Rho-Isp Revisited and Basic Stage Mass Estimating for Launch Vehicle Conceptual Sizing Studies

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A single metric for judging between two candidate propellant combinations for a given application is sought. By using the ideal rocket equation, the essential link between propellant density and specific impulse as the two primary performance drivers can be demonstrated. This is most clearly illustrated for the case of a volume-limited first stage.

$$\Delta V = V_e * \ln\left(\frac{m_i}{m_f}\right)$$

Where ΔV is the change in velocity required of the stage, and V_e is the propellant exhaust exit velocity, equal to the gravitational constant times the Specific Impulse, Isp. The initial mass, m_i, and final mass, m_f, are not always the most useful values, so the equation can be rewritten any number of ways:

$$\Delta V = V_e \ln\left(\frac{m_i}{m_i - m_p}\right) = -V_e \ln\left(1 - \frac{m_p}{m_i}\right) = V_e \ln\left(\frac{m_f + m_p}{m_f}\right) = V_e \ln\left(1 + \frac{m_p}{m_f}\right)$$

A relationship is sought that allows identifying a reference stage, and answering the question "what different stage can deliver the same ΔV?" For first stages and boosters, the latter formulation, containing the propellant mass, m_p, and retaining m_f, is most useful. A more specific question to pose might be, "for the new candidate propellant, can a stage be built in the same volume as the baseline stage?" If the answer is "no," then that would tend to question any claim of a propellant being a "drop-in replacement."

The assumption of volume-limited is not solely for cases in which there is a physical stop to the stage size that can be realized, but could also take into account the desire to maintain the same volume as the reference stage for cost reasons. Perhaps a larger stage could be built, but is likely more costly, and thus less desirable. In addition, the reference stage could be either a real existing stage looking to be upgraded, or a baseline design for a paper study. This assumes that m_f is constant for the evaluation, by assuming that two stages of the same volume have the same mass, thus leaving the same amount of mass available for the stage's payload. Clearly this assumption is not valid across propulsion types, from solids to liquids. Below it is evaluated in more depth and shown adequate within all liquid combinations evaluated except LOX/Hydrogen. The treatment of this for a first stage is important, because for upper stages, if the total stage weight changes, a different ΔV will be required. That result will be looked at later in light of constant initial mass stages.

Defining

$R = \frac{m_f + m_p}{m_f}$ $\qquad r_\rho = \frac{\rho_2}{\rho_0}$

Where ρ is propellant density, case 0 is the reference and case 2 is the candidate replacement,

$$\frac{\Delta V_2}{\Delta V_0} = \frac{V_{e2}}{V_{e0}} \frac{\ln(R_2)}{\ln(R_0)}$$

R_0 is set by the reference vehicle, and along with the ΔV requirement represents the mission, and R_2 can be derived from the known densities.

$m_{p2} = m_{p0} r_\rho \qquad R_2 = 1 + (R_0 - 1) r_\rho$

$$\frac{\Delta V_2}{\Delta V_0} = \frac{V_{e2}}{V_{e0}} \frac{\ln\left(1 + (R_0 - 1) r_\rho\right)}{\ln(R_0)}$$

This was found by Mellish and Gibb[1], and can be used directly by setting the delta velocity ratio equal to 1 and solving for minimum required r_ρ given a change in Isp or *vice versa*.

Gordon[2] identified the usefulness of the following expressions, such that a single performance factor f_p is computed by density, an exponent and Isp, where the maximum f_p identifies the highest performing propellant for the mission.

$$f_p = \rho^n Isp$$

From the above analysis, n is computed by partial differentiation of the above ΔV equation, and ends up itself being a function of both the mission and the change density ratio being evaluated:

$$n = -\frac{\left.\frac{d\Delta V}{d\ln\rho}\right]_{Isp}}{\left.\frac{d\Delta V}{d\ln Isp}\right]_{\rho}} = \frac{r_\rho (R - 1)}{\left(r_\rho (R - 1) + 1\right) \ln\left(r_\rho (R - 1) + 1\right)}$$

It can be approximated based on solely the mission parameter R for small density ratios, at $r_\rho = 1$. This was the only solution examined by Gordon[2].

$$n = \frac{(R - 1)}{\mathrm{R} \cdot \ln(R)}$$

Now the behavior of the exponent n can be examined as a function of the relevant mission and propellant parameters. First, for density ratios like those experienced within varying solid propellant composition with typically used ingredients. The resulting n is plotted against two parameters. R, and $R_{mp} = 1 - 1/R$, which is the ratio of stage propellant mass to total vehicle mass.

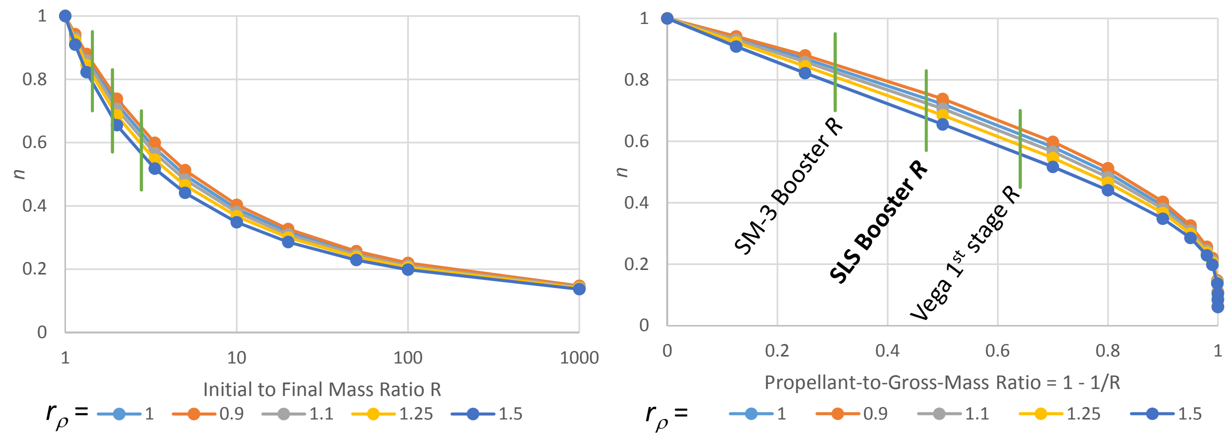
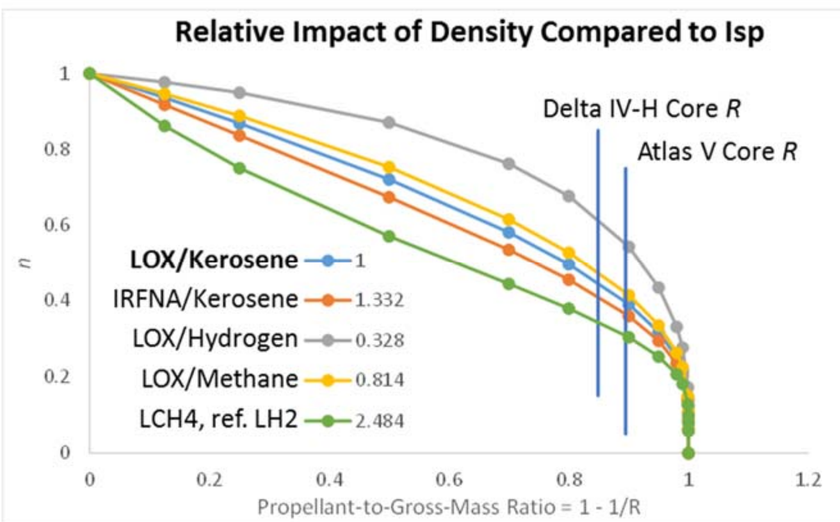


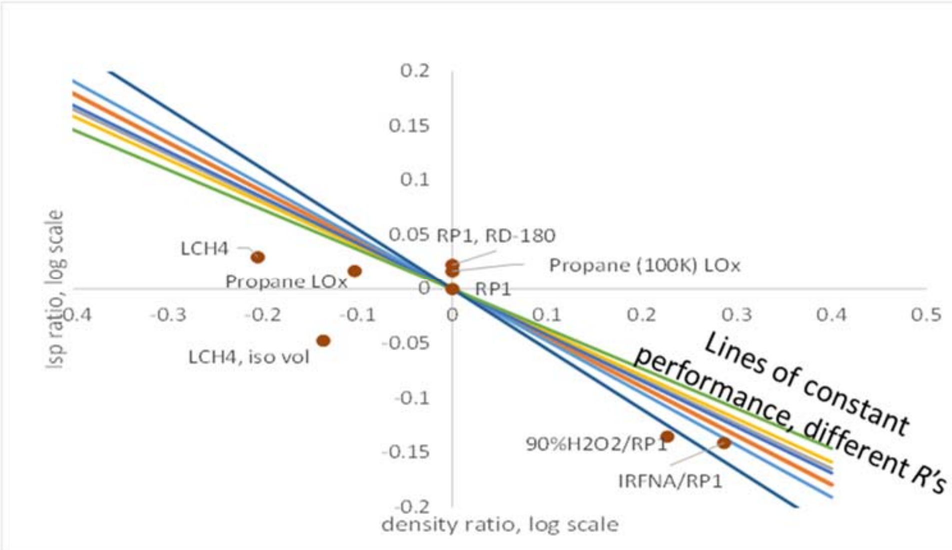
Figure 1: Density Exponent n Demonstrates the Effect of Density Relative to Isp for Different Conditions

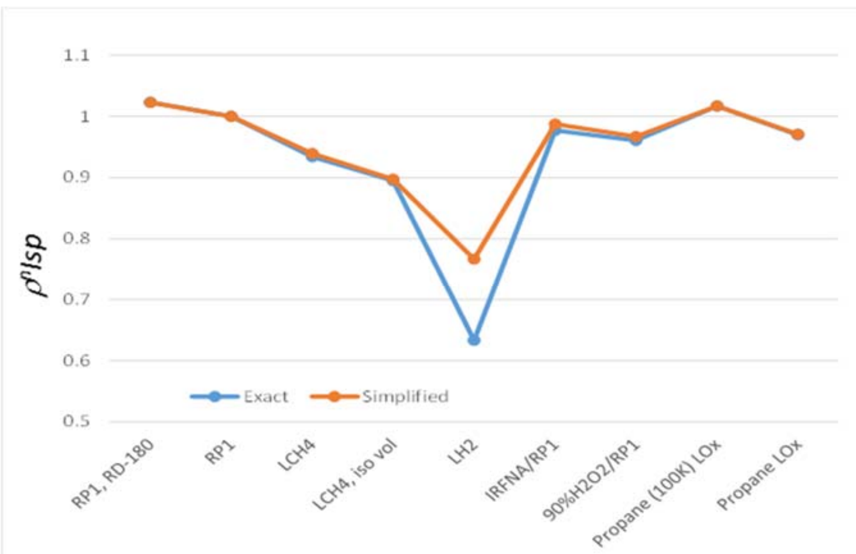
Note that the primary driver is the mission, how much of the reference stage is propellant. Of secondary importance is how different are the densities of the two propellants. Note that the smaller the stage relative to the vehicle, the more important density is, approaching the same importance as Isp on a percentage basis. On the plot are shown three example solid motor systems for reference[3,4,5,6]. An example of how higher density propellants fare in one of these will be shown in the final paper.

For liquid bi-propellants, due to the broader density range, the exact equation is essential for capturing the performance. The figures below show this for the different density ratios. Note that here the relevant R_{mp} zones are much higher, exemplified by the Delta IV-H and Atlas V lines[7].



This can be looked at plotting Isp vs. density relative to the reference for each propellant combo, where a higher-performing propellant combination is one that is above and right of the lines of constant performance. So, with LOx/RP1 as the baseline, it is seen that that liquid methane, LCH4, though higher Isp, loses performance due to its lower density. Conversely, the lower Isp peroxide and IRFNA oxidizers nearly make up for it in density for these cases. In the next plot, the performance measure is plotted directly. There the significant departure from the simplified equation is seen with LOx/LH2.





Departure from "same final mass" assumption

Now a revisit is warranted of the constant final mass assumption. Given the constant stage volume, the propellant mass fraction of the candidate stage can be computed from the reference stage and density ratio, where λ is the propellant mass fraction of the stage:

$$\frac{1}{\lambda_2} = 1 + \frac{1}{r_\rho}\left(\frac{1}{\lambda_0} - 1\right)$$

A good test of the assumptions is to use this to predict the Delta IV Common Booster Core (CBC) based on the Atlas V core stage. Because these stages have similar application, thrust-to-weight, and development era, one would expect the equation above to predict accurately, if indeed the "same volume means same final mass" assumption is valid across that propellant range. Even though they are not the same volume, since they are large enough for scale to not matter, the non-dimensional mass fraction should still work. However, starting with Atlas V's 0.93, the equation predicts a Delta IV CBC mass fraction of 0.82, while its mass fraction is actually published as 0.88[7].

Next order mass model

This suggests that a stage inert mass model is required that depends on more of the relevant parameters. A more complete mass model should account for the individual densities and O/F ratios of the propellants, and also thrust-dependent aspects of the stage mass. The thrust-dependent structure includes the engines and non-wetted structure associated with them, and also the fuel tank, because the loads to accelerate the heavy load of oxidizer above must be transmitted through the walls of the fuel tank.

The strategy here is to identify constants that describe the Atlas V core and then use scaling equations to predict other stages based on propellant density ratios, R, and changes in engine thrust-to-weight and vehicle lift-off thrust-to-weight. The possibilities were adjusted until the Delta IV CBC was predicted with the most parsimonious model, containing two free factors. The derivation will be described in more detail for the final paper, but is summarized below:

Defining f_i as the ratio of component or subsystem inert weight to total propellant weight, the key settings for Atlas V are:

$f'_{i,tank,0}$ = 0.02, for the composite fuel and oxidizer tank and volume-related systems weight.

$f_{i,E\&S,0}$ = 0.0529, for the engine and structure weight.

$f'_{i,tank,0}$ is decomposed into $f'_{i,ft,0}$ for the fuel tank and $f'_{i,ot,0}$ for the oxidizer tank, here apportioning them by volume as follows, based on the oxidizer to fuel mass ratio O/F:

$$f'_{i,tank,0} = f'_{i,ot,0} \frac{\left.\frac{O}{F}\right|_0}{\left.\frac{O}{F}\right|_0 + 1} - f'_{i,ft,0} \frac{1}{\left.\frac{O}{F}\right|_0 + 1}$$

This method assumes the engine and structure weight, as thrust-dependent weight, is a constant multiple, r_F, of the engine weight, and proportional to the launch loads, represented by vehicle thrust-to-weight at launch, Ψ. This could be artificially skewed by using engine thrust-to-weights much different from historical nominal values, but having the dependence on engine weight represents real effects of main propulsion system components masses depending on mass or volume flow rate. The engine thrust-to-weight used here should be representative of the propellant class. The above settings provide the r_F with the associated Atlas V Ψ_0 and engine thrust-to-weight, FW_{Eng}, as follows:

r_F = 2.735

Ψ_0 = 1.28, using vacuum thrust to keep things simple

FW_{Eng} = 78

Then the calculation of engine and structure inert fraction is:

$$f_{i,E\&S} = \frac{m_{E\&S}}{m_p} = r_f \frac{\Psi}{FW_{Eng}} \left(\frac{1}{R_{mp}}\right) = r_f \frac{\Psi}{FW_{Eng}} \left(1 - \frac{1}{r_\rho (R_0 - 1)}\right)$$

The fuel tank inert fraction is given below. This includes not only volume-dependent mass but also loads-dependent mass, as the fuel tank has to support the weight of the oxidizer and stage payload through the launch accelerations.

$$f_{i,ft} = f'_{i,ft,0} \frac{r_\rho}{r_{\rho f}} \frac{1}{\frac{O}{F}+1} \frac{\Psi}{\Psi_0} \frac{\frac{1}{R_{mp}} - \frac{1}{\frac{O}{F}+1}}{\frac{1}{R_{mp,0}} - \frac{1}{\left.\frac{O}{F}\right|_0 + 1}}$$

Where $r_{\rho f}$ is the ratio of fuel density to reference fuel density. Finally, the oxidizer tank mass is assumed to be based solely on the volume of oxidizer, as follows:

$$f_{i,ot} = f'_{i,ot,0} \frac{1}{r_{\rho ox}} \frac{\frac{O}{F}}{\frac{O}{F}+1}$$

Where $r_{\rho ox}$ is the ratio of oxidizer density to reference oxidizer density.

The reference ox and fuel fractions are given as f'_i because they are defined as individual tank mass to individual oxidizer of fuel mass, respectively, while the calculated f_i are individual tank mass to total propellant mass, and are simply summed. The total inert fraction and propellant mass fraction are then

$$f_{i,total} = f_{i,ft} + f_{i,ot} + f_{i,E\&S}$$

$$\lambda = \frac{1}{1 + f_{i,total}}$$

The result is that for any set of stage construction assumptions, i.e., a reference stage, the $f_{i,0}$ can be estimated according to the level of information available. Then comparable other stages mass fractions can be estimated. For instance, one could set the constants according to the existing LOx/LH2 Centaur, and estimate a LOx/LCH4, 1.3 thrust-to-weight "Centaur."

The final paper will compare propellants with these models for boost and modified single-stage-to-orbit applications. It will also demonstrate mass fraction models as a function of stage propellant mass for stages small enough to show significant mass fraction reduction. This is based on the empirical solid rocket motor data, with the scaling law extended to liquids and compared to the less extensive database of small liquid stages. The combination of these models then allows the exploration small launch vehicle design spaces, and the beginnings of design-to-lowest-cost optimization.

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