DEVELOPMENT AND CHARACTERIZATION OF A RATE-DEPENDENT THREE-DIMENSIONAL MACROSCOPIC PLASTICITY MODEL SUITABLE FOR USE IN COMPOSITE IMPACT PROBLEMS

Robert K. Goldberg and Kelly S. Carney, NASA Glenn Research Ctr.
Paul DuBois, George Mason University
Canio Hoffarth and Subramaniam Rajan, Arizona State University
Gunther Blankenhorn, Livermore Software Technology Corporation

SAE INTERNATIONAL

Background and Motivation

- Current LS-DYNA material models have been found to have limitations in the modeling of impact in composites
  - Existing models usually require significant a priori knowledge of damage and failure responses on the structural scale
- Several years ago a new consortium was formed with the goal of creating a composite material model general enough to:
  - Model the wide range of material properties and architectures found in PMC's
  - Recreate all of the behavior that can be found in material property tests (including tests that are not typically performed)

Create a general material model for PMC composites which can predict impact results from mechanical properties, without relying on post impact correlation

FAA/NASA/ASU/LSTC Composite Material Modeling Consortium

FAA
Bill Emmerling
Program and Technical Management

Grant

George Mason University (GMU)
Paul DuBois
Steve Kan
Software Development

Grant

Arizona State University (ASU)
Subramaniam Rajan
Canio Hoffarth
Algorithm and Software Development and Testing

LSTC
Gunther Blankenhorn
Implementation and QA

NASA
Kelly Carney
Robert Goldberg
Charles Lawrence
NASA LaRC
Algorithm and Software Testing
Gary Roberts
Lee Kohlman
Brad Lerch
Jon Salem
Mechanical Property Testing
Mike Pereira
Duane Revilock
Charles Ruggeri
Impact Testing

- Developed model to be implemented into LS-DYNA as MAT_213.
- Initial focus is on development of deformation model.

General Composite Model Requirements

- Continuum Deformation/Damage Model with generalized, tabulated input, stress strain curve for non-damage related behavior (with limited or no curve fitting required by user)
  - Current models use point-wise properties that lead to curve fit approximations to actual material response
  - Tabulated input based on a well defined set of mechanical property tests leads to more accurate representations of actual material behavior
- Input parameters based upon standard mechanical property tests – although alternate specimen test configurations or micro-mechanic analytical approaches producing virtual test results are acceptable
- Effects of strain rate need to be accounted for in a flexible, unified manner accounting for anisotropy of rate effects.
- Temperature dependency
- Strain based damage and failure parameters
- Shell and solid element implementations required (through thickness properties can be important)
- Must be computationally extremely fast

General Approach

- Material nonlinearity in composites can be due to a combination of deformation and damage mechanisms acting independently or simultaneously
  - Most composite models assume that it is one or the other
- The new model will allow both plasticity-like non-linearity and/or non-linearity caused by damage to be defined by the user
  - Tsai-Wu criteria (typically a failure surface) is used to define an orthotropic yield surface
  - Damage laws that model the orthotropic stiffness degradation will be defined by tabulated input

Our goal is to create the capability to model general orthotropic behavior

Theoretical Formulation-Yield Surface

Tsai-Wu failure criteria generalized to a yield function with 12 coefficients

$$f(\boldsymbol{\sigma}) = a + \begin{pmatrix} F_1 & F_2 & F_3 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{pmatrix} + \begin{pmatrix} \sigma_{xx} & \sigma_{yy} & \sigma_{zz} & \sigma_{xy} & \sigma_{yz} & \sigma_{zx} \end{pmatrix} \begin{pmatrix} F_{11} & F_{12} & F_{13} & 0 & 0 & 0 \\ F_{12} & F_{22} & F_{23} & 0 & 0 & 0 \\ F_{13} & F_{23} & F_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & F_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & F_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & F_{66} \end{pmatrix} \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{pmatrix}$$

Coefficients determined from tension, compression, shear and off-axis tests

$$a = -1 \qquad a = -1 \qquad a = -1$$

$$F_1 = \frac{1}{\sigma_{xx}^T} - \frac{1}{\sigma_{xx}^C} \qquad F_{11} = \frac{1}{\sigma_{xx}^T \sigma_{xx}^C} \qquad F_{44} = \frac{1}{\sigma_{xy}^2}$$

$$F_2 = \frac{1}{\sigma_{yy}^T} - \frac{1}{\sigma_{yy}^C} \qquad F_{22} = \frac{1}{\sigma_{yy}^T \sigma_{yy}^C} \qquad F_{55} = \frac{1}{\sigma_{yz}^2}$$

$$F_3 = \frac{1}{\sigma_{zz}^T} - \frac{1}{\sigma_{zz}^C} \qquad F_{33} = \frac{1}{\sigma_{zz}^T \sigma_{zz}^C} \qquad F_{66} = \frac{1}{\sigma_{zx}^2}$$

$$F_{12} = \frac{2}{\left(\sigma_{45}^{xy}\right)^2} - \frac{F_1 + F_2}{\sigma_{45}^{xy}} - \frac{1}{2}(F_{11} + F_{22} + F_{44})$$

Values of coefficients vary as plastic strain evolves. Use tabulated input, not analytical function, to define evolution

Theoretical Formulation-Flow Surface

Non-associative flow rule applied with 9 independent constants

$$h = \sqrt{H_{11}\sigma_{xx}^2 + H_{22}\sigma_{yy}^2 + H_{33}\sigma_{zz}^2 + 2H_{12}\sigma_{xx}\sigma_{yy} + 2H_{23}\sigma_{yy}\sigma_{zz} + 2H_{31}\sigma_{zz}\sigma_{xx} + H_{44}\sigma_{xy}^2 + H_{55}\sigma_{yz}^2 + H_{66}\sigma_{zx}^2}$$

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}\frac{\partial h}{\partial \boldsymbol{\sigma}} \qquad \dot{W}_p = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^p = \boldsymbol{\sigma} : \dot{\lambda}\frac{\partial h}{\partial \boldsymbol{\sigma}} = h\dot{\lambda}$$

Constant coefficients based on plastic Poisson ratios and off-axis tests

$$\sigma_{xx} \neq 0 \qquad \sigma_{yy} \neq 0 \qquad \sigma_{zz} \neq 0$$

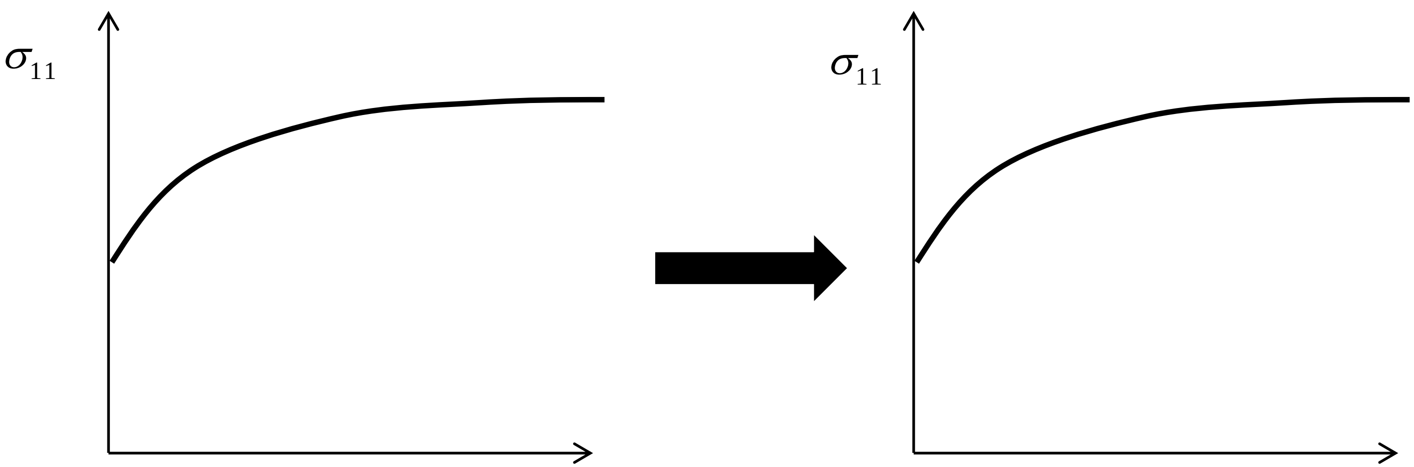
$$\nu_{12}^p = -\frac{\dot{\varepsilon}_{yy}^p}{\dot{\varepsilon}_{xx}^p} = -\frac{H_{12}}{H_{11}} \qquad \nu_{21}^p = -\frac{\dot{\varepsilon}_{xx}^p}{\dot{\varepsilon}_{yy}^p} = -\frac{H_{12}}{H_{22}} \qquad \nu_{32}^p = -\frac{\dot{\varepsilon}_{yy}^p}{\dot{\varepsilon}_{zz}^p} = -\frac{H_{23}}{H_{33}}$$

$$\nu_{13}^p = -\frac{\dot{\varepsilon}_{zz}^p}{\dot{\varepsilon}_{xx}^p} = -\frac{H_{13}}{H_{11}} \qquad \nu_{23}^p = -\frac{\dot{\varepsilon}_{zz}^p}{\dot{\varepsilon}_{yy}^p} = -\frac{H_{23}}{H_{22}} \qquad \nu_{31}^p = -\frac{\dot{\varepsilon}_{xx}^p}{\dot{\varepsilon}_{zz}^p} = -\frac{H_{13}}{H_{33}}$$

$$g = \sigma\sqrt{H_{44}}$$

$$d\varepsilon_{12}^p = 0.5 d\varepsilon_e^p \sqrt{H_{44}}$$

Current values of yield stresses required for evolution of yield surface and calculation of flow surface determined from 12 experimental (actual or virtual) stress-strain curves converted to be a function of effective plastic strain.



$$\varepsilon_{11}^{p} = \varepsilon_{11} - \frac{\sigma_{11}}{E_{11}}$$

$$\varepsilon_{e}^{p} = \int \left(\sigma_{11} d\varepsilon_{11}^{p} / h \right)$$

$$\mathbf{q}^n = \begin{pmatrix} \sigma_{11}^{T}(\lambda^n) \\ \sigma_{11}^{C}(\lambda^n) \\ \sigma_{22}^{T}(\lambda^n) \\ \sigma_{22}^{C}(\lambda^n) \\ \sigma_{33}^{T}(\lambda^n) \\ \sigma_{33}^{C}(\lambda^n) \\ \sigma_{12}(\lambda^n) \\ \sigma_{23}(\lambda^n) \\ \sigma_{31}(\lambda^n) \\ \sigma_{Y45}(\lambda^n) \\ \sigma_{Y45-23}(\lambda^n) \\ \sigma_{Y45-31}(\lambda^n) \end{pmatrix}$$

User provided load curves are true stress versus true plastic strain

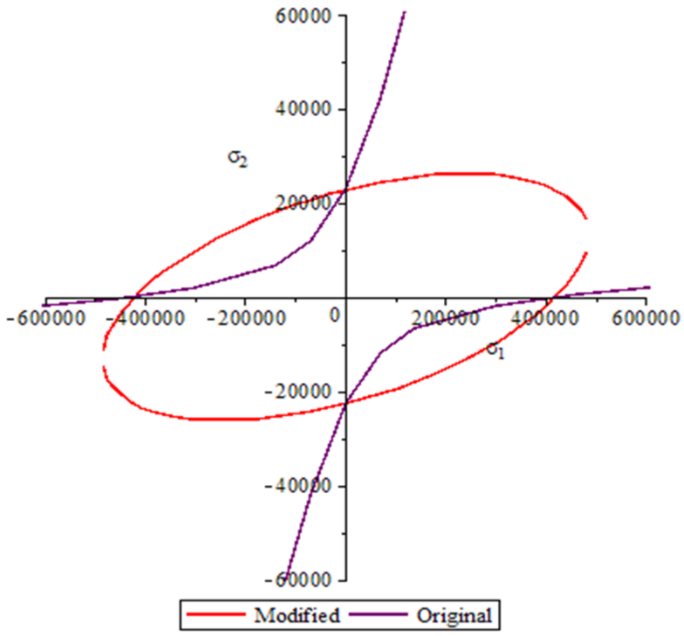
Internally stored is true stress versus effective plastic strain

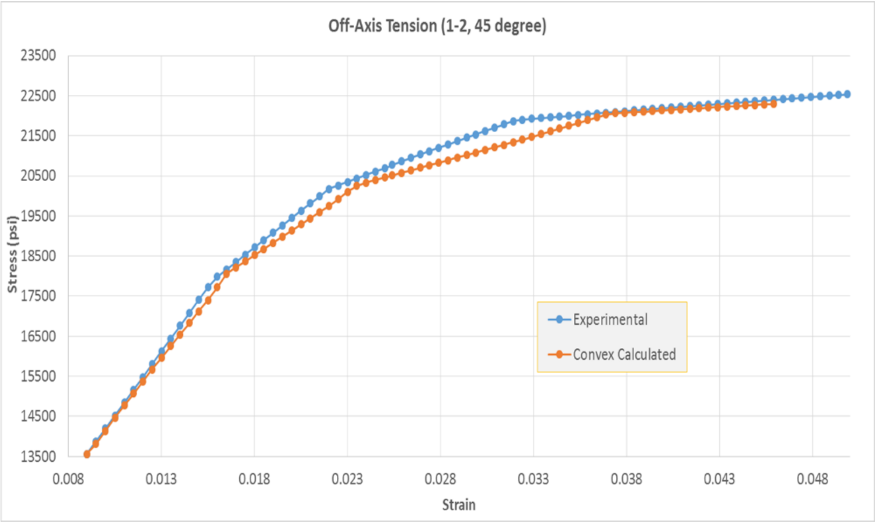
Characterization of off-diagonal terms in yield function

Experimental and/or numerical variability can result in non-convex yield surface using default characterization method.

$$F'_{ij} = -\frac{1}{2}\sqrt{F_{ii}F_{jj}}$$

$$F_{12} = \frac{2}{\left(\sigma_{45}^{xy}\right)^2} - \frac{F_1 + F_2}{\sigma_{45}^{xy}} - \frac{1}{2}(F_{11} + F_{22} + F_{44})$$





Initial curves can lead to hyperbolic yield surface where elliptical surface is required

Off-diagonal terms adjusted to ensure convex yield surface and off-axis input curve adjusted accordingly

Characterization of Flow Law Coefficients for Unidirectional Composite

- H11, H12 and H13 identically equal to zero due to elastic response in composite longitudinal (1) direction.
- H22 arbitrarily set equal to 1 by assuming in-plane transverse tension curve equal to effective stress-effective strain curve.
- H33 equal to 1 due to transverse isotropy of unidirectional composite.
- H23 equal to negative of plastic Poisson ratio v23.
- H44 found by optimizing shear test or 45° off-axis test.
- H66 equal to H44 due to transverse isotropy.
- H55 found by optimizing “23” shear test or by using isotropic relation

Required Mechanical Property Tests to Characterize Composite Model

Material Characterization Tests	ASTM Reference Test
Tension (1-direction)	ASTM D3039/D638
Tension (2-direction)	ASTM D3039/D638
Tension (3-direction)	ASTM D7291/D7291M-07
Compression (1-direction)	ASTM D3410/D3410M-03(2008)
Compression (2-direction)	ASTM D3410/D3410M-03(2008)
Compression (3-direction)	ASTM C365/D695,ASTM D7137
Shear (1-2 plane)	ASTM D5379
Shear (2-3 plane)	ASTM D5379
Shear (1-3 plane)	ASTM D5379
Off-axis tension (45°, 1-2 plane)	ASTM D3039/D638
Off-axis tension (45°, 2-3 plane)	ASTM C365/D695,ASTM D7137
Off-axis tension (45°, 1-3 plane)	ASTM D3039/D638

High Strain Rate Tests
Tension (1-direction)
Tension (2-direction)
Tension (3-direction)
Compression (1-direction)
Compression (2-direction)
Compression (3-direction)
Shear (1-2 plane)

Damage & Failure Characterization Tests
Tension (1 coupled w/2)
Tension (2 coupled w/1)
Tension (1 coupled w/3)
Tension (3 coupled w/1)
Tension (2 coupled w/3)
Tension (3 coupled w/2)
Compression (1 coupled w/2)
Compression (2 coupled w/1)
Compression (1 coupled w/3)
Compression (3 coupled w/1)
Compression (2 coupled w/3)
Compression (3 coupled w/2)

- Not all materials, architectures, and designs will require the full suite of tests for accurate predictions (through thickness properties can be very important in impact response)
- Mat 213 will fully accommodate the resulting test data

Overview of Numerical Implementation of Material Model
Radial Return Based Approach

- Step 1 : Compute elastic trial stress and yield surface coefficients from the current values of the yield stress.
- Step 2 : Check if state is elastic by computing value of yield function.
- Step 3 : If state is plastic (value of yield function greater than zero), start secant iteration to compute effective plastic strain and stress state that leads to yield function being equal to zero (convergence).

$$\sigma^{n+1}(f^1) = \sigma^e - C : \Delta\lambda^1 \left.\frac{\partial h}{\partial \sigma}\right|_e$$

$$q^{n+1}(f^1) = q\left(\lambda^n + \Delta\lambda^1\right)$$

$$\sigma^{n+1}(f^2) = \sigma^e - C : \Delta\lambda^2 \left.\frac{\partial h}{\partial \sigma}\right|_e$$

$$q^{n+1}(f^2) = q\left(\lambda^n + \Delta\lambda^2\right)$$

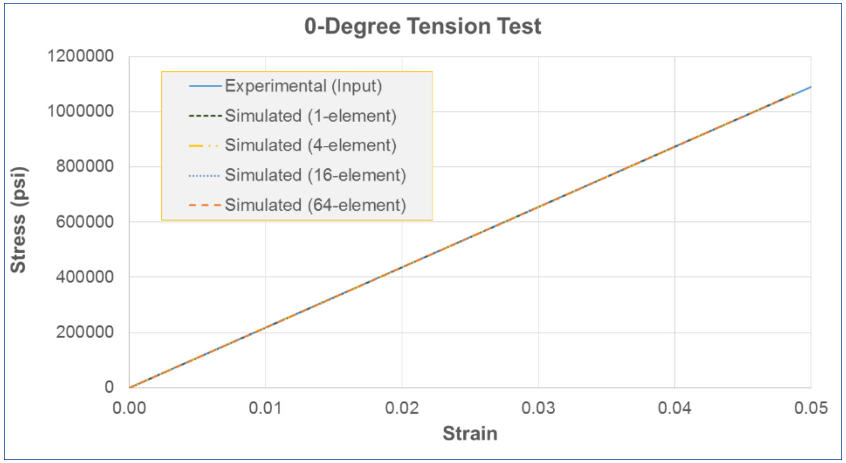
$$\Delta\lambda^3 = \Delta\lambda^1 - f^1 \frac{\Delta\lambda^2 - \Delta\lambda^1}{f^2 - f^1}$$

$$f_3 > 0 \Rightarrow \begin{cases} \Delta\lambda^1 = \Delta\lambda^3 \\ \Delta\lambda^2 = \Delta\lambda^2 \end{cases}$$

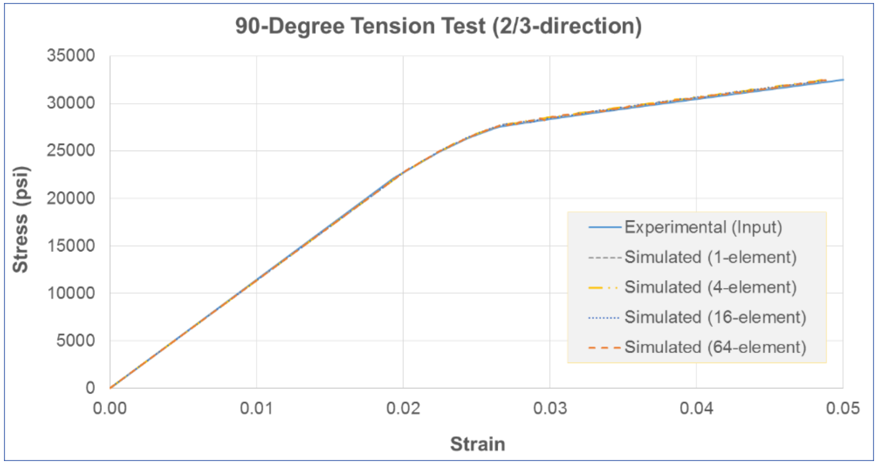
$$f_3 < 0 \Rightarrow \begin{cases} \Delta\lambda^1 = \Delta\lambda^1 \\ \Delta\lambda^2 = \Delta\lambda^3 \end{cases}$$

$$f_3 \approx 0 \Rightarrow \Delta\lambda = \Delta\lambda^3$$

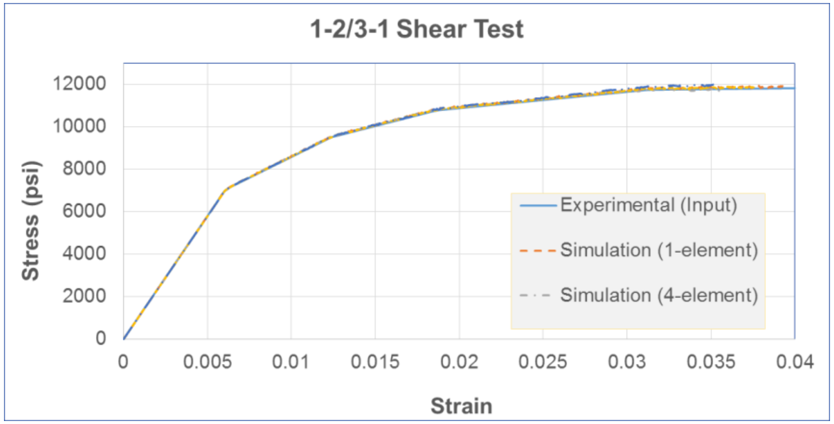
Verification of Material Model



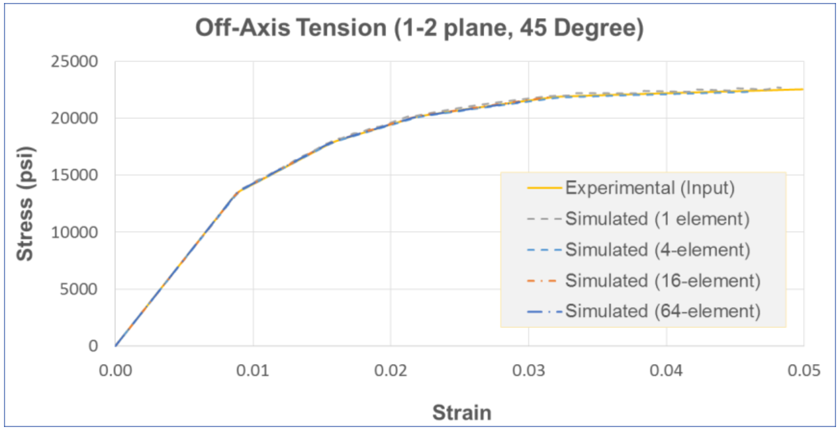
Longitudinal Tension



Transverse Tension

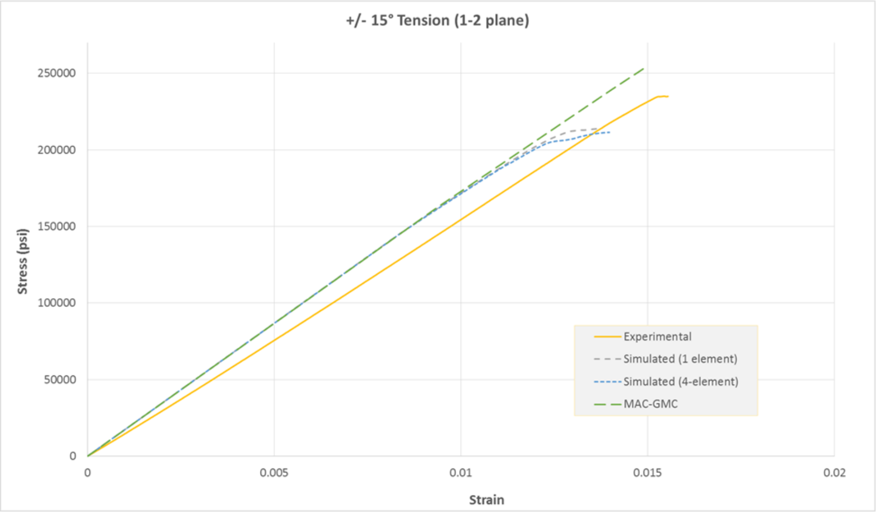


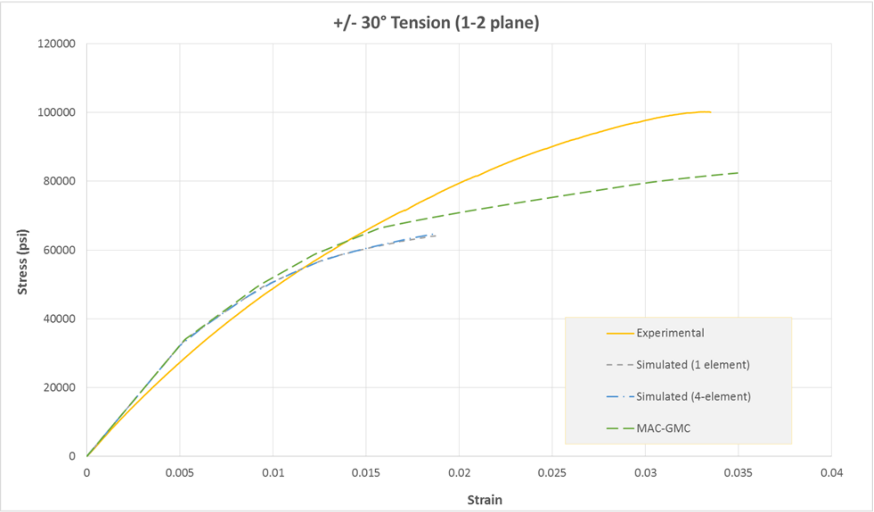
In-Plane Shear

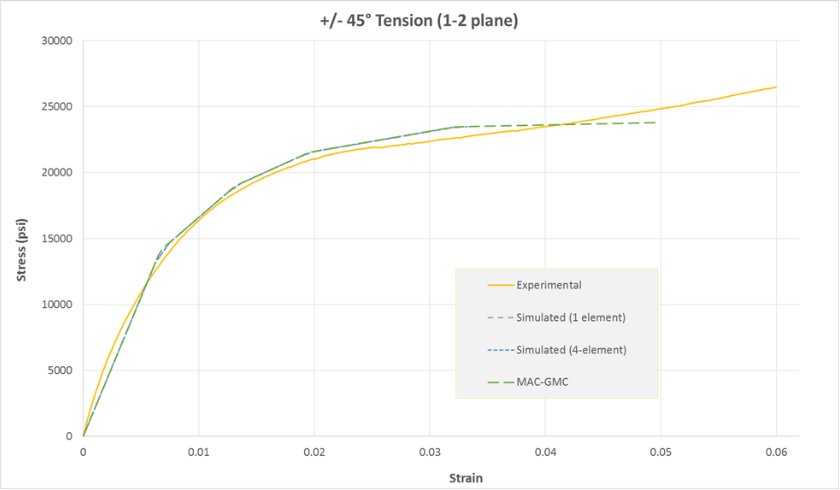


Off-Axis Tension

Laminate Level Verification of Material Model

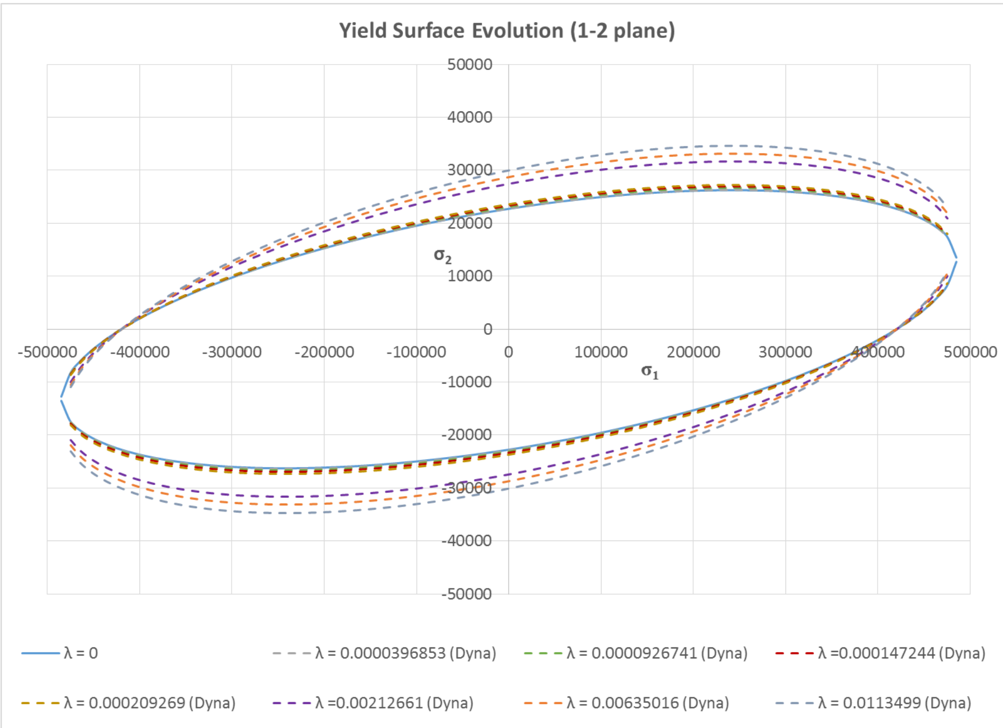






- Simulated curves show higher degree of nonlinearity compared to baseline most likely due to numerical method.
- Discrepancy between experiment and analysis for [+/- 30º] curve most likely due to assumption that compression response equals tension response.

Proposed Revised Numerical Method to Account for Rotation of Yield Surface



$$\Delta\lambda_{n+1}^{i+1} = \Delta\lambda^{1} - f^{1}\,\frac{\Delta\lambda^{2} - \Delta\lambda^{1}}{f^{2} - f^{1}}$$

$$\sigma_{n+1}^{i+1} = \sigma_{n} + C : \left[\Delta\varepsilon - \Delta\lambda_{n+1}^{i+1} \left.\frac{\partial h}{\partial \sigma}\right|_{n+1}^{i} \right]$$

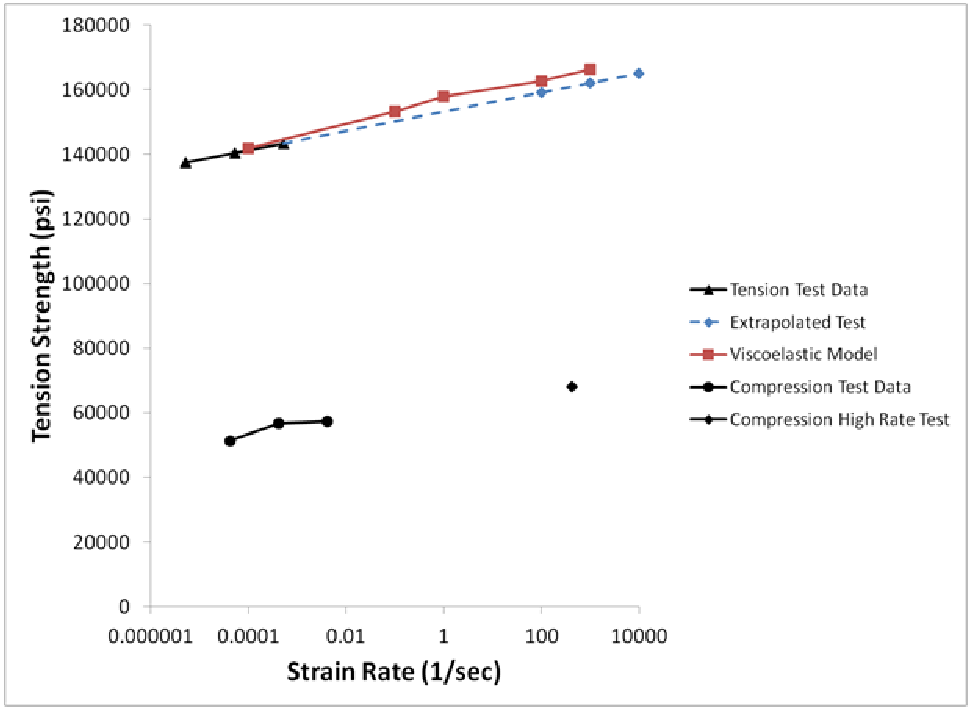
$$q_{n+1}^{i+1} = q\left(\lambda_{n} + \Delta\lambda_{n+1}^{i+1}\right)$$

$$\left.\frac{\partial h}{\partial \sigma}\right|_{n+1}^{i+1} = \frac{\partial h}{\partial \sigma}\left(\sigma_{n+1}^{i+1}\right)$$

Yield surface rotates with evolving plastic strain due to anisotropic yield function with yield stresses evolving in an anisotropic manner.

Radial return algorithm adjusted to dynamically vary direction of plastic strain vector at each iteration and each time step.

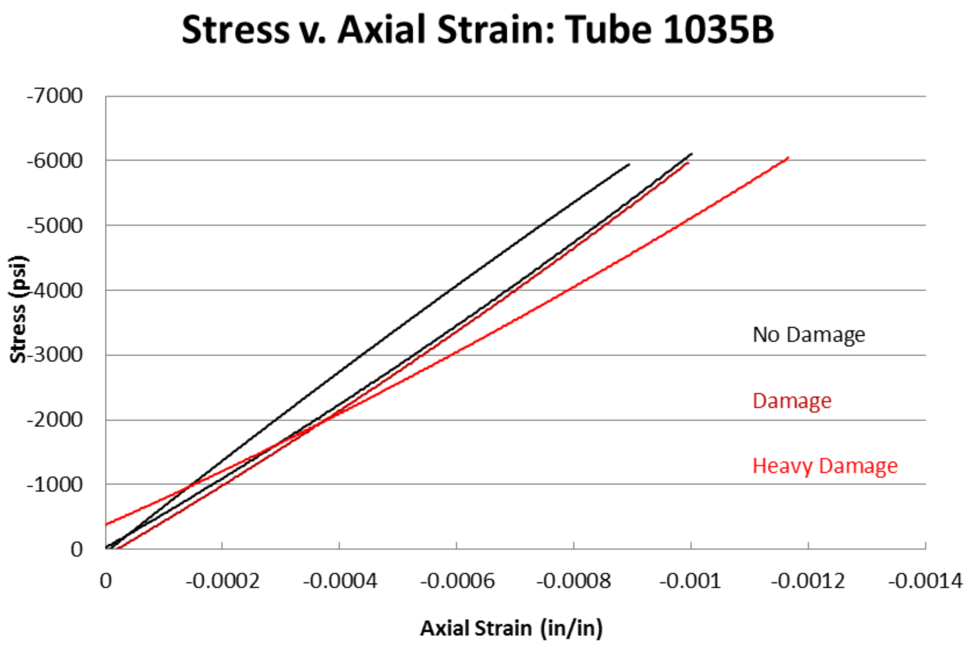
Incorporating Strain Rate Sensitivity into Material Model

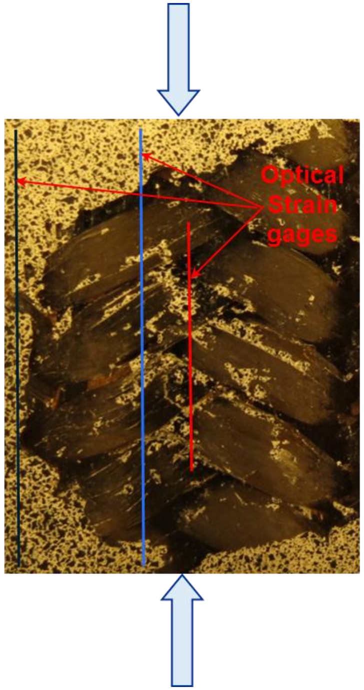


- Composite response can be extremely sensitive to strain rate.
- Response in behavior at different rates of loading cannot be captured merely by scaling stress by rate.
- Strain rate sensitivity will be incorporated into material model by use of tabulated input.

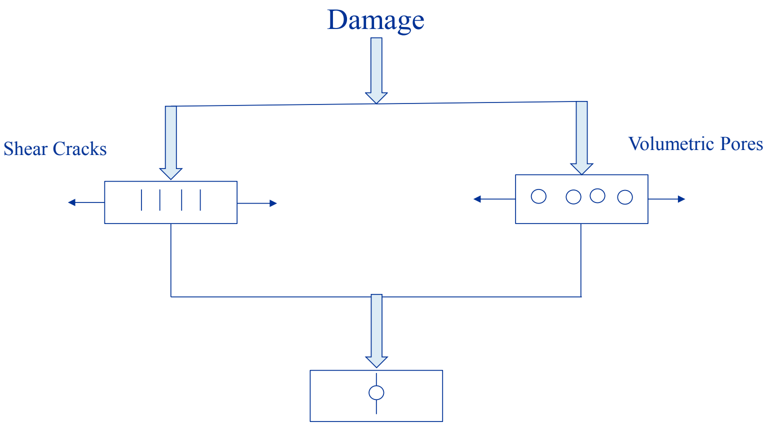
Damage Coupling
Triaxially Braided Composite Example

- Damage created by strain in one direction can create damage (reduction in stiffness) in another, not correlated to the Poisson's effect
- Modeling of this effect will require an anisotropic coupled damage law
  - Most damage models are uncoupled
- Testing performed at NASA GRC by Jon Salem and Nathan Wilmoth

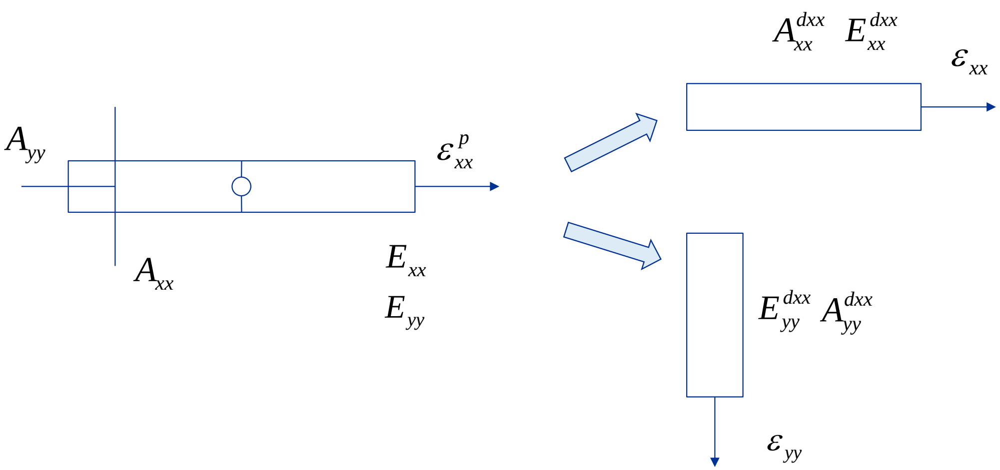




Preliminary Damage Model



- Damage likely due to a combination of mechanisms.
- Most damage models assume only cracks perpendicular to load are present.
- Current model assumes "predamage" already exists due to transverse loads.



$$E_{xx}^{dxx} = \left(1 - d_{xx}^{xx}\left(\varepsilon_{xx}^{p}\right)\right)E_{xx}^{dyy} = \left(1 - d_{xx}^{xx}\left(\varepsilon_{xx}^{p}\right)\right)\left(1 - d_{yy}^{xx}\left(\varepsilon_{yy}^{p}\right)\right)E_{xx}$$

$$A_{xx}^{dxx} = \left(1 - d_{xx}^{xx}\left(\varepsilon_{xx}^{p}\right)\right)A_{xx}^{dyy} = \left(1 - d_{xx}^{xx}\left(\varepsilon_{xx}^{p}\right)\right)\left(1 - d_{yy}^{xx}\left(\varepsilon_{yy}^{p}\right)\right)A_{xx}$$

$$\sigma_{xx} = \left(1 - d_{xx}^{xx}\right)\left(1 - d_{yy}^{xx}\right)\left(1 - d_{xy}^{xx}\right)\sigma_{xx}^{eff}$$

$$\sigma_{yy} = \left(1 - d_{xx}^{yy}\right)\left(1 - d_{yy}^{yy}\right)\left(1 - d_{xy}^{yy}\right)\sigma_{yy}^{eff}$$

$$\sigma_{xy} = \left(1 - d_{xx}^{xy}\right)\left(1 - d_{yy}^{xy}\right)\left(1 - d_{xy}^{xy}\right)\sigma_{xy}^{eff}$$

Required Mechanical Property Tests to Characterize Composite Model

Material Characterization Tests	ASTM Reference Test
Tension (1-direction)	ASTM D3039/D638
Tension (2-direction)	ASTM D3039/D638
Tension (3-direction)	ASTM D7291/D7291M-07
Compression (1-direction)	ASTM D3410/D3410M-03(2008)
Compression (2-direction)	ASTM D3410/D3410M-03(2008)
Compression (3-direction)	ASTM C365/D695,ASTM D7137
Shear (1-2 plane)	ASTM D5379
Shear (2-3 plane)	ASTM D5379
Shear (1-3 plane)	ASTM D5379
Off-axis tension (45°, 1-2 plane)	ASTM D3039/D638
Off-axis tension (45°, 2-3 plane)	ASTM C365/D695,ASTM D7137
Off-axis tension (45°, 1-3 plane)	ASTM D3039/D638

- Not all materials, architectures, and designs will require the full suite of tests for accurate predictions (through thickness properties can be very important in impact response)
- Mat 213 will fully accommodate the resulting test data

High Strain Rate Tests
Tension (1-direction)
Tension (2-direction)
Tension (3-direction)
Compression (1-direction)
Compression (2-direction)
Compression (3-direction)
Shear (1-2 plane)

Damage & Failure Characterization Tests
Tension (1 coupled w/2)
Tension (2 coupled w/1)
Tension (1 coupled w/3)
Tension (3 coupled w/1)
Tension (2 coupled w/3)
Tension (3 coupled w/2)
Compression (1 coupled w/2)
Compression (2 coupled w/1)
Compression (1 coupled w/3)
Compression (3 coupled w/1)
Compression (2 coupled w/3)
Compression (3 coupled w/2)

Conclusions and Future Work

- New composite material model MAT 213 being developed to provide improved predictive capability for LS-DYNA simulations of composite impact.
- Tsai-Wu composite failure model generalized to an orthotropic yield function.
- Tabulated stress-strain curves used to track evolution of coefficients of yield function and stresses for flow law.
- Characterization and numerical implementation of material model adjusted to account for issues related to orthotropic yield function.
- Complementary damage model based on tabulated experimental input under development.
- Strain rate and temperature effects being added to deformation model.
- Further extensive sets of verification and validation studies planned.