Adapting Guidance Methodologies for Trajectory Generation in Entry Shape Optimization

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Motivation

**Flight Feasible Trajectories** will

Model **Realistic In-Flight Thermal States:**

- Allow for increased accuracy in Thermal Protection System sizing (potential mass savings)

- Reduce the number of design cycles required to close an entry spacecraft design (potential cost savings)
Novel Research Objective

Develop a planetary guidance algorithm that is adaptable to:
- Mission Profiles
- Vehicle Shapes

for integration into vehicle optimization.
Sample Concept of Spaceflight Operations

* Adapted graphic from NASA Johnson Space Center

Launch to:
- Earth Orbit
- Planetary Body

Exploration:
Vehicle completes mission over several day or weeks

De-Orbit

Atmospheric Entry

Separation

EDL

Descent

Landing
Planetary Entry Spacecraft Design (cont’d)

**Mid - Low L/D Spacecraft**

> σ – variable bank angle
> α – fixed angle of attack

**High L/D Spacecraft**

> σ – variable bank angle
> α – variable angle of attack

* Orion Capsule
  Prakash et al., NASA JPL

* Ellipsled
  Garcia et al., AIAA Conf. Paper

* MSL Capsule
  AIAA 2006-8013

* Space Shuttle
  AIAA 2006-659

* NASP
  AIAA 2006-239

* Orion Capsule
  www.nasa.gov

* HL-20
  AIAA 2006-239
Multi-Disciplinary Design, Analysis, and Optimization (MDAO)

Minimize:
Heat Rate (Trajectory/Shape)
Ballistic Coefficient (Shape)

Aerodynamic ($C_D$, $C_L$) & Aerothermodynamic ($\dot{q}$) Databases

Decoupled Iterations

Guidance, Navigation, & Control

Flight Feasible Trajectory Database
*(replace Traj. Opt.)*

\[
\text{Minimize:} \\
\text{Heat Rate (Trajectory/Shape), Ballistic Coefficient (Shape)}
\]

Available Descent Technologies

Un/manned Planetary Models

Mission Profile

Computer Generated Spacecraft Models

Vehicle Optimization

Entry Trajectory Modeling

Coupled

Thermal Protection System (TPS) Sizing

Structures

Coupled

Mission Profile
## Trajectory Optimization vs. Guidance

<table>
<thead>
<tr>
<th></th>
<th>Trajectory Optimization</th>
<th>Guidance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraints</td>
<td>Multiple included</td>
<td>Minimal included</td>
</tr>
<tr>
<td>Objective</td>
<td>Any variable of interest</td>
<td>Target specific</td>
</tr>
<tr>
<td>Solution</td>
<td>Purely numerical</td>
<td>Combination of numerical and analytical</td>
</tr>
<tr>
<td>Time to Solution</td>
<td>Minutes to hours</td>
<td>Seconds</td>
</tr>
<tr>
<td>Guaranteed Solution</td>
<td>No</td>
<td>Must enforce that a solution is found</td>
</tr>
<tr>
<td>Parameter Changes</td>
<td>Handles large parameter changes</td>
<td>Handles parameter changes that are relatively small</td>
</tr>
<tr>
<td>Result</td>
<td>Nominal Trajectory – not always realistic control</td>
<td>Flight Feasible Trajectory with realistic controls</td>
</tr>
</tbody>
</table>
Guidance Development Trade-Offs

**Adaptability**
Numerical formulation for adaptability to different vehicles and missions without significant changes

**Rapid Trajectory Generation**
Analytical driving function keep time to a solution low

**Minimize Range Error & Heatload**
Optimal Control theory to introduce heat load as an additional objective
Guidance Development Criteria

Guidance Specific (In-Flight)

• Determine flight feasible control vectors (control rate/acceleration constraints)

• Be highly robust to dispersions and perturbations

• Include a minimal number of mission dependent guidance parameters

Vehicle Design Specific

• Be applicable to multiple mission scenarios and vehicle dispersions

• Manage the entry heat load in addition to achieving a precision landing
Types of Guidance Techniques

Reference Tracking Only – follow a pre-defined track

In-flight Reference Generation & Tracking – Generate a real-time reference trajectory and follow that track

In-flight Controls Search – One dimensional search, usually solving equations of motion numerically

In-flight Optimal Control – Requires numerical methods to meet some cost function
# Types of Guidance Formulations

<table>
<thead>
<tr>
<th></th>
<th>Analytical Guidance</th>
<th>Numerical Guidance</th>
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<tbody>
<tr>
<td><strong>Advantages</strong></td>
<td>• Simple to Implement</td>
<td>• Accurate trajectory solutions</td>
</tr>
<tr>
<td></td>
<td>• Computation time minimal</td>
<td>• No simplifying assumptions</td>
</tr>
<tr>
<td></td>
<td>• Solution Guaranteed</td>
<td>(possibility of multiple entry cases to be simulated with few modifications)</td>
</tr>
<tr>
<td><strong>Disadvantages</strong></td>
<td>• Simplifications reduce accuracy of the trajectory solution</td>
<td>• Convergence is not assured</td>
</tr>
<tr>
<td></td>
<td>• Formulation tied to a specific entry case</td>
<td>• Convergence is not timely</td>
</tr>
</tbody>
</table>
Novel Approach to Guidance for MDAO

Real-Time Trajectory Generation and Tracking

Adaptability

Numerically solve entry equations of motion
Use generalized analytical functions to represent the reference

Rapid Trajectory Generation

Use analytical driving function keep time to a solution low
Use Single Optimal Control Point with Blending

Minimize Range Error & Heatload

Optimal Control theory used to introduce heat load objective
Skip Entry Critical Points

Test Case: Orion Capsule, L/D 0.4

Control: Bank Angle only

Begin with 1st Entry portion of the trajectory and gradually includes remaining phases.
Trajectory Simulation Validation

Truth Model

Simulation of Rocket Trajectories (SORT)
Developed by NASA Johnson Space Center for Space Shuttle Launch/Entry Simulations
Flight Dynamics

$\mathbf{D}$

$\mathbf{L}$

Horizon

$b$ – body fixed coordinate

$\mathbf{V}$

$\mathbf{Z}_{\text{ECF}}$

$\mathbf{X}_{\text{ECF}}$

$\mathbf{Y}_{\text{ECF}}$

$\sigma$ – bank angle

$\theta$ - longitude

$\phi$ - latitude

$\gamma$ - flight path angle

$\psi$ - azimuth

Horizontal Plane Diagrams

Landing Site

$V_{\text{proj}}$
Trajectory Modeling

\[ \dot{r} = V \sin \gamma \]
\[ \dot{\theta} = \frac{V \cos \gamma \sin \psi}{r \cos \phi} \]
\[ \dot{\phi} = \frac{V \cos \gamma \cos \psi}{r} \]
\[ \dot{V} = -D - g \sin \gamma + \Omega^2 r \cos \phi (\sin \gamma \cos \phi - \cos \gamma \sin \phi \cos \psi) \]
\[ \dot{\phi} = \frac{1}{V} \left[ \frac{L \cos \sigma + \cos \gamma \left( \frac{V^2}{r} - g \right)}{r} + 2 \Omega V \cos \phi \sin \psi + \Omega^2 r \cos \phi (\cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi) \right] \]
\[ \dot{\psi} = \frac{1}{V} \left[ \frac{L \sin \sigma}{\cos \gamma} + \frac{V^2}{r} \cos \gamma \sin \psi \tan \phi - 2 \Omega V (\tan \gamma \cos \psi \cos \phi - \sin \phi) + \frac{r \Omega^2}{\cos \gamma} \sin \psi \sin \phi \cos \phi \right] \]

**State Variables**
- r - radial distance
- V - relative velocity
- \( \theta \) - longitude
- \( \phi \) - latitude
- \( \gamma \) - flight path angle
- \( \psi \) - azimuth

**Control Variables**
- \( \sigma \) - bank angle
- \( \alpha \) - angle of attack

**Vehicle and Planet Variables**
- L, D - Lift, Drag Acceleration
- g - gravity
- \( \Omega \) - Earth’s Rotation
- \( \rho \) - atmospheric density
**General Entry Guidance Block Diagram**

**Trajectory Solver**
- **Reference Trajectory:** Analytical functions adapted from Shuttle Entry Guidance
- **Bank Schedule Solution:** \( \tilde{\sigma}_{cmd} \)
- **Range Prediction:** numerically solve equations of motion, range calculation

**Targeting Algorithm**
- **Solver:** Single Point Optimal Control
- **Solution from Energy State Approximation**
- **Purpose:** Targeting for precision landing and minimizing heatload

**Dispersed State:** \( \tilde{y}_{disp} \)

Send \( \tilde{\sigma}_{cmd} \) to flight simulation

\( R_{err} \sim= 0 \)

- **Yes**
- **No**

\( \tilde{\sigma}_{new} \)
Control Solution: Shuttle Entry Guidance Adaptation

Shuttle Entry Guidance (SEG) Concept: Temperature Phase

- Reference Tracking Algorithm, Closed Form Solution

\[ \frac{d}{dt} \left( D = \frac{\rho V_r^2 C_D A}{2m} \right) \]

**Reference Trajectory**

\[ D_{\text{ref}} = C_2 V^2 + C_1 V + C_0 \]
\[ \gamma_{\text{ref}} = \text{constant} \]

**Bank Schedule**

\[ \dot{\rho} = \frac{d}{dt} \left( \rho_o e^{-\frac{h}{h_s}} \right) \Rightarrow \frac{\dot{\rho}}{\rho} = -\frac{\dot{h}}{h_s} \]
Control Solution: Shuttle Entry Guidance Adaptation

Improvements on Shuttle Entry Guidance “Drag Based Approach”

• Increase # of segments
• Increase order of polynomial
• Change Atmospheric Model representation
• Modify flight path angle representation

Challenges with Drag Based Approach

• Discontinuities between segments
• Increasing # of coefficients for storage with increasing segments and/or order
• Effect of small flight path angle assumption unknown
• Formulations are derived from 2DOF Longitudinal EOMs
Control Module: Shuttle Entry Guidance Adaptation

Sensitivity to atmospheric non-linearity is significant during initial and final segments. Need an Alternative Analytical Equation!
Automated Selection of Transition Events

Framework:
- Allows for adaptability
- Automated generation of Reference Trajectory
- Open loop

Study Objective: Define bank profile for trajectory phases

<table>
<thead>
<tr>
<th>Phase</th>
<th>Bank Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry Interface to Guidance Start</td>
<td>Constant Bank</td>
</tr>
<tr>
<td>Guidance Start to Guidance End</td>
<td>Trajectory Solver</td>
</tr>
<tr>
<td>Guidance End to Exit</td>
<td>Linear Transition to Meet 2nd Entry Bank</td>
</tr>
<tr>
<td>Exit to 2nd Entry</td>
<td>Attitude Hold</td>
</tr>
</tbody>
</table>
Automated Selection of Transition Events

- Metric to determine best trajectory: lowest range error, lowest heat load from EI to 2nd Entry, and bank transitions
Automated Selection of Transition Events

Study Results:

<table>
<thead>
<tr>
<th>Phase</th>
<th>Bank Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry Interface to Guidance Start</td>
<td>Constant Bank = 57.95°</td>
</tr>
<tr>
<td>Guidance Start to Guidance End</td>
<td>Trajectory Solver</td>
</tr>
<tr>
<td></td>
<td>{0.12, 0.11} G’s</td>
</tr>
<tr>
<td>Guidance End to Exit</td>
<td>Linear Transition to Meet 2nd Entry Bank</td>
</tr>
<tr>
<td></td>
<td>Linear Transition Velocity: 23,784.65 ft/s</td>
</tr>
<tr>
<td>Exit to 2nd Entry</td>
<td>Bank Attitude Hold = 70°</td>
</tr>
</tbody>
</table>

![Diagram showing transition events and bank angles](image)
General Entry Guidance Block Diagram

**Trajectory Solver**
- **Reference Trajectory:** Analytical functions adapted from Shuttle Entry Guidance
- **Bank Schedule Solution:** \( \vec{\sigma}_{cmd} \)
- **Range Prediction:** numerically solve equations of motion, range calculation

**Dispersed State:** \( \vec{y}_{disp} \)

**Targeting Algorithm**
- **Solver:** Single Point Optimal Control
- **Solution from Energy State Approximation**
- **Purpose:** Targeting for precision landing and minimizing heatload

**Send** \( \vec{\sigma}_{cmd} \) to flight simulation

\[ R_{err} \sim= 0 \]

Yes

\( \vec{\sigma}_{new} \)

No
Targeting Algorithm Development

When is Targeting Activated?

1. Overshoot – Vehicle is predicted to fly way past target
2. Undershoot – Vehicle is predicted to fly short of the target

How to find a set of controls to Correct Over/Underhoot?

*Adapt Energy State Approximation Methods:*  
Optimal control method that replaces altitude and velocity with specific energy height \((e)\)  
\[
e = \frac{V_r^2}{2g_o} + h
\]

**Advantages:** Allows for a compact set of analytical equations  
Add heat load to the range error objective function

**Disadvantage:** Optimal control formulations may not converge to a solution

**Solution:** Derive a localized optimal control point instead and blend back reference trajectory
Targeting Algorithm Development

Must Relate Euler-Lagrange Equation

$$\bar{\lambda} = \frac{\lambda_\psi}{\lambda_\gamma} = \tan \sigma^* \cos \gamma$$

$$\lambda_\gamma \leq 0$$

To Reference Trajectory Variables

$$\left. \frac{L}{D} \right|_{\text{total}} \cos \sigma = \frac{1}{\rho \Phi_{\text{ref}}} \left[ V_r \dot{\gamma}_{\text{ref}} - \cos \gamma \left( \frac{V_r^2}{r} - g \right) - C_y(y) \right]$$

Using trigonometry and other manipulations, the control equation is found

$$\left. \frac{L}{D} \right|_{\text{total}} \sqrt{\frac{1}{1 + \left( \frac{\lambda}{\cos \gamma} \right)^2}} = \frac{\left[ V \gamma_{\text{ref}} - \cos \gamma \left( \frac{V_r^2}{r} - g \right) - 2\Omega V \cos \phi \sin \psi - \Omega^2 r \cos \phi \left( \cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi \right) \right]}{D_{\text{aprxx}}}$$
Targeting Algorithm Development

\[ \Phi_{\text{ref}} = \frac{C_D A}{2m} \left( \frac{V_r}{c_{\text{ref}}} \right)^2 \]

Least Squares Curve Fitting:
3 Interpolation Points

\[ \Phi_{\text{blnd}} = B_2 V^2 + B_1 V + B_0 \]
Targeting Algorithm Development

Targeting Technique 1 – Design Space Interrogation

\[ C_\Phi \] - drag/density ratio coefficient

\[ d\lambda \] - change in Lagrange multiplier

\[ dV \] - change in relative velocity at next point

Targeting Technique 2 – Design Space Interrogation

\[ d\lambda \] - change in Lagrange multiplier

\[ dV_1 \] - change in relative velocity halfway to curve fit end point

\[ \Delta(dE) \] - second order change in energy
Targeting Algorithm Development

Targeting Technique 1 – Design Space Interrogation

<table>
<thead>
<tr>
<th>Case</th>
<th>Dispersion</th>
<th>Target Miss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Increase Entry Flight Path Angle</td>
<td>Undershoot</td>
</tr>
<tr>
<td>2</td>
<td>Decrease Entry Flight Path Angle</td>
<td>Overshoot</td>
</tr>
<tr>
<td>3</td>
<td>L/D Dispersion</td>
<td>Overshoot</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Lower Limit</th>
<th>Upper Limit</th>
<th>Incr.</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_\Phi$</td>
<td>0</td>
<td>1</td>
<td>ND</td>
<td></td>
</tr>
<tr>
<td>$d\lambda$</td>
<td>0</td>
<td>1</td>
<td>0.01</td>
<td>ND</td>
</tr>
<tr>
<td>$dV$</td>
<td>100</td>
<td>1000</td>
<td>100</td>
<td>ft/s</td>
</tr>
</tbody>
</table>
Targeting Algorithm Development

FPA Dispersion - Undershoot
Targeting Algorithm Development

FPA Dispersion - Overshoot
Aerodynamic Dispersion - Overshoot

Targeting Algorithm Development
Shape Optimization Analog

Geometry #1: $C_L = 1.70, C_D = 3.4$

Geometry #2: $C_L = 1.90, C_D = 3.8$

Geometry #3: $C_L = 1.95, C_D = 3.9$

Current Guidance Algorithms – Robust to ~20% aerodynamic dispersions

Must exceed 20% to demonstrate potential for integration into MDAO

**ANALOG:** Changing angle of attack disperses $C_L$ and $C_D$
Targeting Algorithm Development

Guidance Algorithm for Comparison – Apollo Derived Final Phase Guidance

Reference Tracking to a stored trajectory database, function of relative velocity

Performance Results – Threshold Miss Distance, 1 nmi
Targeting Algorithm Development

Targeting Technique 1 – Targeting Procedure

1. Guess a value for \( d\lambda \)
2. Iterate on \( dV \) using secant method to converge on a zero range error trajectory
3. If no solution is found, \( d\lambda \) is incremented and the iteration is repeated
4. Solution is then flown in flight simulation
Targeting Algorithm Development

Targeting Implementation, 1st and 2nd Phase - Results

\( \alpha = 152^\circ, \ L/D = 0.418 \)

\( \alpha = 153^\circ, \ L/D = 0.402 \)

\( \alpha = 154^\circ, \ L/D = 0.386 \)

\( \alpha = 155^\circ, \ L/D = 0.371 \)

\( \alpha = 156^\circ, \ L/D = 0.357 \)

\( \alpha = 157^\circ, \ L/D = 0.343 \)

\( \alpha = 158^\circ, \ L/D = 0.328 \)

\( \alpha = 159^\circ, \ L/D = 0.313 \)

\( \alpha = 160^\circ, \ L/D = 0.299 \)
Targeting Algorithm Development

Targeting Technique 2

Use Energy Height \[ e = \frac{V^2}{2g_o} + h \] to determine Control Point \([V_{new}, \Phi_{new}]\)

Undershoot → energy dissipating (de/dt) too fast

Overshoot → energy dissipating (de/dt) too slow

Since Velocity is an independent variable
and a pseudo control de/dV is examined
Targeting Algorithm Development

Targeting Technique 2

Recall the equation for the ratio of drag acceleration to density: \[
\frac{D}{\rho} = \frac{C_D A}{2m} V_r^2
\]

- Extract altitude and velocity from \([dV_1, \Delta(dE)]\) to find \(\Phi_{new}\)
Targeting Algorithm Development

Targeting Technique 2 – Design Space Interrogation

<table>
<thead>
<tr>
<th></th>
<th>Lower Limit</th>
<th>Upper Limit</th>
<th>Incr.</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d\lambda$</td>
<td>0</td>
<td>$d\lambda_{limit}$</td>
<td>ND</td>
<td></td>
</tr>
<tr>
<td>$dV_1$</td>
<td>0</td>
<td>1524</td>
<td>Predict</td>
<td>m/s</td>
</tr>
<tr>
<td>$\Delta(dE)$</td>
<td>0</td>
<td>$\Delta(dE)_{limit}$</td>
<td>Predict</td>
<td>m</td>
</tr>
</tbody>
</table>

Limit are trajectory dependent and control system dependent

$$\lambda_{min/max} = \tan \sigma_{min/max} \cos \gamma_i$$

$$d\lambda_{limit} = \mp (\lambda_{min/max} - \lambda_{old})$$

Dispersion Cases:

<table>
<thead>
<tr>
<th>$\alpha$ [deg]</th>
<th>$L/D$ Dispersion</th>
<th>Target Miss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>0.4 (0%)</td>
<td></td>
</tr>
<tr>
<td>152</td>
<td>0.42 (+5%)</td>
<td>Undershoot</td>
</tr>
<tr>
<td>162</td>
<td>0.28 (-30%)</td>
<td>Overshoot</td>
</tr>
<tr>
<td>165</td>
<td>0.23 (-43%)</td>
<td>Overshoot</td>
</tr>
<tr>
<td>167</td>
<td>0.2 (-50%)</td>
<td>Undershoot</td>
</tr>
</tbody>
</table>
Targeting Algorithm Development

*Design Space Interrogation, Results: Range Error [%]*

- $\alpha = 152^\circ$, Undershoot
- $\alpha = 162^\circ$, Overshoot
- $\alpha = 165^\circ$, Overshoot
- $\alpha = 167^\circ$, Undershoot
Targeting Algorithm Development

*Design Space Interrogation, Results: Heatload [J/cm^2]*

\( \alpha = 152^\circ, \text{ Undershoot} \)

\( \alpha = 162^\circ, \text{ Overshoot} \)

\( \alpha = 165^\circ, \text{ Overshoot} \)

\( \alpha = 167^\circ, \text{ Undershoot} \)
Targeting Algorithm Development

*Design Space Interrogation, Results: Bank Rate [deg/s]*

- $\alpha = 152^\circ$, Undershoot
- $\alpha = 162^\circ$, Overshoot
- $\alpha = 165^\circ$, Overshoot
- $\alpha = 167^\circ$, Undershoot
Targeting Algorithm Development Results

Dispersions –
Apollo Derived Guidance = -20% dispersion
MDAO Algorithm = -43% dispersion

Managing heatload may be a challenge for dispersions greater than 20%
Conclusions

Guidance Specific (In-Flight)

✓ Determine flight feasible control vectors (control rate/acceleration constraints)

○ Be highly robust to dispersions and perturbations

✓ Include a minimal number of mission dependent guidance parameters

Vehicle Design Specific

• Be applicable to multiple

○/✓ mission scenarios
✓ vehicle dispersions

○ Manage the entry heat load in addition to achieving a precision landing
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UC Davis Mechanical and Aerospace Engineering Faculty

Colleagues in Systems Analysis Office

UC Davis Mechanical and Aerospace Engineering Staff

Thank You !!!
Questions?
Additional Slides (optional)
# Overview

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<th>Elements of Spacecraft Design</th>
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<td>Dissertation Research Plan and Status</td>
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<td>MAPGUID Development</td>
<td>MAPGUID Proposed Approach</td>
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<td></td>
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<td>Aerothermal Management</td>
<td>Proposed Approach</td>
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<tr>
<td>During Guidance</td>
<td>Key Results #1</td>
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<td></td>
<td>Key Results #2</td>
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<td>Guidance/COBRA Integration</td>
<td>Proposed Approach</td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>Key Results #2</td>
</tr>
<tr>
<td>Closing Remarks</td>
<td>Dissertation Findings and Status</td>
</tr>
</tbody>
</table>
Big Picture:
Spacecraft Design Process
Vehicle Optimization and TPS Sizing

Example Objective Function: \( \dot{q}_{conv} = 1.83 \times 10^{-4} \sqrt{\frac{\rho}{R_n}} \left(1 - \frac{h_W}{H_s}\right) V^3 \)

**Results**

- Most studies use a single trajectory to find altitude-velocity corresponding to maximum heat rate.
  - Used for all geometries within optimization to find heat rate.

- Some studies use new trajectories, but there is no accounting for bank constraints or target accuracy.

- **None of these studies incorporated flight feasible trajectories**

**What is Flight Feasible?**

- Reaches Target @ Landing Speeds
- Control does not exceed system limits
- Used for all geometric within optimization to find flight rate
Proposed Approach to MDAO for Spacecraft Design

- Vehicle Optimization
- Planetary Entry Guidance
- Thermal Protection System (TPS) Sizing
- Structures

Available Descent Technologies
Un/manned
Planetary Models
Mission Profile

Computer Generated Spacecraft Models

Aerodynamic \( (C_d, C_L) \) & Aerothermodynamic (\( \dot{q} \)) Databases

Flight Feasible Trajectory Database

Guidance, Navigation, & Control

Reduced Decoupled Iterations
Trajectory Modeling for Design vs. In-Flight Trajectory Modeling
Planetary Entry Guidance Literature Review

- **High L/D, Earth**: Space Shuttle, X-33, X40A
  - Most Robust: In Flight Trajectory Shaping with Reference Tracking
  - Least Robust: Reference Tracking Only
- **Low L/D, Earth**: Apollo, Orion
  - Most Robust: In-Flight Controls Search
  - Least Robust: Reference Tracking Only
- **Other Planetary Entry Vehicles**: MSR, MSL, Biconic
  - Flight Tested algorithms preferred
Planetary Entry Guidance Literature Review (cont’d)

Key Results

Modern guidance algorithms: optimal control is potential framework, but

Adaptability of guidance algorithms: very little among all algorithms

Heat load management: not included
Trajectory Optimization Literature Review

Trajectory Optimization

Traj - Nonlinear constrained optimization

Mission - Sequential Quadratic Programming

Energy State Method – Reduced Order Modeling, one dimensional parameter search

Pseudospectral Methods – Combination indirect and direct method, mapping and discretization of domain
Trajectory Optimization Literature Review (cont’d)

Key Results

- No convergence to a solution
- Fidelity of modeling may be compromised
- Convergence time increases with dimensionality
Introduction to Planetary Entry Guidance
Guidance Development Process

*Guidance must be robust to many dispersions: (Atmospheric properties, Aerodynamics properties, Navigational Inputs, Entry Interface Conditions, Mass, Control System performance, and many others)
## Case Study Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vehicle</strong></td>
<td>Orion Capsule, L/D = 0.4</td>
</tr>
<tr>
<td><strong>Trajectory</strong></td>
<td>Skip Entry for Lunar Return</td>
</tr>
<tr>
<td><strong>Control</strong></td>
<td>Bank Angle only</td>
</tr>
<tr>
<td><strong>Atmospheric Model</strong></td>
<td>1976 Standard Atmosphere</td>
</tr>
<tr>
<td><strong>Gravity Model</strong></td>
<td>Central Force + Zonal Harmonics</td>
</tr>
<tr>
<td><strong>Aerodynamics</strong></td>
<td>$C_L$, $C_D$ corresponding to Mach #</td>
</tr>
<tr>
<td></td>
<td>CBAERO Databases, function of Mach #, Dynamic Pressure, and Angle of Attack</td>
</tr>
<tr>
<td><strong>Trajectory Simulation</strong></td>
<td>MATLAB Simulation validated against SORT Trajectories</td>
</tr>
</tbody>
</table>
Trajectory Simulations Developed

Open Loop Numerical Predictor- Corrector (NPC) Simulation
Used to test guidance formulations

3DOF Rotating Spherical Planet

\[
\begin{align*}
\dot{r} &= V \sin \gamma \\
\dot{\theta} &= \frac{V \cos \gamma \sin \psi}{r \cos \phi} \\
\dot{\phi} &= \frac{V \cos \gamma \cos \psi}{r}
\end{align*}
\]

\[
\begin{align*}
\dot{V} &= -D - g \sin \gamma + \Omega^2 r \cos \phi (\sin \gamma \cos \phi - \cos \gamma \sin \phi \cos \psi) \\
\dot{\psi} &= \frac{1}{V} \left[ L \cos \sigma + \cos \gamma \left( \frac{V^2}{r} - g \right) + 2\Omega V \cos \phi \sin \psi + \Omega^2 r \cos \phi (\cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi) \right] \\
\dot{\sigma} &= \frac{1}{V} \left[ \frac{L \sin \sigma}{\cos \gamma} + \frac{V^2}{r} \cos \gamma \sin \psi \tan \phi - 2\Omega V (\tan \gamma \cos \psi \cos \phi - \sin \phi) + \frac{r \Omega^2}{\cos \gamma} \sin \psi \sin \phi \cos \phi \right]
\end{align*}
\]

Flight Simulation - Closed Loop Guidance Testing
Using equations derived from Newton’s 2nd Law, dynamics of relative motion, and Earth Centered Inertial (ECI) coordinate system
Trajectory Solver Development
Control Solution: Shuttle Entry Guidance Adaptation

Drag Curve Fit Accuracy

<table>
<thead>
<tr>
<th>Segments</th>
<th>Order</th>
<th># of stored coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 (3)</td>
<td>Irrational</td>
<td>168</td>
</tr>
<tr>
<td>7 (5)</td>
<td>Irrational</td>
<td>105</td>
</tr>
<tr>
<td>14</td>
<td>5</td>
<td>84</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>21</td>
</tr>
</tbody>
</table>

\[ D_{ref} = C_2V^{x_2} + C_1V^{x_1} + C_0V^{x_0} \]
Control Solution: Shuttle Entry Guidance Adaptation

Would Cubic Spline Interpolation work?

\[ h_s = \left( \frac{1}{P \frac{dP}{dh}} - \frac{1}{T \frac{dT}{dh}} \right)^{-1} \]
Targeting Algorithm Development
Targeting Algorithm Development

Targeting Technique 1 – Trajectory Behavior to Full Set of Aerodynamic Dispersion

Can Technique 1 find a trajectory that points toward correcting the range error?
General Conclusions
Targeting Algorithm Development

Pontryagin’s Principle in Optimal Control

Find Optimal Control $\vec{u}^*$, $\sigma^*(t)$ and $V^*(t)$

for dynamic system $\dot{x} = f(\vec{x}, \vec{u}, t)$

The optimal control satisfies several constraints including the Euler-Lagrange Equation:

$$\dot{e} = \frac{V}{g_0} + \frac{h_{geo}^2}{h^2_0} \dot{h}$$

$$\left. \frac{\partial H}{\partial u} \right|_{u=u^*} = 0$$

$$\dot{\phi} = \frac{V \cos \gamma}{r} \quad \frac{\lambda}{\lambda_\gamma} = \tan \sigma^* \cos \gamma$$

$$\dot{\gamma} = \frac{1}{V} \left[ L \cos \sigma + \cos \gamma \left( \frac{r}{r} - g \right) + 2\Omega \nu \cos \phi \sin \psi + \Omega^2 r \cos \phi (\cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi) \right]$$

$$\dot{\psi} = \frac{1}{V} \left[ \frac{L \sin \sigma}{\cos \gamma} + \frac{V^2}{r} \cos \gamma \sin \psi \tan \phi - 2\Omega V (\tan \gamma \cos \psi \cos \phi - \sin \phi) + \frac{r \Omega^2}{\cos \gamma} \sin \psi \sin \phi \cos \phi \right]$$
Targeting Algorithm Development

Targeting Technique 1

\[ \dot{\lambda}_{new} = \dot{\lambda}_{old} \pm d\lambda \]

Determines new bank angle at current time step

\[ \dot{\gamma} = \frac{1}{V} \left[ L \cos \sigma + \cos \gamma \left( \frac{V^2}{r} - g \right) + 2\Omega V \cos \phi \sin \psi + \Omega^2 r \cos \phi (\cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi) \right] \]

Calibrated for Each Dispersed Case

\[ \Phi_{new,\text{bound}} = \sqrt{1 + \left( \frac{\dot{\lambda}}{\cos \gamma} \right)^2 \left[ V \dot{\gamma}_{ref} - \cos \gamma \left( \frac{V^2}{r} - g \right) - 2\Omega V \cos \phi \sin \psi - \Omega^2 r \cos \phi (\cos \gamma \cos \phi + \sin \gamma \cos \psi \sin \phi) \right]} \]

Determines Blended Trajectory that nulls range error

\[ \Phi_{new} = \Phi_{old} \pm C_\Phi \left| \Phi_{old} - \Phi_{new,\text{bound}} \right| \]

\[ V_{initial} = V_{current} + 0.01 dV \]

\[ V_{ref,f} = V_{current} - (1 - 0.01) dV \]
Targeting Algorithm Development
Targeting Technique 1 – Design Space Interrogation

- The blending technique exhibits potential to find new bank profiles that null the range error

- The design space is constrained by control system limitations

- There is a zero range error solution for each change in $d\lambda$
Targeting Algorithm Development

Targeting Technique 1 – Trajectory Behavior to Full Set of Aerodynamic Dispersion

Why did this not follow the Expected Behavior?

The reference bank profile over-corrects with respect to the dispersion of L/D
Targeting Algorithm Development

**Targeting Technique 2**

Now that the blended function is fully defined, \( \Phi_{\text{blnd}} = Bb_2 V^2 + Bb_1 V + Bb_0 \)

The following equation can be used to solve for:

\[
\dot{\gamma}_{i,\text{new}} = \frac{1}{V_i} \left[ \frac{L}{D_{i,\text{total}}} \sqrt{\frac{1}{\left( \frac{1}{\cos \gamma_i} \right)^2 \rho_i \Phi_{\text{blnd},i} + 2 \Omega V_i \cos \phi_i \sin \psi_i + \cos \gamma_i \left( \frac{V_i^2}{r_i} - g_i \right) + \Omega^2 r_i \cos \phi_i (\cos \gamma_i \cos \phi_i + \sin \gamma_i \cos \psi_i \sin \phi_i)} \right]
\]

The FPA rate table is shifted accordingly.
Targeting Algorithm Development

*Design Space Interrogation, Results: Bank Acceleration [deg/s^2]*

\[ \alpha = 152^\circ, \text{ Undershoot} \]

\[ \alpha = 162^\circ, \text{ Overshoot} \]

\[ \alpha = 165^\circ, \text{ Overshoot} \]

\[ \alpha = 167^\circ, \text{ Undershoot} \]
Targeting Algorithm Development

Targeting Technique 1 – Targeting Implementation, 1st and 2nd Phase

1. Guess a value for $d\lambda$
2. Iterate on $dV$ using secant method to converge on a zero range error trajectory
3. If no solution is found $d\lambda$ is incremented and the iteration is repeated
4. Solution is then flown in flight simulation

Performance Metric –

Compare range of aerodynamic dispersions this algorithm can handle to the range of aerodynamic dispersions a heritage algorithm can handle.
Trajectory Solver Research Questions

Can a simplification in the equations of motion be made without loss of accuracy?

Can a simplification on flight path angle be made without loss of accuracy?
Simplified Equations of Motion Study

3DOF Rotating, Spherical Earth (3RSP)
\[
\dot{\psi} = \frac{1}{V} \left[ \frac{L \sin \sigma}{\cos \gamma} + \frac{V^2}{r} \cos \gamma \sin \psi \tan \phi - 2\Omega V (\tan \gamma \cos \psi \cos \phi - \sin \phi) + \frac{r\Omega^2}{\cos \gamma} \sin \psi \sin \phi \cos \phi \right]
\]

3DOF Non-Rotating Spherical Planet
\[
\dot{\psi} = \frac{1}{V} \left[ \frac{L \sin \sigma}{\cos \gamma} + \frac{V^2}{r} \cos \gamma \sin \psi \tan \phi \right]
\]

3DOF Non-Rotating Flat Planet
\[
\dot{\psi} = \frac{1}{V} \left[ \frac{L \sin \sigma}{\cos \gamma} \right]
\]

2DOF Longitudinal Equations (2LON)
\[
\begin{align*}
\dot{h} &= V \sin \gamma \\
\dot{s} &= V \cos \gamma \\
\dot{V} &= -D - g \sin \gamma \\
\dot{\gamma} &= \frac{1}{V} \left[ L \cos \sigma + \cos \gamma \left( \frac{V^2}{r} - g \right) \right]
\end{align*}
\]

Coriolis and Centripetal Acceleration
Apollo and Shuttle Entry Guidance
Simplified Equations of Motion Study (cont’ d)
Simplified Equations of Motion Study (cont’d)
Trajectory Solver Research Questions

Can a simplification in the EOMs be made without loss of accuracy?
Not for a skip trajectory

Can a simplification on flight path angle be made without loss of accuracy?
Control Solution: Shuttle Entry Guidance Adaptation

\[ \dot{h}_{ref} = -h_s \left[ \frac{\dot{D}_{ref}}{D_{ref}} - \frac{2\dot{V}}{V} - \frac{\dot{C}_D}{C_D} \right] \]
Control Solution: Shuttle Entry Guidance Adaptation
Control Solution: Shuttle Entry Guidance Adaptation

\[ P = \rho RT \]

\[ \frac{\dot{\rho}}{\rho} = \frac{\dot{P}}{P} - \frac{\dot{T}}{T} \]

\[ h_s = \left( \frac{1}{P \frac{dP}{dh}} - \frac{1}{T \frac{dT}{dh}} \right)^{-1} \]
Control Solution: Shuttle Entry Guidance Adaptation

Need to Resolve 1st Segment to Capture Atmospheric Non-Linearity

IDEA: Curve fit drag with Mach Number

\[ D_{ref} = \sum_{i=1}^{n} C_i M a^i \]

\[ \frac{1}{H_a} = (\frac{dP}{dh} \ast P^{-1}) - (\frac{dT}{dh} \ast T^{-1}) \]

9 Segments (~15 pnts/seg)  6th Order Drag Curve Fit
Control Solution: Shuttle Entry Guidance Adaptation

Check Altitude Acceleration Approximation
Trajectory Solver Research Questions

Can a simplification in the EOMs be made without loss of accuracy?  
**Not for a skip trajectory**

Can a simplification on flight path angle be made without loss of accuracy?
Range Prediction Sensitivity to Flight Path Angle Assumption

• Apollo and Shuttle Entry guidance formulations approximate flight path angle (FPA) to be small:
  \[ \gamma \ll 1 \text{ rad} \quad \text{and/or} \quad \dot{\gamma} \ll 1 \text{ rad/s} \]

Why does this matter?
• If predicted range does not equal the range to landing site then targeting is erroneously active
• Are model reductions in the Trajectory Module and Control Module valid based on the nominal case?
Range Prediction Sensitivity to Flight Path Angle Assumption

Case Studies:

A. Apply $\gamma << 1 \text{ rad}$ to Trajectory Module only

B. Apply $\gamma << 1 \text{ rad}$ to Controls Module only

C. Apply $\dot{\gamma} << 1 \text{ rad} / s$ to bank equation only

\[ \frac{L}{D_{v,ref}} = \frac{1}{\rho \Phi_{ref}} \left[ V_r \dot{\gamma}_{ref} - \cos \gamma \left( \frac{V_r^2}{r} - g \right) - C_{\gamma} (y) \right] \]

Trajectory Module
NPC Solves 3DOF EOMs

Controls Module
Drag and FPA Rate Reference Trajectories
Range Prediction Sensitivity to Flight Path Angle Assumption

Nominal 661.73 [nmi]

<table>
<thead>
<tr>
<th>Case</th>
<th>Total Range [nmi]</th>
<th>% Range Error</th>
<th>Termination</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>662.39</td>
<td>0.099%</td>
<td>Drag Limit</td>
</tr>
<tr>
<td>B</td>
<td>649.74</td>
<td>1.813%</td>
<td>Drag Limit</td>
</tr>
<tr>
<td>C</td>
<td>632.13</td>
<td>4.474%</td>
<td>Velocity Limit</td>
</tr>
</tbody>
</table>

Conclusion  

*FPA* approximation can be applied to the **trajectory module**, but not to the **control module**