Solutions of the Taylor-Green Vortex Problem Using High-Resolution Explicit Finite Difference Methods

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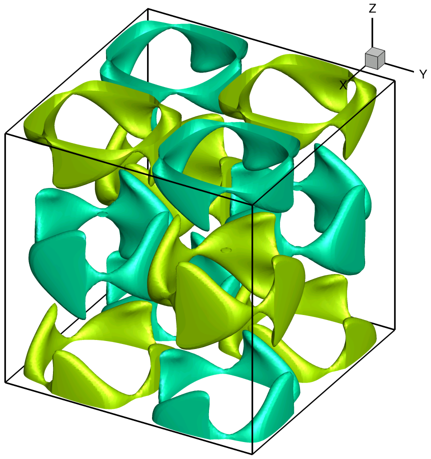
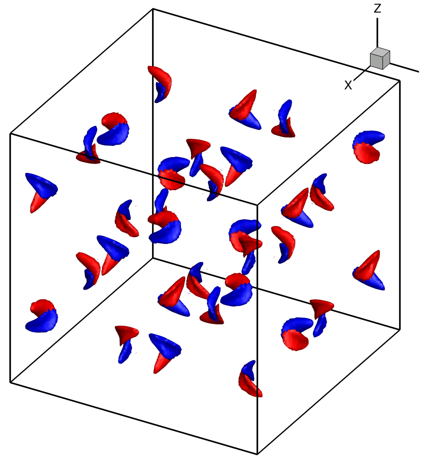
Taylor-Green Vortex

- Simple benchmark case to study vortical flow, transition and turbulence
- Test case from 1st International Workshop on High-Order CFD Methods at 2012 AIAA Aerospace Science Meeting
- Time accurate
- Incompressible
- Flow Conditions
  - Re = 1600
  - M = 0.1
- Periodic domain
  - Simple cartesian grids
  - No complicated boundary conditions

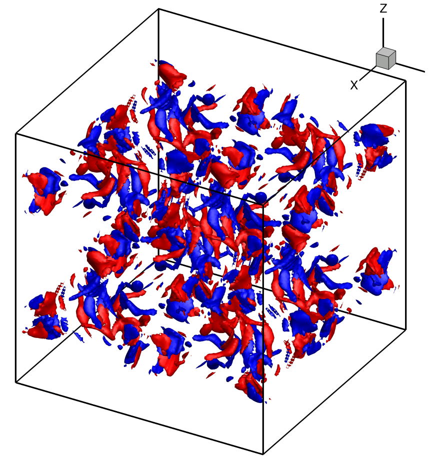
Z-Vorticity Evolution



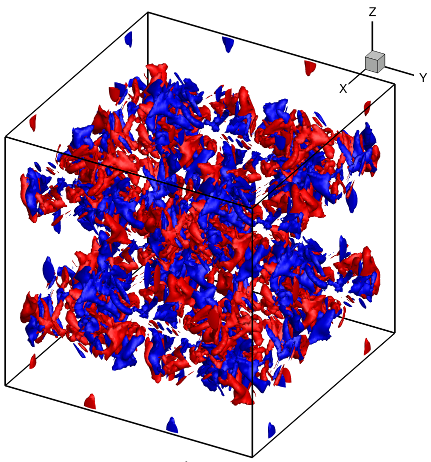
$t^*<3$
inviscid



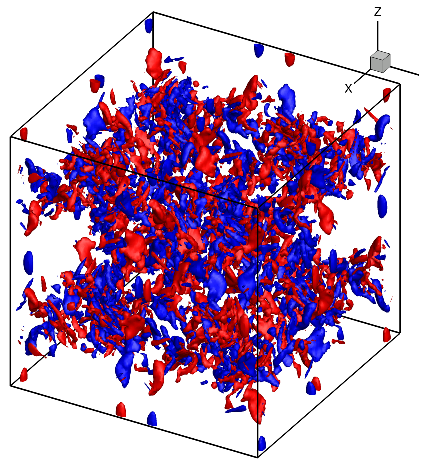
$t^*=5$
vortex roll-up



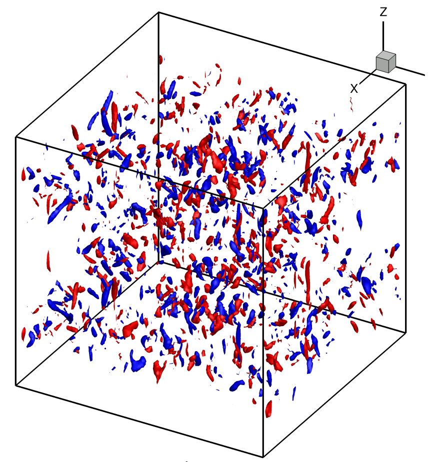
$t^*=7$
structure changes



$t^*=9$
coherent breakdown



$t^*=11$
fully turbulent

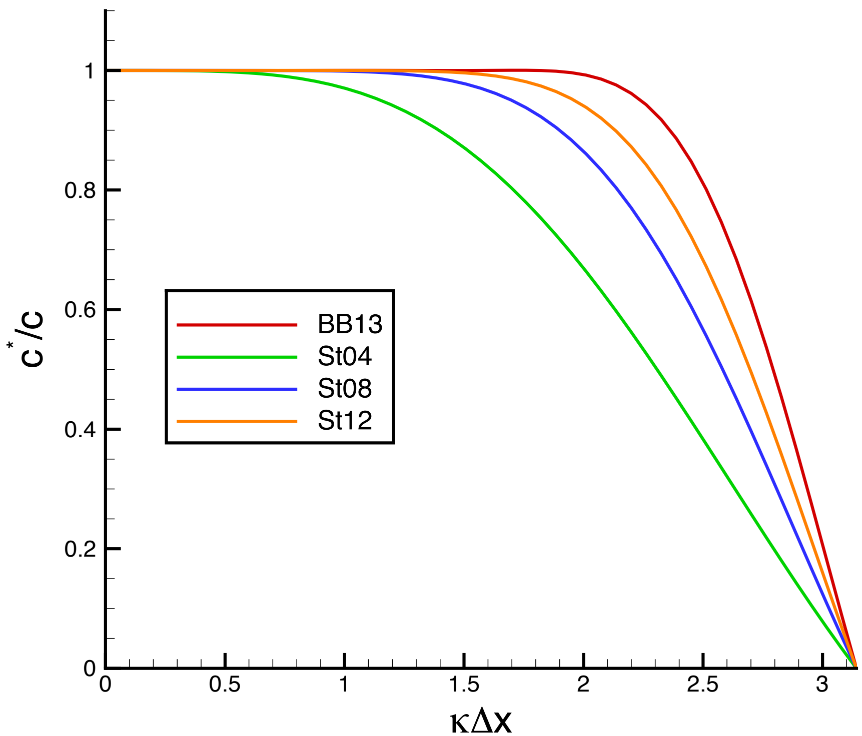


$t^*>11$
turbulent decay

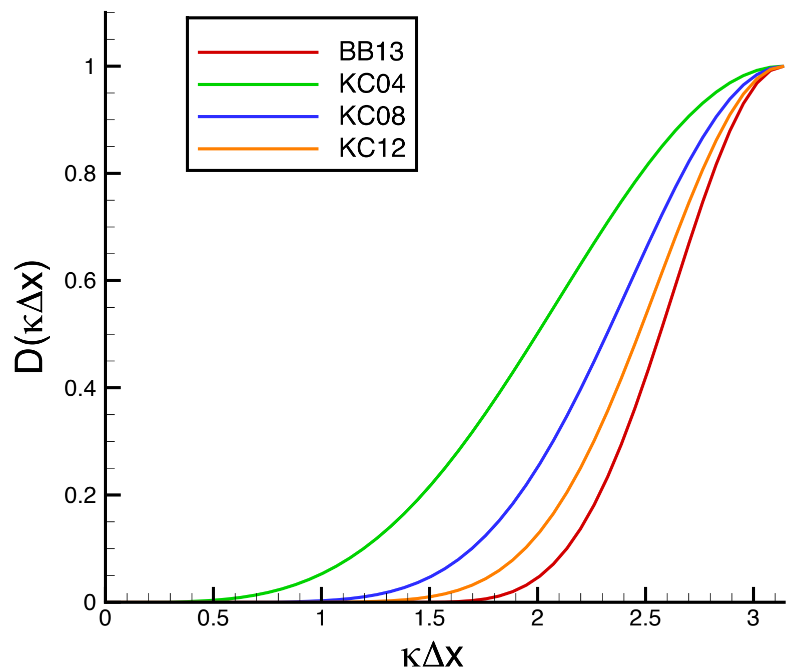
WaveResolvingLES

- Compressible Navier-Stokes equations
- Generalized curvilinear coordinates
- Temporal discretization
  - Low Dispersion Runge-Kutta
- Spatial discretization
  - Standard central differencing, 2^{nd} - 12^{th} order
  - Dispersion relation preserving (DRP)
    - Tam & Webb’s 7-point scheme
    - Bogey & Bailley’s 9-, 11- and 13-point schemes
  - Solution filtering for stability
    - Kennedy & Carpenter filters, 2^{nd} – 12^{th} order
    - DRP filters
  - 4^{th}–order viscous terms
- Sub-grid models
  - Smagorinsky
  - Dynamic Smagorinsky

Fourier Analysis of the Schemes



Error in phase speed for cent. diff.



Damping function for filters

Kinetic Energy Dissipation Rate (KEDR)

- Directly computed KEDR

$$E_k = \frac{1}{\rho_0 \Omega} \int_\Omega \rho \frac{\mathbf{v} \cdot \mathbf{v}}{2} d\Omega \qquad \varepsilon(E_k) = -\frac{dE_k}{dt}$$

- Enstrophy based KEDR

$$\zeta = \frac{1}{\rho_0 \Omega} \int_\Omega \rho \frac{\omega \cdot \omega}{2} d\Omega \qquad \varepsilon(\zeta) = 2\frac{\mu}{\rho_0}\zeta$$

- Comparison to ref. solution
  - 512^3 grid, spectral method
  - van Rees et al, v. 230, J. Comp. Phys., 2011

Computing

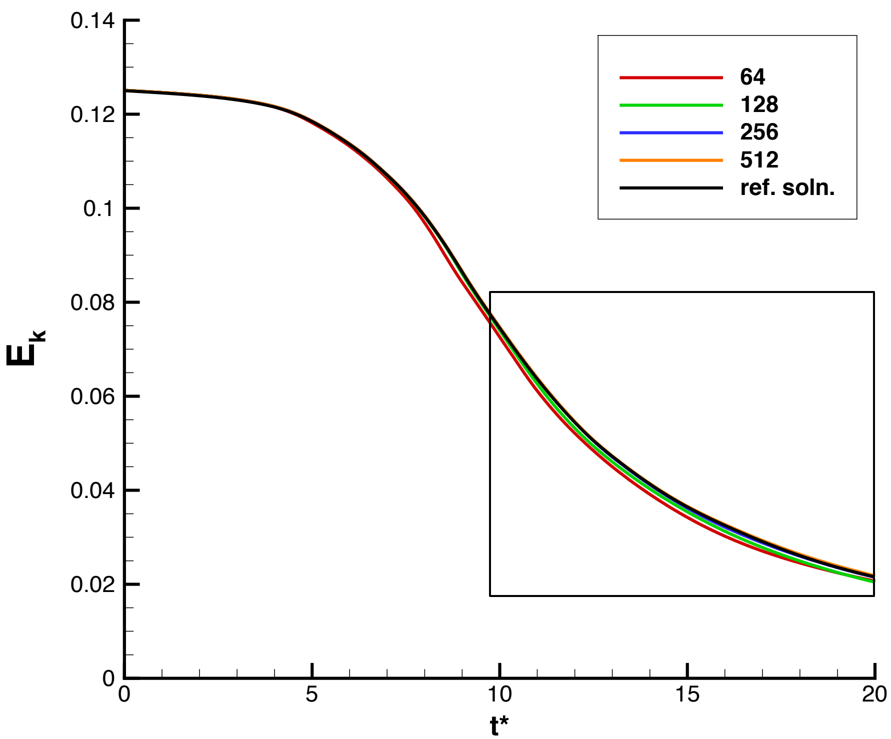
- 64^3 and 128^3 cases
  - Six core single processor desktop
  - 1 grid block
  - 6 OpenMP processes
- 256^3 and 512^3 cases
  - NASA Pleiades system
  - 256^3 – 8 nodes
  - 512^3 – 46 nodes
  - 8 processors/OpenMP processes per node

grid size	time step	machine	cores	wallclock time
64^3	$3.385 \cdot 10^{-3}$	desktop	6	.5
128^3	$1.693 \cdot 10^{-3}$	desktop	6	9
256^3	$8.463 \cdot 10^{-4}$	Pleiades	64	40
512^3	$4.231 \cdot 10^{-4}$	Pleiades	368	130

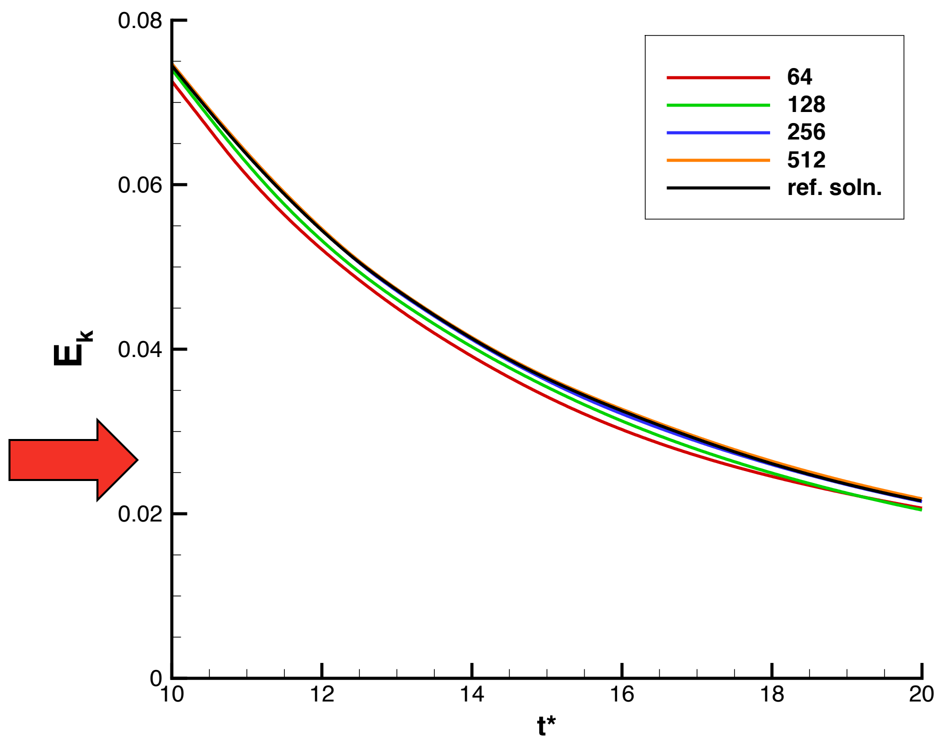
Baseline Case

- Numerical Scheme
  - Temporal Discretization
    - Carpenter and Kennedy's 4-stage, 3rd-order
  - Spatial Discretization
    - Bogey & Bailly's 13-point DRP scheme, BB13
  - Filter
    - Bogey & Bailly's 13-point filter, BB13
    - Filter coefficient halved until minimum stable value was found
    - Min. stable coefficient, $\sigma = 0.05$
- Grids
  - 64^3, 128^3, 256^3 and 512^3

BB13 – 4 grid resolutions

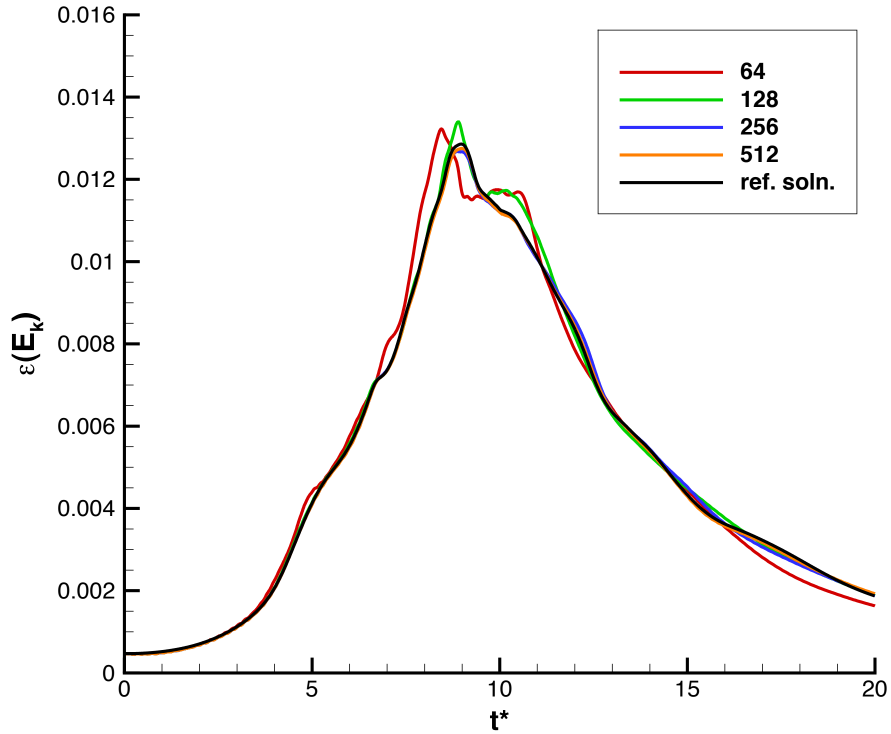


Evolution of kinetic energy

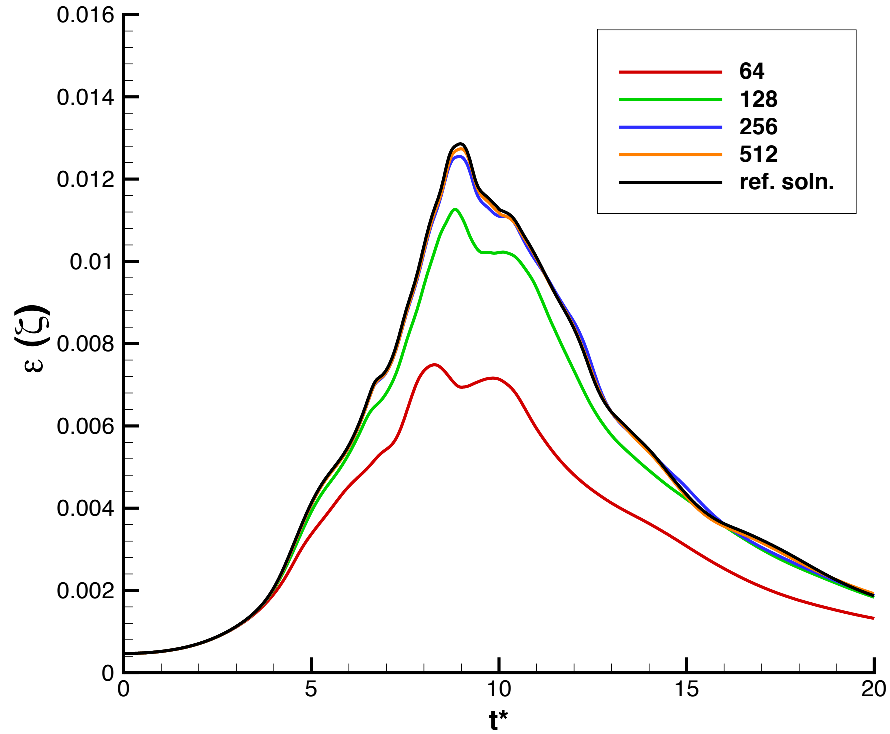


Closep-up of
Evolution of kinetic energy

BB13 – 4 grid resolutions



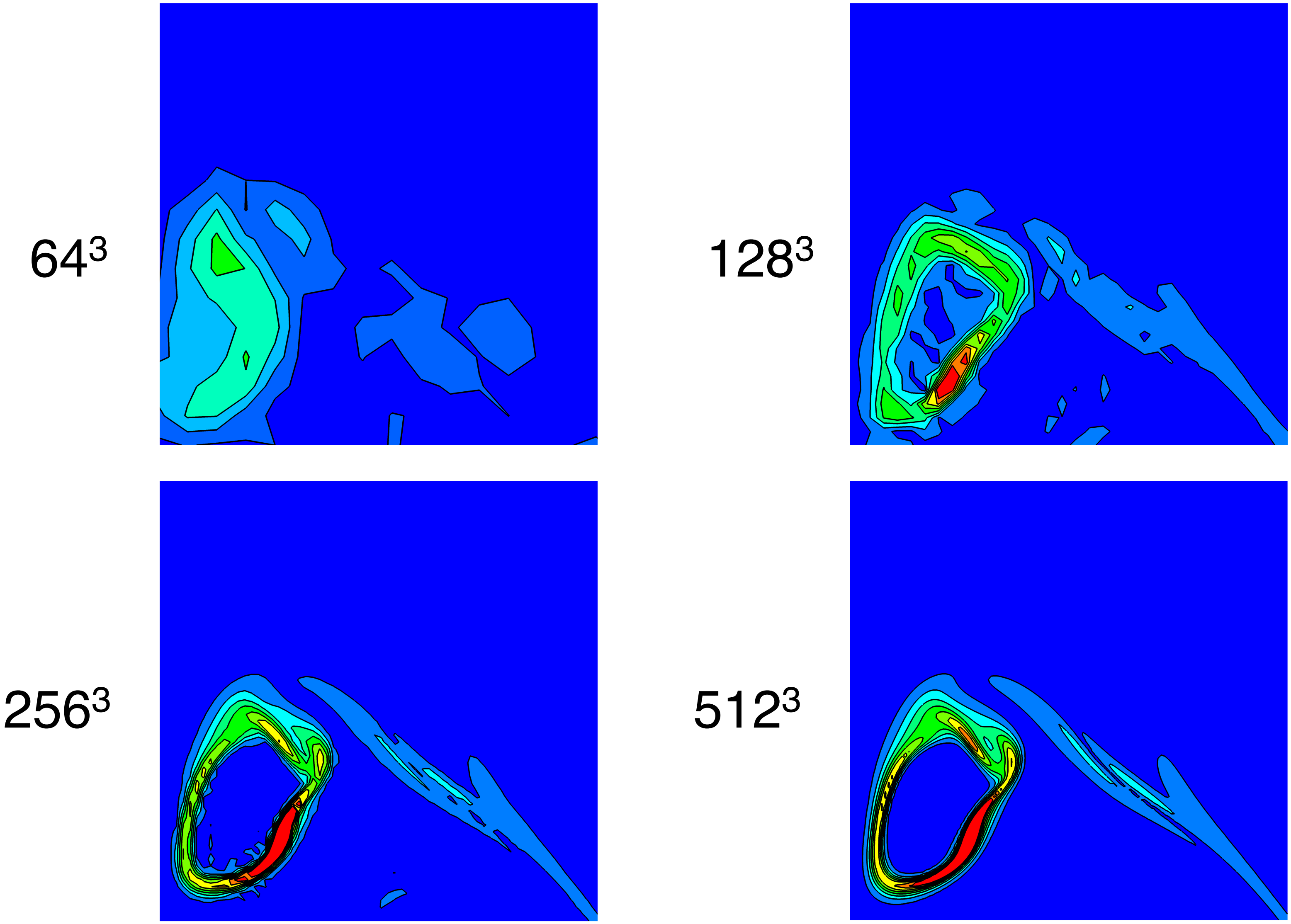
Directly computed KEDR



Enstrophy based KEDR

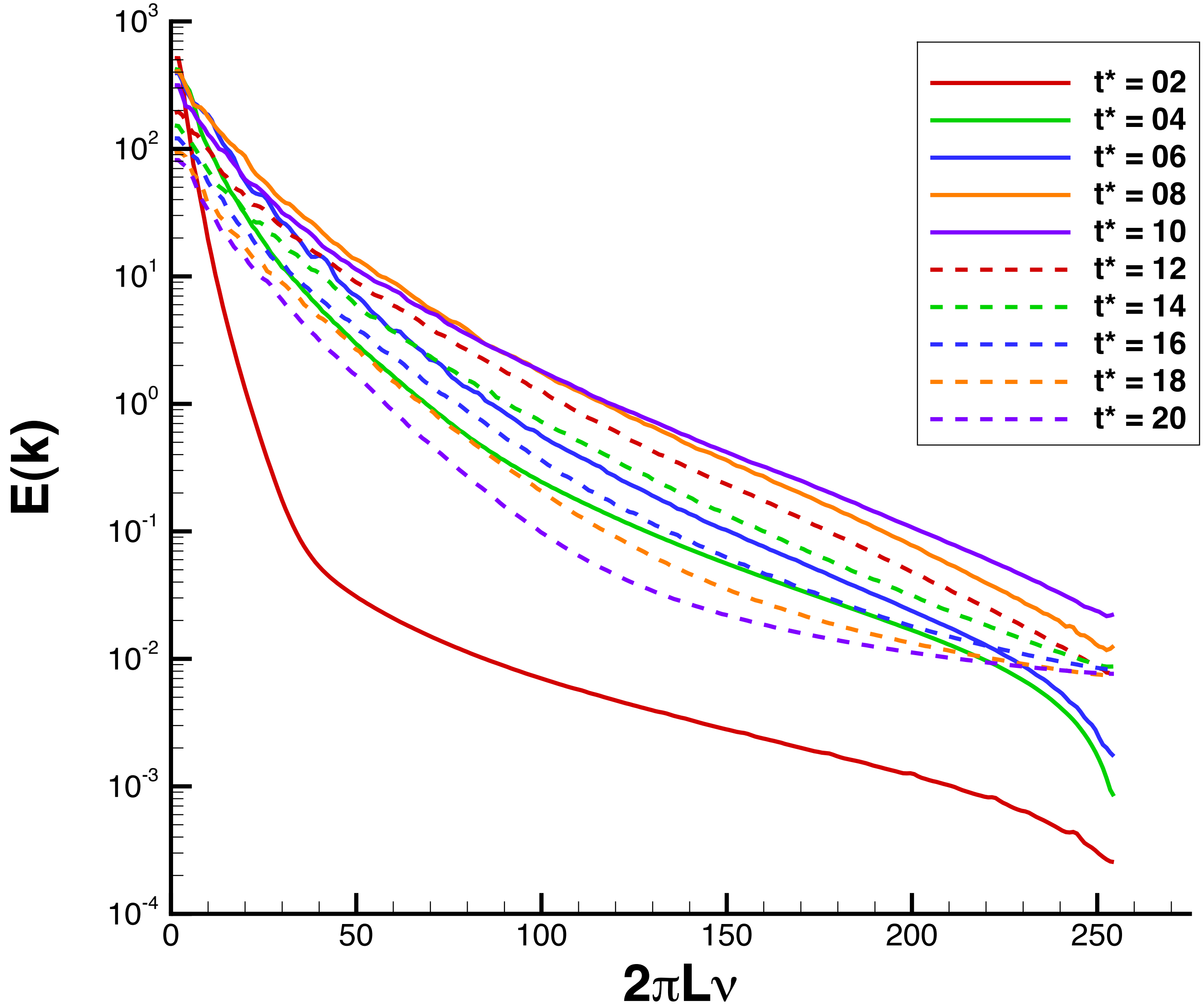
Vorticity Contours at $x = -\pi L$

$t^* = 8$



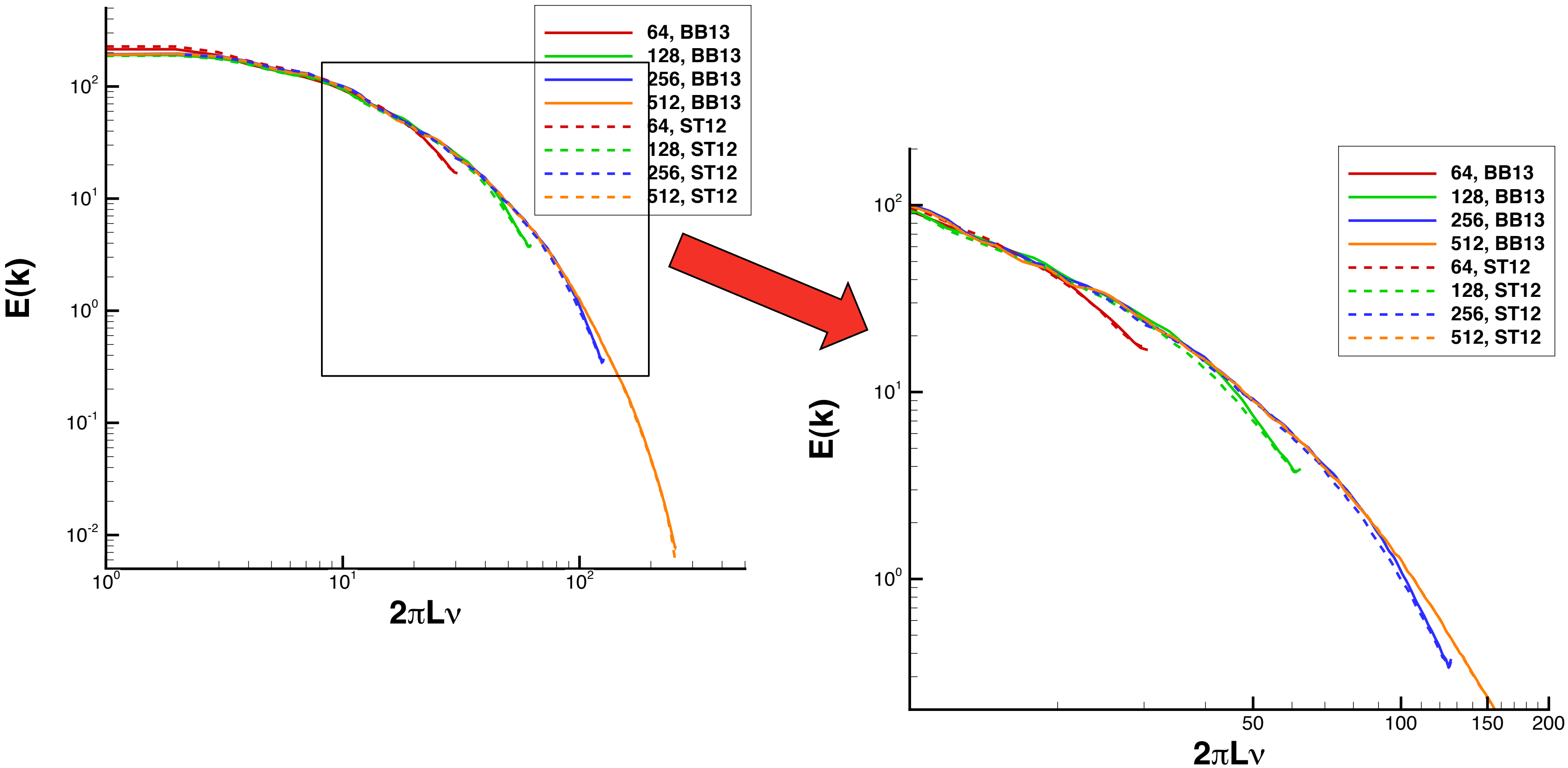
Kinetic Energy Spectra, 512^3

BB13 Scheme



Kinetic Energy Spectra, t* = 12

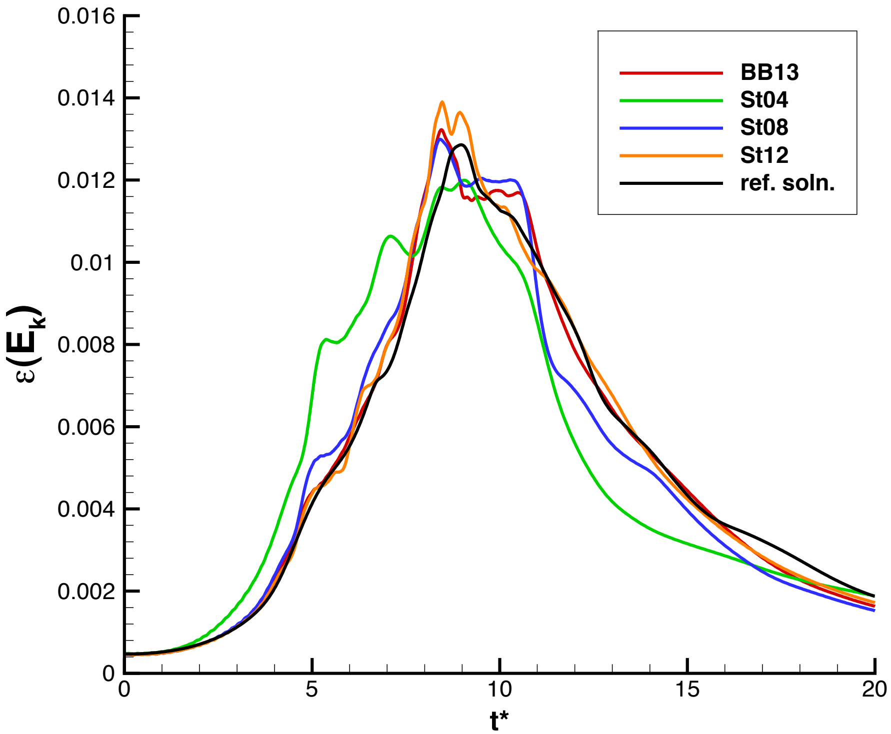
BB13 & St12 Schemes



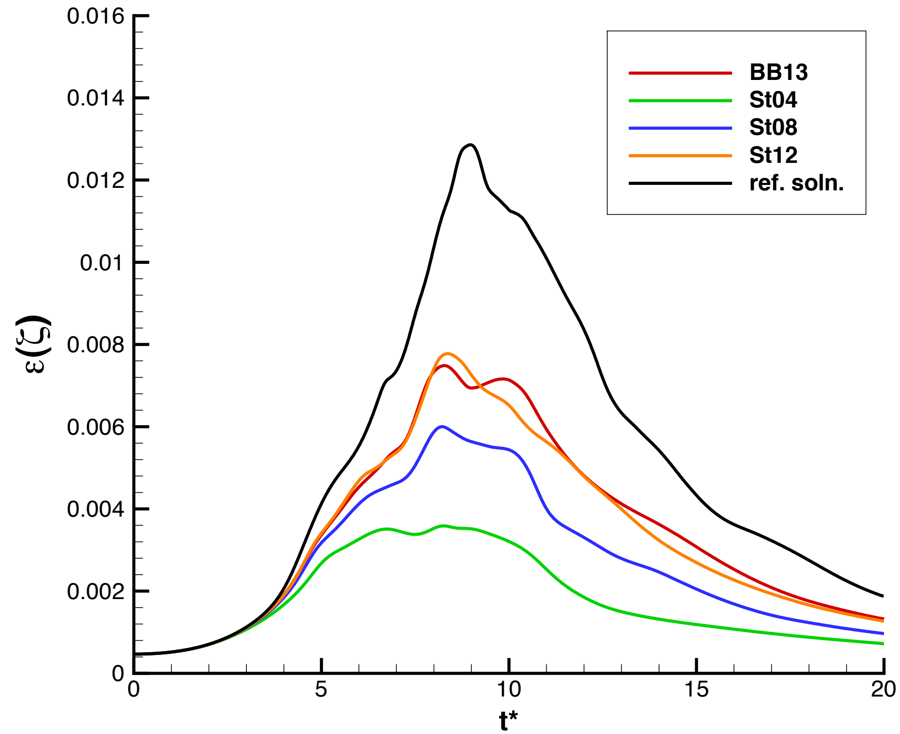
Scheme Comparison

- Numerical Scheme
  - Temporal Discretization
    - Carpenter and Kennedy's 4-stage, 3rd-order
  - Spatial Discretization
    - Bogey & Bailly's 13-point DRP scheme, BB13
    - 4th-, 8th- and 12th-order standard schemes: St04, St08 and St12
  - Filter
    - Bogey & Bailly's 13-point filter, BB13
    - 4th-, 8th- and 12th-order Kennedy & Carpenter filters: KC04, KC08 and KC12
    - Filter coefficient halved until minimum stable value was found
- Grids
  - BB13 & St12: 64^3, 128^3, 256^3 and 512^3
  - St04 & St08: 64^3, 128^3 and 256^3

Scheme Comparison - 64^3

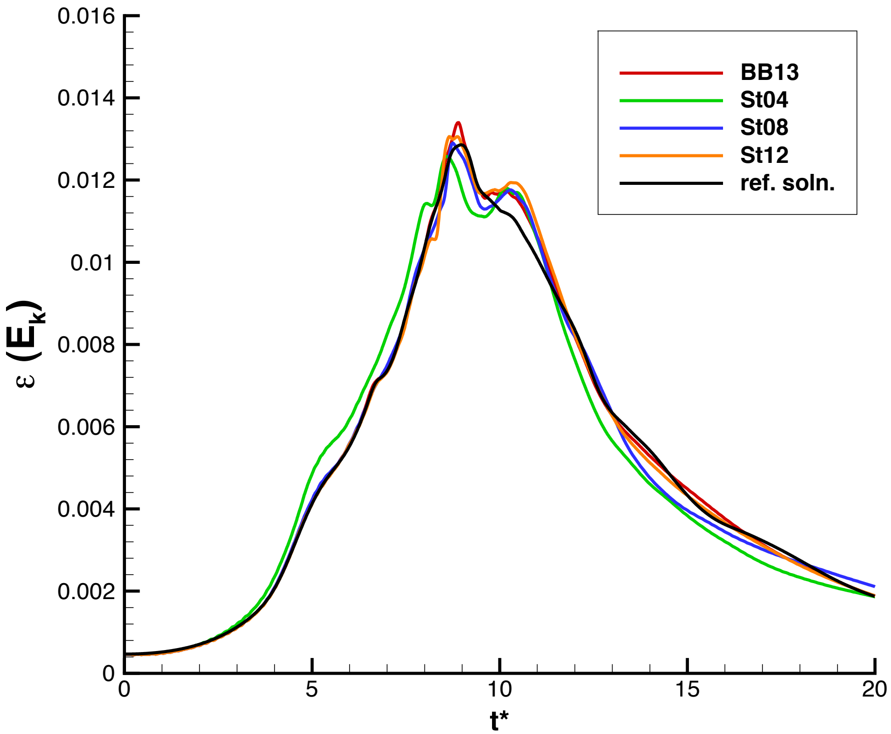


Directly computed KEDR

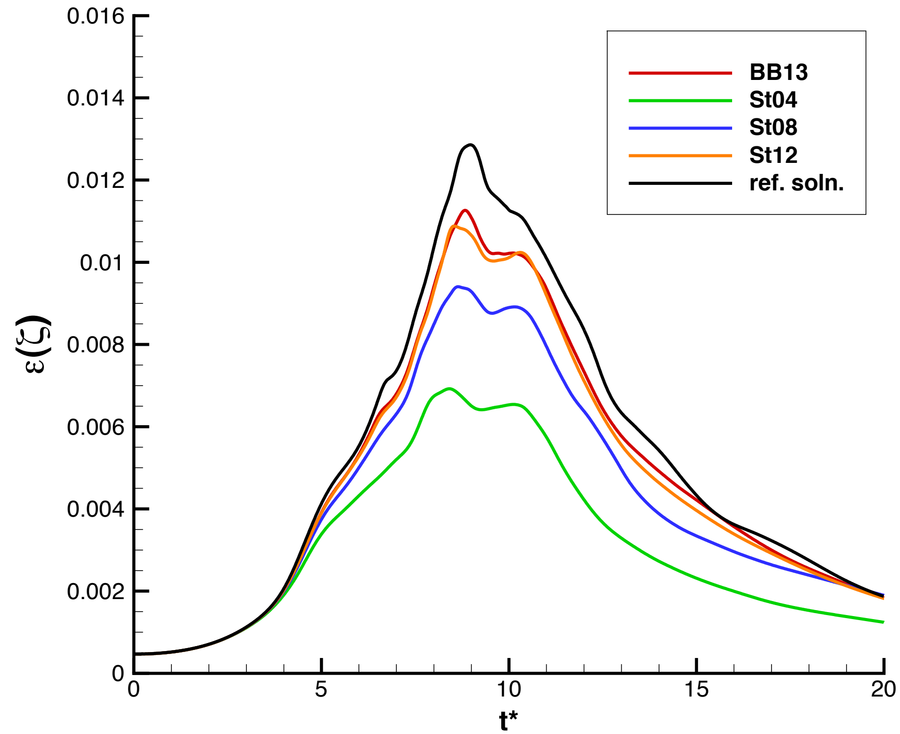


Enstrophy based KEDR

Scheme Comparison - 128^3

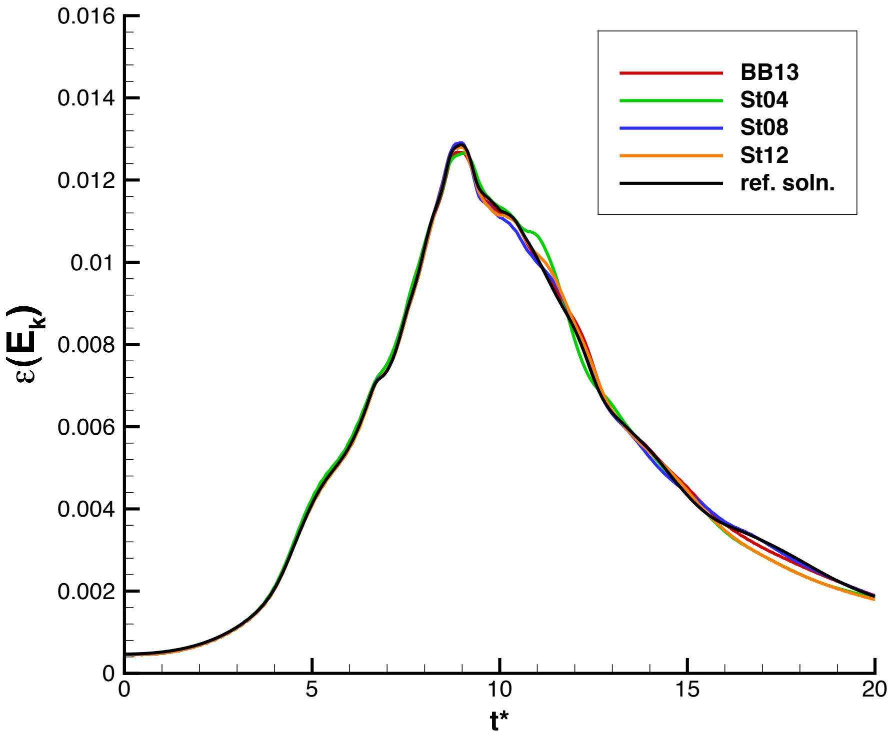


Directly computed KEDR

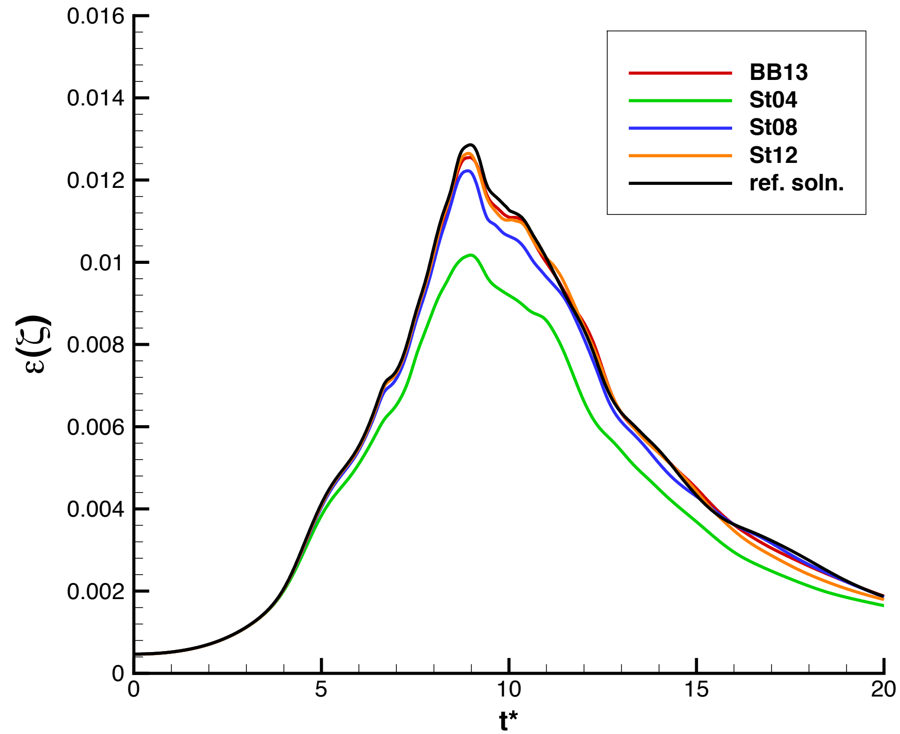


Enstrophy based KEDR

Scheme Comparison - 256^3

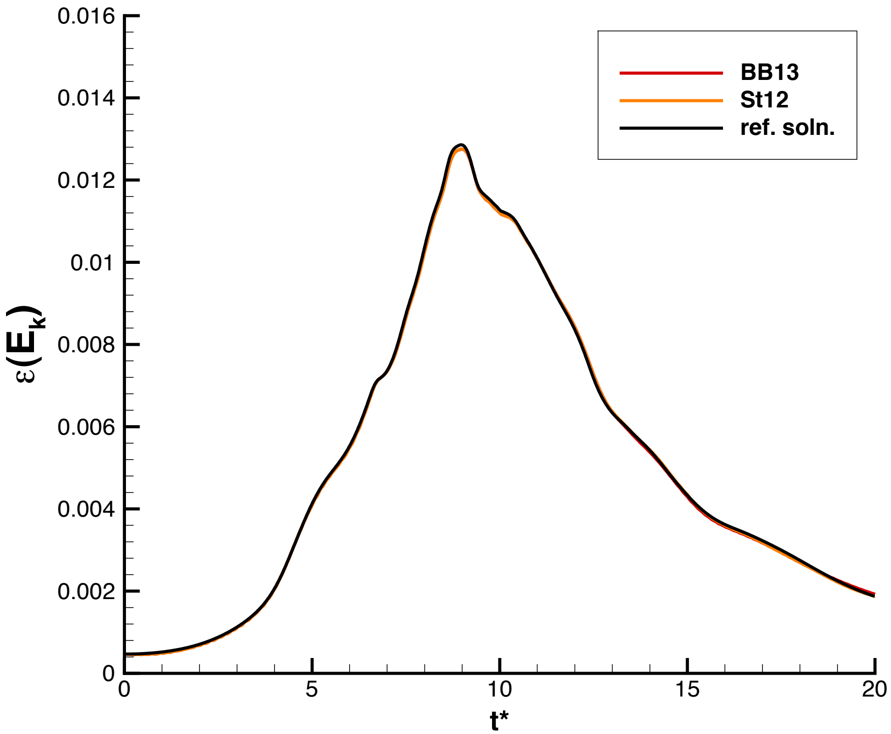


Directly computed KEDR

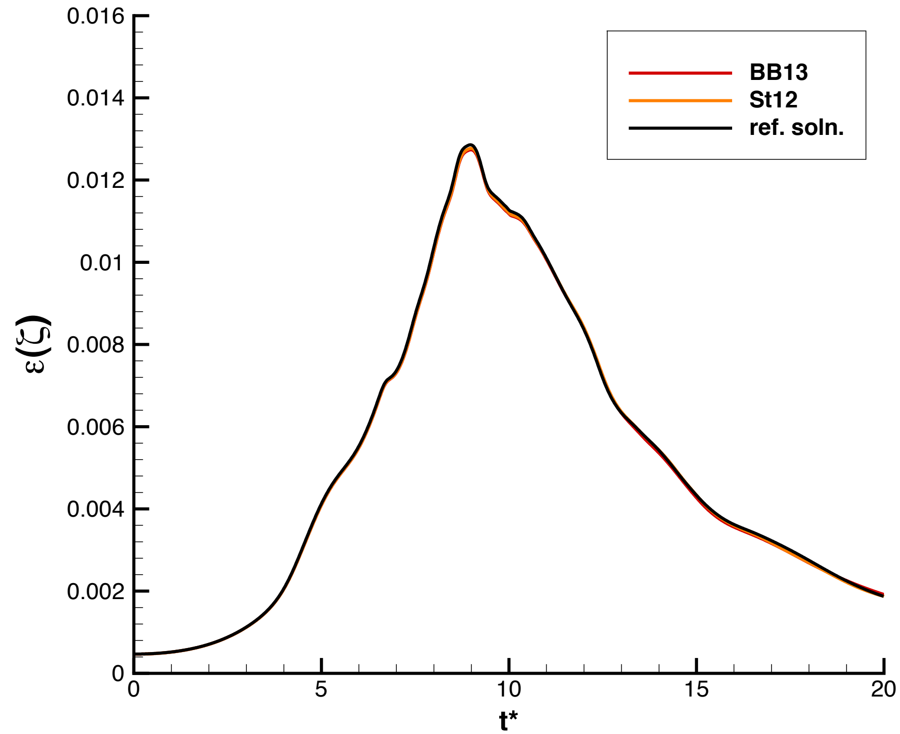


Enstrophy based KEDR

Scheme Comparison - 512^3



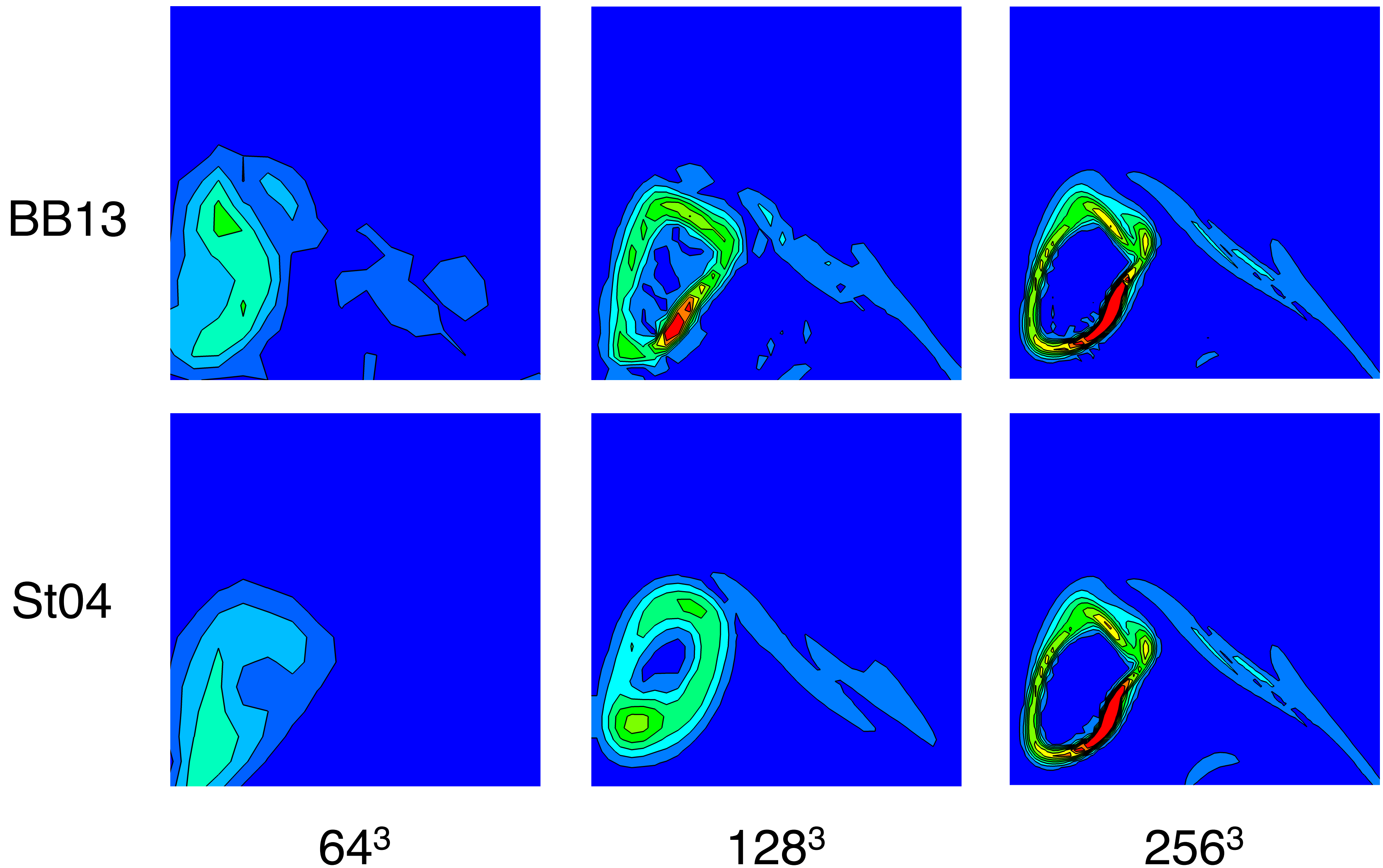
Directly computed KEDR



Enstrophy based KEDR

Vorticity Contours at $x = -\Pi L$

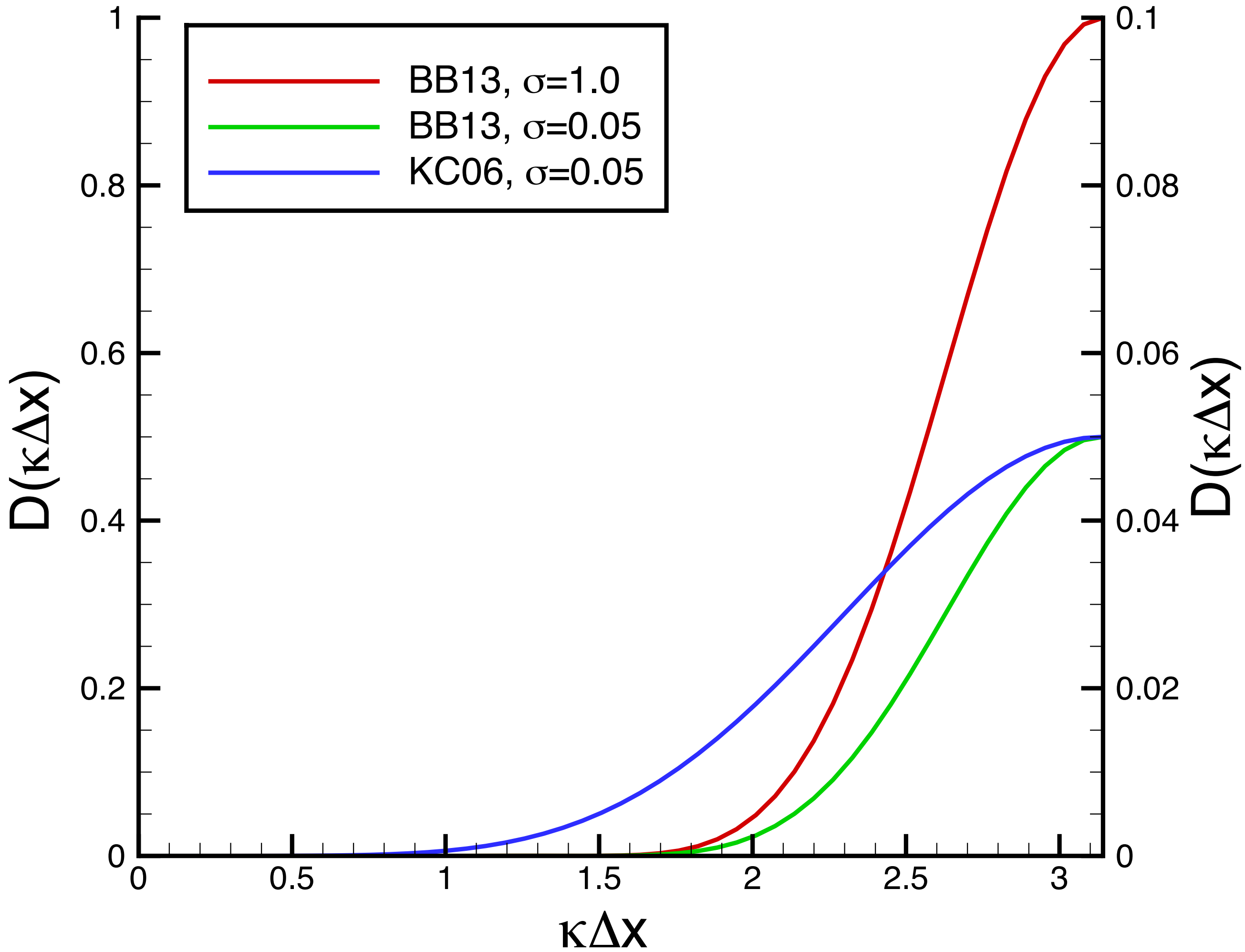
$t^* = 8$



Effect of Filter

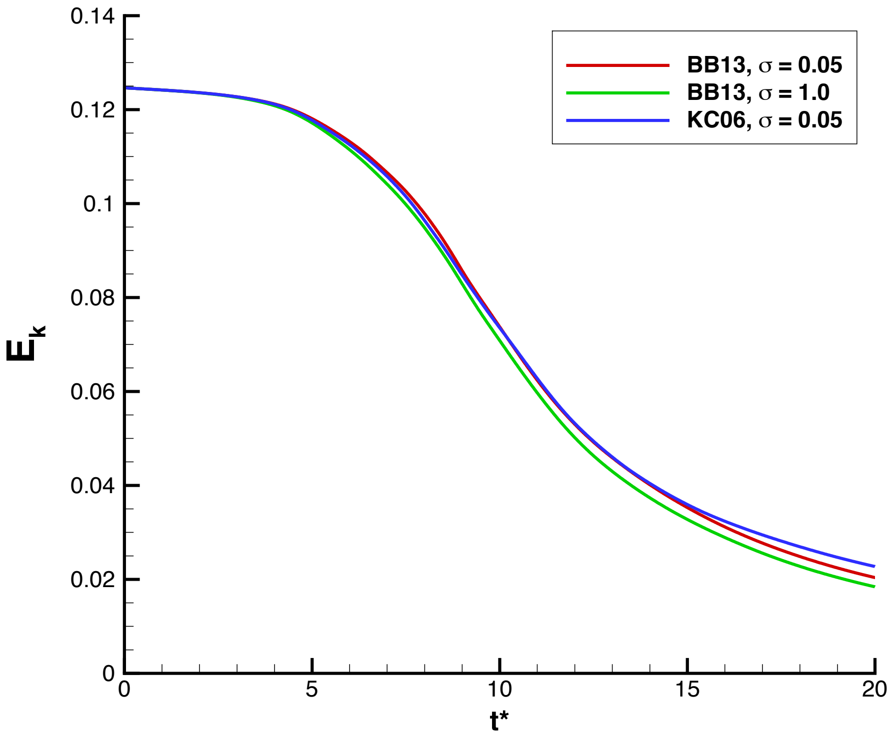
- Numerical Scheme
  - Temporal Discretization
    - Carpenter and Kennedy's 4-stage, 3rd-order
  - Spatial Discretization
    - Bogey & Bailly's 13-point DRP scheme, BB13
  - Filter
    - Bogey & Bailly's 13-point filter, BB13
      - Min. stable coefficient, $\sigma = 0.05$
      - Max. coefficient, $\sigma = 1.0$
    - Kennedy and Carpenter's 6th-order filter (7-point)
      - Coefficient, $\sigma = 0.05$
- Grids
  - 128^3

Filter Damping Functions

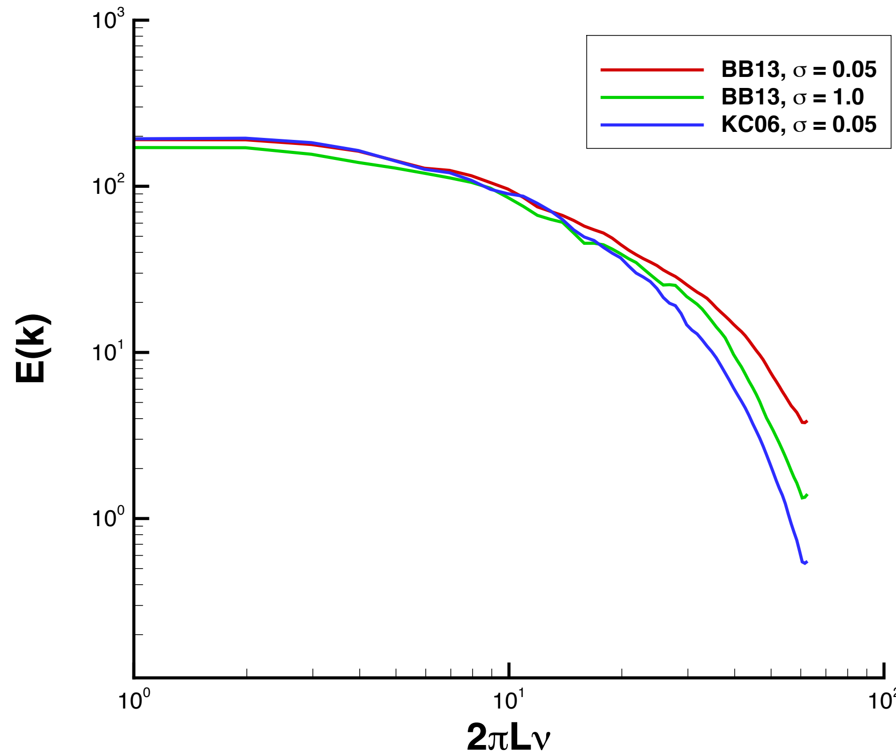


Effect of the Filter

BB13 Scheme, 128^3 grid



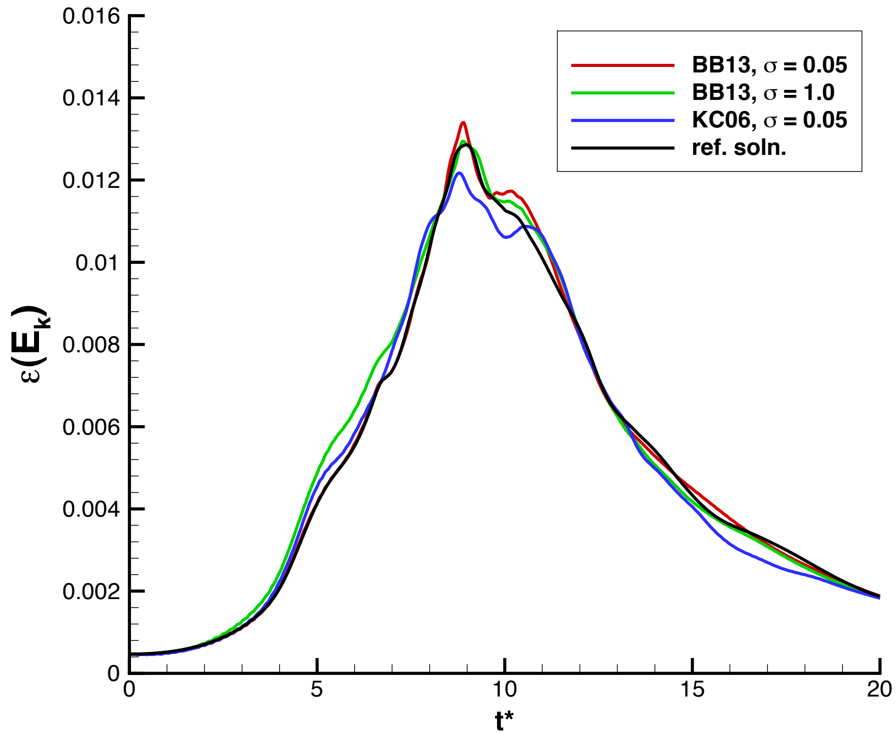
Evolution of kinetic energy



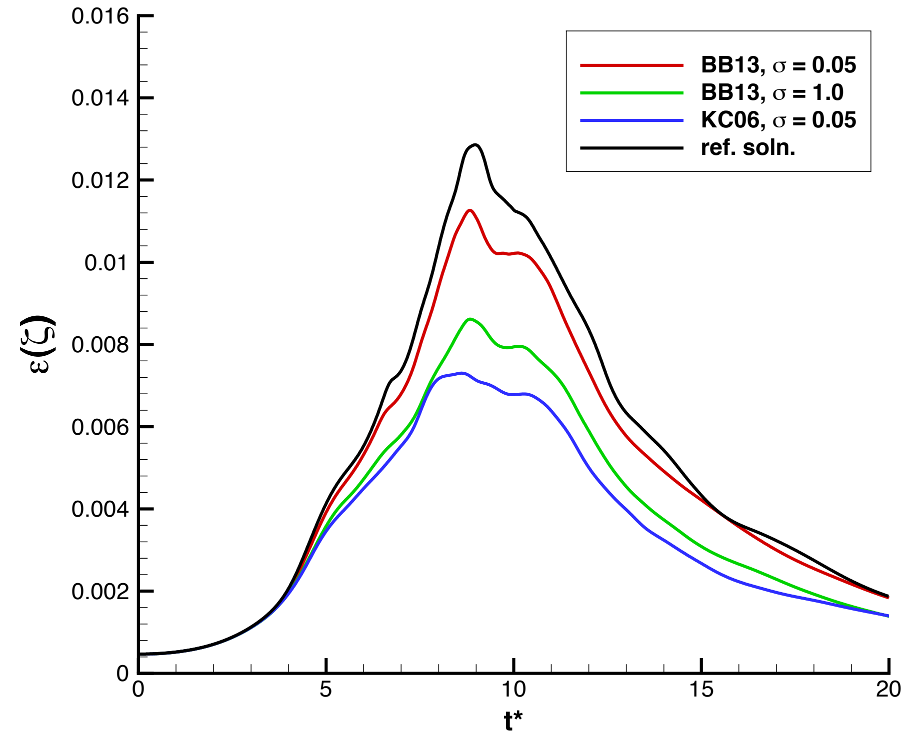
Kinetic energy spectra, t*=12

Effect of the Filter

BB13 Scheme, 128^3 grid



Directly computed KEDR



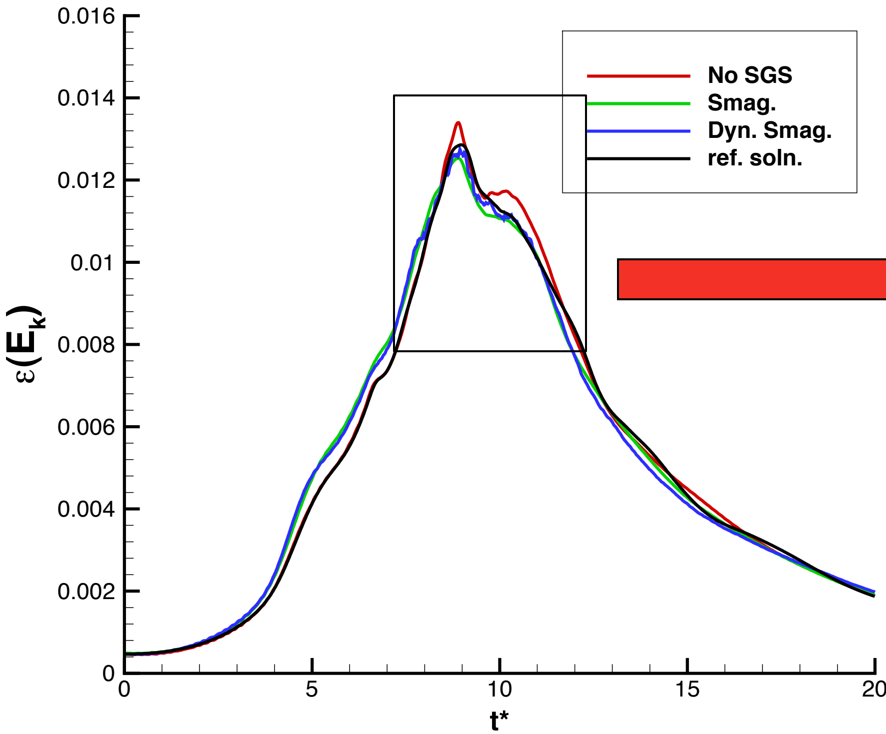
Enstrophy based KEDR

Effect of Sub-Grid Model

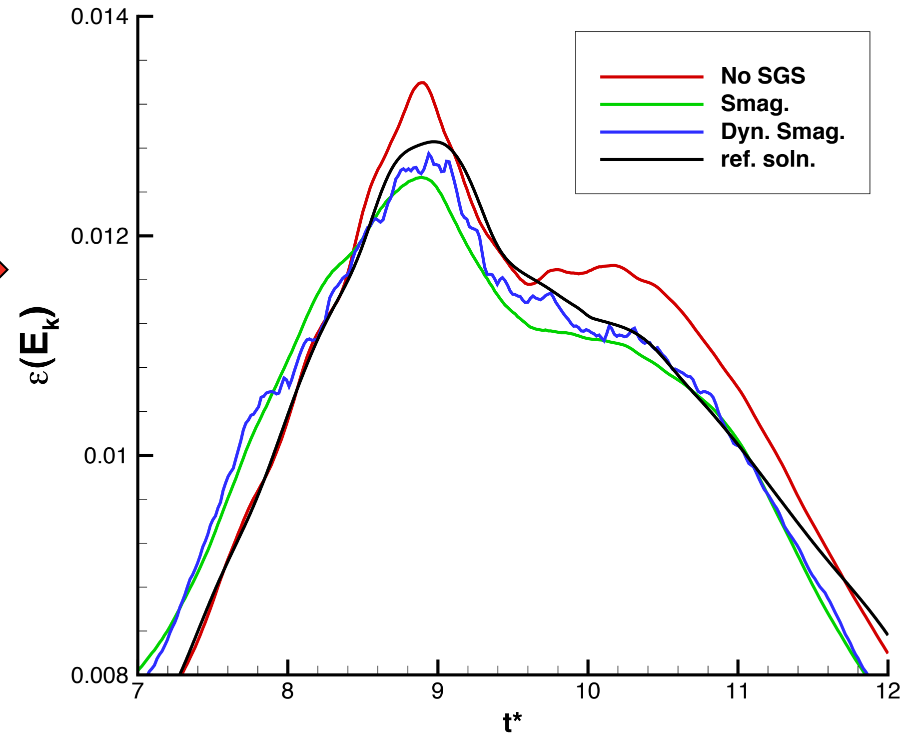
- Numerical Scheme
  - Temporal Discretization
    - Carpenter and Kennedy's 4-stage, 3^{rd}-order
  - Spatial Discretization
    - Bogey & Bailly's 13-point DRP scheme, BB13
  - Filter
    - Bogey & Bailly's 13-point filter, BB13
    - Filter coefficient halved until minimum stable value was found
    - Min. stable coefficient, $\sigma = 0.05$
- Sub-grid model
  - Smagorinsky
  - Dynamic Smagorinsky
- Grids
  - 128^3

Sub-grid Models

BB13, 128^3 grid



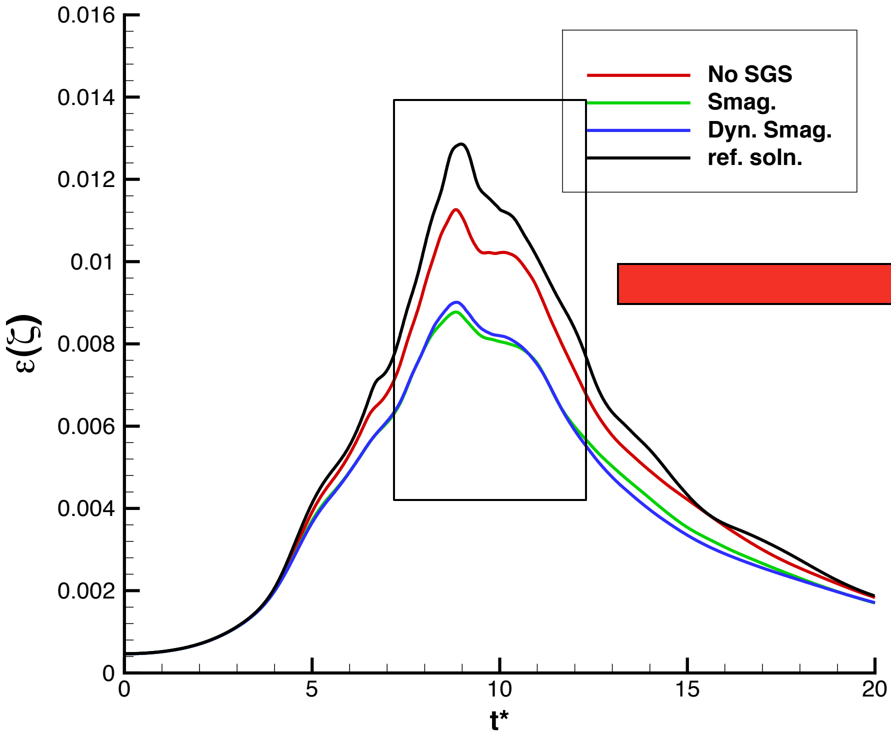
Directly computed KEDR



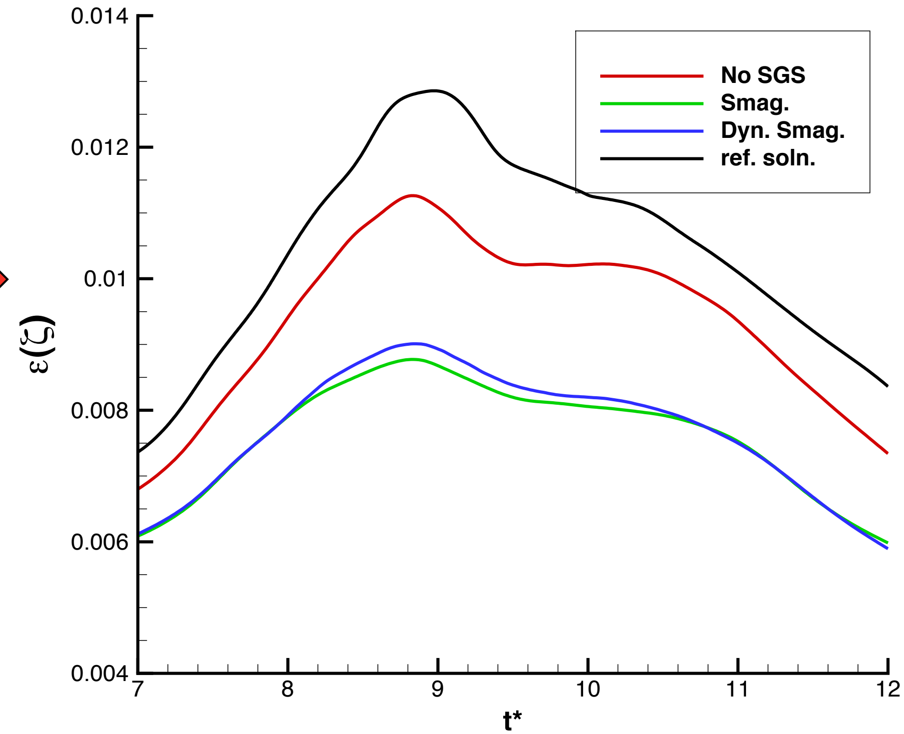
Close-up of
directly computed KEDR

Sub-grid Models

BB13, 128^3 grid



Enstrophy based KEDR



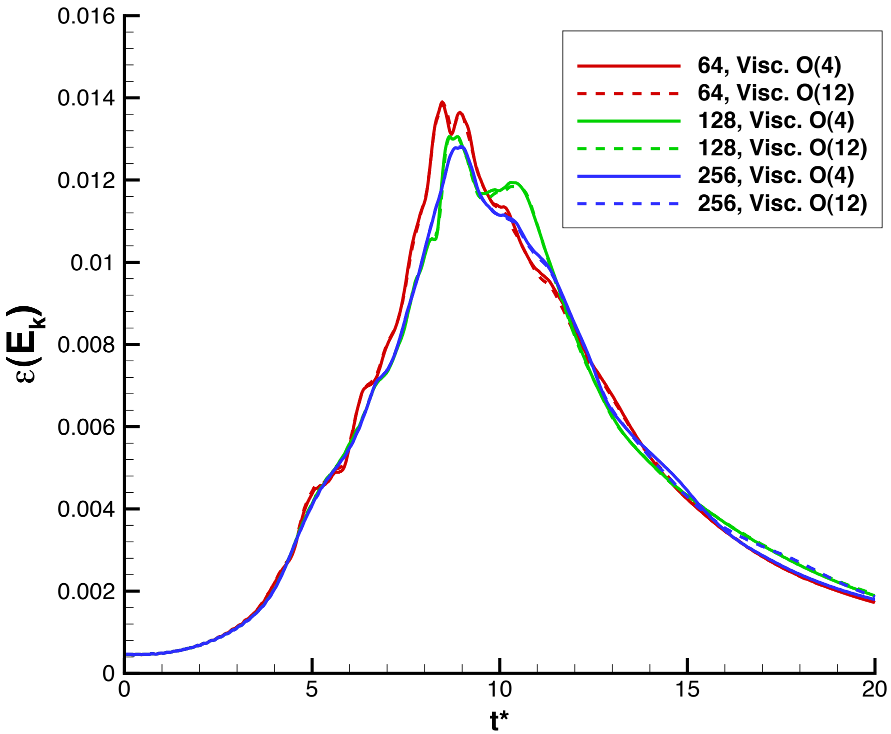
Close-up of
enstrophy based KEDR

Effect of Viscous Discretization

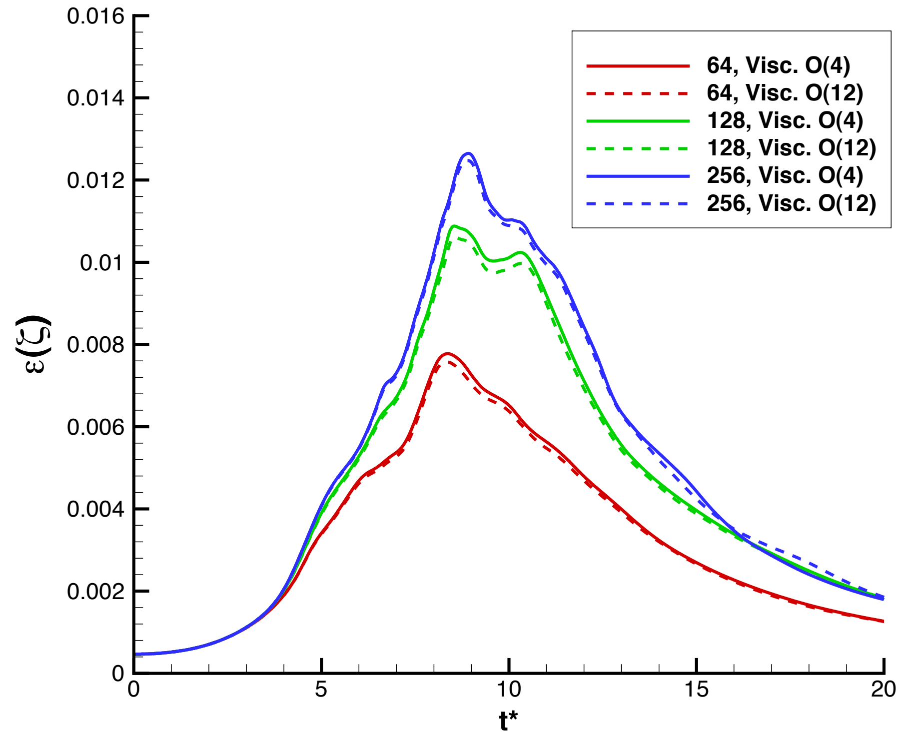
- Numerical Scheme
  - Temporal Discretization
    - Carpenter and Kennedy's 4-stage, 3rd-order
  - Spatial Discretization
    - Standard 12th-order central differencing, St12
  - Filter
    - Kennedy & Carpenter 12th-order filter, KC12
    - Filter coefficient halved until minimum stable value was found
    - Min. stable coefficient, $\sigma = 0.025$
  - Viscous Terms
    - 4th-order - standard practice
    - 12th-order
- Grids
  - 64^3, 128^3 and 256^3

Effect of Viscous Discretization

St12 Scheme



Directly computed KEDR



Enstrophy based KEDR

Summary and Conclusions

- Typically directly computed KEDR well predicted and enstrophy based KEDR under-predicted
  - Turbulent structures not well resolved
  - Numerical dissipation has significant role
- Largest discrepancies at peak dissipation rates
- Numerical scheme
  - High-order/resolution schemes most efficient
  - Low-order schemes adequate for directly computed KEDR
  - High-order schemes necessary for resolution of turbulent structures
  - DRP scheme has slight advantage in resolution of spectra

Summary and Conclusions

- Solution filtering
  - Creates a pronounced “tailing off” of spectra at the highest resolved wave numbers
  - Increasing damping coefficient uniformly increases dissipation and increases KEDR
  - Lowering the cut-off of the filter removes larger structures and can actually reduce KEDR
- Sub-grid model
  - Too dissipative where the flow is not fully turbulent
  - Improves the solution near the peak dissipation rates,
  - Dissipates the resolved turbulent structures
  - Increases the numerical dissipation
  - Dynamic model also shows evidence of backscatter

Summary and Conclusions

- Discretization of Viscous Terms
  - High-order representations are slightly more dissipative
  - 4^{th}-order discretization under-predicts magnitude of the viscous terms