Rho-Isp Revisited and Basic Stage Mass Estimating for Launch Vehicle Conceptual Sizing Studies

Timothy P. Kibbey[1]
Jacobs, ESSSA Group, Huntsville, AL, 35806

The ideal rocket equation is manipulated to demonstrate the essential link between propellant density and specific impulse as the two primary stage performance drivers for a launch vehicle. This is illustrated by examining volume-limited stages such as first stages and boosters. This proves to be a good approximation for first-order or Phase A vehicle design studies for solid rocket motors and for liquid stages, except when comparing to hydrogen-fueled stages. A next-order mass model is developed that is able to model the mass differences between hydrogen-fueled and other stages. Propellants considered range in density from liquid methane to inhibited red fuming nitric acid. Calculated comparisons are shown for solid rocket boosters, liquid first stages, liquid upper stages, and a balloon-deployed single-stage-to-orbit concept. The derived relationships are ripe for inclusion in a multi-stage design space exploration and optimization algorithm, as well as for single-parameter comparisons such as those shown herein.

Nomenclature

Symbols:

f_i	Inert Fraction
f_p	Performance Factor
f_v	Tank mass per unit Volume contained
FW_{Eng}	Engine Thrust-to-Weight ratio
g	Gravitational acceleration constant
Isp	Specific Impulse
L	Loads
m	Mass
m_f	Final vehicle mass after stage burn
m_i	Initial vehicle mass at stage burn
n	Exponent on density
O/F	Oxidizer-to-Fuel mass ratio
R	Ratio of initial mass to final mass
R_{mp}	Ratio of propellant to initial mass
r_F	Engine weight multiplication factor
r_ρ	Ratio of propellant density to reference
V_e	Exit velocity

Greek:

ΔV	Change in velocity
λ	Propellant mass fraction
ρ	Density
Ψ	Overall Thrust-to-Weight ratio

Abbreviations:

90%H2O2	Hydrogen Peroxide, 90% concentration
CBC	Common Booster Core (Delta IV)
IRFNA	Inhibited Red Fuming Nitric Acid
LCH4	Liquid Methane
LH2	Liquid Hydrogen
LOX	Liquid Oxygen
LWT	Space Shuttle Lightweight Tank
Pro100K	Liquid Propane, temperature of 100 K
RP, RP1	Kerosene, Rocket Grade
SLWT	Space Shuttle Super Lightweight Tank
SSTO	Single Stage to Orbit

Subscripts:

0	Reference
fu	Fuel
Ox	Oxidizer
p	Propellant
tank	Pertaining to both tanks
ft	Pertaining to the fuel tank
ot	Pertaining to the oxidizer tank
E&S	Pertaining to the engine and structures

Superscripts:

'	Referenced to individual oxidizer or fuel mass, rather than total propellant mass

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[1] Senior Engineer, Propulsion Design, Analysis and Test, MSFC ER51

I. Introduction

A single metric for judging between two candidate propellant combinations for a given rocket propulsion application is sought. By using the ideal rocket equation (Eq. 1), the essential link between propellant density and specific impulse as the two primary performance drivers can be demonstrated. This will be illustrated for the case of a volume-limited first stage, and for a mass-limited upper stage.

$$\Delta V = V_e * \ln\left(\frac{m_i}{m_f}\right) \tag{1}$$

Here ΔV is the change in velocity required of the stage, and V_e is the propellant exhaust exit velocity, equal to the product of the gravitational constant and the Specific Impulse, Isp. The initial mass, m_i, and final mass, m_f, are not always the most useful values, so the equation can be rewritten a number of ways, as shown in Equation (2). A relationship is sought that allows identifying a reference stage and answering the question, "what different stage can deliver the same ΔV?"

$$\Delta V = V_e \ln\left(\frac{m_i}{m_i-m_p}\right) = -V_e \ln\left(1 - \frac{m_p}{m_i}\right) = V_e \ln\left(\frac{m_f+m_p}{m_f}\right) = V_e \ln\left(1 + \frac{m_p}{m_f}\right) \tag{2}$$

The latter formulation, containing the propellant mass, m_p, and retaining m_f, is most useful or first stages and boosters. A more specific question to pose might be, "for the new candidate propellant, can a stage be built in the same volume as the baseline stage?" If the answer is "no," then that would tend to question any implication of a propellant being a "drop-in replacement."

II. Volume-Limited Treatment and $\rho^n \cdot Isp$

The assumption of volume-limited is not solely for scenarios in which there is a physical limit to the stage size that can be realized, but could also take into account the desire to maintain the same volume as the reference stage for cost reasons. Perhaps a larger stage could be built, but is likely more costly, and thus less desirable. The reference stage could be either a real existing stage looking to be upgraded, or a baseline design for a paper study. This evaluation assumes that m_f is constant, by assuming that two stages of the same volume have the same dry mass, thus leaving the same amount of mass available for the payload atop the stage. Clearly this assumption is not valid across propulsion types, from solids to liquids. Below it is evaluated in more depth and shown adequate within all liquid combinations evaluated except LOX/Hydrogen. That this treatment is for a first stage is important, because for upper stages, if the total stage weight changes, a different ΔV will be required. That result will be looked at later in light of constant initial mass stages.

A. Derivation
Defining

$$R = \frac{m_f+m_p}{m_f} \qquad r_\rho = \frac{\rho}{\rho_0} \tag{3}$$

where ρ is propellant density, case 0 is the reference and the candidate replacement is unsubscripted,

$$\frac{\Delta V}{\Delta V_0} = \frac{V_e}{V_{e0}}\frac{\ln(R)}{\ln(R_0)}. \tag{4}$$

R_0, set by the reference vehicle, along with the ΔV requirement represents the mission, and R can be derived from the known densities.

$$m_p = m_{p0}r_\rho \qquad R = 1 + (R_0 - 1)r_\rho \tag{5}$$

$$\frac{\Delta V}{\Delta V_0} = \frac{V_e}{V_{e0}}\frac{\ln\left(1+(R_0-1)r_\rho\right)}{\ln(R_0)} \tag{6}$$

Equation (6) was found by Mellish and Gibb[1], and can be used directly by setting the ΔV ratio equal to 1 and solving for minimum required r_ρ given a change in Isp or *vice versa*.

Gordon[2] identified the usefulness of the following expressions, such that a single performance factor f_p is computed by density, an exponent and Isp, where the maximum f_p identifies the highest performing propellant for the mission. Herein, it is represented relative to the reference vehicle, as in Equation (7).

$$f_p = \frac{\rho^n Isp}{\rho_0^n Isp_0} \tag{7}$$

From the above analysis, n is computed by partial differentiation of the Equation (6), and ends up itself being a function of both the mission and the density ratio being evaluated (Eq. 8). It can be approximated based on solely the mission parameter R_0 for small density ratios, at $r_\rho = 1$, as shown in Equation (9). This was the only solution examined by Gordon[2].

$$n = -\frac{\frac{d\Delta V}{d(ln\rho)}\big]_{Isp}}{\frac{d\Delta V}{d(lnIsp)}\big]_\rho} = -\frac{ln\left[\frac{ln(R_0)}{ln(r_\rho(R_0-1)+1)}\right]}{ln(r_\rho)} \tag{8}$$

$$n = \frac{(R_0-1)}{R_0 \cdot ln(R_0)} \tag{9}$$

Now the behavior of the exponent n can be examined as a function of the relevant mission and propellant parameters: first, for density ratios like those experienced within varying solid propellant composition with typically used ingredients. The resulting n is plotted against two parameters in Figure 1, R and $R_{mp} = 1 - 1/R$, which is the ratio of stage propellant mass to total vehicle mass.
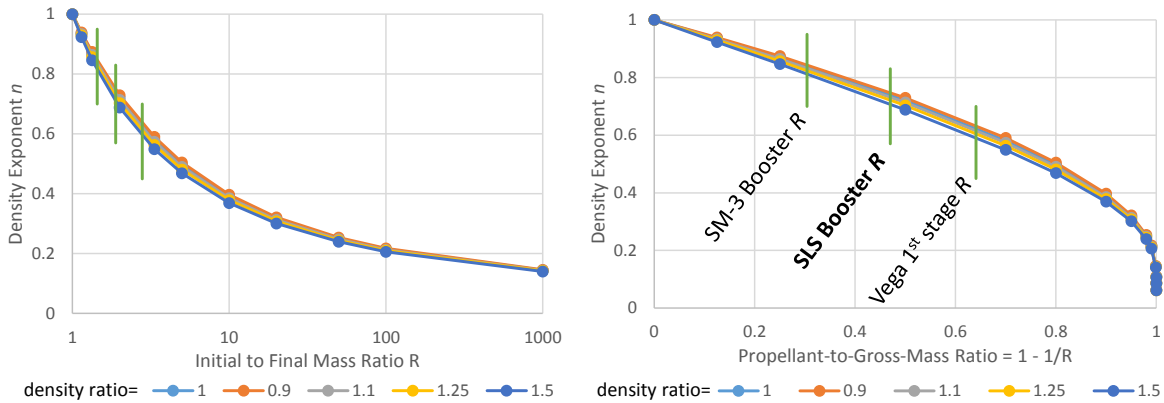

Figure 1. Density Exponent *n* as the Effect of Density Relative to Isp for Different Solid Motor Cases

Note that the primary driver is the mission: that is, how much of the reference vehicle is stage 1 propellant. Of secondary importance is the difference in the densities of the two propellants. Note that the smaller the stage relative to the vehicle, the more important density is, approaching the same importance as Isp on a percentage basis. On the plot are shown three example solid motor systems for reference[3,4,5,6]: the Standard Missile-3 (SM-3) boost stage, the Space Launch System (SLS) boosters, and the Vega launch vehicle first stage.

B. Example: Solid Propellant Densification

To illustrate the effect of solid propellant chemistry changes, consider the substitution of bismuth oxide as a portion of the oxidizer in the SLS booster system. The addition of bismuth oxide increases the density while decreasing the specific impulse. Thermochemical calculations provided the relative Isps of a range of bismuth oxide loadings. Then Equation (6) was used to estimate the performance impact for the range of loadings, over several different mission values, shown in Figure 2 as the different propellant-to-vehicle mass ratios, R_{mp}. From the baseline SLS vehicle, the ratio of booster propellant mass to total initial mass is 0.463. Even though the boosters burn in parallel with the core, this value for R_{mp} was used for initial estimation. Overlaid on the plot are the ratios of payload

delivered by sets of boosters at different bismuth oxide loadings, in the "SLS Booster Designs Analyzed" series. These were analyzed by modifying the reference motor thrust curve and performing a trajectory analysis. The shape of the curve best matches the modeled performance trend with R_{mp} of 0.42, rather than the calculated 0.462. This is rather a small difference for this fidelity of analysis, especially since any impacts of the thrust curve shape and the parallel core burning are not included in Equations (6) through (9).

Next, a larger booster was analyzed in the same way. The "Larger Designs" had a reference R_{mp} of 0.48, with the actual performance needing 0.5 to at least 0.6 to explain. In the case of this study, even the gains made were judged too small to justify the complexity of the added propellant ingredients and process change. So the density-Isp modeling proved a useful tool, leading to the same conclusion as the eventual design and analysis process.
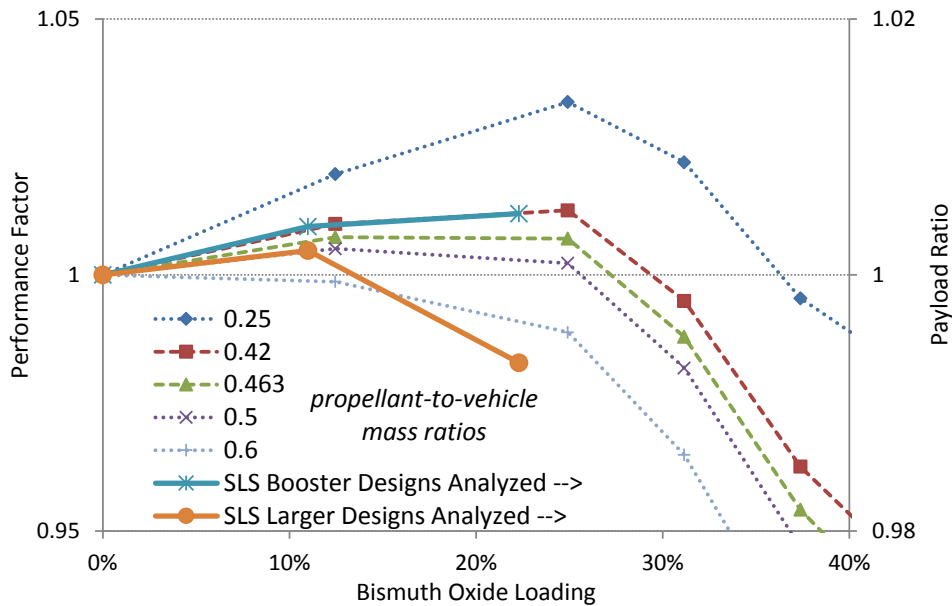

Figure 2. Modeled and Analyzed Performance Impact of Density Increase with Isp Decrease

C. Liquid Bipropellants Constant-Volume Results

For liquid bipropellants, due to the broader density range, the exact equation is more important for capturing the performance. Figure 3 shows the calculated n over several propellant combinations for the whole mission range.
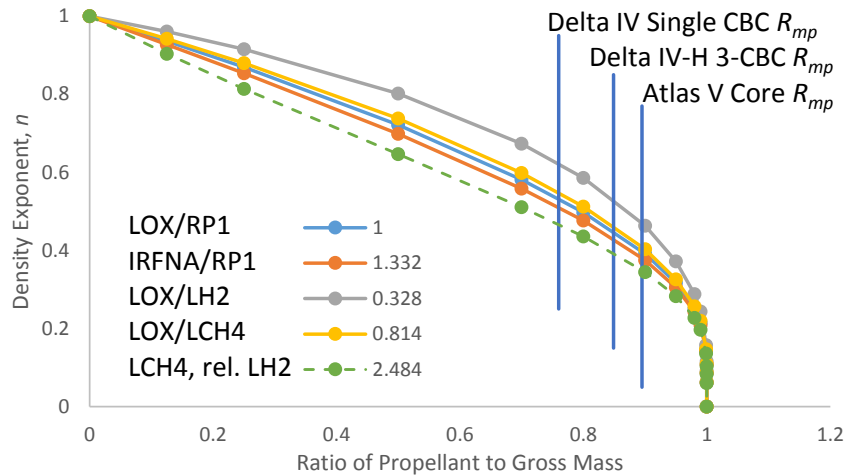

Figure 3. Effect of Density Relative to Isp by Solving n for Liquid Stages Relative to LOX/RP1 (–) or Relative to LOX/LH2 (- -)

Note that here the relevant R_{mp} zones are much higher, exemplified by the Delta IV-H and Atlas V lines[7]. A convenient way to judge the interchangeability of stages is to plot Isp versus density relative to the reference for each propellant combination, as in Figure 4. A higher performing propellant combination is one that is to the upper-right of the lines of constant performance. So, with LOX/RP1 as the baseline, it is seen that switching to methane fuel, LCH4, costs performance due to its lower density, in spite of its higher Isp. Conversely, the peroxide and IRFNA oxidizers with RP1 also cost performance slightly, but mostly make up for their lower Isp in density for these cases. The liquid hydrogen fuel result was not included on this plot, for reasons discussed below. The "LCH4, same tanks" point explores a "drop-in replacement engine," where the LOX and fuel tanks are kept the same size. Methane's lower density drives up the O/F to 5.1, leading to lower Isp with higher density.

In Figure 5, the performance measure is plotted directly, where "Exact" uses Equation (6), and "Simplified" uses
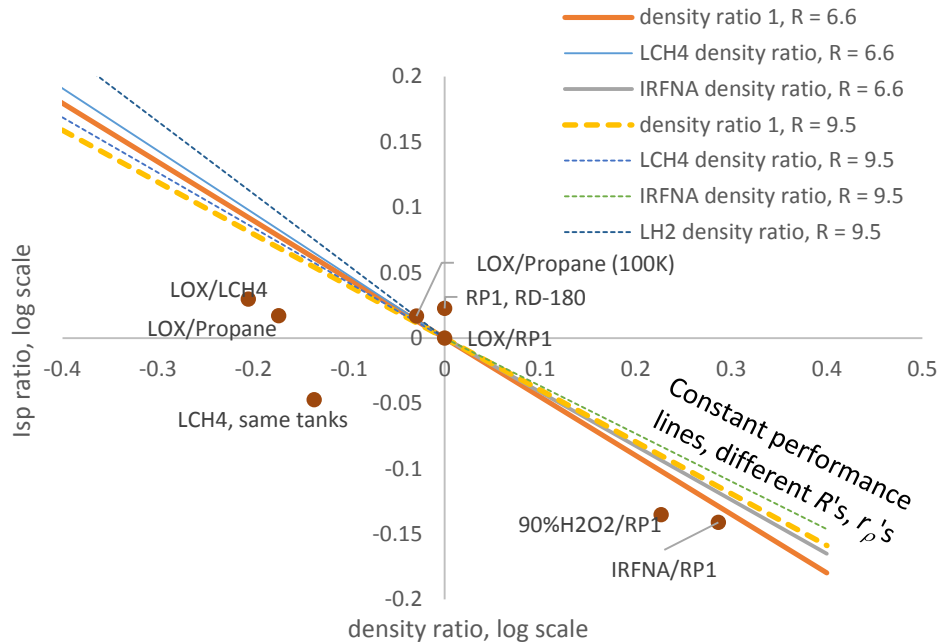

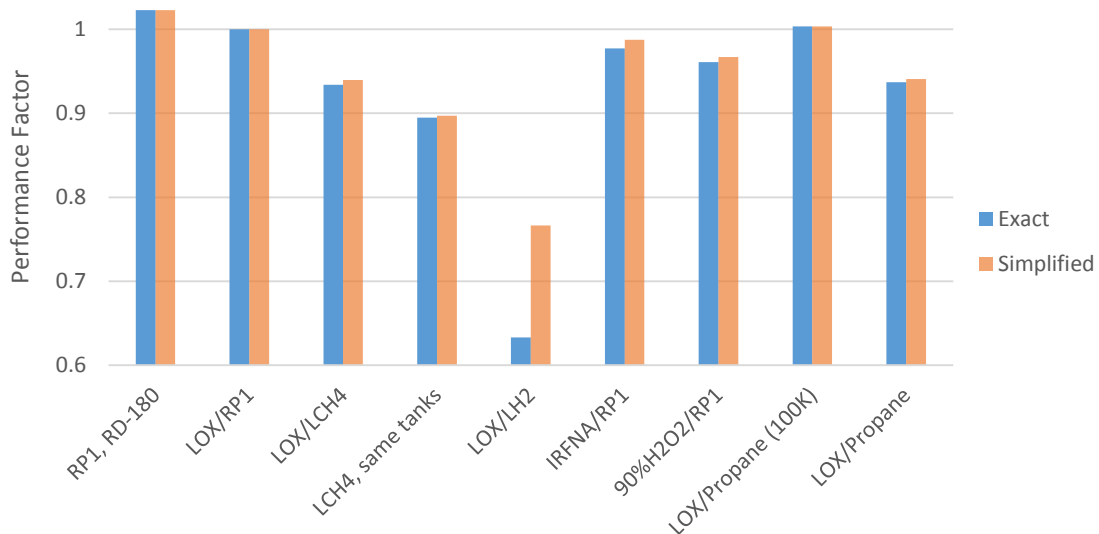**Figure 4. Interchangeability of Constant-Volume Stages Compared to Generic LOX/RP1 Reference Stage**

Figure 5. Performance of Constant-Volume Stages Compared to Generic LOX/RP1 Reference Stage

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Equations (7) and (9). The difference between the two methods is most pronounced for LOX/LH2.

The preceeding analysis can also lead to the propellant mass fraction. The above assumed the reference and candidate stages had the same inert mass if they had the same volume. So the propellant mass fraction of the candidate stage can be computed from the reference stage and density ratio, where λ is the propellant mass fraction of the stage, as in Equation (10).

$$\frac{1}{\lambda} = 1 + \frac{1}{r_\rho}\left(\frac{1}{\lambda_0} - 1\right) \tag{10}$$

D. Departure From "Same Final Mass" Assumption

Now the assumption that constant volume leads to constant final mass warrants revisiting. A good test of the assumption is to predict the Delta IV Common Booster Core (CBC) based on the Atlas V core stage. Because these stages have similar application, thrust-to-weight, and development era, one would expect the equation above to predict accurately, if indeed the "same volume means same final mass" assumption is valid across that propellant range. Even though they are not the same volume, they are large enough for scale to not matter, therefore the non-dimensional propellant mass fraction should still work. However, starting with Atlas V's 0.93, the equation predicts a Delta IV CBC propellant mass fraction of 0.82, while its published mass fraction is actually 0.88[7].

III. Next-Order Mass Model

This suggests that a stage inert mass model is needed that depends on more of the relevant parameters. A more complete mass model should account for the individual densities and *O/F* of the propellants, as well as thrust-dependent aspects of the stage mass. The thrust-dependent structure includes the engines and non-wetted structure, and also the fuel tank, because its walls bear the loads necessary to accelerate the heavy oxidizer above. Table 1 summarizes the breakdown of mass effects.

Table 1. Mass Model Dependencies

Dependent Mass:	Propellant Mass	Thrust	Oxidizer Density	Fuel Density	Bulk Density	Oxidizer to Fuel Ratio
Engine & Structure		X				
Oxidizer Tank	X		X			X
Fuel Tank	X	X		X	X	X

The strategy here is to identify constants that describe the Atlas V core and then use scaling equations to predict other stages based on propellant density ratios, changes in engine thrust-to-weight and vehicle lift-off thrust-to-weight, and *R*. Candidate models were evaluated until the Delta IV CBC was predicted with the most parsimonious model, containing two free factors. The controlling constants could alternatively be varied to investigate more or less mass-efficient reference stages. For example, reducing the tank reference inert fraction is akin to paying for better technology that results in lighter-weight tanks. The parameters are kept non-dimensional so that the analysis can be applied as broadly as possible.

The controlling constants are defined as follows, with the values calibrated to Atlas V:

1. $f_{i,tank,0}$, the inert fraction of both tank masses: 0.02
2. r_F, the multiplier on engine mass that determines engine-and-structure mass: 2.735

The 3 dependent mass variables are:

1. $f_{i,ft}$, the fuel tank inert mass per total propellant mass
2. $f_{i,ot}$, the oxidizer tank inert mass per total propellant mass
3. $f_{i,E\&S}$, the engine-and-structure inert mass per total propellant mass

The independent variables, with the values for Atlas V, are:

1. Mission-based independent variables:
   a. *R*, the initial to final mass ratio: 6.62
   b. Ψ, the initial vehicle vacuum thrust-to-weight: 1.28
2. Propellant-based independent variables:
   a. *O/F*, the oxidizer to fuel mass ratio: 2.7
   b. $r_{\rho fu}$, the candidate to reference fuel density ratio
   c. $r_{\rho ox}$, the candidate to reference oxidizer density ratio
   d. FW_{Eng}, the engine thrust to weight ratio: 78

The results are three inert fractions, f_i, that when summed can be converted into the stage propellant mass fraction. Each of these f_i are defined as the inert mass of the component divided by the total propellant mass.

$$f_{i,total} = f_{i,ft} + f_{i,ot} + f_{i,E\&S} = f_{i,tank} + f_{i,E\&S} \tag{11}$$

$$\lambda = \frac{1}{1+f_{i,total}} \tag{12}$$

These three will be derived in turn.

A. Inert Fraction Due To Engine And Structure

This method assumes the engine and structure weight, because it is thrust-dependent weight, is a constant multiple, r_F, of the engine weight, and proportional to the launch loads, represented by vehicle thrust-to-weight at launch, Ψ. This could be artificially skewed by using engine thrust-to-weights much different from historical nominal values, but having the dependence on engine weight represents real effects of mass or volume flow rate on main propulsion system components' masses. The engine thrust-to-weight used here should be representative of the propellant class. For now, thrust-to-weight values for representative LOX/LH2 and LOX/RP1 engines, 51 and 78, respectively, have been simply correlated with Equation (13) to provide a value for any propellant combination based on bulk propellant density.

$$FW_{Eng} = 39.83 \frac{\rho}{\rho_{LOX/RP1}} + 38.17 \tag{13}$$

The calculation of engine and structure inert fraction is:

$$f_{i,E\&S} = \frac{m_{E\&S}}{m_p} = \frac{r_F W_{Eng}}{m_p g} = r_F \frac{\Psi}{FW_{Eng}} \left(\frac{1}{R_{mp}}\right) \tag{14}$$

B. Inert Fraction Due To Tanks

The reference tank inert fraction now must be decomposed into useful fuel and oxidizer values. These are slightly different than the f_i above, because they are the ratio of oxidizer or fuel tank mass to oxidizer or fuel mass, respectively, rather than to total propellant mass. Assuming the reference vehicle tanks have the same wall thickness and material, the tanks share a single reference mass per unit volume, $f_{v,tank}$. Then the following equations can be solved together for the reference stage oxidizer and fuel tank specific inert fractions $f'_{i,ot,0}$ and $f'_{i,ft,0}$:

$$f_{v,tank,0} = f_{v,ot,0} = f_{v,ft,0} \tag{15a}$$

$$f'_{i,ot,0} = \frac{f_{v,ot,0}}{\rho_{ox}} \qquad f'_{i,ft,0} = \frac{f_{v,ft,0}}{\rho_{fu}} \tag{15b, c}$$

$$f'_{i,tank,0} = f'_{i,ot,0} \frac{\left.\frac{O}{F}\right|_0}{\left.\frac{O}{F}\right|_0 + 1} - f'_{i,ft,0} \frac{1}{\left.\frac{O}{F}\right|_0 + 1} \tag{16}$$

This results in $f'_{i,ot,0} = 0.0180$ and $f'_{i,ft,0} = 0.0255$. These non-dimensional parameters are preferred in the following equations instead of $f_{v,tank}$ which has density units.

It is also convenient to define density ratios in terms of individual propellant densities, rather than just bulk propellant combination densities, for comparing to reference propellants, with Equations (17) and (18).

$$r_{\rho fu} = \frac{\rho_{fu}}{\rho_{fu,0}} \qquad\qquad r_{\rho ox} = \frac{\rho_{ox}}{\rho_{ox,0}} \qquad\qquad \rho = \frac{\frac{O}{F}+1}{\frac{\frac{O}{F}}{\rho_{ox}}+\frac{1}{\rho_{fu}}} \tag{17a, b, c}$$

$$r_\rho = \frac{\rho}{\rho_0} = \left(\frac{\frac{O}{F}+1}{\frac{O}{F}\big|_0+1}\right)\left(\frac{\frac{\frac{O}{F}\big|_0}{\rho_{ox,0}}+\frac{1}{\rho_{fu,0}}}{\frac{\frac{O}{F}}{\rho_{ox}}+\frac{1}{\rho_{fu}}}\right) \tag{18}$$

C. Inert Fraction Due To Oxidizer Tank

It is assumed the oxidizer tank mass depends solely on its volume. Thus Equations (19) and (20):

$$f_{i,ot}m_p = m_{inert,ot} = f_{v,ot}Volume_{ot} = f'_{i,ot,0}\rho_{ox,0}\frac{m_{ox}}{\rho_{ox}} = f'_{i,ot,0}\frac{1}{r_{\rho ox}}\left(\frac{\frac{O}{F}}{\frac{O}{F}+1}\right)m_p \tag{19}$$

$$f_{i,ot} = f'_{i,ot,0}\frac{1}{r_{\rho ox}}\left(\frac{\frac{O}{F}}{\frac{O}{F}+1}\right) \tag{20}$$

D. Inert Fraction Due To Fuel Tank

The fuel tank inert fraction depends not only volume but also on loads, as the fuel tank supports the mass of the oxidizer and stage payload through the launch acceleration. This is modeled simply as the product of volume-dependent mass and the ratio of loads to reference loads:

$$m_{inert,ft} = f_{i,ft}m_p = f_{v,ft}Volume_{ft}\frac{L}{L_0} \tag{21}$$

$$f_{i,ft} = f'_{i,ft,0}\frac{1}{r_{\rho fu}}\left(\frac{1}{\frac{O}{F}+1}\right)\frac{L}{L_0} \tag{22}$$

The loads-dependent mass of the fuel tank is primarily related to the wall thickness transmitting thrust load to the upper (as a rule) oxidizer tank and payload. To approximately capture the drivers, the force required to accelerate the fuel itself is subtracted off from the total thrust:

Load = thrust – (mass fuel)*(acceleration)

$$L = F - m_{fu}g\,\Psi = F\left(1 - R_{mp}\frac{1}{\frac{O}{F}+1}\right) \tag{23}$$

$$\frac{L}{L_0} = \frac{F\,R_{mp}\left(\frac{1}{R_{mp}}-\frac{1}{\frac{O}{F}+1}\right)}{F_0\,R_{mp,0}\left(\frac{1}{R_{mp,0}}-\frac{1}{\frac{O}{F}\big|_0+1}\right)} = \frac{\Psi\,m_i g\,R_{mp}\left(\frac{1}{R_{mp}}-\frac{1}{\frac{O}{F}+1}\right)}{\Psi_0\,m_{i,0}g\,R_{mp,0}\left(\frac{1}{R_{mp,0}}-\frac{1}{\frac{O}{F}\big|_0+1}\right)} \tag{24}$$

With $m_iR_{mp} = m_p$ and $m_p/m_{p0} = r_\rho$,

$$f_{i,ft} = f'_{i,ft,0}\frac{r_\rho}{r_{\rho fu}}\frac{1}{\frac{O}{F}+1}\frac{\Psi}{\Psi_0}\frac{\left(\frac{1}{R_{mp}}-\frac{1}{\frac{O}{F}+1}\right)}{\left(\frac{1}{R_{mp,0}}-\frac{1}{\frac{O}{F}\big|_0+1}\right)}. \tag{25}$$

The result is that for any set of stage construction assumptions, i.e., a real or imagined reference stage, the $f_{i,0}$ can be estimated according to the level of information available. Then comparable other stages' mass fractions can be estimated. For instance, one could set the constants according to the existing LOX/LH2 Centaur, and estimate a LOX/LCH4, 1.4 Ψ "Centaur."

IV. Next-Order Mass Model Results

A. Comparison Between Models and Data

The mass model was used to predict the Delta IV CBC propellant mass fraction, and extended to several other propellant combinations. The mass model results are shown in Figure 6 along with the Equation (10) results dependent only on the density and *Isp* derivation. For the Delta IV, the mass model computes a 0.884 mass fraction, compared to an actual value of 0.882, representing a huge improvement over the Equation (10) model. For further validation, the Space Shuttle External Tank was predicted simply by setting $f_{i,E\&S}$ to 0. This computation is shown compared to the mass fraction values of the Lightweight Tank (LWT)[8] and Super-Lightweight Tank (SLWT)[7] variants. So, the mass model satisfied its goal of predicting LOX/LH2 performance far better than the constant final mass assumptions. For the other propellant combinations, the two models are much closer together. In the absence of additional data, one can merely say that the trend makes sense. Compared to LOX/RP1, lower density propellant combinations would be disadvantaged by the density-Isp-only model, and higher density propellant combinations would be improperly advantaged, as the plot shows. However, the difference may not be significant for comparing stage performance, and the mass model for the high density propellant combinations is less robust, as it required extrapolation with the engine thrust-to-weight correlation.
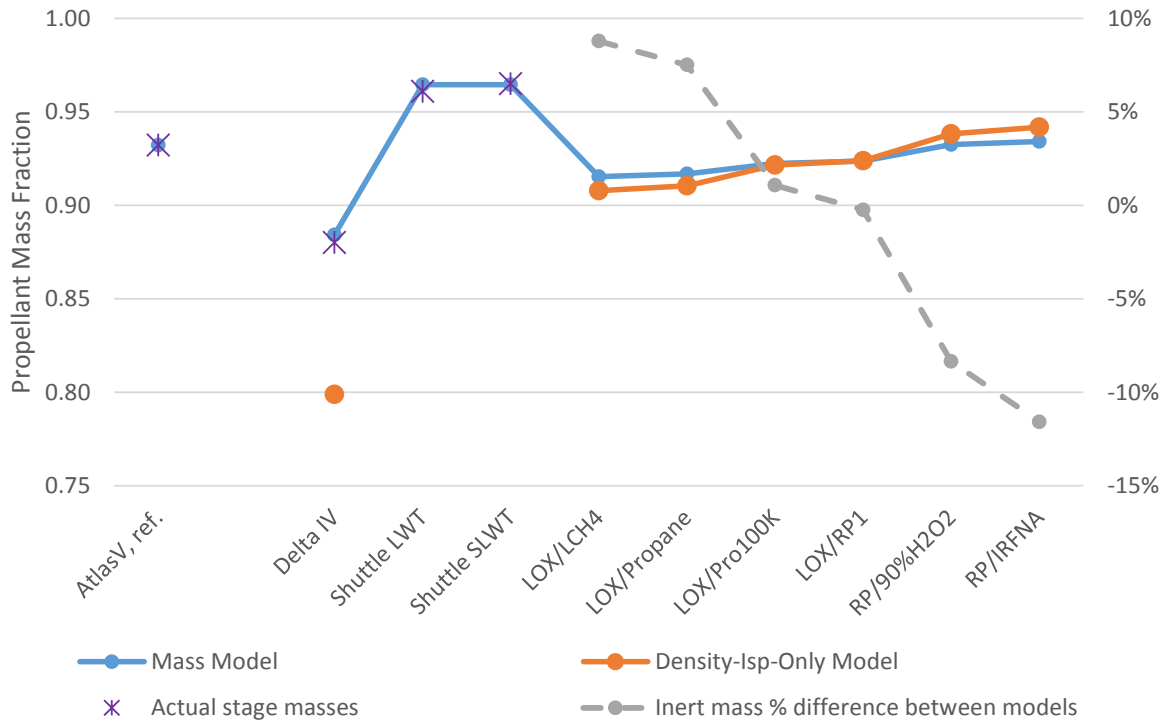

Figure 6. Mass Model Showing Good Mass Fraction Prediction for LOX/LH2

B. Example: Stage Candidates Compared to Atlas V or Delta IV

To compare the performance of these stages, Equation (4) still provides the performance factor, but the *R* calculation requires the following derivation. It is based on the stage's payload being the same as that of the reference stage.

$$R = \frac{m_i}{m_f} \cdot \frac{m_{p0}}{m_{p0}} = \frac{\frac{m_{payload}}{m_{p0}} + \frac{r_\rho}{\lambda}}{\frac{m_{payload}}{m_{p0}} + r_\rho\left(\frac{1}{\lambda}-1\right)} \tag{26}$$

The payload mass ratio is defined from the reference vehicle. Then the *R* can be calculated from known parameters.

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$$\frac{m_{payload}}{m_{p0}} = \frac{m_{i,0}}{m_{p0}} - \frac{1}{\lambda_0} = \frac{1}{R_{mp}} - \frac{1}{\lambda_0} \qquad (27)$$

$$1 - R_{mp} = \frac{1}{R} = 1 - \frac{r_\rho}{\frac{1}{R_{mp}} - \frac{1}{\lambda_0} + \frac{r_\rho}{\lambda}} \qquad (28)$$

Thus, when computing the performance factor to compare to the density-Isp-only model, the mass model becomes iterative, with R_{mp} and λ dependent upon each other.

Figure 7 shows how the various propellant combinations would perform in the same volume as the Atlas V core at the same mission R value of 6.62. Not surprisingly, the hydrogen stage falls well short; that is why the Delta IV CBC is significantly larger for a similar performance. For the other propellants, the conclusions do not really change from those of Figures 4 and 5 with the density-Isp-only model. The primary observations are IRFNA/RP1 not being quite as close to parity with LOX/RP1, and methane and standard propane being about even. However, densified propane, cooled by the LOX to a temperature of 100 K, maintains the volume-constrained performance of
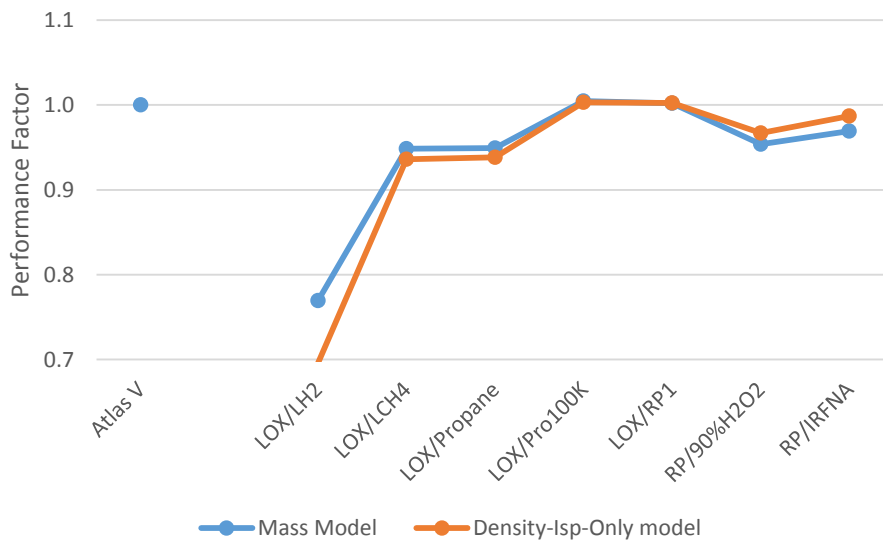

Figure 7. Performance Comparison of Stages with Same Volume as Atlas V Core for Same Mission
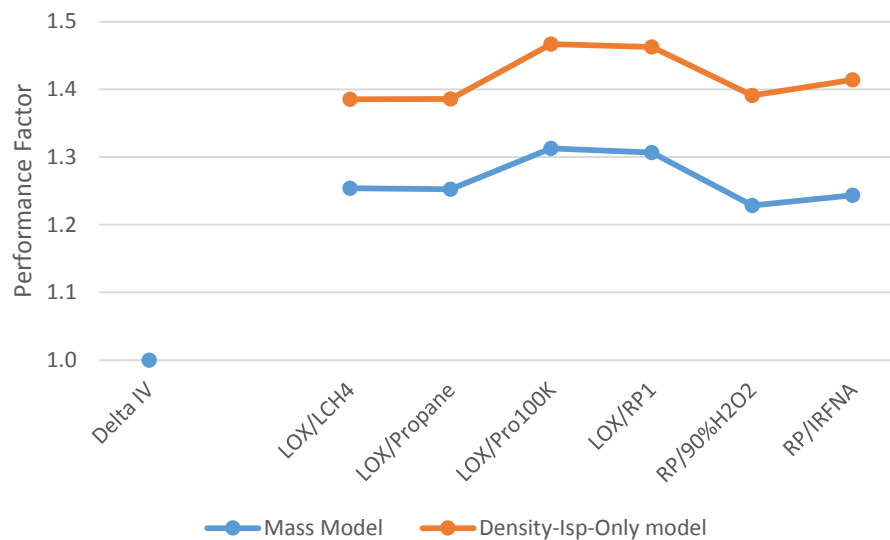

Figure 8. Performance Comparison of Stages with Same Volume as Delta IV CBC for Same Mission

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LOX/RP1. Chilled methane was not considered, because its density at 100 K is not much higher than the analyzed density based on normal boiling point, so there is not a significant densification benefit[9]. In fact, if properties for methane at closer to standard temperature were used, its performance would be even worse than that shown.

Figure 8 shows how the various propellant combinations would perform in the same volume as the Atlas V core at the same mission R value of 5.78. The accuracy gain from the mass model is huge across the board here. So, when starting from hydrogen, the switch to methane fuel looks like a 25% or so upgrade: more than a drop-in replacement, but a genuine upgrade. Interestingly, 100 K propane comes out on top, with LOX/RP1 similar, but now the dense oxidizer combinations have fallen behind the light hydrocarbon combinations.

C. The Effect of Mission Value R

To illustrate how important the mission R is in trading performance for a constant-volume first stage design space, different reference stage R's are shown in Figure 9. In a), stepping up from the single core Delta IV mission to the three-core mission significantly reduces the benefit of the denser propellants. Plot b) demonstrates a reasonable upper limit for $R_{mp,0}$, when using LOX/RP1 as the reference stage. Plots c) and d) show how the R can be reduced until the denser oxidizer combinations begin to exceed the reference performance. IRFNA and RP1 can outperform a LOX/RP1 stage with a propellant mass as much as 75% of the initial mass. 90% hydrogen peroxide and RP1 can outperform a LOX/RP1 stage with a propellant mass as much as 65% of the initial mass.

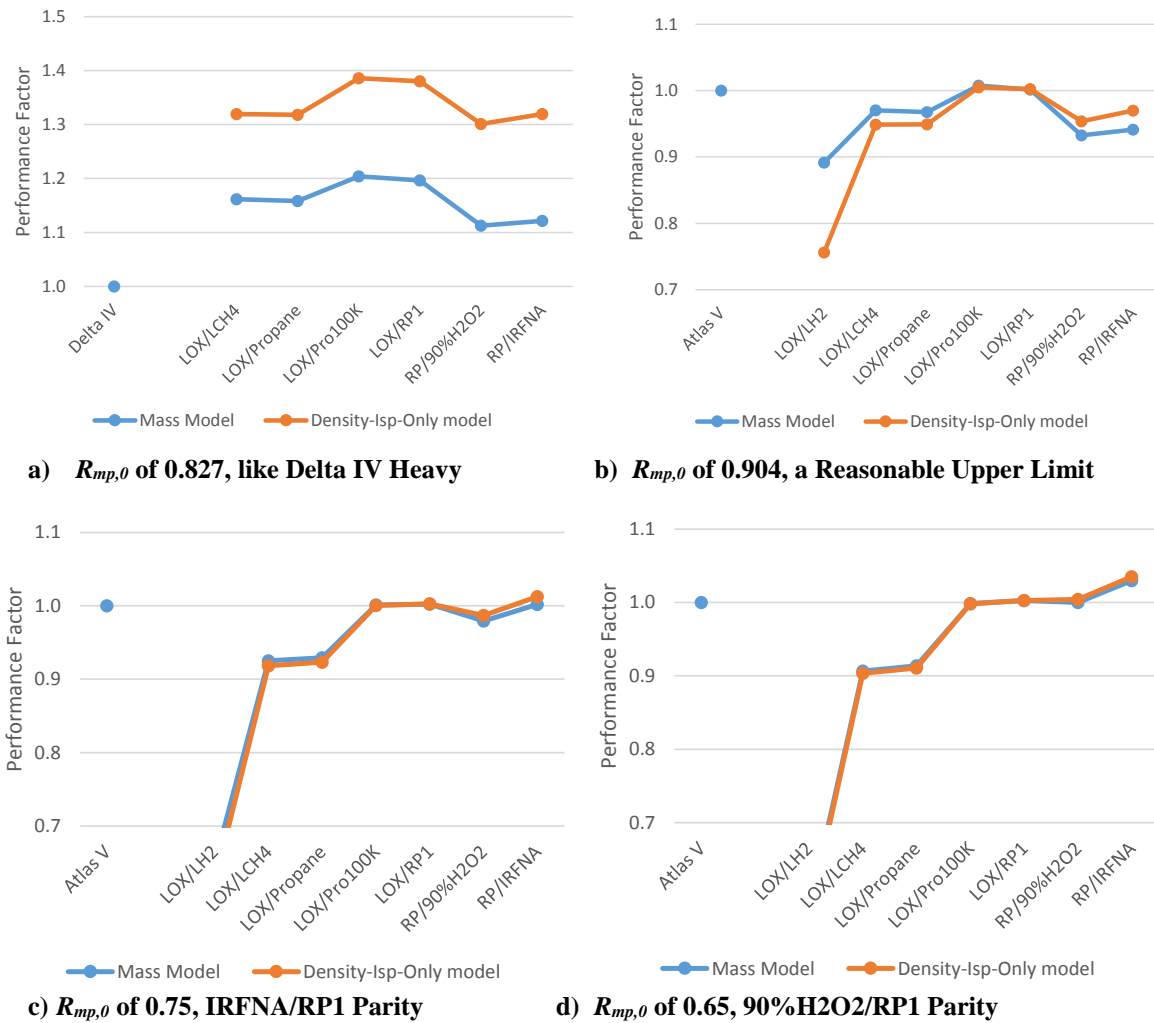a) $R_{mp,0}$ of 0.827, like Delta IV Heavy

b) $R_{mp,0}$ of 0.904, a Reasonable Upper Limit

c) $R_{mp,0}$ of 0.75, IRFNA/RP1 Parity

d) $R_{mp,0}$ of 0.65, 90%H2O2/RP1 Parity

Figure 9. Effect of Mission Value R on the Propellant Interplay

D. Example: Upper Stage

Now, consider an upper stage. These tools could be used with a multi-stage optimization just as well. However, when comparing directly to an existing stage in the absence of such an optimization, it is convenient to treat it as a constant initial mass case, so that the ΔV imparted by the previous stages is unchanged. The performance factor gets modified again by the calculation of R, this time computing the ΔV based on constant initial mass and constant payload. Therefore, the total stage mass is constant, so that Equation (29) can substitute to generate Equation (30).

$$\frac{m_p}{\lambda} = \frac{m_{p0}}{\lambda_0} \tag{29}$$

$$R = \frac{m_i}{m_f} = \frac{m_i}{m_i - m_p} = \frac{m_i}{m_i - m_{p0}\frac{\lambda}{\lambda_0}} = \frac{1}{1 - R_{mp0}\frac{\lambda}{\lambda_0}} \tag{30}$$

This allows computation with either the mass model λ or the density-Isp derived λ. For the reference stage, the dual-engine Centaur was chosen. The previous tank inert mass factors were modified to match Centaur's mass fraction, while the r_F was kept the same. Table 2 shows the comparison to the hydrocarbon cases. For a reference upper stage mission of 15,000 ft/s, the hydrocarbons only attain about 7/8 of the ΔV, now with the methane slightly leading the way. For a high ΔV of 28,000 ft/s, with a small payload, the gap to hydrogen performance is narrowed by half, now with chilled propane having a slight edge among the hydrocarbons. So this is the opposite behavior from the volume-constrained first stage, where an increase in ΔV led to an increased hydrogen performance factor.

Next, observe the sensitivity to the technology factor. If the Centaur tank inert mass factors are increased, as if a lower-budget replacement stage were designed, Table 3 shows the hydrocarbons become more competitive. This makes sense: if heavier tank walls are to be used, the penalty is less if the volume is decreased by using higher density propellants.

Table 2. Mass Model Performance Factors for Centaur-Based Technology Levels

	ΔV_0 = 15000 ft/s	ΔV_0 = 28000 ft/s
Centaur, λ = 0.91[7]	1	1
LOX/Methane	0.871	0.930
LOX/Pro100K	0.868	0.937
LOX/RP1	0.858	0.929

Table 3. Mass Model Performance Factors for Lower than Centaur-Based Technology Levels

	ΔV_0 = 15000 ft/s	ΔV_0 = 28000 ft/s
LOX/LH2, λ = 0.88	1	1
LOX/Methane	0.900	0.997
LOX/Pro100K	0.901	1.016
LOX/RP1	0.891	1.010

E. Example: Single Stage to Orbit from Balloon

The 28,000 ft/s ΔV reported above would be enough to get from a 120,000 ft balloon to low earth orbit, with an initial thrust-to-weight of 1.43. For a LOX/LH2 stage like the Centaur with Isp of 451 sec, this lets R_{mp} = 0.855. However, the actual dual-engine Centaur at that R_{mp} only has a thrust-to-weight of 0.99, so this is akin to adding approximately another engine. The mass model predicts a mass fraction drop from 0.91 to 0.878. Because this is above the R for the mission, there is indeed room for positive payload. Non-dimensional payload results are summarized in Table 4. Dimensionally, for the LOX/LH2 based on Centaur, this provides a payload a bit above the Pegasus or Minotaur I capability. It is difficult to see a Centaur-style stage being built and fielded on a new, large enough balloon system for less cost than those systems, but perhaps a new, more affordable stage could be produced that would make the endeavor reasonable.

Table 4. Single Stage to Orbit from Balloon Mass Model Results and Capability

	λ	*Isp*	*R*	Payload per initial mass	Payload relative to LOX/LH2
Centaur, Ψ = 0.99	0.91	451	--	--	--
LOX/LH2, Ψ = 1.43	0.878	451	0.855	0.029	1
LOX/Methane	0.920	365	0.908	0.013	0.45
LOX/Pro100K	0.923	360	0.911	0.014	0.48
LOX/RP1	0.930	355	0.914	0.017	0.59

V. Conclusions

A simple stage comparison method based only on the density and *Isp* of propellants proved adequate for a range of concept studies 1. Across hydrocarbon fuels, 2. Between LOX and heavy oxidizers, and 3. Across solid propellant formulations. A mass model slightly more detailed, but still simple enough to be tractable for exploring large design spaces and quick trade studies, proved valid for including hydrogen in the comparisons. It also appears to improve all the other liquid propellant estimates, and provides a tool for predicting mass fraction as a function of thrust-to-weight and oxidizer-to-fuel ratios.

The calculations show, for example:
1) Densifying solid propellant at the expense of specific impulse provides marginal performance gains for Space Launch System boosters, and thus is not likely to be worth the trouble of additional ingredients and process changes;
2) Liquid methane fuel improves upon hydrogen performance for a volume-limited first stage, but fails to meet the performance of kerosene;
3) Among liquid upper stages, hydrogen fuel will typically perform highest, but the gap is narrowed by higher ΔV missions or heavier, lower cost tank materials and methods. It is possible in those cases that cost or convenience could trump performance in a full-up vehicle trade.

Recommended next steps are:
1) Set up a multistage design space computation and optimization utilizing the mass model;
2) Include an appropriate model relating mass fraction to stage size for smaller stage sizes;
3) Consider how the technology factors as well as size can be built into a cost model.

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