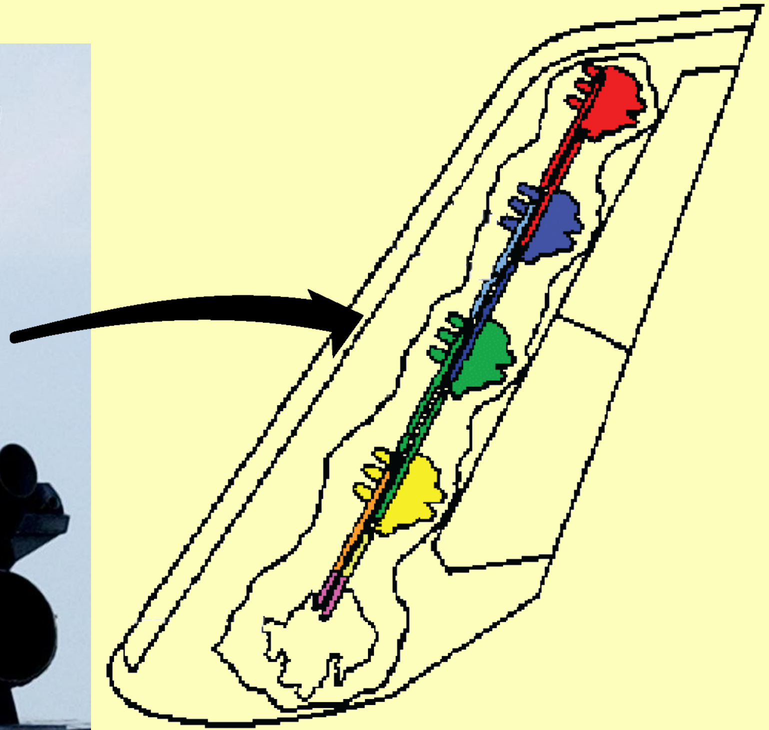
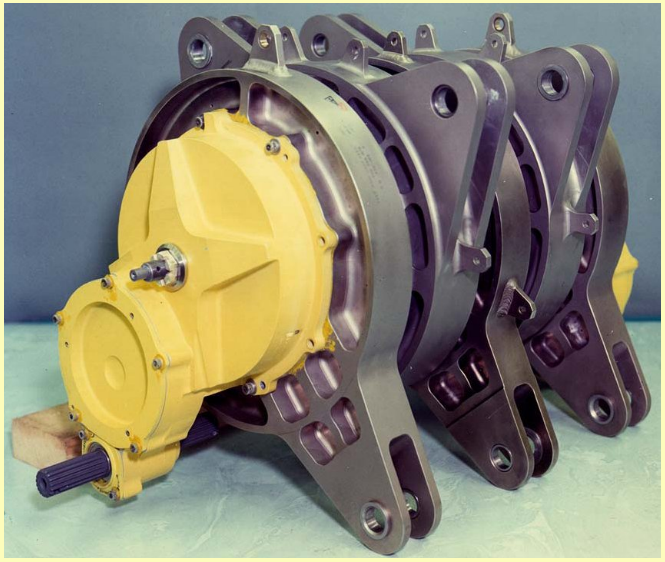
Rudder/Speed Brake Actuator

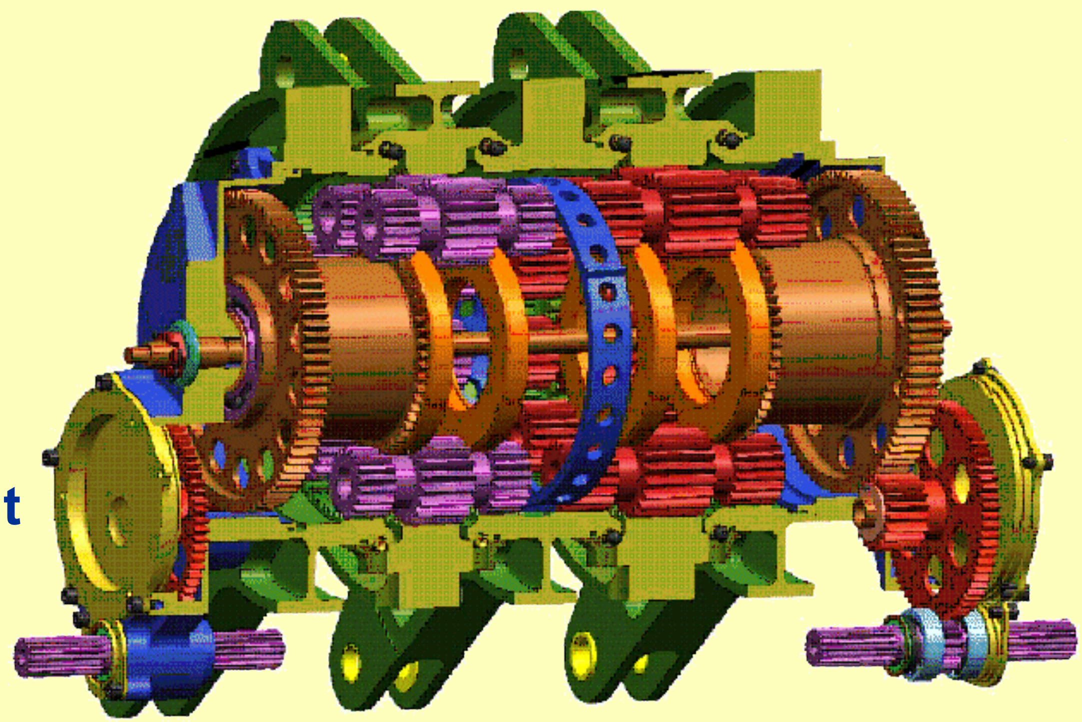
Four actuators, each with two gear trains



Rudder/Speed Brake Actuator



Actuator has 2 gear trains — each with 17 gears & 10 bearings



- Overall Reduction
  474 : 1
- Two Stage Spur Gear Input
  19.75 : 1
- Differential Planetary
  24 : 1

Background

- Space Shuttle designed for 100 missions in 10 years
- First Shuttle “Columbia” entered service Apr. 1981
- Actuators from Columbia were inspected during vehicle modification after sixth flight in 1984.
- No other actuator inspections or maintenance was planned during shuttle program

Background

Post-Shuttle Columbia crash Feb 2003 . . . .

- **"Flight leader" Discovery had 30 flights in 19 years**
- **Corrosion & cracks were found on Discovery body flap actuator shaft spline**
- **Wear, damage & red-colored grease were found in rudder/speed brake actuator**

Issues

1. **What are the risks from fatigue, wear, lubrication breakdown for 20 or more flights per actuator?**

2. **What is an acceptable risk for single actuator and four actuators as a system?**

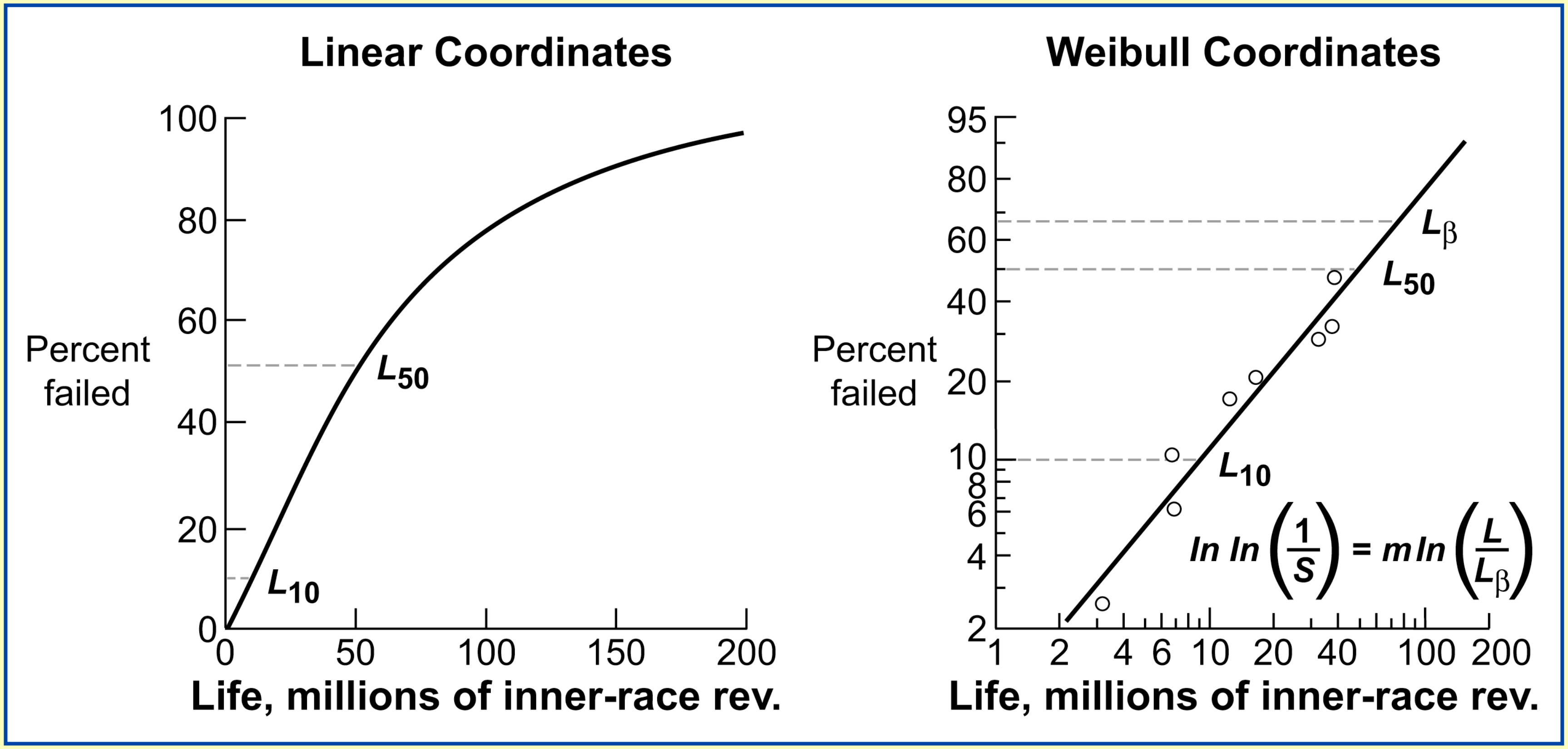
Method of Analysis

1. Load analysis & cycle counts on bearings & gears
2. Probabilistic analysis for life & reliability using Lundberg-Palmgren theory applied to bearings & gears
3. Component life for operating profile in each actuator based on linear damage rule (Palmgren-Langer-Miner rule)
4. System life for single actuator & four actuators based on strict series reliability and Weibull theory

Analysis Tasks

- Relate Shuttle mission spectrum to cycles of actuator components
- Compute gear surface Hertzian fatigue life
- Compute gear tooth bending fatigue life
- Life analysis for gearbox bearings (supplied by actuator manufacturer)
- Combine lives to calculate life of actuator system(s)

Weibull Distribution Function



2-parameter Weibull Distribution Function

$$\ln\ln\left(\frac{1}{S}\right) = m\ln\left(\frac{L}{L_\beta}\right)$$

where:

S **= probability of survival or reliability**

m **= Weibull slope (shape factor)**

L **= life @ reliability S**

L_β = Characteristic life (life @ 63.2% failure rate)

Lundberg-Palmgren Theory
System Life for Multiple Components

Strict Series Reliability

$$S_S = S_1 \times S_2 \times S_3 \times \ldots S_n$$

where all survival probabilities are at the same time increment

Lundberg-Palmgren Theory (1947)

based on Weibull Distribution Function (1939)

Gives:
$$\frac{1}{L_S^m} = \left(\frac{1}{L_1}\right)^{m_1} + \left(\frac{1}{L_2}\right)^{m_2} + \left(\frac{1}{L_3}\right)^{m_3} + \ldots + \left(\frac{1}{L_n}\right)^{m_n}$$

where all lives are at the same probability of survival

Linear Damage Rule

- A. Palmgren — 1924
- B. Langer — 1937
- M. Miner — 1945

$$\frac{1}{L} = \frac{X_1}{L_1} + \frac{X_2}{L_2} + \frac{X_3}{L_3} + \ldots + \frac{X_n}{L_n}$$

$$X_1 + X_2 + X_3 + \ldots + X_n = 1$$

L ~ Life

X ~ time fraction

System L_{10} Life

$$L_{10} = \left[\frac{\text{System Dynamic Capacity}}{\text{Equivalent Output Torque}} \right]^{p}$$

L_{10} = Output cycles @ 90% probability of survival

From Linear Damage Rule:

Equivalent Output Torque =

$$T = \left[\frac{X_1 T_1^p + X_2 T_2^p + X_3 T_3^p + \ldots + X_n T_n^p}{X_1 + X_2 + X_3 + \ldots + X_n} \right]^{1/p}$$

X_n = time @ condition n

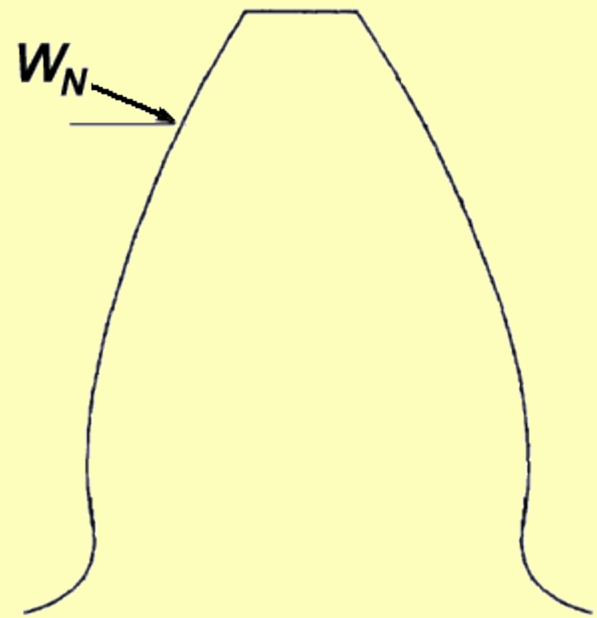
T_n = torque @ condition n

Gear Tooth Surface Fatigue Life

$$L_{10tc} = C_{10} W_N^{-4.3} F_e^{3.9} \rho^{-5.0} \ell^{-0.4}$$

where:

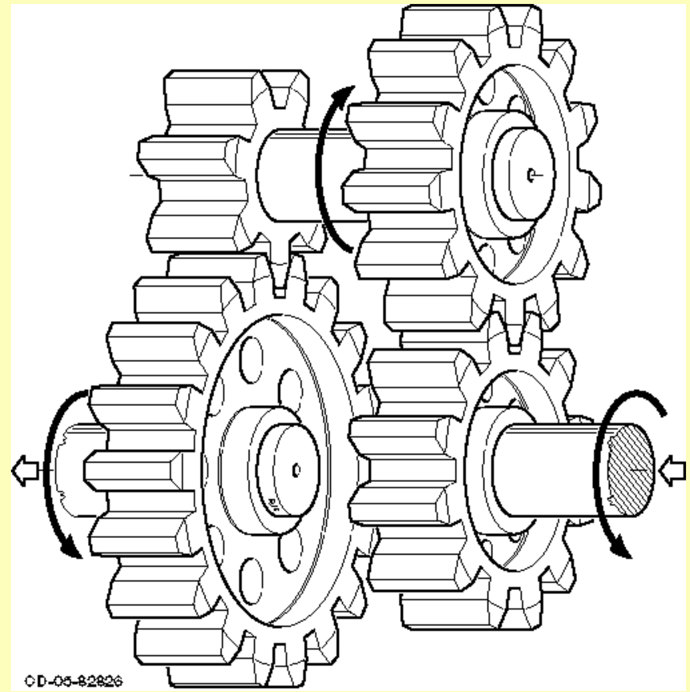
- L_{10tc} = Contact fatigue 10% life for a single tooth
- C_{10} = Gear tooth dynamic load capacity
- W_N = Normal load on tooth
- F_e = Effective tooth width
- ρ = Curvature sum
- ℓ = Length of loaded tooth profile



Gear Surface Fatigue Life

$$L_{10tc} = C_{10} W_N^{-4.3} F_e^{3.9} \rho^{-5.0} \square^{-0.4}$$

$$L_{10gc} = L_{10tc} n^{-1/m}$$



where:

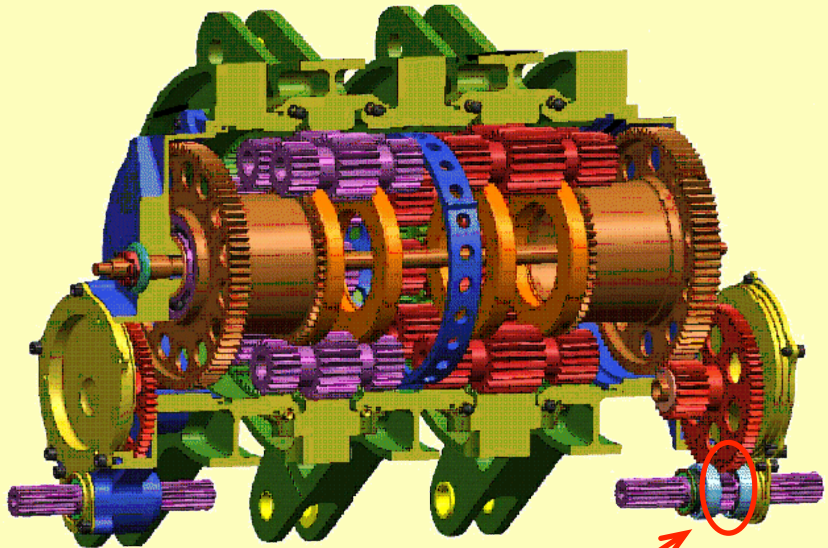
L_{10tc} = Contact fatigue 10% life for a single tooth

L_{10gc} = Surface fatigue 10% life for gear

n = Number of teeth

m = Weibull slope = 2.5

Gear Surface Fatigue Life



$$L_{10gc} = L_{10tc} n^{-1/m}$$

Lowest-life gear: 16 tooth input pinion

Max Hertz stress 1818 MPa (263.7 ksi)

Pinion $L_{10} = 1.807 * 10^{12}$ cycles

based on weighted output torque 29.8 Nm (263.7 in-lb)

Gear system life (contact): $$\frac{1}{L_{Sc}^{m}} = \left(\frac{1}{L_{1c}}\right)^{m_1} + \left(\frac{1}{L_{2c}}\right)^{m_2} + \left(\frac{1}{L_{3c}}\right)^{m_3} + \ldots + \left(\frac{1}{L_{nc}}\right)^{m_n}$$

Gear Tooth Bending Fatigue Life

AGMA Bending Stress Number

$$s_t = W_t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

where:

s_t	= Bending stress number, lb/in^2
W_t	= Tangential load, lb
K_o	= Overload factor
K_v	= Dynamic factor
K_s	= Size factor
P_d	= Diametral pitch, in^{-1}
F	= Face width, in
K_m	= Load distribution factor
K_B	= Rim thickness factor
J	= Geometry factor

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Gear Tooth Bending Fatigue Life

$$L_{10gb} = \left(\frac{S}{s_t}\right)^p$$

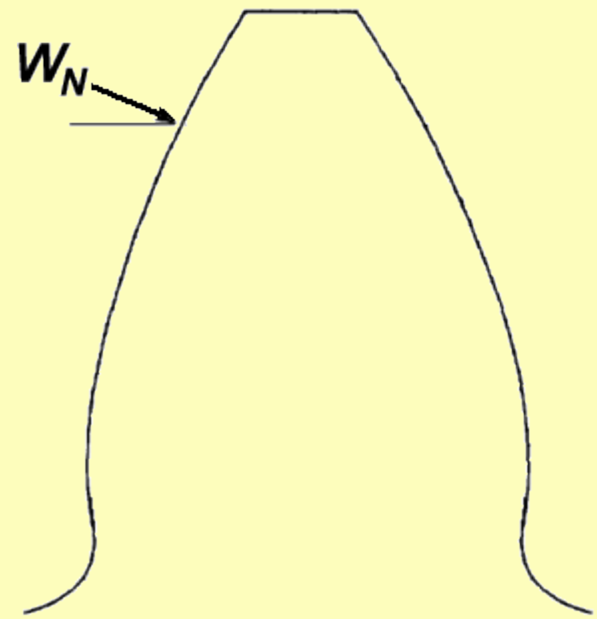
where:

L_{10gb} = Bending stress life for a single tooth

S = Gear tooth strength (database)

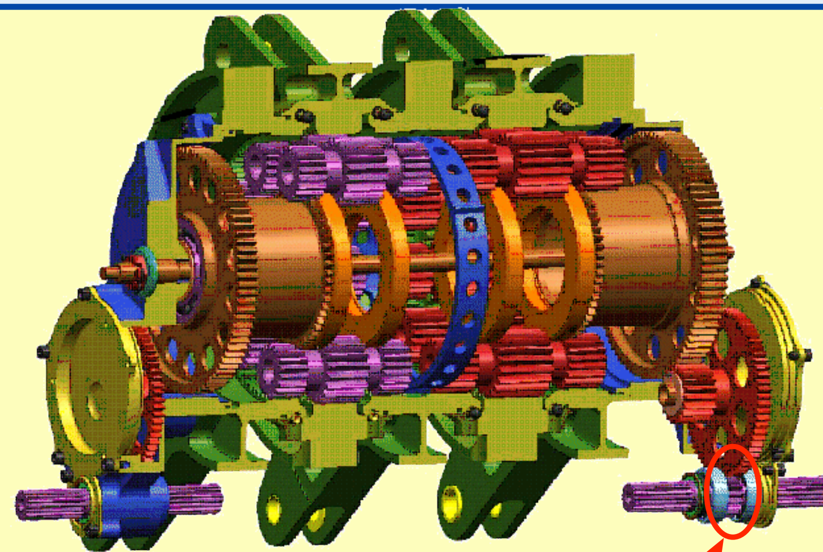
s_t = Bending stress

p = Stress-life exponent



Gear Tooth Bending Fatigue Life

$$L_{10gb} = \left(\frac{S}{s_t}\right)^p$$



Lowest-life gear for tooth bending 16 t input pinion

Bending stress 1070 MPa (155 ksi),

L_{10b} life = 13 x 10^{12} cycles

Gear system life (bending):

$$\frac{1}{L_{Sb}^m} = \left(\frac{1}{L_{1b}}\right)^{m_1} + \left(\frac{1}{L_{2b}}\right)^{m_2} + \left(\frac{1}{L_{3b}}\right)^{m_3} + \ldots + \left(\frac{1}{L_{nb}}\right)^{m_n}$$

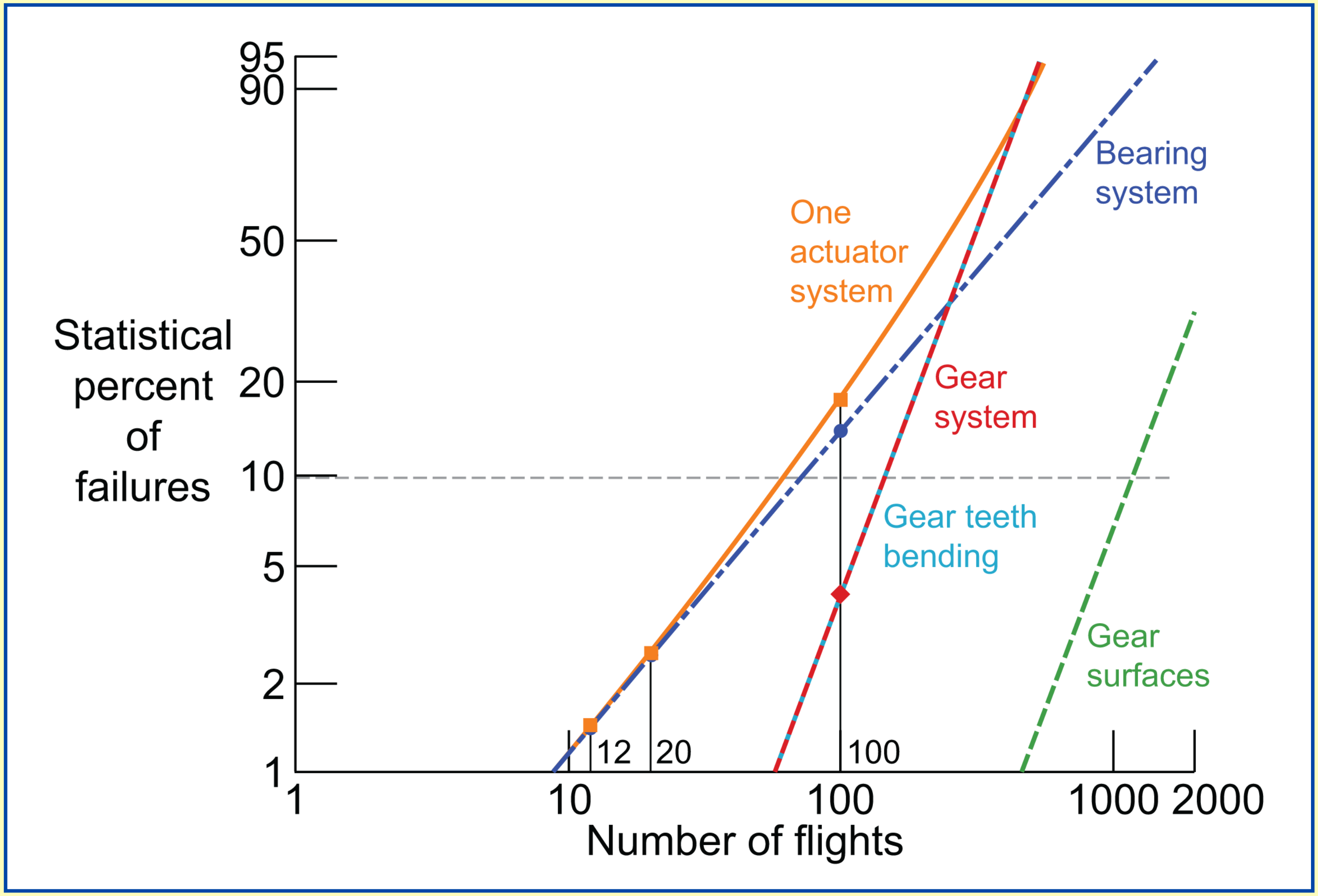
Gear Tooth Load Cycles (for 1 tooth @ Output)

Gear	No Teeth	Teeth – load cycles	Mesh Cycles
Output Ring	81	9 – 1	9
Output Planet	18	9 – 1	9
Fixed Ring	90	9 – 1	9
Fixed Planets	18	9 – 1	9
Sun	54	36-1	36
Sun-Planets	18	36-1	36
Output Spur	87	26-1	26
Intermed. Output	19	12-1 + 7-2	26
Intermed. Input	69	44-1+ 25-2	94
Input Spur	16	2-5 + 14-6	94

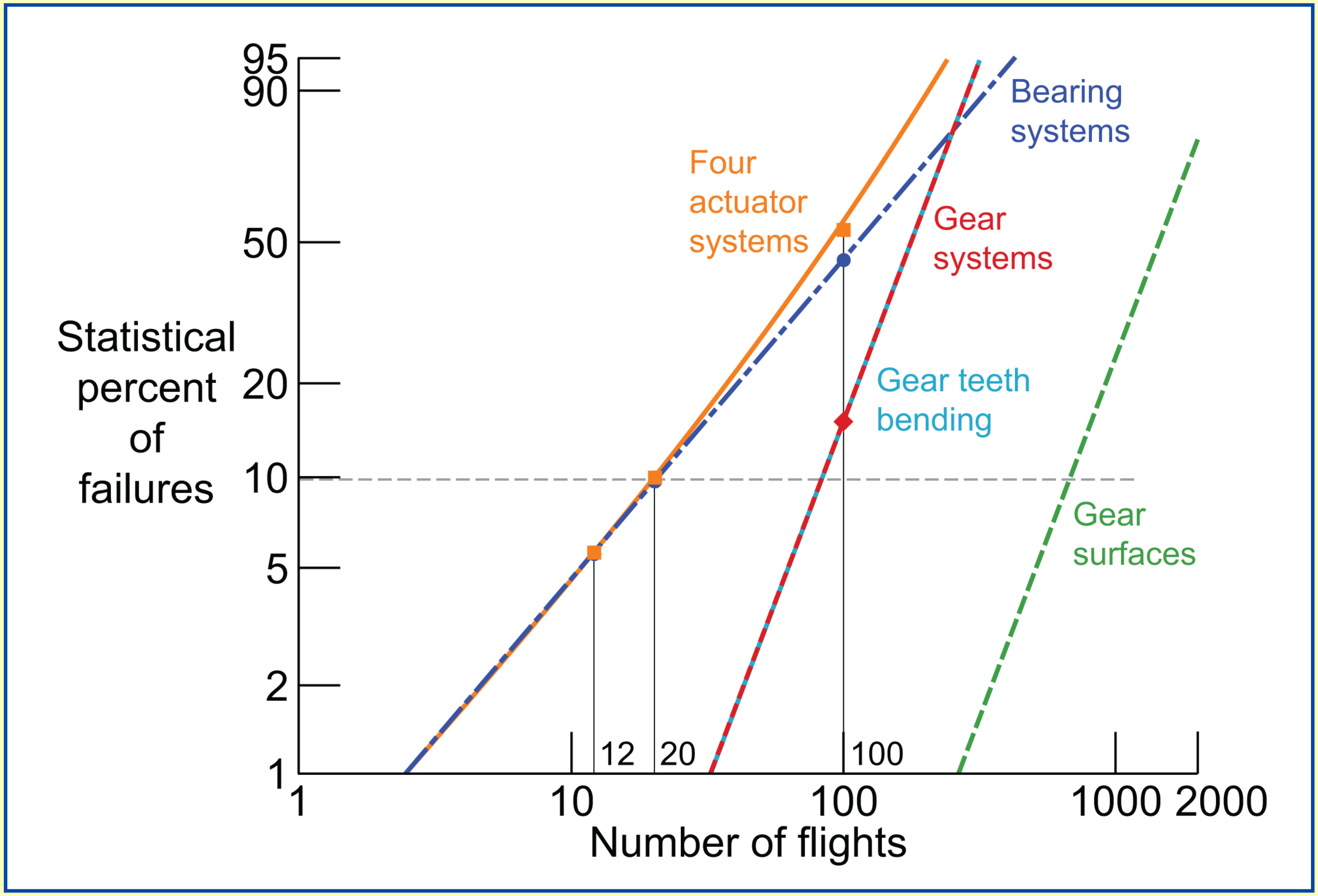
Load Spectrum (for 100 flights)

Event	Moment N-m (in-lb)	Duration (Minutes)	Output gear teeth loading cycles
Ferry	560 (5,000)	420	32,392,514
Ferry	24,000 (212,000)	0.25	19,281
Ascent	9,000 (80,000)	0.2	15,425
Ascent	5,200 (46,000)	1.7	131,113
Ascent	11,000 (98,000)	0.1	7,713
Descent	16,000 (140,000)	33.0	2,545,126
Descent	27,000 (240,000)	1.0	77,125

Life vs. Reliability – 1 Actuator



Life vs. Reliability – 4 Actuators



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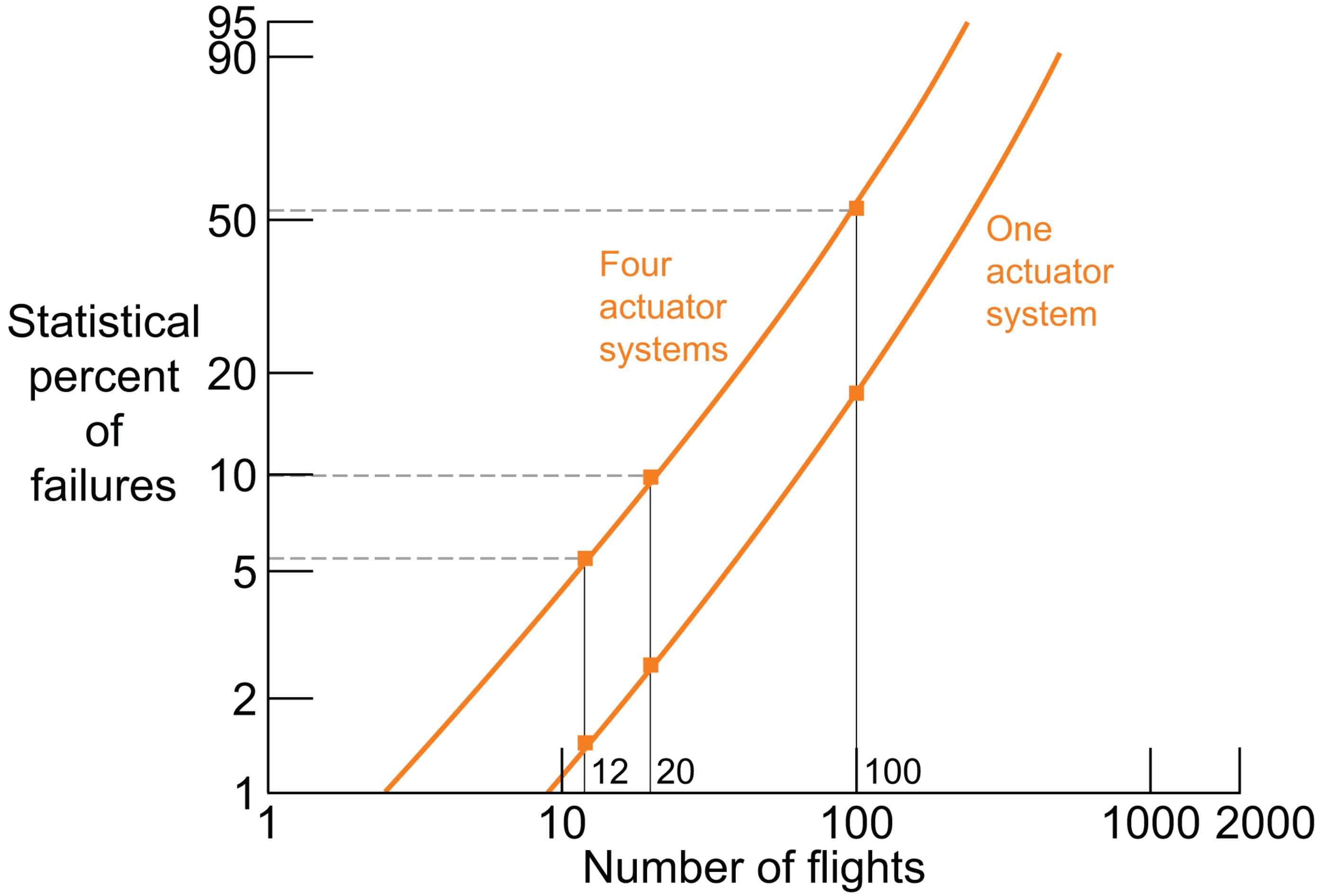
Comparison of Life & Reliability for 1 & 4 Actuators



Predicted Lives of Rudder/Speed Brake Actuator Bearings

Bearing No.	1a	1b	2	3a	3b	4	6a	6b
Bearing type	roller	roller	roller	roller	roller	ball	ball	ball
L_{10B} life (hrs)	647,000	647,000	15,823	4,875	4,875	54,973	1,509	1,089

L_{10B} life data provided for bearings by actuator manufacturer

No bearing failure expected before 5.3% of min. L_{10B} life:

L_{10B} = 0.053(1089 hrs) = 58 hrs or 7 flights

Rudder/Speed Brake Actuator Probability of Survival

Number of flights	Number of actuators	Gear Reliability, percent			Actuator Bearing Reliability, percent	Total System Reliability, percent
		Tooth Bending Fatigue	Tooth Surface Fatigue	Combined Bending & Surface		
100	1	95.9	99.98	95.9	86.0	82.4
100	4	84.71	99.91	84.7	54.6	46.2
20	1	99.93	99.999+	99.93	97.5	97.4
20	4	99.7	99.998	99.7	90.3	90.1
12	1	99.98	99.999+	99.98	98.6	98.6
12	4	99.92	99.998	99.92	94.4	94.3

Rudder/Speed Brake Actuator Probability of Survival

Number of Actuators	Number of Flights			
	7	12	20	100
	Probability of Survival (failure), %			
1	100	98.6	97.4	82
4	100	94	90	46
Bearing System	100	99.8	97.5	86
Weakest Bearing	100	99.3	98.8	93

90% reliability (10% failure) for 20 flights unacceptable

New goal 12 flights, 94% reliability (6% failure)

No failures predicted for up to 7 flights

Summary of Results

- Rudder/Speed Brake Actuators limited to 12 flights
- Reliability for 12 flights: 98.6% for one actuator, 94% for four actuators
- No failures expected for up to 7 flights
- Four actuator system reliability: 90% for 20 flights and 46% for 100 flights
- Life & reliability of actuator system dominated by bearings

Epilogue

- Shuttle Discovery returned to flight, July 2005
- Shuttle Atlantis concluded program, July 2011
- Space Shuttle Program had a total of 135 missions from 1981 — 2011
- At retirement, flight leader, Discovery had 39 missions in 27 years (9 after return to flight)
- 22 missions for 3 remaining shuttles from return to flight until end of program