DEVELOPMENT OF A COUPLED AIR AND PARTICLE THERMAL MODEL FOR ENGINE ICING TEST FACILITIES

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Many engine power-loss events reported since the 1990’s
Mason et al.¹ hypothesized how power-loss events can result from ice crystals entering the engine core
Ingestion of ice into engine is studied
NASA conducted tests at NRC’s RATFac
Observed environmental conditions changed with cloud activation
- Air temperature change
- Air humidity change
- Particle water content change
Hypothesis: Thermal interaction between air and cloud particles

Thermal Model General Description

- Model couples air and cloud particle conservation equations
  - Mass, energy and momentum
- Simulates icing tunnel (applicable to engines too)
- Ice, water, and humid air mass broken into fundamental CV units
  - Uniform particle size (MVD representation)
  - Full particle size distribution
Assumptions

- Air and water vapor are treated as ideal gases
- Air is continually well mixed
- No supersaturation
- 1-D air and particle flow
- Dilute system (no particle interaction)
- Particles are spherical
- Discrete particle size distribution (bins)
- Uniform temperature within the particle
- Supercooling can occur
- Mixed phase particles are not spatially resolved
- Phase change occurs at particle surface at particle temperature
- Adiabatic tunnel walls
- The flow of particles and air is a continuous stream
- The fundamental CV is adiabatic and mass is conserved
  - Provisions for heat and mass transfer added using source terms
Particle Conservation of Energy
Single Particle Formulation (1/2)

- Change in the particle’s enthalpy, is due to the convective heat transfer and latent energy exchange due to mass transfer

\[
\frac{\partial H_p}{\partial t} = q_{\text{conv}} + q_{\text{latent}}
\]

- Rate of mass change with a water surface is proportional to the difference in vapor pressure between the particle surface and the ambient air

**Ice:** \( \rho_p C_p \frac{\pi d^3}{6} \frac{\partial T_p}{\partial t} = \pi d^2 h(T_{\text{air}} - T_p) + \pi d^2 h_m \rho_{\text{air}} L_{\text{subl}}(\omega_{\text{air}} - \omega_p) \)

**Mix:** \( \rho_p L_{\text{melt}} \frac{\pi d^3}{6} \frac{\partial \eta_p}{\partial t} = \pi d^2 h(T_{\text{air}} - T_p) + \pi d^2 h_m \rho_{\text{air}} L_{\text{subl/evap}}(\omega_{\text{air}} - \omega_p) \)

**Water:** \( \rho_p C_p \frac{\pi d^3}{6} \frac{\partial T_p}{\partial t} = \pi d^2 h(T_{\text{air}} - T_p) + \pi d^2 h_m \rho_{\text{air}} L_{\text{evap}}(\omega_{\text{air}} - \omega_p) \)
Particle Conservation of Energy
Single Particle Formulation (2/2)

- Empirical heat and mass transfer expressions for flow over a sphere\(^2\)
  \[ \text{Nu} = \frac{h d}{k_{\text{air}}} = 2 + 0.6 R e^{1/2} P r^{1/3} \]
  \[ \text{Sh} = \frac{h_{m d}}{D_{ab}} = 2 + 0.6 R e^{1/2} S c^{1/3} \]
  \[ \text{Re} = \frac{\rho_{\text{air}}|v_{\text{air}} - v_p|d}{\mu_{\text{air}}} \]
  \[ \text{Pr} = \frac{C_{\text{air}} \mu_{\text{air}}}{k_{\text{air}}} \]
  \[ \text{Sc} = \frac{\mu_{\text{air}}}{\rho_{\text{air}} D_{ab}} \]
  \[ \omega_{\text{air}} = \frac{M W_{\text{water}} P_{w v, \text{air}}}{M W_{\text{air}} P_{\text{air}}} \]
  \[ \omega_{\text{surf}} = \frac{M W_{\text{water}} P_{w v, \text{surf}}}{M W_{\text{air}} P_{\text{air}}} \]

Change in the air enthalpy is due to the convective heat transfer and the sensible energy change of the water vapor mass that has changed phase

\[- \frac{\partial H_{\text{air}}}{\partial t} = q_{\text{conv}} + q_{\text{wv,sens}}\]

- $q_{\text{wv,sens}}$ is thermal mixing

\[- m_{\text{air}} C_{\text{air}} \frac{\partial T_{\text{air}}}{\partial t} = \pi d^2 h (T_p - T_{\text{air}}) + \frac{\partial m_{\text{wv}}}{\partial t} \int_{T_{\text{air}}}^{T_p} C_{\text{wv}} \partial T\]

- $m_{\text{wv}}$ can be mass from evaporation or mass to condense
Change in particle mass due to vapor phase change
\[- \frac{\partial m_p}{\partial t} = \rho_p \frac{\partial}{\partial t} \left( \frac{\pi d^3}{6} \right) = \pi d^2 h_m \rho_{air} (\omega_{air} - \omega_p)\]

Change in air mass is opposite the change in particle mass
\[- \frac{\partial m_{air}}{\partial t} = - \frac{\partial m_p}{\partial t} = \pi d^2 h_m \rho_{air} (\omega_p - \omega_{air})\]
Conservation of Energy
Particle Distribution Formulation

• MVD vs particle distribution
  – Smaller particles have greater SA/Vol ratio → faster transfer response
  – Cumulative differences will add up to a different final result
  – Fraction of the total water mass calculated for every particle size bin

• An energy balance equation for every particle size \( i \)
  \[
  \rho_{p,i} C_{p,i} \frac{\pi d_{i}^3}{6} \frac{\partial T_{p,i}}{\partial t} = \pi d_{i}^2 h_{i} (T_{\text{air}} - T_{p,i}) + \pi d_{i}^2 h_{m,i} \rho_{\text{air}} L_{i} (\omega_{\text{air}} - \omega_{p,i})
  \]

• One air energy equation contains the sum of all the particle heat transfers and vapor sensible energy transfers.
  \[
  m_{\text{air}} C_{\text{air}} \frac{\partial T_{\text{air}}}{\partial t} = \sum_{i=1}^{n} \left[ \pi d_{i}^2 h_{i} (T_{p,i} - T_{\text{air}}) + \frac{\partial m_{wv,i}}{\partial t} \int_{T_{\text{air}}}^{T_{p,i}} C_{wv,i} \partial T \right] (#_{i})
  \]
  • \( n \) = number of particle size bins
  • \( #_{i} \) = number of particles in the \( i^{th} \) bin
A mass balance equation for every particle size $i$

$$- \frac{\partial m_{p,i}}{\partial t} = \pi d_i^2 h_{m,i} \rho_{air}(\omega_{air} - \omega_{p,i})$$

One air mass equation contains the sum of all the particle mass transfers.

$$- \frac{\partial m_{air}}{\partial t} = - \sum_{i=1}^{n} \left[ \frac{\partial m_{p,i}}{\partial t} \right] (\#_i)$$
Conservation of Momentum

- Conservation of momentum equation is solved in reference to particle

\[ F = F_{drag} + F_g = m_p a = \rho_p \frac{\pi d^3}{6} \frac{\partial v_p}{\partial t} \]

- \( F_g = 0 \)
- \( F_{drag} = \frac{1}{2} \rho_{air} U^2 A C_D \)
- \( U = v_{air} - v_p \)
- \( A = \pi d^2 \)

\[ C_D = \frac{24}{Re} + \frac{2.6\left(\frac{Re}{5.0}\right)^{1.52}}{1+\left(\frac{Re}{5.0}\right)^{1.52}} + \frac{0.411\left(\frac{Re}{263,000}\right)^{-7.94}}{1+\left(\frac{Re}{263,000}\right)^{-8.00}} + \frac{Re^{0.80}}{461,000} \]  \hspace{1cm} \text{(Morrison correlation)}^3

\[ \frac{\partial v_p}{\partial t} = \frac{3}{4} \frac{\rho_{air} C_D (v_{air} - v_p)^2}{\rho_p d} \]

Supercooled Freezing Formulation

- Four stages in supercooled freezing
  1. Sensible liquid cooling (below 273.15K)
  2. Latent heat release
  3. Latent freezing (at 273.15K)
  4. Sensible ice cooling

- Homogeneous crystallization temperature as a function of diameter
  \[ T_{hc} = 7.2015 \ln(d) + 214.64 \]

- Expressions to determine melt fraction after latent heat release
  \[ H_{sens,super} = m_p \int_{T_{hc}}^{273.15} C_p(T) dT \]
  \[ \eta_p = 1 - \frac{m_p L_{melt} - H_{sens,super}}{m_p L_{melt}} \]

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• Written in MATLAB version R2014a
• Solves conservation equations using built-in ODE45 solver
• Relative and absolute convergence tolerance of $10^{-12}$
• Mass transferred between the air and particle(s) balanced to $10^{-15}$
• Energy transferred between the air and particle(s) balanced to $10^{-4}$
  – Model accuracy dependent on accuracy of property values ($C_p$, $L_{heat}$, etc.)
Parametric Analysis
Baseline 1 Tests

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<tr>
<th>Parameter</th>
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<table>
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Takeaway: Wet-bulb temperature determines state of particle
### Parametric Analysis

#### Baseline 2 Tests

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**Takeaway:** Air thermal mass >> Particle thermal mass
Air velocity, air temperature, and vapor content vary from injection to test section at RATFac (no icing cloud).

CFD model previously written to approximate \( v_{\text{air}} \) and \( T_{\text{air}} \).

Centerline values normalized.

Water vapor profile analogous to normalized air temperature profile.

Approximate changing values as sources in expressions (no cloud).

- Valid for tests Mach \( \sim 0.2 \) and \( P_{\text{total}} \sim 44000 \) Pa at test section.
Model – Experiment Comparison

### Conditions
- Scan 887: Water sprayed into low P, low RH, very warm air
- Scan 983: Ice sprayed into medium P, medium RH, warm air
- Scan 1003: Water & Ice sprayed into medium P, medium RH, warm air

**Takeaway:** Model accounts for ~20% of experimentally observed changes
Sources for Model – Experiment Discrepancy

- Experimentally observed water/ice film on tunnel walls
  - Increased residence time for evaporation
- Poor approximation for the vapor mass source term
- Non-spherical particles increase heat and mass transfer
- Turbulence at spray nozzle may enhance transfer
- Uncertainties with experimentally measured values
  - Independent corroborating measurements minimize this possibility
Thermal model that couples particle and air conservation equations

- Simulates:
  - All phase change types
  - Supercooled and normal freezing
  - Single particle and full particle distribution sprays (ice, water, combined)
  - Complicated icing tunnels with energy and mass sources
- Air temperature, pressure (air mass), and RH dominate \( T_{wb} \)
- Model compared to experiments conducted at NRC
  - Simulated ~ 20% of the cloud and air changes observed experimentally
  - Reasons for discrepancy are offered
- Future work to determine sources for discrepancy
- Model can be modified to simulate other icing facilities (PSL)