On Hilbert-Huang Transform Based Synthesis of a Signal Contaminated by Radio Frequency Interference or Fringes

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Abstract—Norden E. Huang et al. had proposed and published the Hilbert-Huang Transform (HHT) concept correspondently in 1996, 1998 [1]. The HHT is a novel method for adaptive spectral analysis of non-linear and non-stationary signals. The HHT comprises two components: – the Huang Empirical Mode Decomposition (EMD), resulting in an adaptive data-derived basis of Intrinsic Mode functions (IMFs), and the Hilbert Spectral Analysis (HSA1) based on the Hilbert Transform for 1-dimension (1D) applied to the EMD IMF's outcome. Although paper [1] describes the HHT concept in great depth, it does not contain all needed methodology to implement the HHT computer code [2]. In 2004, Semion Kizhner and Karin Blank implemented the reference digital HHT real-time data processing system for 1D (HHT-DPS Version 1.4) [3-4]. The case for 2-Dimension (2D) (HHT2) proved to be difficult due to the computational complexity of EMD for 2D (EMD2) and absence of a suitable Hilbert Transform for 2D spectral analysis (HSA2) [7], [9]. The real-time EMD2 and HSA2 comprise the real-time HHT2. Kizhner completed the real-time EMD2 and the HSA2 reference digital implementations respectively in 2013 & 2014 [5-6]. Still, the HHT2 outcome synthesis remains an active research area. This paper presents the initial concepts and preliminary results of HHT2-based synthesis and its application to processing of signals contaminated by Radio-Frequency Interference (RFI), as well as optical systems' fringe detection and mitigation at design stage. The Soil Moisture Active Passive (SMAP mission (SMAP) carries a radiometer instrument that measures Earth soil moisture at L1 frequency (1.4 GHz polarimetric - H, V, 3rd and 4th Stokes parameters). There is abundant RFI at L1 and because soil moisture is a strategic parameter, it is important to be able to recover the RFI-contaminated measurement samples (15% of telemetry). State–of-the-art only allows RFI detection and removes RFI-contaminated measurements. The HHT-based analysis and synthesis facilitates recovery of measurements contaminated by all kinds of RFI, including jamming [7-8]. The fringes are inherent in optical systems and multi-layer complex contour expensive coatings are employed to remove the unwanted fringes. HHT2-based analysis allows test image decomposition to analyze and detect fringes, and HHT2-based synthesis of useful image.

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1. INTRODUCTION

The HHT generated considerable interest in the worldwide engineering and scientific community. Multitudes of both research and engineering papers were published reporting HHT interpretation and application results based on HHT algorithm [2], [3]. The HHT is the first direct method of spectral analysis of non-linear and non-stationary signals since Napoleonic times.

Firstly, the HHT EMD component allows decomposition of a non-linear and non-stationary signal s(t), under most general conditions, into a finite set of IMFs. The IMF is a generalization of heritage transforms' decomposition artificial basis such as polynomial terms in Taylor expansion, sine and cosine trigonometric functions basis in Fourier Transform or wavelets in the Wavelet Transform. Unlike its century old and most recent predecessors, the HHT's EMD produces an outcome *adaptive IMFs basis* from the input data itself without any a priori assumptions about the data.

Secondly, the Hilbert Transform (H) is applied to each IMF (2) yielding the complex part of the analytic signal defined by Dennis Gabor in [7].

The HHT IMFs basis is obtained by the following procedure called EMD, and EMD1 for the 1D input vector.

(a) Finding the local maxima and local minima points for the input function s(t)
(b) Predicting these (a) sets of points outside the input function domain boundaries
(c) Building the input function upper envelope Uen out of the local maxima and its predicted points
(d) Constructing lower envelope Len out of the local minima and its predicted points (like a, b, c)
(e) Computing the Median = (Uen + Len)/2
(f) Finding the first IMF candidate by computing the difference IMF1=s(t) − Median over the input function domain.
(g) Checking the IMF against the IMF criteria –
(h) An IMF satisfies two conditions:
 - that the number of extrema and the number of zero crossings must either equal or differ at most by one in the whole data set and
 - that the mean value of the envelope defined by the local maximum and the envelope defined by the local minima at any point is zero
(i) Repeating the procedure beginning at (a) by setting s(t) = IMF or sifting.

Denoting a specific IMF function as d(t), the analytic signal and its spectral characteristics are defined by equations (1)-(6): analytic signal for d(t) (1) Hilbert Transform as convolution (*) of d(t) and $1/\pi t$ (2), analytic signal representation in polar coordinates (3) allows to obtain instantaneous amplitude of the analytic signal (4), its local phase (5) and instantaneous frequency (6). Equations (1) − (6) form the HHT theoretical foundation for HHT1:

$$a(t) = d(t) + iH(d(t)) \qquad (1)$$

$$d_H(t) = d(t) * 1/\pi t \qquad (2)$$

$$a(t) = A(t)e^{i\,\theta(t)} \qquad (3)$$

$$A(a(t)) = \sqrt{d^2 + d^2{}_H} \qquad (4)$$

$$\theta(t) = \arctan(d_H / d) \qquad (5)$$

$$f(t) = (1/2\pi\,)\theta(t)^` \qquad (6)$$

The HHT was selected as the NASA invention of the year 2003 and its digital implementation, the HHT-DPS V1.4, received the Regional Excellence in Technology Transfer Award in 2006 from the Federal Laboratory Consortium Mid-Atlantic Region. The HHT-DPS implemented the HHT for 1D and became a real-time reference system with which all other implementations and results could be compared. It was maintained at the Goddard Space Flight Center (GSFC) where it was commercialized and licensed to DynaDx Corporation in 2009, leading to its development into a commercial product. Within the GSFC Internal Research and Development Program (IRAD), the HHT was under continuous research which resulted in development of the real-time EMD2 and HSA2 prototype technologies in 2013 and 2014 correspondingly. The complexity of HHT2 development required abstraction from applications to some degree. Presently, in view of the HHT2 completion, we are turning our attention to the HHT2 applications, namely the synthesis based on HHT2. Syneren Technologies Corporation further implemented the EMD2 as a reference real-time system commercial product and its applications are being explored under the 2013-2014 Space Act Agreement between NASA/GSFC and Syneren Technologies Corporation. These applications include hyper-spectral image analysis, fringe detection and mitigation and RFI noise detection and science data recovery using HHT2-based synthesis.

Following in Figures 1-2 is the presentation of HHT1 EMD1 application to the analysis of length-of-day file LOD78.csv comprising 3250 data points that were collected in 1978 by the Jet Propulsion Laboratory (JPL).
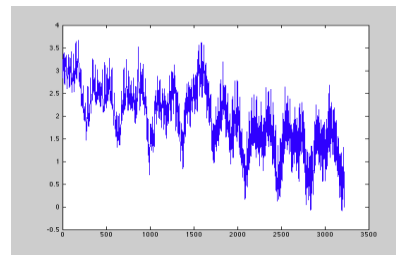

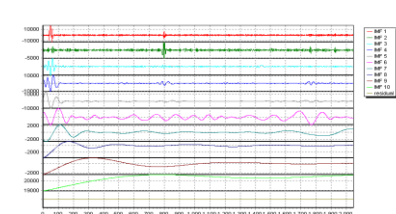
Figure 1. Graph of input file LOD78.csv

Figure 2. HHT1 EMD1 IMFs graph for length-

of-day variations in microseconds LOD78.csv

The 7 IMFs and 1 residual were obtained running HHT-DPS version 1.4 on a Lenovo PC under Windows 7 OS and EMD1 configuration with such parameters: Sifting Criteria 3, Maximum IMF 100, Maximum # Sifts 50,000, Sift Type - Extrema Sift, Endpoint Selection – Pattern Prediction.

2. REFERENCE REAL-TIME EMD2

The HHT2, similar to HHT1, comprises two modules the EMD2 and HSA2:

$$HHT2 = \{EMD2, HSA2\}$$

The problem of HHT2 digital implementation is two-fold:

- EMD2 computational complexity has a formidable order of $O(N^4)$, which for an NxN image, where N=2048, is $O(10^{14})$. It is complex because it involves constructing image surface envelopes, envelope prediction at $4(N - 1)$ boundary points, and envelope smoothing.

- HSA2 requires a suitable Hilbert Transform for 2D, which doesn't exist in a form that allows reconstructing the 2D analytic signal.

The computational complexity of EMD2 was resolved by developing its algorithm in a way that allows parallelism in execution. The input N x N image is then broken down into 64 smaller sub-arrays (Figure 2) and run on an NVIDIA GTX 690 Card. The EMD2 run time for a 2K x 2K image with the number of Bidimensional Intrinsic Mode Functions (BIMF) set to 10, is a fraction of a second. The run time also depends on the number of extrema points in the input image. The performance evaluation was observed using image of raindrops on a lake (Figure 3), and the first BIMF (BIMF1) is depicted in Figure 4.
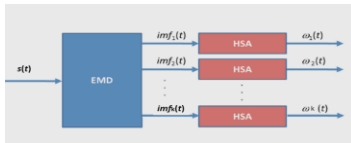

Figure 2. EMD2 parallelism implemented on the NVIDIA GTX 690 PC/Win7 Card

The NVIDIA card code was developed in C++ and the 640 x 640 images in Figure 4 were obtained by Syneren Technologies Corporation with a run time of 0.1 seconds.
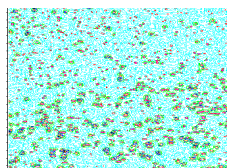

Figure 3. Lake raindrops.   **Figure 4. The BIMF1.**

3. REFERENCE REAL-TIME HSA2

Spacecraft advanced passive microwave radiometry makes use of natural thermal emissions to remotely sense Earth phenomena of interest to science in the L-band (1.4GHz surface moisture, for example). Here, a terrestrial signal-to-space suffers less attenuation by the intervening canopy or atmosphere. Unfortunately, relative insensitivity of the L-band region to atmospheric effects also makes it an extremely attractive spectral range for wireless radar & cell communications that are causing RFI with the spaceflight science radiometer instrument's terrestrial phenomenon signal of interest. State-of-the-art technology detects an RFI event and deletes the entire contaminated measurement point. Excision of just the RFI component from the composite signal is presently a tall challenge. The three goals described in this paper related to research and development of the HHT2 HSA2 components are:

1) Develop the engineering tool EMD2 for a direct decomposition of the RFI-contaminated signal into 2D (Bi-dimensional) IMFs (BIMFs) in real-time and use IMFs and BIMFs to recover RFI-contaminated science data points

2) Research and develop the Hilbert-Huang Transform for 2D (HHT2) last component – the Hilbert Spectral Analysis for 2D (HSA2) and complete the HHT2 theory. The HSA2 is required for reconstructing the 2D analytic signal (monogenic signal or *monogenic*) from which the 2D image instantaneous amplitude and phase are obtained.

3) Verify the new technology tool prototype by a controlled drone experiment at Wallops Flight Facility, comprising a miniature radiometer, a controlled RFI source, and science data processing on the ground.

We constructed a bi-dimensional By-orthant Hilbert Transform (BHT or Rietz Transform) digital implementation that allows construction of the 2D analytic signal a(x, t) for the input image f(x, y). This was accomplished by using a combination of directed Hilbert Transforms along the OX and OY axis or Hx and Hy, as well as the total Hilbert Transform Ht =HxHy of f(x, y):

$$BHT(f(x, y)) = Hx + Hy(f(x, y) - Ht(Ht)) \quad (7)$$

Then

$$a(x, t) = f(x, y) + i*BHT(f(x, y)) \quad (8)$$

The instantaneous amplitude, phase, and frequency are derived for 2D as follows:

$$a(x, t) = A(x, t)e^{i}\theta(x, t) \quad (9)$$

$$A(a(x, t)) = \sqrt{d^2 - 2dxd_H + d^2}_H \qquad (10)$$

$$\theta(x, t) = \arctan(d_H / d) \qquad (11)$$

$$f(v) = (1/2\pi)\theta(x,t,v)' \qquad (12)$$

This completes the research and development of the HHT2, where:

$$HHT2 = \{EMD2, HSA2\} \qquad (13)$$

4. HERITAGE DECOMPOSITIONS AND ANALYSIS

4.1 Taylor Series Functional Analysis

In mathematics, a Taylor series is a representation of a function as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point. A one-dimensional Taylor series is an expansion of a real function, f(x), about a point, x=a, is given by:

$$f(x) = f(a) + f'(a)(x-a) + f''(a)(x-a)^2/2! + \ldots + f^{(n)}(x-a)^n/n! \\ +\ldots \qquad (14)$$

The analysis of f(x) Taylor series and synthesis of its approximation to f(x) is done in the polynomial terms domain and is straight forward.

4.2 FFT1 and FFT2 Spectral Analysis

The digital signal s(t) fft1 outcome-based analysis is depicted by the following sequence:

s(t) -> fft1(s(t)) -> frequency spectrum vector -> spectrum vector processing (filtering) -> g(t) (15)

4.3 HHT1 and HHT2 Analysis

The digital signal s(t) HHT1 outcome-based analysis is depicted by the following sequence:

s(t) -> $\qquad\qquad$ (16)
$\quad$ EMD1 $\{IMF_1(t), IMF_2(t), \ldots, IMF_k(t)\}$ ->
$\quad$ HSA1 $\{a(IMF_1(t)), a(IMF_2(t)), \ldots, a(IMF_k(t))\}$ ->
$\qquad A_1(t_i), A_2(t_i), \ldots, A_k(t_i)$ ->
$\qquad f_1(t_i), f_2(t_i), \ldots, f_k(t_i)$

where $f_i(t)$ is the instantaneous spectrogram for $IMF_i(t)$ and $A_i(t)$ is the instantaneous amplitude of the instantaneous frequency component $f_i(t)$ for $1<=i<=k$.

The analysis, performed in HHT1 frequency domain, is similar to the analysis in frequency domain performed for heritage transforms, such as the fft1 depicted above.

It is important to notice that HHT1 instantaneous characteristics of a signal are all bounded to the time t_i vertical slice and its synthesis is directly based on this time slice t_i.

5. HERITAGE FFT BASED SYNTHESIS

The fft based synthesis comprises processing of the frequency spectrum vector resulting in frequency synthesized vector g(t). The synthesis of the approximation to the input signal s(t) is then obtained by applying the inverse Fourier Transform or ifft(g(t)).

5.1 FFT1 Outcome Synthesis

Given a time variable function s(t), the FFT1 is used over a sequence of short time periods, Δt, to construct two-dimensional spectro-temporal "scene". This image is then processed using image-based filtering and the approximation to s(t) is synthesized using IFFT1.

The case for 2D is described in the following Section 5.2.

5.2 FFT2 Outcome Synthesis

An example of an fft2-based synthesis is presented in [9]. The concept of the two-dimensional spectro-temporal modulation filtering of the auditory model is implemented for the fft spectrogram (2-D signal analyses). A 2-D invert fft (fft2) is then used to synthesize the denoised audio signal.

6. HHT2 EMD2 AND HSA2 OUTCOME SYNTHESIS

The HHT based synthesis is anchored in the fact that all spectral characteristics of the input signal are tied to the unique time slice t, and after processing of the instantaneous spectrum, the synthesized approximation to the original input signal is obtained by direct selection of corresponding to t_i instantaneous frequency components such as $A_l(t_i)$ for $1<=l<=k$, $1<=i<n$.

The HHT based synthesis is two-fold:

- EMD IMF qualitative level and

- HSA spectrum quantitative level.

The HHT-based synthesis concepts and terminology are as follows:

- The HHT-based synthesis is time-slice t_i oriented

- The HHT-based synthesis is direct and accomplished by

- moving up from frequency component of interest $g_l(t_i)$ along the time-slice t_i to the analytic signal

- $a_l(t_i)$) amplitude and

- selecting the $IMF_l(t)$ the $IMF_l(t_i)$ datum from which this frequency entity of interest originated from.

Once these HHT-based synthesis' concepts and the terminology are established, we can move to the next step

of HHT applications, such as fringe phenomena analysis based on HHT2 analysis, and mitigation based on HHT2 synthesis.

6.1 EMD2 Outcome Based Synthesis
The EMD based synthesis is performed in the IMFs time domain similar to that of Taylor series extension. The IMF components of qualitative interest are selected to represent components of the synthesized signal approximation

$$s(t) \rightarrow \qquad (17)$$
$$EMD1 \ \{IMF_1(t), IMF_2(t), \ldots, IMF_k(t)\} \rightarrow$$
$$\{IMF_{l1}(t), IMFl_{l2}(t), \ldots, IMF_{lm}(t)\}$$

6.2 HSA2 Spectrum Based Synthesis
The development of the real-time EMD2 and the HSA2 required a great deal of abstraction from the pressures of a specific application. With the completion of the real-time HHT2 implementation, it is time to turn to applications of HHT2 to real problems of spaceflight remote sensing data analysis and synthesis.

7. HHT BASED APPLICATIONS
Our interest is in applications related to remote sensing data processing and recognition of its man-made signal components, such as RFI signals. There are a few emerging methodologies in discriminating man-made signals – information theory based negative entropy, statistical kurtosis. Applications of HHT1 and HHT2-based analysis and synthesis to RFI-contaminated signal reconstruction is an important application for advanced radiometer remote sensing missions. The outline of the HHT-based synthesis for such an application is presented below in Section 7.1 and 7.2.

7.1 HHT2 ANALYSIS OF RFI CONTAMINATED SIGNAL
The HHT2-based RFI detection and mitigation is as follows:

(1) Perform SMAP RFI Detection Algorithms on a SMAP measurements file f1 and on-board statistically derived from oversampling products of size N (fm11, fm12, fm13, fm14) generating a vector RFI detect of size N with 0s and 1s, where indices to 1s point to RFI contaminated samples. This is state-of-the-art in RFI detection and RF_ samples removal. Perform the HHT2 EMD1/EMD2 on SMAP measurements and products files of size N
{f1, fm11, fm12, fm13, fm14} applying SMAP RFI algorithms to IMFs sets:
{IMF$_i$(f1),IMF$_i$(fm11),IMF$_i$(fm12),IMF$_i$(fm13), IMFi(fm4)} for i=1 to k.
Synthesize the i-indices into a single RFI-index file and compare with above state-of-the-art.

(2) RFI event persists in a few IMFs across coincidental maxima spatial locations. Determine, for each IMF vertical cross-cuts, the IMFs with coincidental maxima points. Combine them and subtract the total from input signal to recover science measurements.

(3) Apply HSA2 to each BIMF of drone imaging radiometer and synthesize the RFI spectra (Figure 5).

(4) Apply the Principal Component Analysis (PCA) to the k x n IMFs to synthesize the SMAP measurements contaminated by RFI.

7.2 HHT2-based synthesis of RFI decontaminated signal
The recovery of RFI-contaminated data points (synthesis) is accomplished by applying steps (1)-(4) above and the results are depicted below in Figures 5 and 6.
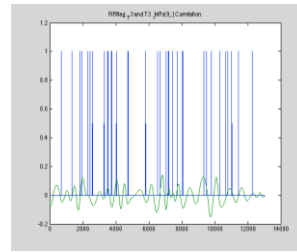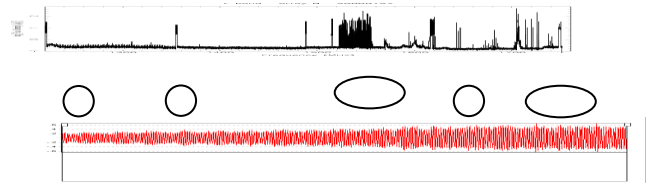

Figure 5. RFI events detected by HHT

Figure 6. RFI spectrum with RFI-events removed and HHT spectrum with RFI-events recovered data.

8. SYNTHESIS OF FRINGE DECONTAMINATED IMAGES
In physics, interference is a *phenomenon* in which two waves superimpose to form a complex wave, and *fringes* are observed wherever the two or more waves overlap. The *fringe* analysis has a wide range of applications in physical and engineering *measurement* based on fringes by design (interferometers) and analyses of unwanted fringes aims at their detection and mitigation. Fringe analysis' state-of-the-art processing is using linearity-based Fourier Transform that is inadequate due to fringe phenomenon non-linear and non-stationary nature. The breakthrough technology for fringe analysis presented in this paper is based on the HHT2, allowing fringe detection and mitigation and phase untangling in real-time by firmware assisted HHT.
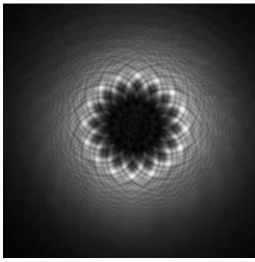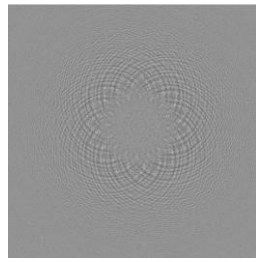
Figure 7. EMD2 input test image   **Figure 8. Fringe BIMF1**

CONCLUSIONS

We have described the HHT2 reference digital implementation in real-time and initial solution methodology for HHT2-based monogenic signal analysis synthesis. The HHT2 is a reference real-time data processing system. It processes a 2K x 2K image under a second for number of BIMFs larger than 10. It is a reference system in respect that it provides the HHT2 computer code and its hardware platform specification so that an application can be run on an equivalent system and the result compared to this reference system maintained by Syneren Technologies Corporation under a Space Act Agreement with NASA. We have developed the EMD2 algorithm that allows parallelism and implemented HSA2 the Rietz Transform. We applied the HH2-based analysis to RFI detection and RFI-contaminated science data synthesis. We demonstrated the HHT2 applicability to fringe patterns decomposition analysis and image synthesis.

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BIOGRAPHY

Semion Kizhner is an aerospace engineer with the National Aeronautics and Space Administration at the Goddard Space Flight Center. He proposed the development of the HHT-DPS1-D and had been leading the HHT-DPS development team. He also proposed in 2013-2014 and, as Principal Investigator, developed the HHT2 system for 2D. Recently, he participated in the evaluation of the NASA Advanced Space Technology proposals. Also, he was the instrument electrical and electronics subsystem lead for the DESTINY/Joint Dark Energy and WFIRST concept studies and the Mars-2020 PING instrument proposal. He is currently the lead for the SMAP radiometer science data processing code development, scheduled for launch

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Meg Vootukuru is President and Chief Executive of Syneren Technologies Co. She obtained a Bachelors degree in Electronics and Communications from Osmania University in Hyderabad, India. She later earned a Master's degree in Electrical and Computer Engineering from the University of Cincinnati. Her interests are in noise reduction and image processing using HHT2.

Alessandro (Alex) Coletti serves as senior scientist at the Syneren Technologies Corporation, where he advises and coordinates the company's R&D activities in areas such as machine vision, new algorithm development, and decision support. He received a doctorate degree in Astrophysics from the University of Rome, Italy, and spent the early part of his career developing space flight instruments and contributing to scientific space flight missions planning. While in Italy, he contributed to space missions for the European Space Agency. While at Georgia Institute of Technology he developed and tested remote sensing instruments that flew on-board NASA airplanes in occasion of several experimental missions. In the second part of his career, Alex Coletti served as research scientist for IT companies working on projects for NASA and