Conjunction Assessment Risk Analysis

Collision Avoidance “Short Course”

Part I: Theory

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Part I Contents

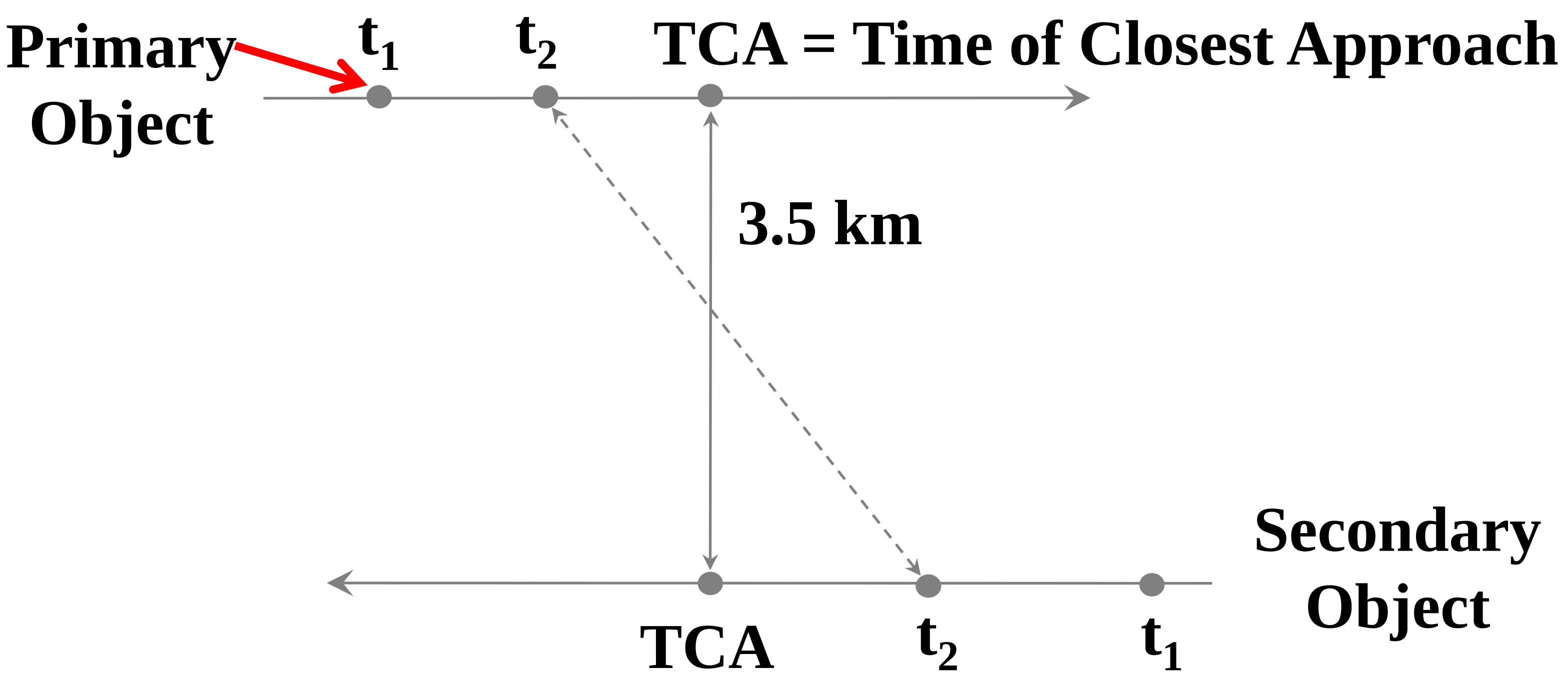
- **CA terminology and very high level concepts**
- **Space catalogue maintenance basics**
  - Collecting satellite position data
  - Updating and propagating orbits
- **OD uncertainty modeling through covariance**
- **Probability of collision computation**
- **CA screenings**
- **Conjunction Data Message contents**

CA TERMINOLOGY

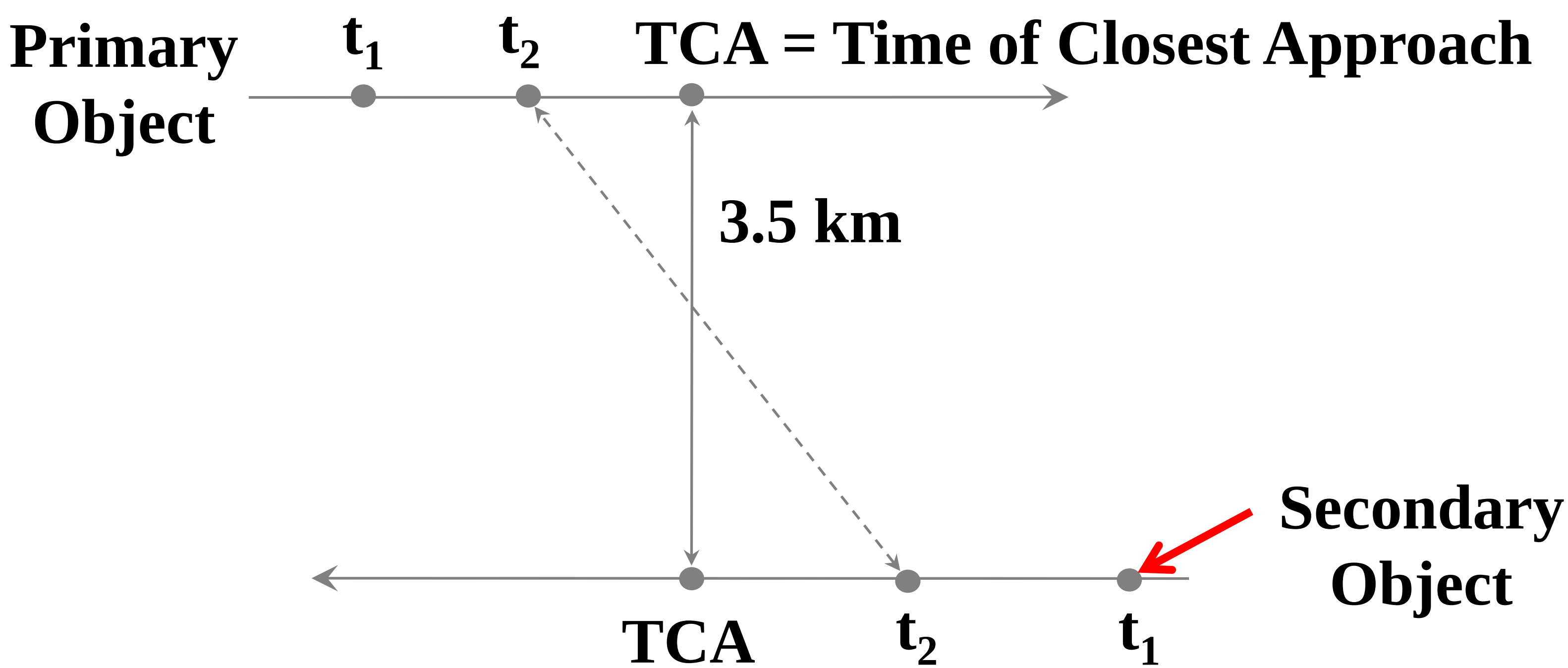
CA Terms (1 of 7)

- **Conjunction Assessment (CA)**
  - An iterative process for determining the Point of Closest Approach (PCA) and Time of Closest Approach (TCA) of two tracked orbiting objects or between a tracked orbiting object and a launch vehicle (including spent stages) or payload
    - PCA and TCA will be defined shortly
  - Further activities to identify high-interest conjunction events
- **Conjunction**
  - When the predicted miss distance between two on-orbit objects, or between a launch vehicle and an orbiting object, is less than a specified reporting volume
- **On-Orbit CA (On-Orbit Screening)**
  - The process of determining the closest approach of two on-orbit satellites

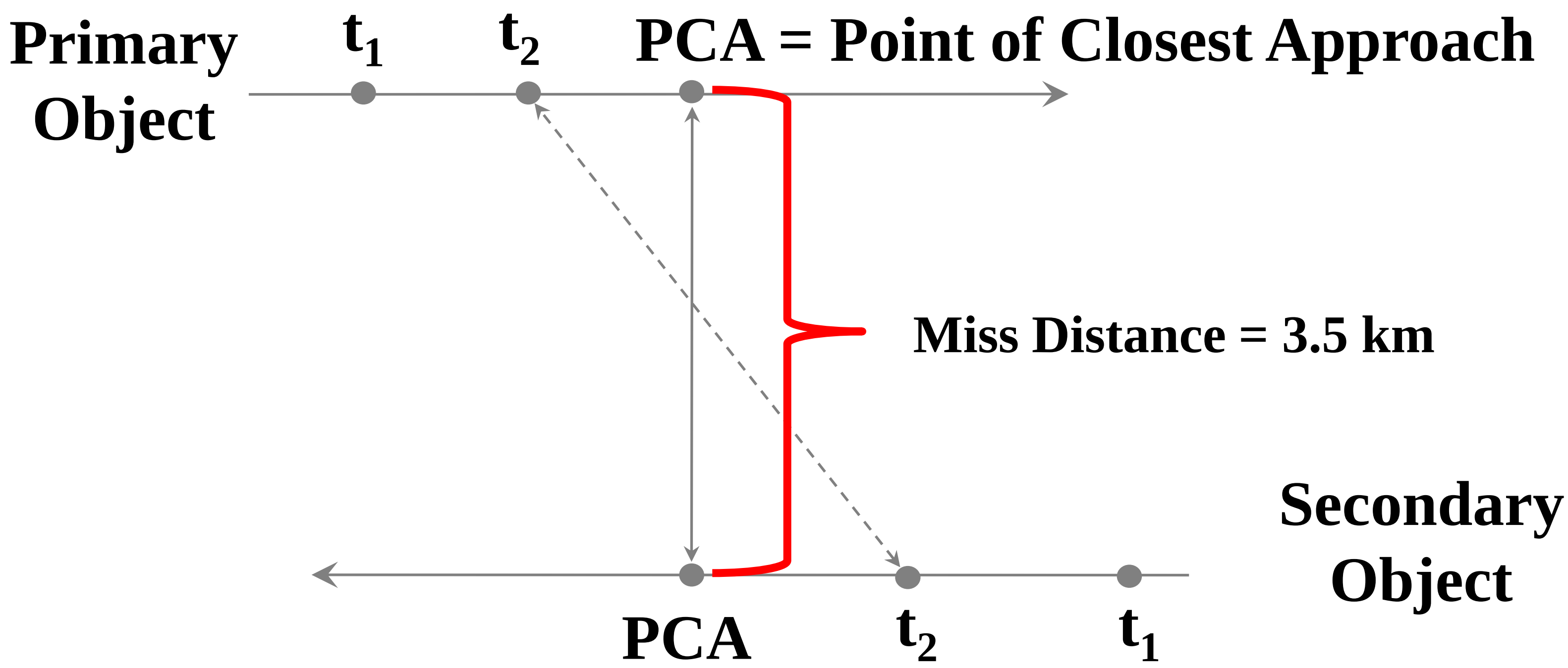
- **Primary Object**
  - The satellite asset, launched object or the ephemeris file that is being screened for potential conjunctions



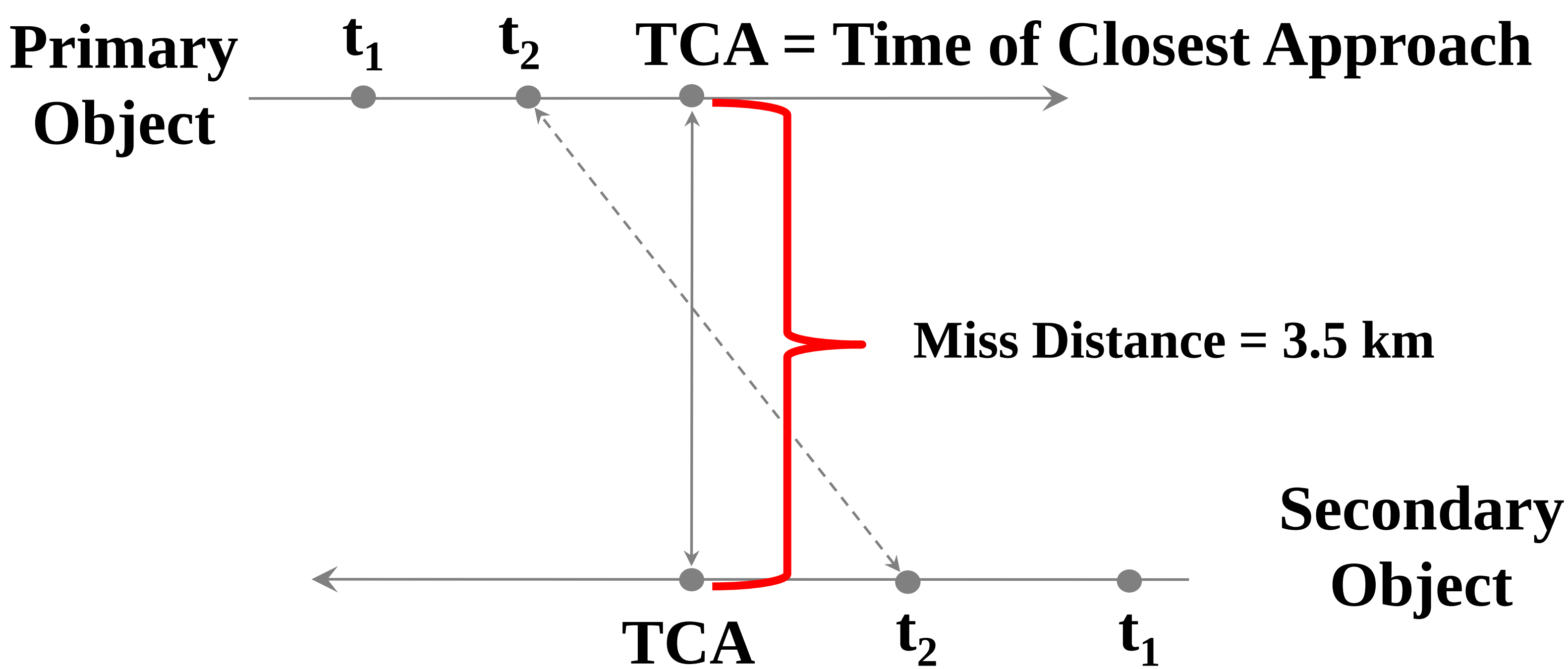
- **Secondary Object**
  - All other satellite objects (examples: payloads, debris, R/B, or analyst satellites) against which the primary object is being screened for potential conjunctions



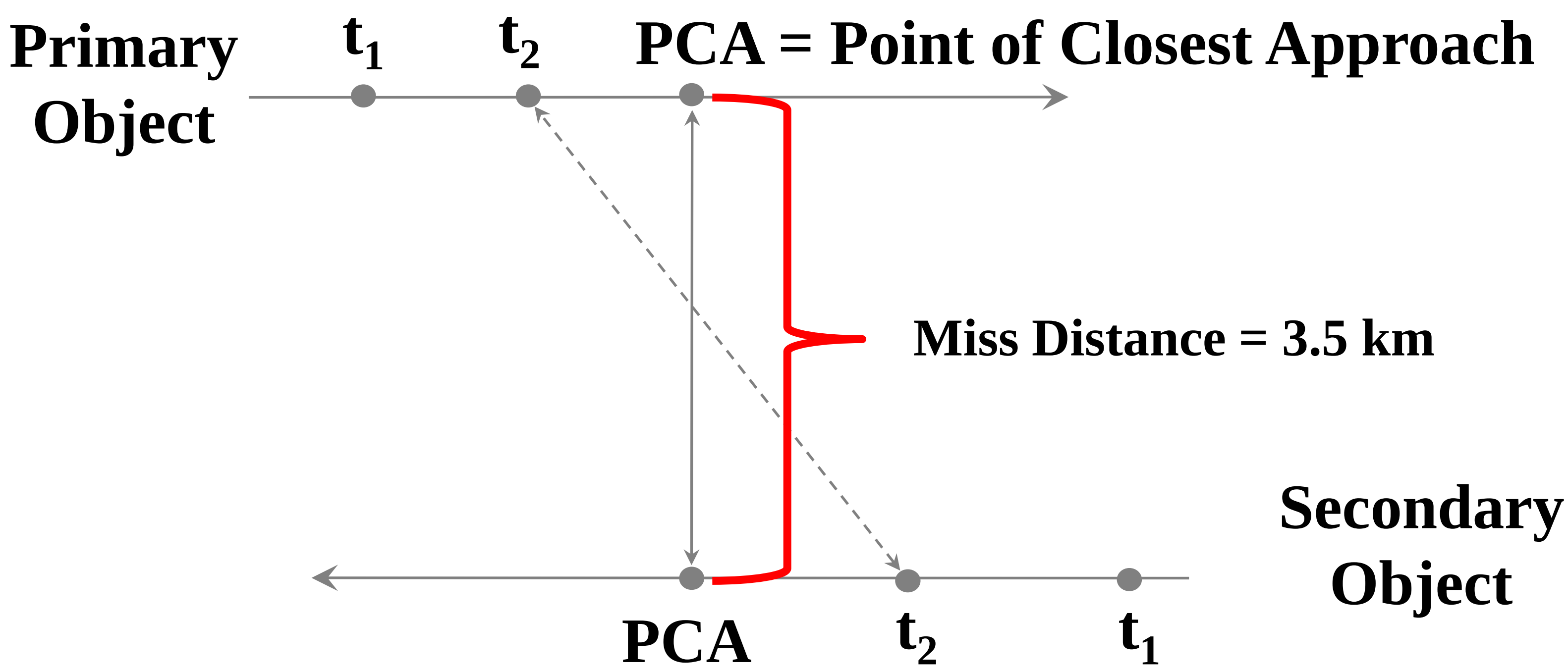
- **Point of Closest Approach (PCA)**
  - The point in each object's orbit where the magnitude of the relative position vector (i.e., miss distance) between the 2 objects is a minimum
  - The PCA occurs at the Time of Closest Approach (TCA)



- **Time of Closest Approach (TCA)**
  - The time at which the minimum miss distance between two objects occurs
    - This occurs when the relative position vector is perpendicular to the relative velocity vector for the two objects involved in a conjunction



- **Overall Miss Distance**
  - The PCA of one object relative to another; i.e., the minimum range, miss distance, or relative position magnitude between two satellites at TCA
    - Can also be expressed by individual three-dimensional component



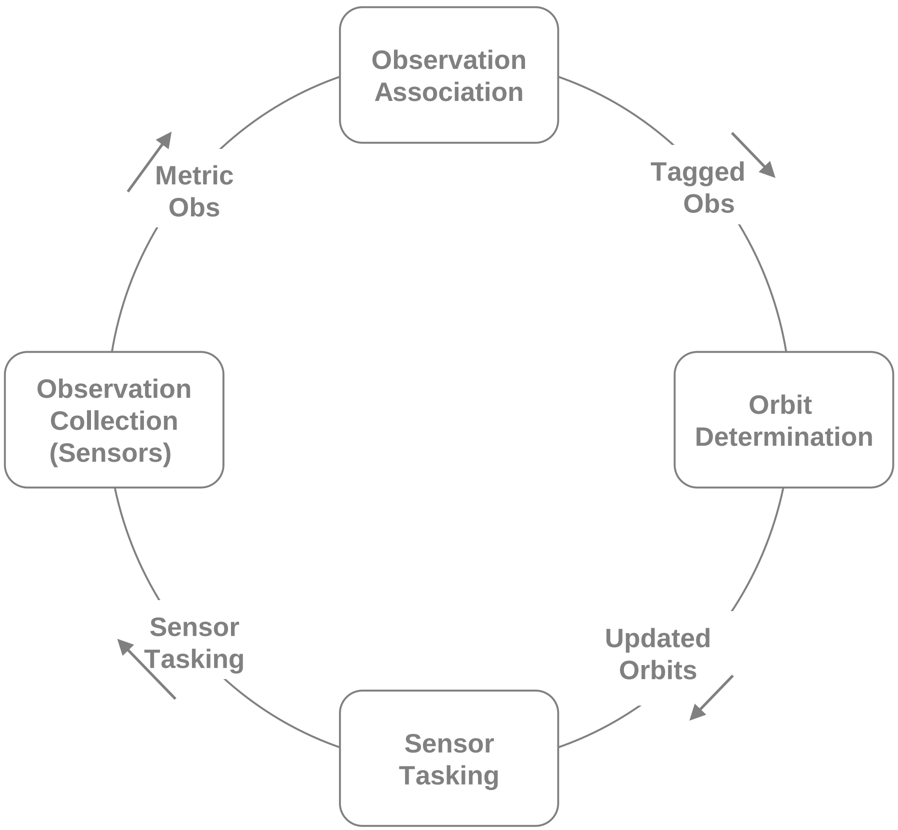
CA Terms (7 of 7)

- **Probability of Collision (Pc)**
  - Statistical measure of the likelihood that two objects' centers-of-mass will come within a specified distance of each other
  - Pc calculation requires covariance data (i.e., uncertainty data) on each object; will be discussed later
  - Pc values usually expressed in scientific notation, e.g., 1E-05
    - Large values are 1E-04 and higher
    - Small values are perhaps 1E-06 and lower
- **Screening Volume**
  - A spherical or ellipsoidal volume around the primary and secondary objects used to determine if a satellite pair is a conjunction candidate
- **Collision on Launch Assessments (COLA)**
  - Screening performed on powered flight trajectory
  - Some entities use "COLA" to mean collision avoidance, or implementation of a risk mitigating actin such as a maneuver. This is separate from CA.

CATALOGUE MAINTENANCE

The Catalog Maintenance Cycle

- Cycle in use since the late 50’s, in many forms
- Sensors collect observations and send them to JSpOC
- JSpOC associates submitted observations to objects
- Orbits are updated using observations
- Tasking tells sensors how many observations should be collected to maintain desired orbital accuracy



SENSOR OBS COLLECTION

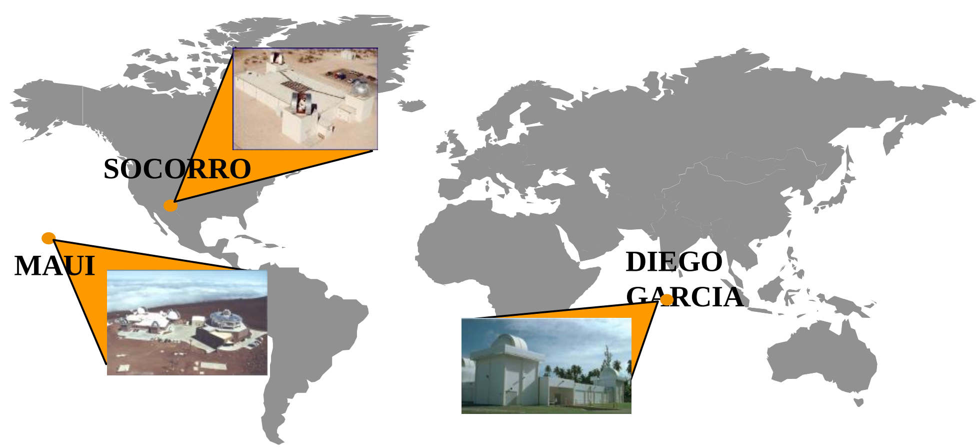
Current 'Find' Capability

Near Earth (NE) 'Find'

- Cavalier, Eglin and Shemya radars have some limited un-cued NE 'Find' capability

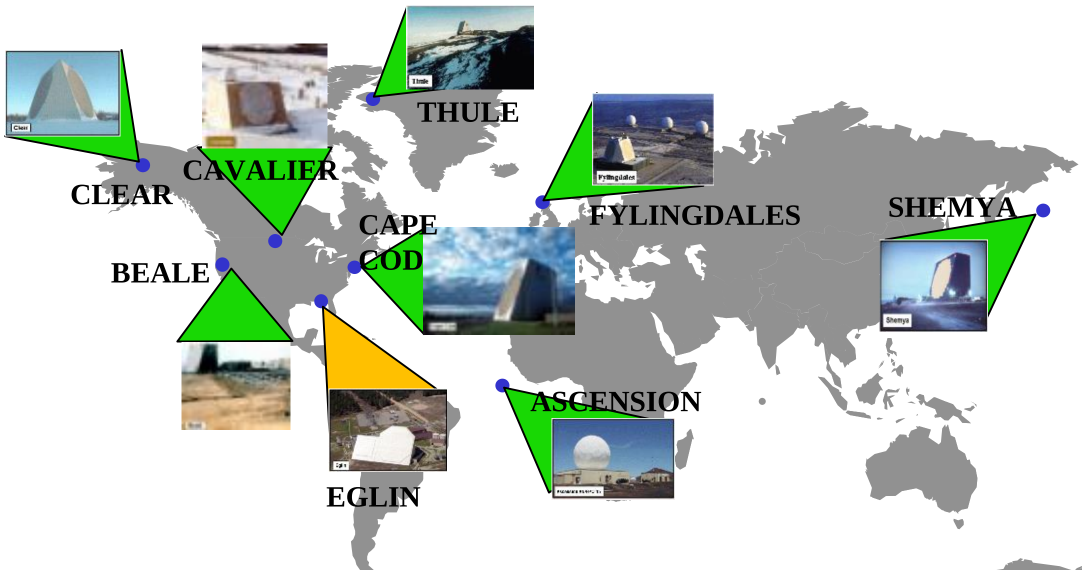
Deep Space (DS) 'Find'

- The 3 GEODDS sites are the only dedicated DS 'Find' capability, and they have limiting factors



Current ‘Fix and Track’ Capability

Near Earth ‘Fix & Track’

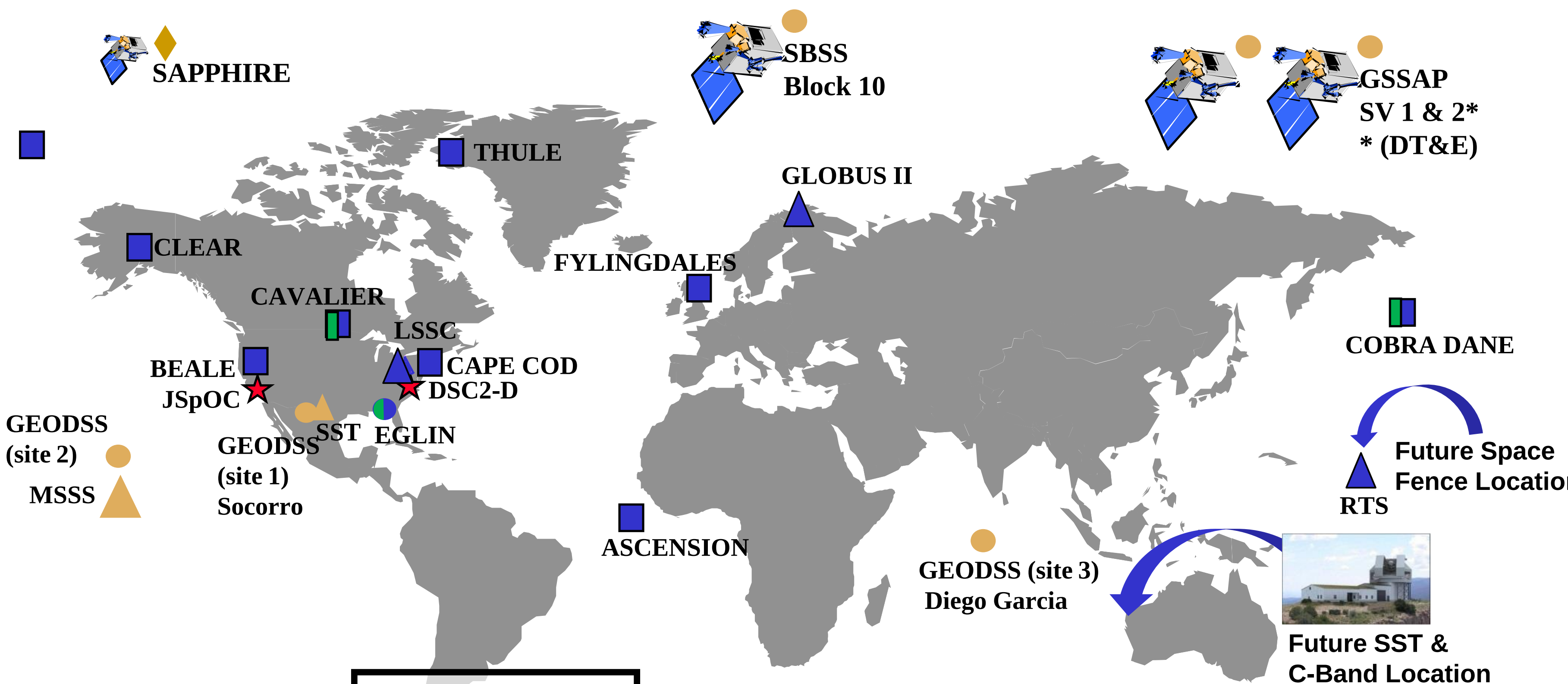


- Eglin Provides Dedicated NE ‘Fix and Track’ Capability
- Missile Warning & Contributing Sensors Provide Non-Dedicated NE ‘Fix and Track’ Capability

Deep Space ‘Fix & Track’

- Ground Based Optical Sensors Provide Dedicated DS ‘Fix and Track’ Capability
- Radars Provide Limited DS ‘Fix and Track’ Capability

Space Surveillance Network



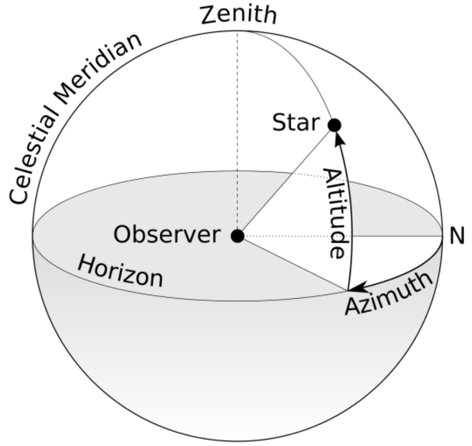
Tracking Radar
Detection Radar
Optical Telescope
SSN C2

- Dedicated
- Collateral
- Contributing
- SSN C2
- Dedicated International

JSpOC = Joint Space Operations Center
LSSC = Lincoln Space Surveillance Complex (Millstone, Haystack, HAX)
MSSS = Maui Space Surveillance System
RTS = Reagan Test Site
SBSS = Space Based Surveillance System
SST = Space Surveillance Telescope

Observation Types

- **Radars typically provide three observables**
  - Range to target (the most useful of the measurements)
  - Two angles to target, typically azimuth and elevation
- **Optical sensors report only two observables, both angles**
  - If azimuth mount (axis normal to earth), then report azimuth and elevation
  - If ra/dec mount (axis points to north star), then report right ascension and declination
    - Inertial system better suited to fixed background of stars

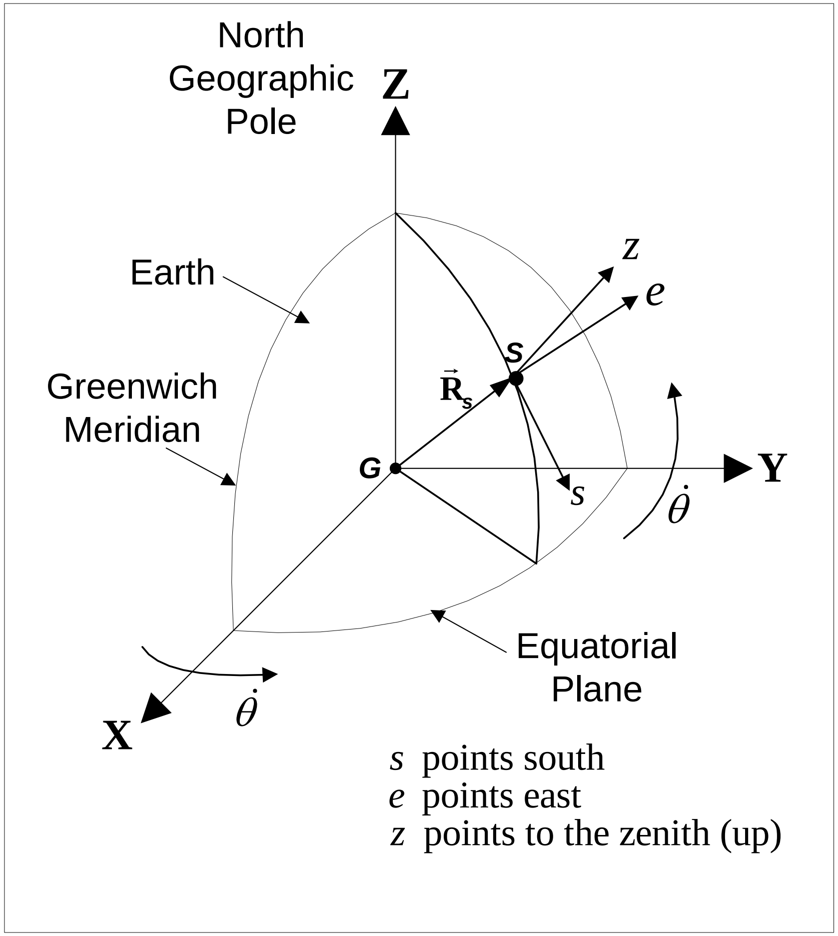


Topocentric Horizon (SEZ)

- **Origin: at sensor**
- **Fundamental plane: established by local horizontal plane**
- **Principal direction: points south**
- **When valid/applicable:**
  - At a radar's search (acquisition) time or when time tagging an observation
  - Used to locate objects relative to a mechanical or phased array radar sensor (e.g., Eglin)
- **Unit vectors: *S, E, Z***
  - ***S*** points south

    E points east

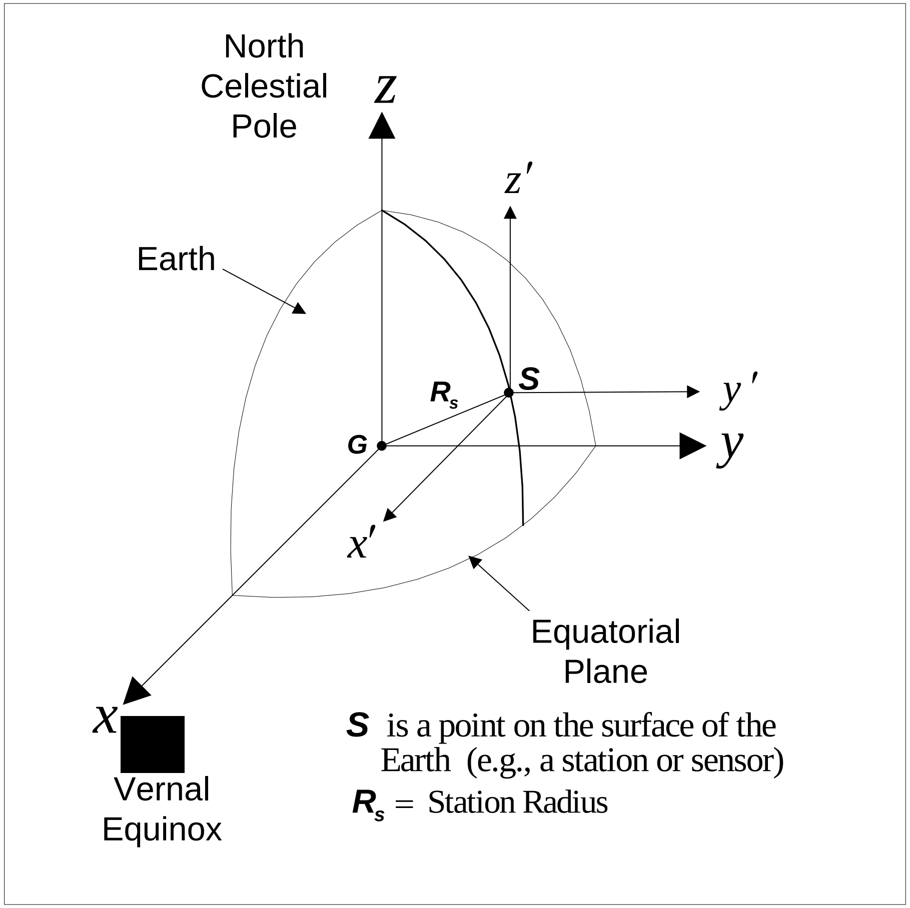
    Z points up (zenith)



From ASTRODYNAMICS CONCEPTS and TERMINOLOGY

Topocentric Inertial

- **Origin: at sensor S**
- **Fundamental plane: parallel to the equatorial plane**
- **Principal direction: points towards the vernal equinox of J2000 MEME frame**
- **When valid/applicable:**
  - At a radar's search (acquisition) time or when time tagging an observation
  - Used to locate objects relative to a GEODSS optical sensor
- **Unit vectors: None**
  - **Origin S** moves with sensor but the x'y'z' axes do not rotate



From ASTRODYNAMICS CONCEPTS And TERMINOLOGY

Sensor Tasking

- **Sensor capacity is a limited resource**
- **Tasking function determines collection requirements**
  - Object type, mission determines tasking priority (category, values 1-5)
    - Tasking priority is also affected by OD age
  - Minimum tracks, obs/day to maintain each satellite (suffix, large # of values)
- **Tasking allocates satellites to sensors (SP Tasker)**
  - First determine sensor/satellite visibility
  - Then estimate sensor response (detectability) for each satellite with visibility
  - Specify the number of obs/tracks for each satellite/sensor pair
  - Establish tracking priority for each satellite
- **Composite Tasking List (CTL) sent to all tasked sensors**
- **Tasking operates on a 24-hour cycle; only one tasking request set per day**

Site Mission Planning

- **Sites receive the CTL from JSpOC and plan data collection**
- **Mission planning allocates limited sensor resources to specific passes**
  - Calculate passes using Two-Line ELSETs from local catalog
  - Estimate sensor response using radar range equation (radars) or visual magnitude (optical)
  - Resource conflicts resolved by tasking category, i.e., when a conflict exists, go after the higher priority satellite
- **Observations are collected according to mission plan**
  - Plan may be superseded by special tasking in support of Space Situational Awareness (SSA)

Will All Tasked Satellites be Tracked? NO!

- **Sensor may experience an outage**
- **Sensor may have bad value for satellite "size" in database**
  - Presume cannot be tracked or allocate too little energy for detection
- **Sensor may not have enough energy/capacity to track object**
  - Tracking of higher-priority objects took more energy or time than expected
- **Position information from JSpOC may be so poor that satellite not acquired by sensor**
- **Observation quality may be so poor (large obs covariance) that the track is discarded**
- **Sensor may misassign observations to a different satellite, thus "losing" the tracking information**

What does all of this have to do with Conjunction Assessment?

- **CA events become known only by sensors' discovering the conjuncting objects in the first place**
  - Need for wide-area surveillance systems
  - No proposed systems to track down to the 1cm level, which is the hardening level for most spacecraft
- **As events develop, additional tracking is desired in order to refine the OD and refine the risk assessment**
  - Small objects can be tracked only by certain sensors, so much of the "fix-track" capability not helpful here
  - Conjuncting objects often have tasking increased to improve tracking, but this is subjected to the vicissitudes of the tasking process

ORBIT DETERMINATION

OD Concept Description

- **OD applies a set of force models to a pre-existing orbit estimate and satellite tracking observations to produce an estimate of the orbital state (a "state estimate") at a particular time (called the epoch time)**
- **This state estimate can then be propagated forward to estimate the satellite's position and velocity at a future time**
- **CA processes involve predicting primary and secondary satellite states forward in time to find the PCA and TCA**
  - This process only as good as the underlying OD that produces the epoch state estimates
  - Thus, some familiarity with OD specifics is necessary to understand CA subtleties

OD Force Models

ORBIT DETERMINATION

OD Force Modeling: 2-Body Motion

$$\ddot{\vec{r}} = \boxed{\ddot{\vec{r}}_{2B}} + \ddot{\vec{r}}_G + \ddot{\vec{r}}_D + \ddot{\vec{r}}_{LS} + \ddot{\vec{r}}_{RP}$$

$$\ddot{\vec{r}}_{2B} = -\frac{\mu\vec{r}}{r^3}$$

where

$\vec{r}$ = Vector from the center of the earth to the object

μ = Gravitational parameter (a constant)

r = Magnitude (length) of the vector

OD Force Modeling: Non-Spherical Earth

$$\ddot{\vec{r}} = \ddot{\vec{r}}_{2B} + \boxed{\ddot{\vec{r}}_G} + \ddot{\vec{r}}_D + \ddot{\vec{r}}_{LS} + \ddot{\vec{r}}_{RP}$$

$$\ddot{\vec{r}}_G = \left(\frac{\partial V}{\partial \vec{r}}\right)^T$$

where

$$V = \frac{\mu}{r}\left(\sum_{n=2}^{n_{\max}}\left(\frac{a_e}{r}\right)^n \sum_{m=0}^{n} P_{nm}(\sin\phi)\left[C_{nm}\cos m\lambda + S_{nm}\sin m\lambda\right]\right)$$

and

μ = *GM*
G = *Universal Constant of Gravitation*
M = *Mass of earth*
a_e = *Mean equatorial radius of the earth*
r = *Distance from center of earth to the object*
P_{nm} = *Legendre polynomials*
φ & λ = *latitude and longitude of sub-point*
C_{nm} *and* S_{nm} = *Constants called spherical harmonics whose values depend on the earth model selected*

OD Force Modeling: Atmospheric Drag

$$\ddot{\vec{r}} = \ddot{\vec{r}}_{2B} + \ddot{\vec{r}}_{G} + \boxed{\ddot{\vec{r}}_{D}} + \ddot{\vec{r}}_{LS} + \ddot{\vec{r}}_{RP}$$

$$\ddot{\vec{r}}_D = -\frac{1}{2}\frac{C_d A}{m}\rho v_a \vec{v}_a$$

where

$B_c = C_d A / m$ = Ballistic Coefficient = The DC solved-for Drag Term

C_d = Coefficient of drag, a constant between 1.0 and 4.0

A = Frontal area of the object that’s exposed to the atmosphere

m = Mass of the object

ρ = Local atmospheric density

$\vec{v}_a$ = Vector velocity of the object relative to the atmosphere

v_a = Magnitude of $\vec{v}_a$

OD Force Modeling: Third Body Effects (Solar and Lunar Gravity)

$$\ddot{\vec{r}} = \ddot{\vec{r}}_{2B} + \ddot{\vec{r}}_{G} + \ddot{\vec{r}}_{D} + \boxed{\ddot{\vec{r}}_{LS}} + \ddot{\vec{r}}_{RP}$$

$$\ddot{\vec{r}}_{LS} = -\mu_m\left(\frac{\vec{r}_{mb}}{\left|\vec{r}_{mb}\right|^3} + \frac{\vec{r}_{em}}{\left|\vec{r}_{em}\right|^3}\right) - \mu_s\left(\frac{\vec{r}_{sb}}{\left|\vec{r}_{sb}\right|^3} + \frac{\vec{r}_{es}}{\left|\vec{r}_{es}\right|^3}\right)$$

where

μ_m = Gravitational constant of the Moon

μ_s = Gravitational constant of the Sun

$\vec{r}_{mb}$ = Position vector from Moon to satellite

$\vec{r}_{sb}$ = Position vector from Sun to satellite

$\vec{r}_{em}$ = Position vector from Earth to Moon

$\vec{r}_{es}$ = Position vector from Earth to Sun

OD Force Modeling: Solar Radiation Pressure

$$\ddot{\vec{r}} = \ddot{\vec{r}}_{2B} + \ddot{\vec{r}}_G + \ddot{\vec{r}}_D + \ddot{\vec{r}}_{LS} + \boxed{\ddot{\vec{r}}_{RP}}$$

$$\ddot{\vec{r}}_{RP} = \Gamma \frac{\vec{r}_{sb}}{r_{sb}^{\ 3}}$$

where

$\Gamma = \gamma A / m$ = Solar radiation pressure coefficient (ASW DC solve-for parameter)

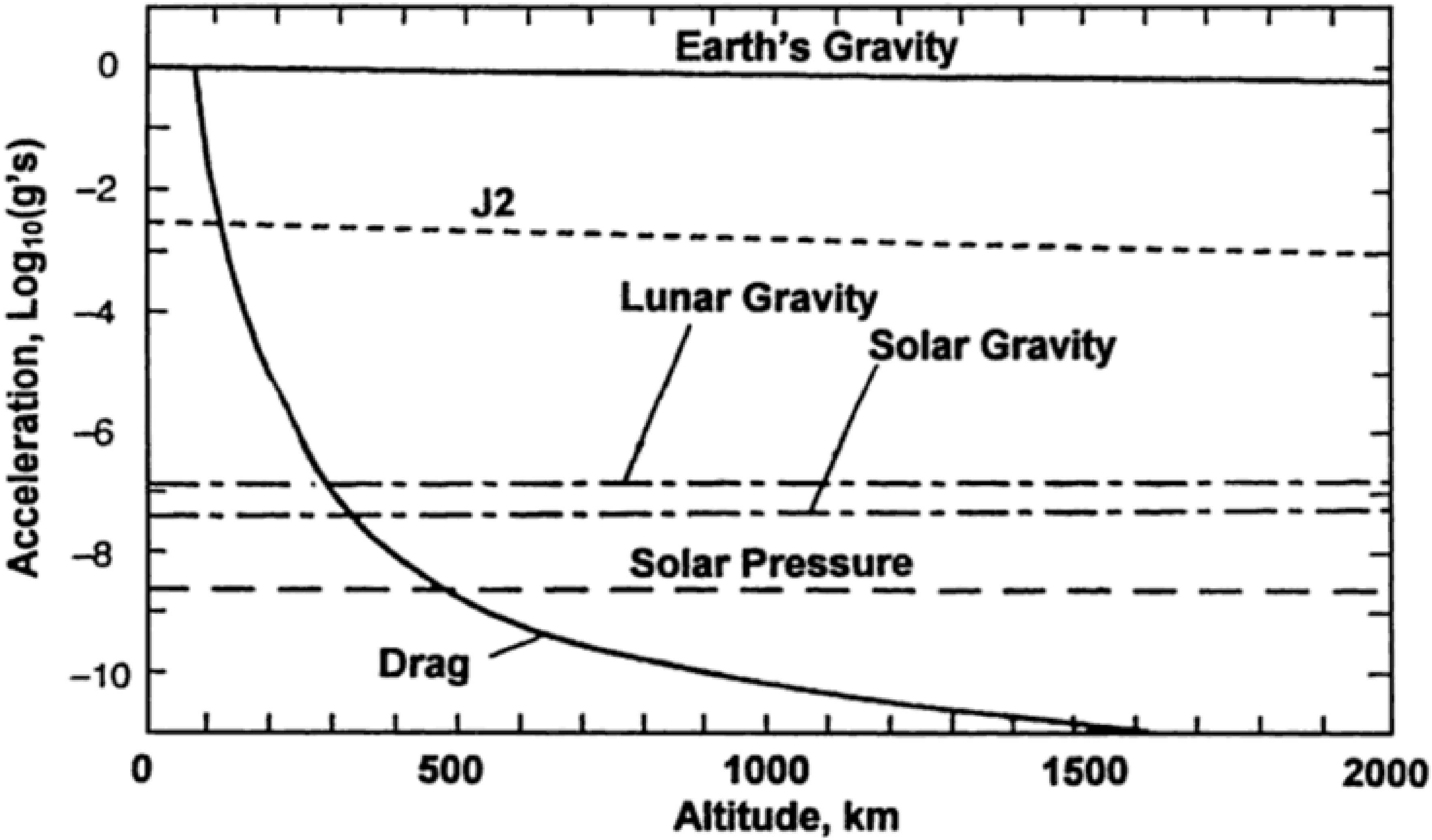
γ = Unit-less reflectivity coefficient of the satellite

A = Projected cross-sectional area perpendicular to the vector towards the sun

m = Satellite mass

$\vec{r}_{sb}$ = Inertial position vector from Sun to the satellite

Force Model Effects vs Altitude (normalized to force of Earth's gravity)



Reference: Spacecraft Systems Engineering, Fortescue and Stark

General vs Special Perturbations

- **General Perturbations (GP): the theory of TLEs**
  - Used for most of the space catalogue for most of SSA history, due to computer processing limitations
  - Simplified geopotential (J2) and analytic atmospheric drag models
  - Some truncated expressions throughout to simplify calculations
  - No solar radiation pressure or third-body effects modeled
  - Fast but imprecise
- **Special Perturbations (SP): the theory of SP vectors**
  - All above perturbations represented and handled numerically
  - All integration numeric
  - Relatively slow but quite precise
- **Originally, TLEs used for CA products**
  - Not precise enough to drive risk assessment and mitigation
- **Now SP-based products available**
  - Much better situation

OD Coordinate Systems

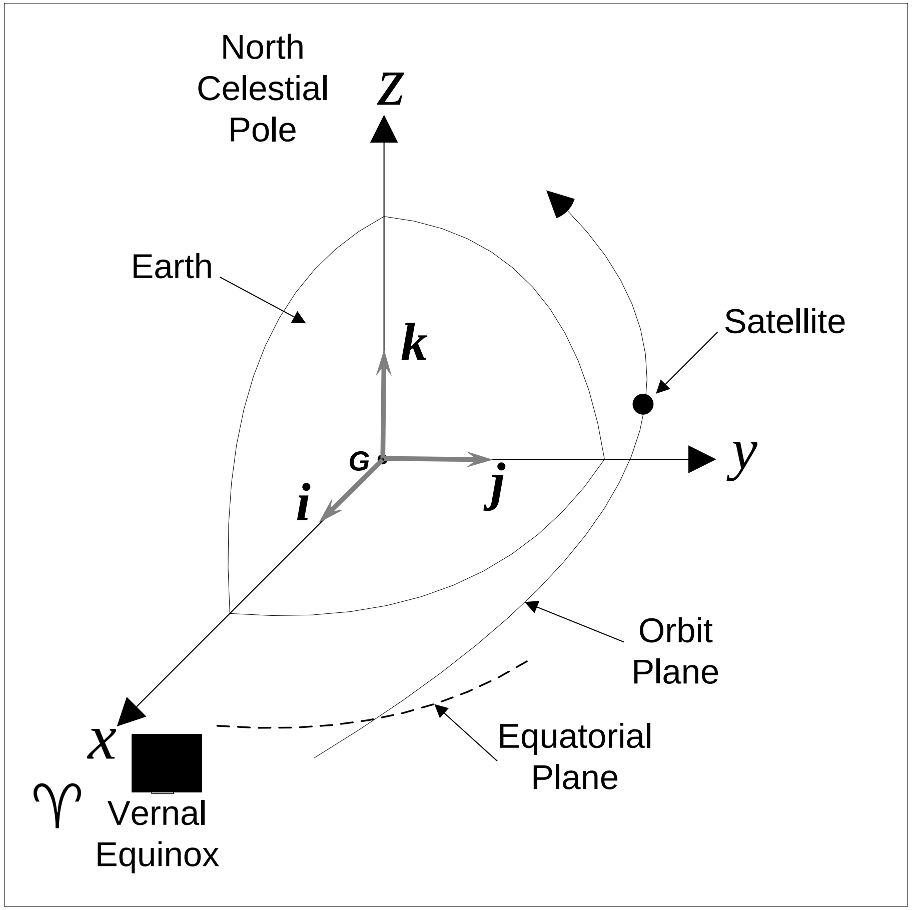
ORBIT DETERMINATION

Using Sensor Observations in OD Updates

- **Sensor radar observations are taken in a topocentric rotating coordinate system**
  - Optical measurements are generally taken in topocentric intertial
- **OD generally conducted in an inertial framework**
  - Earth-centered Inertial, either in Cartesian or Equinoctial elements
- **Coordinate transformation thus required in order to transform sensor observations into usable data in OD**

Earth Centered Inertial (ECI) Reference Frame

- **Origin: at center of Earth**
- **Fundamental plane: is the plane of the equator**
- **Principal direction: along the line formed by the intersection of the equatorial plane and the ecliptic plane**
- **When valid/applicable:**
  - **At epoch (fixed instant) of the coordinate system**
  - **Used to (1) depict motion using Newton's laws and (2) represent points in an ephemeris file**
- **Associated unit vectors: *i, j, k***
  - ***k* along Earth's rotational axis**
  - ***i* points to vernal equinox**



Coordinate frame pictures from ASTRODYNAMICS CONCEPTS and TERMINOLOGY (Author: William N. Barker, Omitron, Inc.)

OD General Description and Errors

ORBIT DETERMINATION

General Description of Batch OD

- **For simplicity, presume solving in Cartesian coordinates (X, Y, Z, Xdot, Ydot, Zdot, all in ECI)**
- **Collect set of observations taken throughout fit-span**
- **Calculate “predicted” ECI positions at point of each observation using a linearization of the force models explained previously**
- **Calculate the residuals at each of these points**
- **Set the partial derivatives of the equations for the squared residual values equal to zero (this approach used to define a maximum)**
- **Solve the non-linear equations and thus determine the “differential” amounts to be added to the position and velocity values**
- **Continue this iterative process until the weighted residual RMS changes less than a specified tolerance**
  - This completes the “differential correction” of the orbit

Drag Solution: Largest Source of OD Error

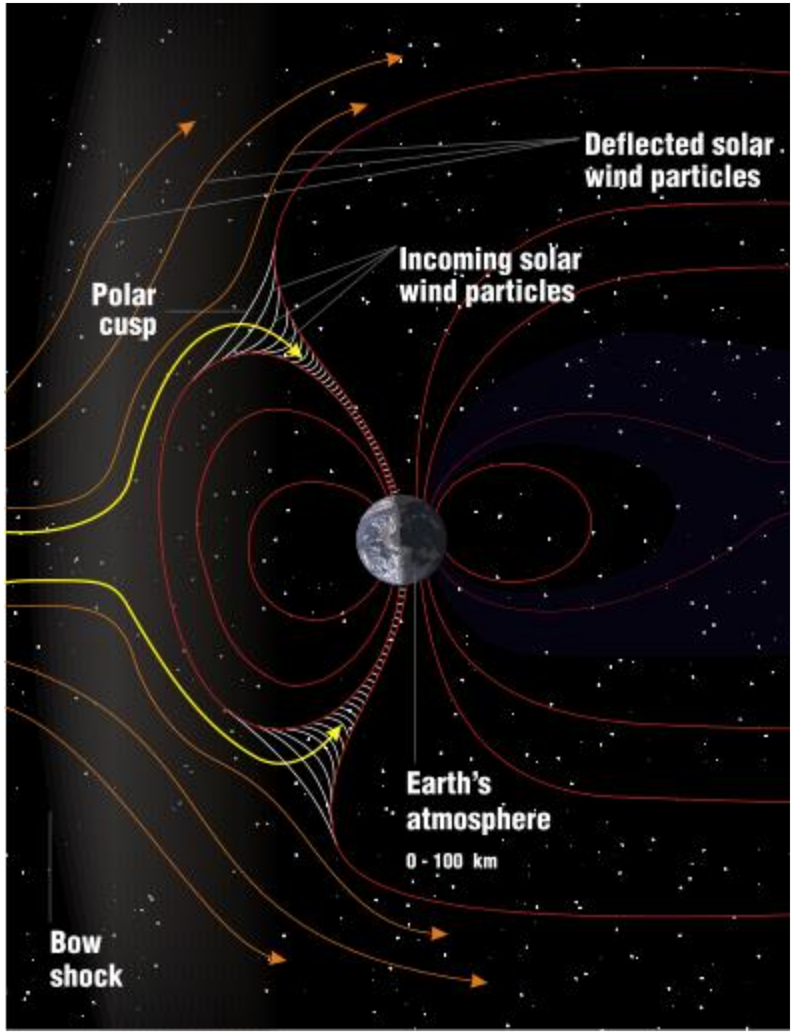
- **Mostly due to difficulty in predicting atmospheric density**
  - Uncertainties based on poor drag coefficient solution a distant second
- **This in turn due to difficulties in estimating atmospheric temperature**
  - Temperature and density related through ideal gas law (remember high school chemistry?) and hydrostatic pressure law
  - Bottom line: if can estimate temperature, can calculate expected density

Thermospheric Heating: Earth Conduction and EUV Solar Heating

- **Diurnal variations**
  - Day-to-night variations in the heating of the spherical Earth
  - Heat reaches bottom of Thermosphere via conduction/convection; heats remainder of Thermosphere by conduction
- **Semiannual variations**
  - Uneven heating of spherical earth at the solstices
  - Changes relative densities of the different Thermosphere gases
- **Solar activity**
  - Radiative heating of atomic, ionic, and molecular nitrogen, oxygen, hydrogen, and some helium/argon
  - Extreme ultraviolet and x-ray radiation most strongly absorbed by these gases
  - Sun temporally uniform in visible band; notably variant in EUV/X bands
    - 27-day solar rotation causes pockets of activity to move in and out of visibility
    - 11-year "solar cycle" brings peaks/troughs in overall level of activity
  - Measurements of EUV/X activity are good proxies of amount of heat absorbed

Thermospheric Heating: Joule Heating through Solar Ejecta (Storms)

- **Geomagnetic activity**
  - Sun constantly ejecting charged particles: solar wind
  - Most prevented from encountering Earth by planet's magnetic field
    - Small percentage can enter at the poles through "polar cusps"
  - Solar storms produce bursts of such particles
    - Those that enter the atmosphere cause ionization and other interactions; both produce atmospheric heating
    - Can cause very large short-term density variations
  - Measurements of irregularities in Earth's magnetic field can determine level of such activity



Solar Radiation Pressure Effects

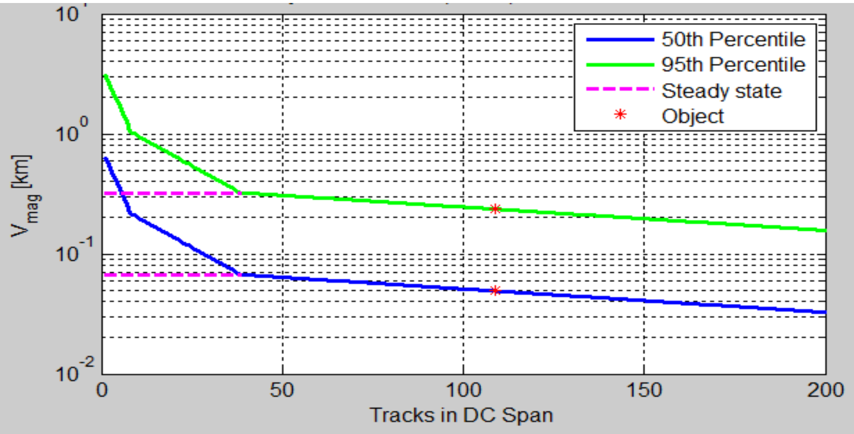
- **SRP effects an issue for deep-space satellites, where drag effect is small(er)**
- **Force is always in anti-solar direction and depends on satellite illumination and area/mass ratio**
  - High area-to-mass ratio satellites can be heavily influenced by SRP (factor of 10 greater than drag effects) and can be very difficult to correct or predict accurately

OD Quality Factors

ORBIT DETERMINATION

OD Quality Determinant: Tracking Adequacy

- **General relationship between amount of tracking and resultant OD quality**
  - “Hybrid” relationship: exponential relationship with smaller amounts of tracking; linear to almost zero-slope relationship with large tracking amounts
- **For CA, would like tracking for secondaries to be in the “flatter” part of the curve, which represents the main part of the distribution**
  - Once CA event is identified, increased tasking can be used (if necessary) to try to accomplish this



OD Quality Determinant: Fit Statistics

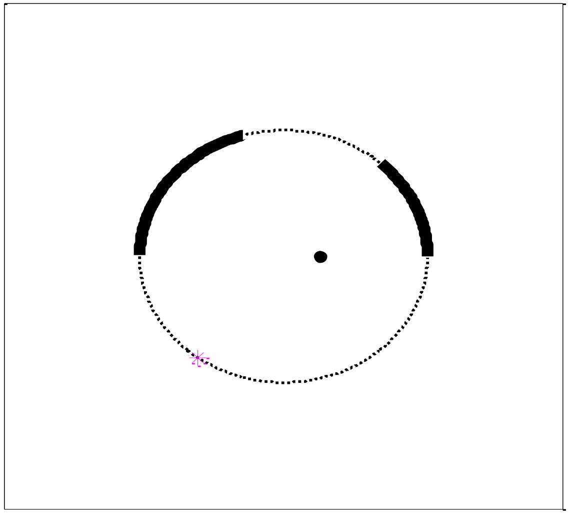
- **Typically, quality of a fit represented by average size of residuals**
- **JSpOC ODs weight individual observables by the expected error in those observables**
  - Determined by evaluating sensor observation errors against reference orbits
- **Therefore, weighted root-mean square (WRMS) method to use to evaluate fit quality**
  - Mean of the squares of the weighted residuals (residuals divided by standard deviations of their expected errors

$$Weighted\ RMS = \sqrt{\frac{\sum_{i=1}^{n}\left(\frac{r_i}{\sigma_i}\right)^2}{n}}$$

- **Values close to unity indicate a good fit**
  - Very large or small values indicate questionable fit
  - For CA purposes, requested that such fits be re-executed manually

OD Quality Determinant: Orbit Distribution

- **Tracking Distribution**
  - Poor distribution affects OD quality
  - Once 50% of the orbit arc is tracked, any additional distribution has rather little additional benefit
- **Evaluation method**
  - Divide orbit into sectors (usually 6)
  - Determine the number of sectors that contain observations in the present fit-span
- **If only one or two sectors, additional tracking should be considered**
- **Also desirable to have tracking in sector in which TCA will occur**



What does all of this have to do with Conjunction Assessment?

- **Accuracy of close-approach prediction dependent on quality of OD for primary and secondary objects**
  - Primary usually more orbitally stable object and tracked more thoroughly
  - OD quality issues arise more frequently with secondaries
- **Problems in modeling of atmospheric drag and solar radiation pressure frequent cause of OD difficulties for CA**
  - Solar storms, particularly those that arise in the middle of a CA event, cause particular difficulties
  - Solar radiation pressure is relatively new problem for CA but does influence deep-space CA state estimates and covariances
- **If solution is poor, consider remediation approaches**
  - Requests for additional tracking
  - Manual execution of questionable ODs

OD UNCERTAINTY: COVARIANCE

OD Solutions

- **Purpose of OD**
  - Generate estimate of the object's state at a given time (called the *epoch time*)
  - Generate additional parameters and constructs to allow object's future states to be predicted (accomplished through orbit *propagation*)
  - Generate a statement of the estimation error, both at epoch and for any predicted state (usually accomplished by means of a *covariance matrix*)
- **Error types**
  - OD approaches (either batch or filter) presume that they solve for all significant systematic errors
  - Remaining solution error is thus presumed to be random (Gaussian) error
  - Sometimes this error can be intentionally inflated to try to improve the fidelity of the error modeling
  - Nonetheless, presumed to be Gaussian in form and unbiased

OD Parameters Generated by ASW Solutions

- **Solved for: State parameters**
  - Six parameters needed to determine 3-d state fully
  - Cartesian: three position and three velocity parameters in orthogonal system
  - Element: six orbital elements that describe the geometry of the orbit
- **Solved for: Non-conservative force parameters**
  - Ballistic coefficient (C_DA/m); describes vulnerability of spacecraft state to atmospheric drag
  - Solar radiation pressure (SRP) coefficient (C_RA/m); describes vulnerability of spacecraft state to visible light momentum from sun
- **Considered: ballistic coefficient and SRP consider parameter**
  - Not solved for but “considered” as part of the solution
  - Derived from information outside of the OD itself
  - Discussed later

OD Uncertainty Modeling

- **Characterizes the overall uncertainty of the OD epoch and/or propagated state**
  - Uncertainty of each estimated parameter and their interactions
- **This is a characterization of a multivariate statistical distribution**
- **In general, need the four cumulants to characterize the distribution**
  - Mean, variance, skewness, and kurtosis; and their mutual interactions
  - Requires higher-order tensors to do this for a multivariate distribution
- **Assumptions about error distribution can simplify situation substantially**
  - Presuming the solution is unbiased places the mean error values at zero
  - Presuming the error distribution is Gaussian eliminates the need for the third and fourth cumulants
  - Error distribution can thus be expressed by means of variances of each solved-for component and their cross-correlations
  - Thus, error can be fully represented by means of a covariance matrix

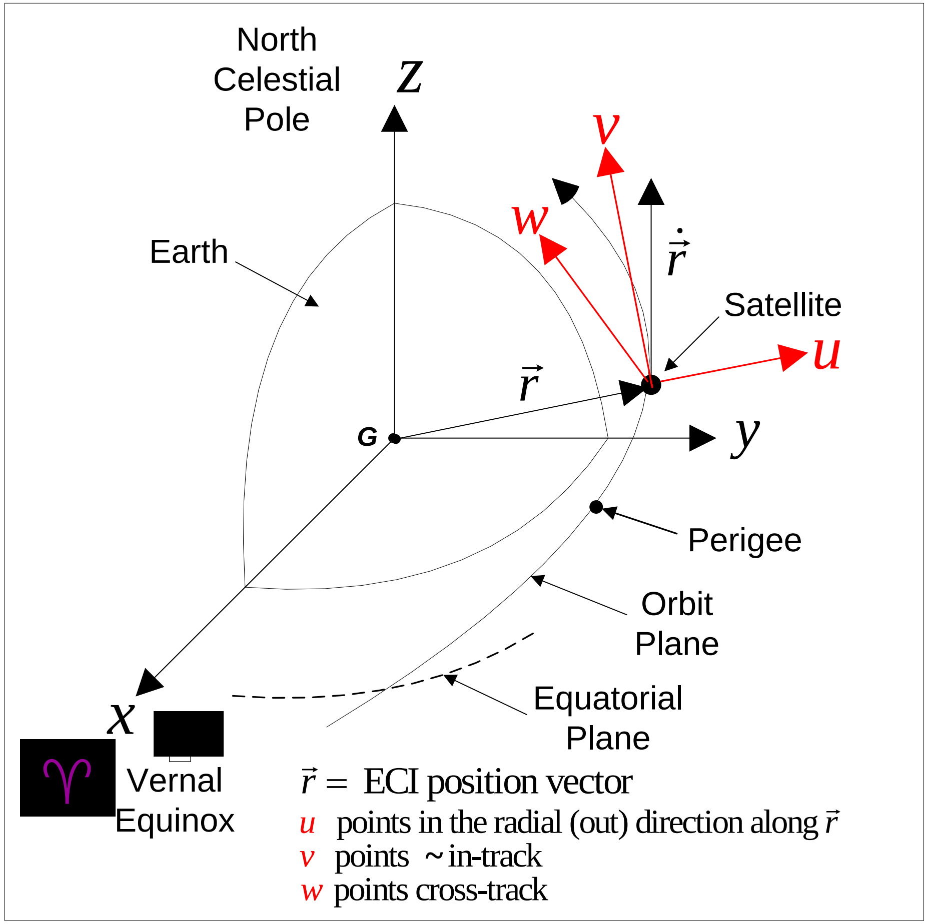
Covariance Matrix Construction: Symbolic Example

- Three estimated parameters (a, b, and c)
- Variances of each along diagonal
- Off-diagonal terms the product of two standard deviations and the correlation coefficient (ρ); matrix is symmetric

	a	**b**	**c**	…
a	σ_a^2	$\rho_{ab}\sigma_a\sigma_b$	$\rho_{ac}\sigma_a\sigma_c$	…
b	$\rho_{ab}\sigma_a\sigma_b$	σ_b^2	$\rho_{bc}\sigma_a\sigma_c$	…
c	$\rho_{ac}\sigma_a\sigma_c$	$\rho_{bc}\sigma_a\sigma_c$	σ_c^2	…
…	…	…	…	…

Covariance often Expressed in Satellite Centered (UVW) Coordinate Frame

- **Origin: at satellite**
- **Fundamental plane: established by the instantaneous position and velocity vectors of the satellite**
- **Principal direction: along the radius vector to the satellite**
- **When valid/applicable:**
  - Valid at time tag for the point
  - Used to represent miss distances relative to the Primary in an Orbital Conjunction Message (OCM)
- **Unit vectors: *u*, *v*, *w***
  - ***w*** is perpendicular to the position and velocity vectors
  - ***v*** established by the right hand rule ***w*** X ***u*** = ***v***



Coordinate frame pictures from ASTRODYNAMICS CONCEPTS and TERMINOLOGY (Author: William N. Barker, Omitron, Inc.)

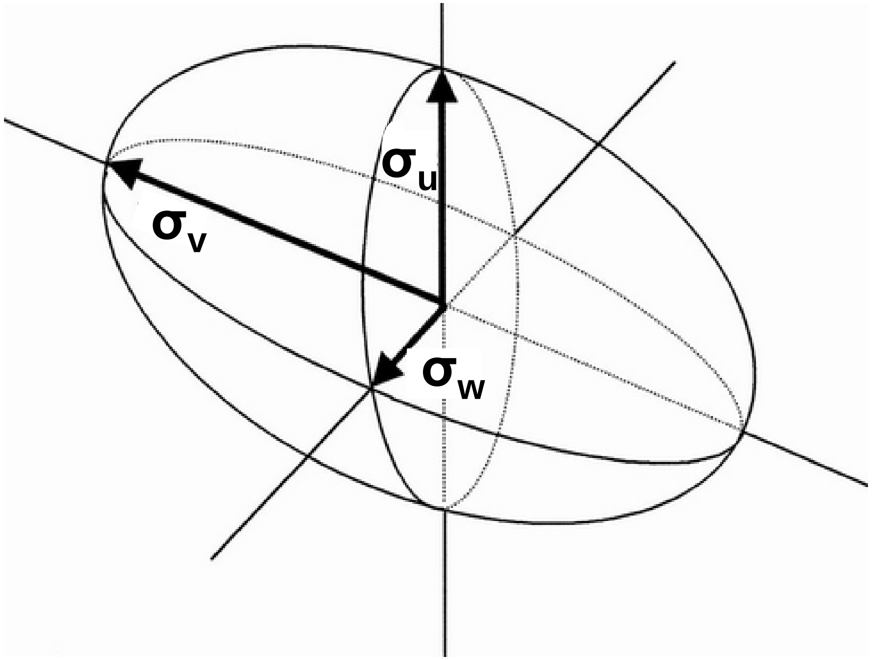
Example Covariance from CDM

- **8 x 8 matrix typical of most ASW updates**
  - Some orbit regimes not suited to solution for both drag and SRP; these covariances 7 x 7
- **Mix of different units often creates poorly conditioned matrices**
  - Condition number of matrix at right is 9.8E+11—terrible!
- **Often better numerically (and more intuitive) to separate matrix into sections**
- **First 3 x 3 portion (amber) is *position* covariance—often considered separately**

	U (m)	V (m)	W (m)	Udot (m/s)	Vdot (m/s)	Wdot (m/s)	B (m2/kg)	AGOM (m2/kg)
U	6.84E+01	-2.73E+02	6.38E+00	2.76E-01	-7.14E-02	8.75E-03	-3.83E-02	-3.83E-02
V	-2.73E+02	1.10E+05	3.23E+01	-1.17E+02	-8.99E-02	2.51E-02	-1.28E-01	-1.28E-01
W	6.38E+00	3.23E+01	4.47E+00	-3.26E-02	-6.83E-03	1.81E-03	-3.73E-03	-3.73E-03
Udot	2.76E-01	-1.17E+02	-3.26E-02	1.24E-01	1.10E-04	-2.47E-05	1.46E-04	1.46E-04
Vdot	-7.14E-02	-8.99E-02	-6.83E-03	1.10E-04	7.57E-05	-9.39E-06	4.10E-05	4.10E-05
Wdot	8.75E-03	2.51E-02	1.81E-03	-2.47E-05	-9.39E-06	2.06E-05	-4.39E-06	-4.39E-06
B	-5.07E-03	1.30E+00	4.34E-05	-1.38E-03	7.97E-07	7.26E-07	1.64E-05	-6.28E-07
AGOM	-3.83E-02	-1.28E-01	-3.73E-03	1.46E-04	4.10E-05	-4.39E-06	-6.28E-07	2.31E-05

Position Covariance Ellipse

- **Position covariance defines an "error ellipsoid"**
  - Placed at predicted satellite position
  - Square root of variance in each direction defines each semi-major axis (UVW system used here)
  - Off-diagonal terms rotate the ellipse from the nominal position shown
- **Ellipse of a certain "sigma" value contains a given percentage of the expected data points**
  - 1-σ: 19.9%
  - 2-σ: 73.9%
  - 3-σ: 97.1%
  - Note how much lower these are than the univariate normal percentage points



Batch Epoch Covariance Generation (1 of 2)

- **Batch least-squares update (ASW method) uses the following minimization equation**
  - $dx = (A^T W A)^{-1} A^T W b$
    - dx is the vector of corrections to the state estimate
    - A is the time-enabled partial derivative matrix, used to map the residuals into state-space
    - W is the "weighting" matrix that provides relative weights of observation quality (usually 1/σ, where σ is the standard deviation generated by the sensor calibration process)
    - b is the vector of residuals (observations – predictions from existing state estimate)
- **Covariance is the collected term $(A^T W A)^{-1}$**
  - A the product of two partial derivative matrices:
    - $A = \frac{\partial(obs)}{\partial X_0} = \frac{\partial(obs)}{\partial X}\frac{\partial X}{\partial X_0}$
    - First term: partial derivatives of observations with respect to state at obs time
    - Second term: partial derivatives of state at obs time with respect to epoch state

Batch Epoch Covariance Generation (2 of 2)

- **Formulated this way, this covariance matrix is called an *a priori* covariance**
  - A does not contain actual residuals, only transformational partial derivatives
  - So $(A^TWA)^{-1}$ is a function only of the amount of tracking, times of tracks, and sensor calibration relative weights among those tracks
    - Not a function of the actual residuals from the correction
- **Limitations of *a priori* covariance**
  - Does not account well for unmodeled errors, such as transient atmospheric density prediction errors
    - Because not examining actual fit residuals
  - W-matrix only as good as sensor calibration process
    - Principal weakness of present process, but expected to be improved eventually with JSpOC Mission System (JMS) upgrades

Covariance Propagation Methods

- **Full Monte Carlo**
  - Perturb state at epoch (using covariance), propagate each point forward to t_n with full non-linear dynamics, and summarize distribution at t_n
- **Sigma point propagation**
  - Define small number of states to represent covariance statistically, propagate set forward by time-steps, reformulate sigma point set at each time-step, and use sigma point set at t_n to formulate covariance at t_n
- **Linear mapping**
  - Create a state-transition matrix by linearization of the dynamics and use it to propagate the covariance to t_n by pre- and post-multiplication
- **All three of above methods legitimate**
  - List moves from highest to lowest fidelity and computational intensity
  - JSpOC uses linear mapping approach

Covariance Tuning

- **For CA, position covariance needs to be a realistic representation of the state uncertainty volume at the propagation point of interest**
- **Two aspects to this requirement**
  - Does the position error volume conform to a trivariate Gaussian distribution?
  - If so, is it of the proper dimensions and orientation?
- **Regarding the first item, extensive study has confirmed that this is not an issue for high-PC events (Pc>1E-04)**
  - Ghrist and Plakalovic (2012)
  - 248 cases examined in different orbit regimes, with prop times of 2 to 7 days
  - 2-d Pc calculation compared to Monte Carlo (with 4E+07 trials)
  - Only one case of more than 10% deviation between 2-d and MC calculation
    - And 10% deviation not considered operationally significant
  - Explanation: high Pc requires covariance overlap near the centers of the covariances—a part that is not affected by non-Gaussian alterations
- **Second item is area of legitimate concern**

Covariance Tuning: Covariance Realism Evaluation Method

- **Presume reference orbit (or precision observation) available for a satellite**
- **Position differences between predicted ephemeris and precision position (from reference orbit or observation) are dU, dV, and dW**
  - Can be collected into vector ε
- **Mahalanobis distance ($\varepsilon * C^{-1} * \varepsilon^T$) represents the ratio of the difference to the covariance's prediction**
  - For a trivariate distribution, expected value is 3
- **A group of such calculations should conform to a chi-squared distribution with three degrees of freedom**
- **This method (distribution testing of groups of such calculations) used to determine if covariance properly sized**

Covariance Tuning: Covariance Irrealism Remediation

- **Examine individual component performance of covariance modeling to determine principal sources of the irrealism**
  - Deviation probably stems from non-conservative force modeling (drag and/or solar radiation pressure)
- **If using process noise, tune/modify process noise matrix to attempt to compensate**
  - Originally directed at geopotential mismodeling; but with common use of higher-order theories, no longer the principal source of errors
- **If using batch methods, include consider parameters**
  - Additive value applied to either the drag or solar radiation pressure variances (or both) in order to make them larger
    - Poor modeling of these phenomena requires larger uncertainty estimate
  - Through cross-correlation terms, these variances will affect the other covariance parameters through the linear state transition
- **Continue tuning process until proper distribution of calculated Mahalanobis distances achieved**

What does all of this have to do with Conjunction Assessment?

- **The covariance is an integral part of the computation of the probability of collision (Pc)**
  - Pc is single metric that encapsulates the collision risk
- **Reliable covariances for primary and secondary objects almost as important as reliable state estimates for determining Pc and therefore collision risk**
- **Covariance production and tuning matters of great interest to CA enterprise**
- **Methods to compensate for covariance determination issues discussed in Part 2 of this course**

2-D PC COMPUTATION

Calculating Probability of Collision (Pc): 3D Situation at Time of Closest Approach (TCA)

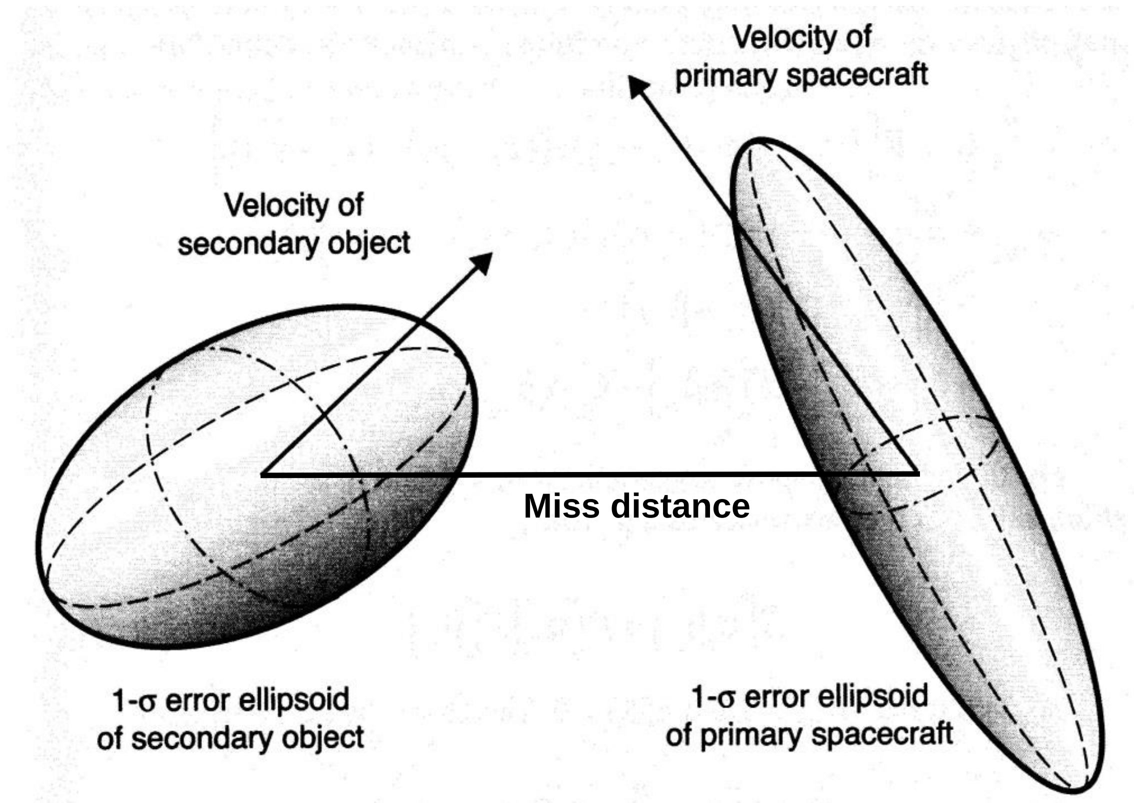


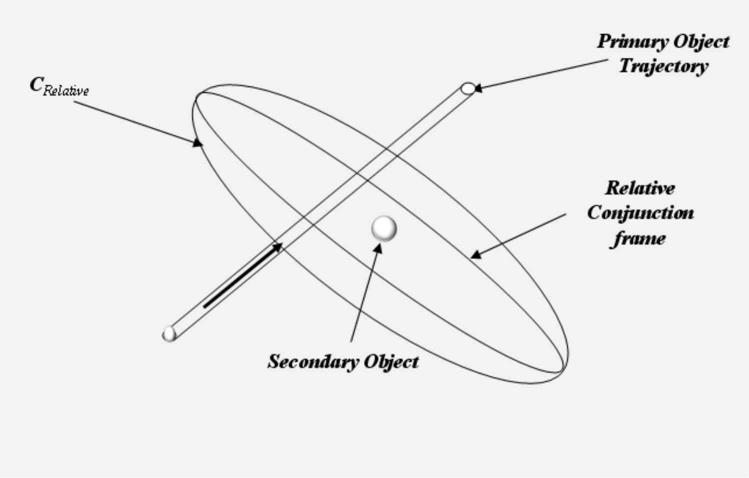
Figure taken from Chan (2008)

Calculating Pc: 2-D Approximation (1 of 3) Combining Error Volumes

- **Assumptions**
  - Error volumes (position random variables about the mean) are uncorrelated
- **Result**
  - All of the relative position error can be centered at one of the two satellite positions
    - Secondary satellite is typically used
  - Relative position error can be expressed as the additive combination of the two satellite position covariances (proof given in Chan 2008)
    - $C_a + C_b = C_c$
  - Must be transformed into a common coordinate system, combined, and then transformed back

Calculating Pc: 2-D Approximation (2 of 3) Projection to Conjunction Plane

- **Combined covariance centered at position of secondary at TCA**
- **Primary path shown as "soda straw"**
- **If conjunction duration is very short**
  - Motion can be considered to be rectilinear—soda straw is straight
  - Conjunction will take place in 2-d plane normal to the relative velocity vector and containing the secondary position
  - Problem can thus be reduced in dimensionality from 3 to 2
- **Need to project covariance and primary path into "conjunction plane"**



Calculating Pc: 2-D Approximation (3 of 3) Conjunction Plane Construction

- **Combined covariance projected into plane normal to the relative velocity vector and placed at origin**
- **Primary placed on x-axis at (miss distance, 0) and represented by circle of radius equal to sum of both spacecraft circumscribing radii**
- **Z-axis perpendicular to x-axis in conjunction plane**

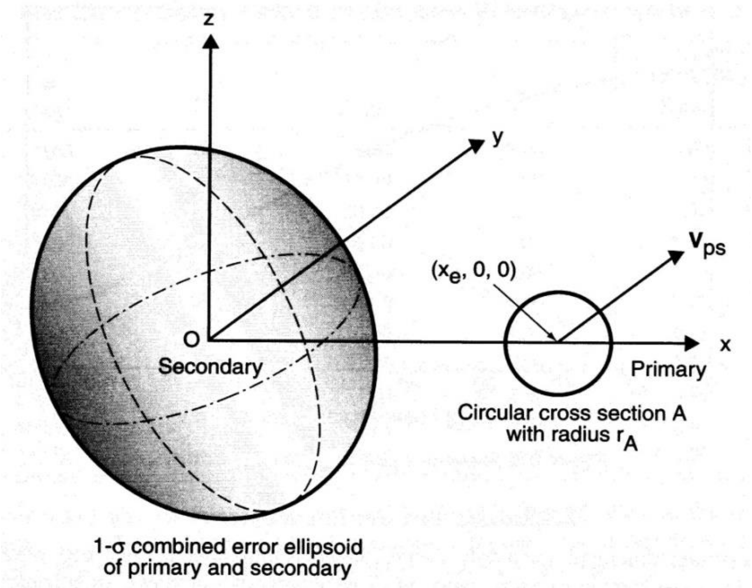
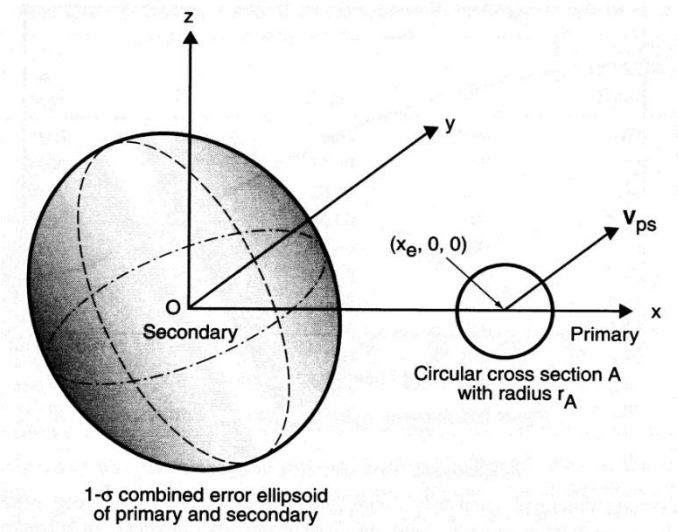


Figure taken from Chan (2008)

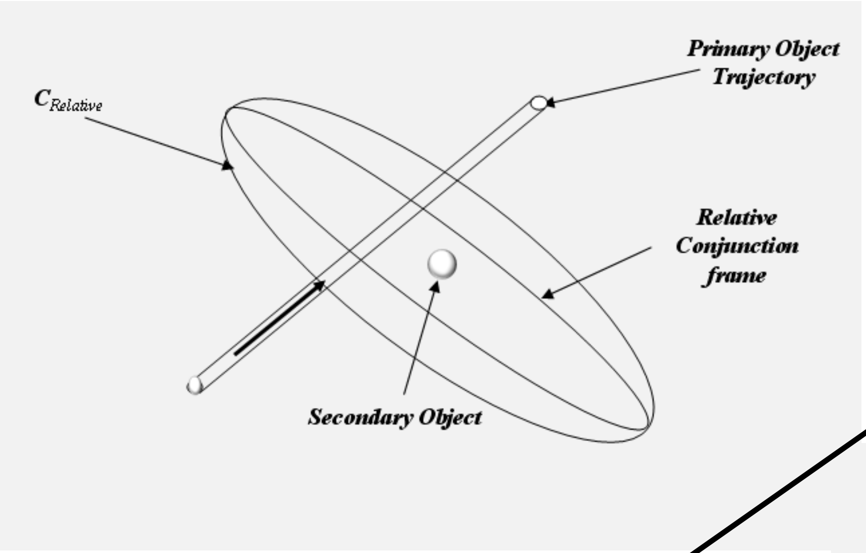
2-D Probability of Collision Computation

- **Rotate axes until they align with principal axes of projected covariance ellipse**
- **Pc is then the portion of the density that falls within the HBR circle**
  - r is [x z] and C* is the projected covariance

$$P_C = \frac{1}{\sqrt{(2\pi)^2 \left|C^*\right|}} \iint_A \exp\left(-\frac{1}{2}\vec{r}^{\,T} C^{*-1} \vec{r}\right) dX dZ$$

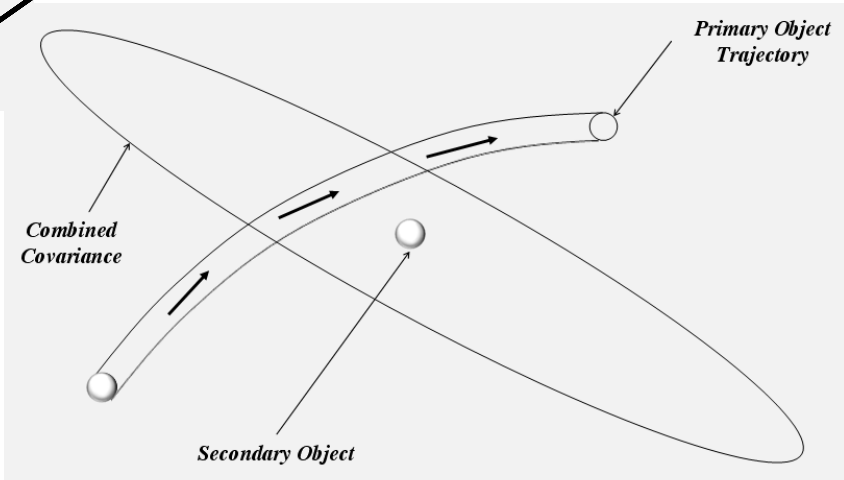


2-D vs. 3-D Conjunction Geometry



2-D Geometry

3-D Geometry



Monte Carlo Description

- **If relative velocity between primary and secondary too small (< 10 m/s, or encounter durations longer than 500s), 2-D rectilinear assumption breaks down**
- **Best alternative in this case is to use Monte Carlo approach**
  - TCA may not be point of highest risk in low-velocity cases
- **Full, propagated Monte Carlo procedure**
  - Perturb primary and secondary positions (and perhaps velocities) at vector epochs, using epoch covariances for each
  - Propagate each forward until region of close approach passed
  - Determine whether the two trajectories come within a proximity tolerance of each other
  - Divide number of proximity violations by number of overall trials; this quotient is an empirical Pc
  - Lower-risk situations may require a large number of trials to produce meaningful results

What does all of this have to do with Conjunction Assessment?

- **The Pc calculation is the core of Conjunction Assessment risk evaluation**
- **The 2-D Pc calculation approach is adequate for most close approaches**
- **Monte Carlo necessary for those few cases that do not conform to the short-duration assumption**

JSPOC SCREENINGS

JSpOC Screening Fundamentals

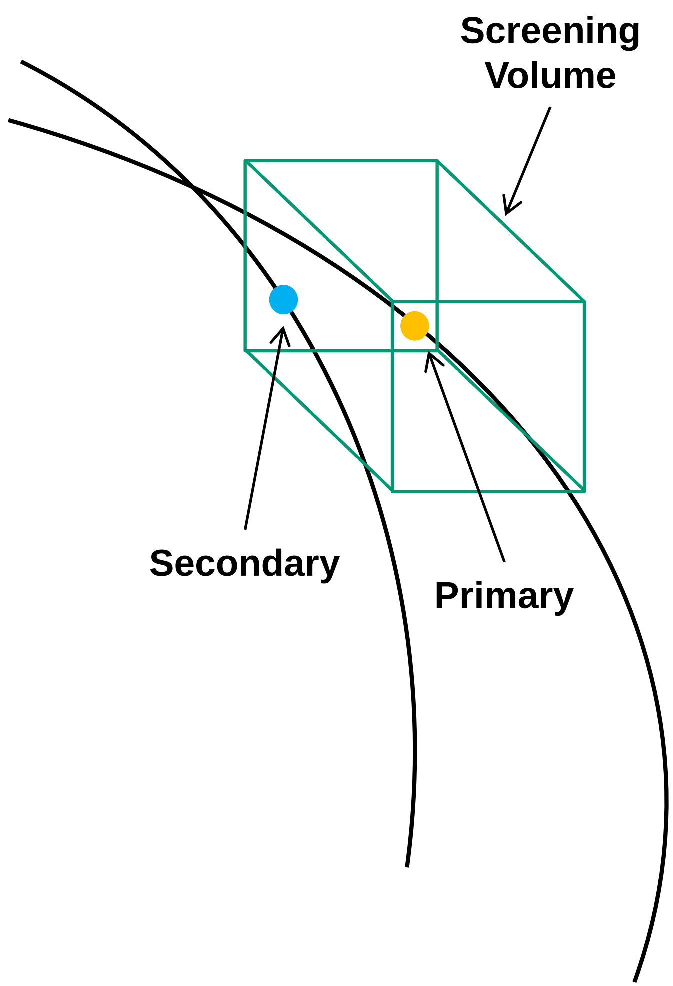
- **Screening is a JSpOC process that determines which secondary satellites will pass within a specified distance of a primary (protected) asset**
- **Screening consists of four parts:**
  - Filtering out secondary satellites that cannot possibly collide with the primary and thus do not need further analysis
  - Of the remaining satellites, comparing ephemerides of primary and secondary to determine whether a secondary represents a penetration of the screening volume
  - Of the "penetrating satellites," determining which have componentized miss distances smaller than set thresholds
  - Of these satellites that violate these thresholds, generating a Conjunction Data Message (CDM) that gives states and covariances of both objects at TCA, as well as other conjunction and OD information

Screening Filtering

- **The following three filters are commonly used (derived from Hoots 1984)**
  - Perigee-apogee comparisons between primary and secondary—identify cases in which difference exceeds a threshold that indicates no possibility of collision
  - Closest point between both elliptical trajectories—analytic method to find closest point between the two orbits and, if larger than a threshold, dismiss pair as extremely unlikely to collide
  - Closest approach between two reasonably close orbits—analytical method to consider orbital positions (treated as angles) and determine if these remain large enough to eliminate pairing as conjunctors
- **Pairings remaining after filtering are subjected to the “fly by” test (next chart)**

"Fly By" Ephemeris Comparison

- **Generate ephemerides for primary and secondaries that are possible threats**
- **Construct screening volume box (or ellipsoid) about primary**
- **"Fly" the box along the primary's ephemeris**
- **Any penetrations of box constitute possible conjunctions**
- **For these conjunctions, generate CDM**
  - State estimates and covariances at TCA
  - Relative encounter information
  - OD information



CDM CONTENTS

CDM Contents:
Conjunction (rather than object) Information

```
CCSDS_CDM_VERS              =1.0
CREATION_DATE               =2015-106T18:19:13.000
ORIGINATOR                  =JSPOC
MESSAGE_FOR                 =                    NASA/GSFC
MESSAGE_ID                  =12345_conj_45678_2015107235948
TCA                         =2015-107T23:59:48.867
MISS_DISTANCE               =8083                     [m]
RELATIVE_SPEED              =12067                    [m/s]
RELATIVE_POSITION_R         =-184.5                   [m]
RELATIVE_POSITION_T         =4764.9                   [m]
RELATIVE_POSITION_N         =6526.6                   [m]
RELATIVE_VELOCITY_R         =-21.6                    [m/s]
RELATIVE_VELOCITY_T         =-9745.0                  [m/s]
RELATIVE_VELOCITY_N         =7118.0                   [m/s]
```

- **Creation time – not necessarily the time of either OD**
  - **Time of closest approach (will change slightly with updates)**
    - **Overall miss distance and relative speed**
      - **Relative position/velocity in RTN coordinates (another name for RIC or UVW, previously defined)**

CDM Contents:
Object OD Information—Force Model Settings

```
OBJECT                            =OBJECT1
OBJECT_DESIGNATOR                 =12345
CATALOG_NAME                      =SATCAT
OBJECT_NAME                       =NASASat
INTERNATIONAL_DESIGNATOR          =2015-001
EPHEMERIS_NAME                    =NONE
COVARIANCE_METHOD                 =CALCULATED
MANEUVERABLE                      =N/A
REF_FRAME                         =ITRF
GRAVITY_MODEL                     =EGM-96: 36D 36O
ATMOSPHERIC_MODEL                 =JBH09
N_BODY_PERTURBATIONS              =MOON,SUN
SOLAR_RAD_PRESSURE                =YES
EARTH_TIDES                       =YES
INTRACK_THRUST                    =NO
```

- **Object/Ephemeris identification information**

- **Force model settings (geopotential, atmosphere, third-body effects, SRP, solid earth tides, and thrust.**

CDM Contents: Object OD Information—OD Factors and Quality

TIME_LASTOB_START	=2015-105T18:19:13.000	
TIME_LASTOB_END	=2015-106T18:19:13.000	
RECOMMENDED_OD_SPAN	=3.92	[d]
ACTUAL_OD_SPAN	=0.98	[d]
OBS_AVAILABLE	=1187	
OBS_USED	=242	
RESIDUALS_ACCEPTED	=94.8	[%]
WEIGHTED_RMS	=1.219	
AREA_PC	=7.8760	[m**2]
CD_AREA_OVER_MASS	=0.035393	[m**2/kg]
CR_AREA_OVER_MASS	=0.048694	[m**2/kg]
THRUST_ACCELERATION	=0.00000E+00	[m/s**2]
SEDR	=3.68502E-04	[W/kg]

- **Obs span – given in actual times if allowed; if not, the ob span coming from the Dynamic LUPI algorithm and the actual obs span used (in days) is reported**
  - **The total number of obs in the recommend obs span, the total actually used, and of those the % of residuals actually accepted**
    - **The weighted RMS of the OD (ideal value is unity)**
      - **Cross-sectional area of satellite (estimated by RCS), ballistic coefficient, SRP coefficient, thrust, and energy dissipation rate**

CDM Contents:
Object OD Information—State Estimate at TCA

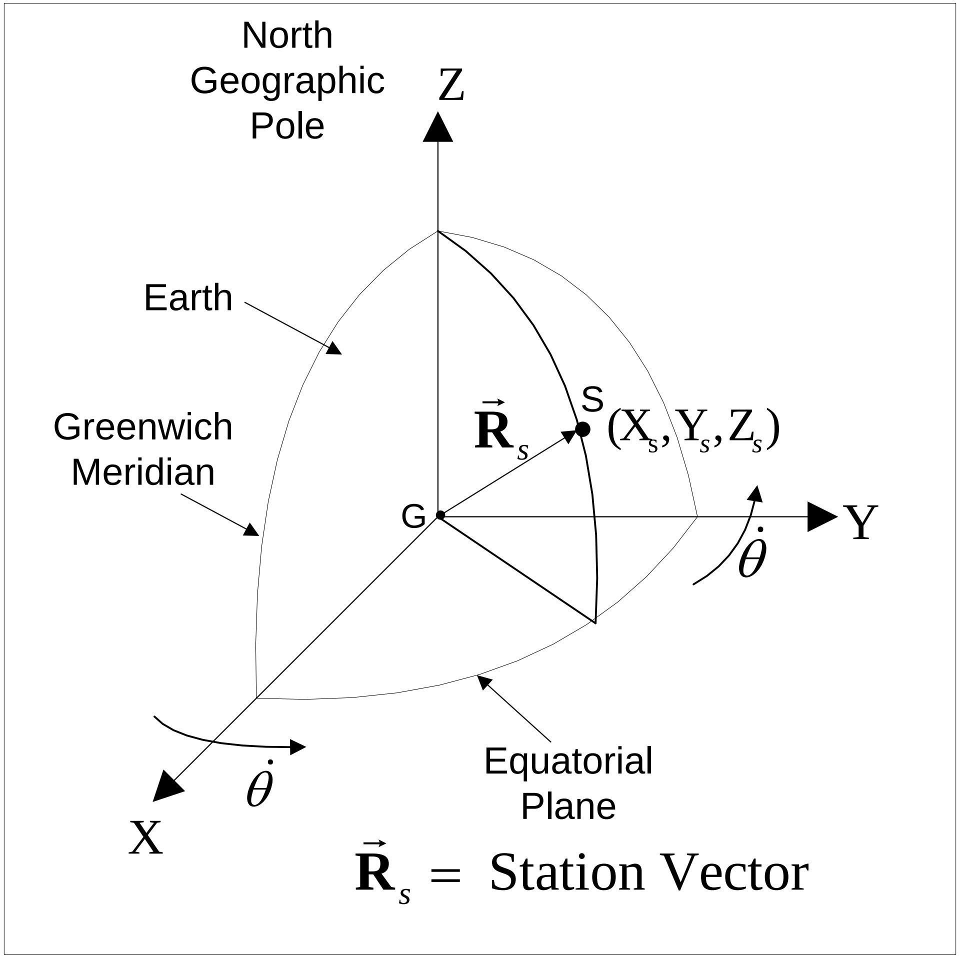
```
X          =-957.341241     [km]
Y          =-1513.787587    [km]
Z          =-6859.189678    [km]
X_DOT      =-6.880520613    [km/s]
Y_DOT      =-2.721926454    [km/s]
Z_DOT      =1.562396855     [km/s]
CR_R       =1.082903E+03    [m**2]
CT_R       =-3.623001E+03   [m**2]
CT_T       =9.930017E+04    [m**2]
CN_R       =1.256933E+02    [m**2]
CN_T       =-2.656842E+02   [m**2]
CN_N       =5.868137E+01    [m**2]
```

- **Position and velocity at TCA (in EDR coordinates: fixed to rotating earth but with only four nutation terms)**
  - **Covariance elements at TCA (a_a is diagonal element; a_b is covariance element between a and b)**
    - **Velocity, drag, and SRP covariance parameters also available if populated**

Earth Centered Rotating (ECR) Coordinate System

- **Origin:** **at center of Earth**
- **Fundamental plane:** **established by the equatorial plane**
- **Principal direction:** **at 0° longitude (through Greenwich meridian)**
- **When valid/applicable:**
  - Always and forever. A sensor does not move relative to the crust of the Earth
  - Used to represent locations of sensors ***fixed to the Earth's crust***
  - $\boldsymbol{R_S}$ can be represented by $\boldsymbol{X_S}$, $\boldsymbol{Y_S}$, and $\boldsymbol{Z_S}$ or by longitude, (geodetic) latitude, and height above/below the reference Earth ellipsoid
- **Unit vectors:** **None. Axes labeled X, Y, and Z**

- Rotates with ***crust of Earth***



From ASTRODYNAMICS CONCEPTS and TERMINOLOGY