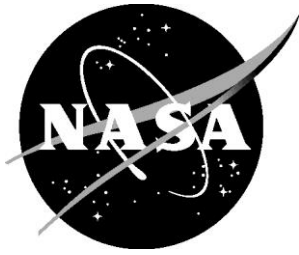


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Multidimensional Risk Analysis: MRISK

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Introduction

Informed decision making rests on the assessment of potential risks which allows projects and programs to focus their efforts on effective risk management (Kaplan and Garrick 1981). Accurate quantification of risks facilitates systematic and rigorous practice of risk management, which is particularly useful in project management. A relatively quantitative and generally accepted definition of risk is probability times consequence (Kaplan and Garrick, 1981; Rosa 1998). Using this definition, quantifying risk is straightforward but flawed. Consequence is oftentimes a multidimensional proposition. The overall consequence may include a schedule dimension, safety dimension, financial dimension, etc. When faced with this issue, organizations typically average (weighted or otherwise) the consequence scores (Zayed et al. 2007); use Euclidean distance to combine the scores (Garvey 2008); or choose the maximum score (Oracle, 1999; Curtis and Carey, 2012).

This paper focuses on an alternative multidimensional method using Statistical science to combine multiple dimensions together into one risk metric, called MRISK. Multidimensional Risk (MRISK) calculates the combined multidimensional score using Mahalanobis distance. MRISK quantifies and accounts for the least to most hazardous risks and, thus, better prioritizes risks. Likewise, MRISK accounts for covariance between consequence dimensions, since each consequence dimension may impact the other dimension. This helps differentiate when and why one should focus on certain risks. Accounting for covariance de-conflicts the interdependencies of consequence dimensions, providing a clearer depiction of risks. Additionally, in the event the dimensions are not correlated, Mahalanobis distance reduces to Euclidean distance normalized by the variance (Wilson and Martinez, 1997) and, therefore, represents the most flexible and optimal method to combine dimensions.

MRISK is currently being used in National Aeronautics and Space Administration's (NASA's) Environmentally Responsible Aviation (ERA) project to assess risk and prioritize scarce resources. The ERA case study is presented alongside specific examples with an explanation of MRISK, to demonstrate its effectiveness.

Multidimensional Risk

Industry accepted risk management practices use a Likert scale to translate qualitative assessments into a quantitative scale for consequence impact measure. (Hillson and Simon, 2012). This paper will utilize Likert-scale data to illustrate MRISK although MRISK is applicable to all forms of quantitative data. In the given examples, three risk factors will be scored using a Likert scale of one to five, with one being associated with the least hazardous consequence and five being associated with the most hazardous consequence. It should be noted that scoring the risk factors appropriately is a vital step in a proper risk assessment. If the risk factor scores are inaccurate, MRISK will be no better than any other method. However, defining a risk score assessment process is beyond the scope of this paper.

As previously mentioned, some alternatives to MRISK include choosing the maximum score (which will be referred to as the Maximization method), Euclidean method, and Averaging method.

The Maximization method assigns the highest consequence score amongst all the possible consequence dimensions to the consequence. For example, if three factors are rated as (1,5,1) the rating for the consequence is 5, and it is then combined with the probability score to determine the final risk rating. This implies that the consequence has a relatively high severity. The Maximization method may incorrectly take into account the weights of the other factors associated with the overall risk. More specifically, the Maximization method assumes all the risk

components are highly correlated. By definition, using the highest consequence dimension score requires the assumption that the other consequence dimensions are equally high. This is an assumption of absolute correlation for all dimensions. For example, the risk rating for (1,5,1) is the same as (5,5,5). If the three consequences are perfectly correlated, then theoretically only (1,1,1), (2,2,2), (3,3,3), (4,4,4), and (5,5,5) would ever be observed. Likewise, highly correlated factors would have most scores with only marginal differences between them, such as (4,3,4) or (5,5,3). Larger differences such as (1,5,1) suggest the possibility of a weak correlation between the three factors. Risk management as a practice seeks to highlight risks. Therefore, the Maximization method may be excessively conservative. However, as the number of risk factors increase towards infinity, the logic of excessive conservatism falls apart, as seen in Table 1.

(1,4)	An overall consequence of 4 might be deemed acceptable for lower numbers of risk dimensions
(1,4,1)	
(1,4,1,1)	
(1,4,1,1,1)	
...	As number of dimensions rise absolute correlation as in the case of Maximization method has reduced merit
(1,4,1,1,.....,1,1,1,1,1)	

Table 1: The Maximization method moves from conservatism to a biased estimate

The Averaging method assigns the mean consequence score among all the consequence dimensions to the consequence. For example, if three factors are rated as (1,5,1) the consequence is rated as a 2, suggesting relatively low severity. The weighted average of $X = \{x_1, x_2, \dots, x_n\}$ is given as,

$$\sum_{i=1}^n W_i * x_i, \text{ where } 0 < W_i < 1 \text{ and } \sum W_i = 1$$

If $W_i=1/n$ for all i , then the formula reduces to the non-weighted average from Elementary Statistics (Triola, 2012). The Averaging method is a common practice among risk managers (Zayed et al., 2007). The average is a univariate parameter, and thus does not account for covariance of dimensions. Unlike the maximization method which has disproportional tendencies towards a single extreme, Averaging accounts for high and low extremes. For example, schedule increases generally go along with cost increases (PMBOK, 2000), and failure to capture covariance leaves important information out of risk analysis. Therefore, without taking covariance into consideration, risk managers may not have an accurate outlook on the risks at hand. Additionally, the Averaging method has disproportional tendencies towards the middle. Due to these tendencies, some risks could be deflated and considered a low severity risk. On the other hand, some risks could be inflated and considered a high severity risk. Both possibilities could be detrimental to a project.

Dimension Scores	Average	Many seemingly different combinations result in strikingly similar scores when using the Averaging method
(1,1,5)	2.33333	
(3,3,2)	2.66667	
(2,1,5)	2.66667	
(1,4,3)	2.66667	
(3,3,1)	2.33333	

Table 2. The Average method has disproportional tendencies towards the center of the scale

The Euclidean method uses Euclidean distance to combine the consequence dimensions (Garvey, 2008). The Euclidean method assigns co-planar distance amongst all the consequence dimensions to the consequence. For example, if three factors are rated at (1,5,1), the consequence would be rated as 3, suggesting a medium severity. The Euclidean distance for two vectors, \mathbf{x} and \mathbf{y} , each of length n is given as,

$$e(\mathbf{x}, \mathbf{y}) = \sqrt{(\mathbf{x} - \mathbf{y})(\mathbf{x} - \mathbf{y})^T} = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \dots + (x_n - y_n)^2}$$

For this paper, the Euclidean method uses a ratio of the relative distances of the dimensions from the minimum, \mathbf{x}_{min} , and maximum, \mathbf{x}_{max} of the dimensions. The Euclidean method value, E , is found as follows.

$$e_{min} = \sqrt{(\mathbf{x} - \mathbf{x}_{min})(\mathbf{x} - \mathbf{x}_{min})^T}$$

$$e_{max} = \sqrt{(\mathbf{x} - \mathbf{x}_{max})(\mathbf{x} - \mathbf{x}_{max})^T}$$

$$E = a \left(\frac{e_{min}}{e_{min} + e_{max}} \right) + b$$

Where, the dimensions are scored on a scale of $[b,c]$ such that $a=b-c$.

The Euclidean method is meant for multidimensional situations; therefore it is better suited than the Averaging method for multidimensional risk analysis. Also, the Euclidean method does not have disproportional tendencies towards a single extreme as in the case of the Maximization method. However, the Euclidean method assumes the dimensions occupy the same plane, which may not be accurate.

This paper focuses on conducting Risk Analysis in the presence of multiple dimensions. With only one or two dimensions the conservative propensities present in the Maximization method may not be a problem. Likewise, the Averaging method's disproportional tendency to the middle and ignorance of covariance could be acceptable with a low number of dimensions. Furthermore, assuming the dimensions are co-planar, as in the Euclidean method, might not make a difference with one or two dimensions. In the 1990s, risk was strictly deemed a two dimensional problem, namely probability and consequence, where consequence was one dimension as opposed to multidimensional (Rosa 1998, Renn 1998). However, risk management has come to recognize over the last two decades that multiple dimensions are a part of risk. In particular, NASA has expanded consequence to Cost, Schedule, and Technical (NASA/SP-2011-3421). Other areas like Finance often use five factors and the Department of Homeland Security (DHS) National Infrastructure Protection Plan (NIPP) and Sector Specific Plans have moved to three risk factors and are starting to consider a fourth Resilience risk factor (DHS, 2013).

The MRISK Procedure

MRISK is a procedure which uses multivariate metric, Mahalanobis distance in particular, to measure impact across several dimensions. In his 1936 paper, *On the Generalized Distance in Statistics*, P.C. Mahalanobis introduced this innovative method to measure distance between two vectors. This method takes into consideration the correlation of the vectors. Mahalanobis distance essentially measures how many standard deviations away from a distribution a given point is; Mahalanobis also generalized the concept to multiple dimensions. MRISK is founded on Mahalanobis distance. The three primary advantages of using this generalized distance are:

- accounting for correlation between variables
- reverting to normalized Euclidean distance when correlation does not exist or when vectors occupy the same plane
- the ability to scale to infinite dimensions. In other words, the procedure will never lose validity as the dimensions grow.

Mahalanobis distance for two random vectors, \mathbf{x} and \mathbf{y} , of the same distribution with covariance matrix, \mathbf{S} , is defined as

$$d(\mathbf{x}, \mathbf{y}) = \sqrt{(\mathbf{x} - \mathbf{y})\mathbf{S}^{-1}(\mathbf{x} - \mathbf{y})^T}$$

\mathbf{S} is a multidimensional matrix where each row is a consequence dimension and each column is also a consequence dimension. Take note that Mahalanobis distance simplifies to Euclidean distance when the covariance matrix is the identity matrix. Subsequently, for any scale, $[b,c]$, where a is the range of the consequence scale and b is the minimum of the consequence scale so that $a=c-b$, the MRISK formula is defined as follows.

$$MRISK = a \left(\frac{d_{min}}{d_{min} + d_{max}} \right) + b$$

where,

$$d_{min} = \sqrt{(\mathbf{x} - \mathbf{x}_{min})\mathbf{S}^{-1}(\mathbf{x} - \mathbf{x}_{min})^T}$$

$$d_{max} = \sqrt{(\mathbf{x} - \mathbf{x}_{max})\mathbf{S}^{-1}(\mathbf{x} - \mathbf{x}_{max})^T}$$

So, d_{min} is the Mahalanobis distance between the vector of consequence scores, \mathbf{x} , and the vector of minimum possible consequence scores, \mathbf{x}_{min} , and d_{max} is the Mahalanobis distances between the vector of the consequence scores, \mathbf{x} , and the vector of maximum possible consequence scores, \mathbf{x}_{max} . When using a Likert scale of one to five as previously mentioned and three consequence factors, $\mathbf{x}_{min} = (1,1,1)$ and $\mathbf{x}_{max} = (5,5,5)$. The number of factors will change based on an organization's preferences, but the range of one to five is representative of risk management as evidenced by ERA. MRISK is not dependent on any one scale. Therefore, $\frac{d_{min}}{d_{min}+d_{max}}$ always has a range of [0,1]; so in this case (1,1,1) maps to 0 and (5,5,5) to 1. Take note, that d_{min} is the numerator so that values near the top of the scale (e.g., (5,5,5)) reflect the highest score.

NASA's Environmentally Responsible Aviation, A Case Study

The Environmentally Responsible Aviation (ERA) project has provided an effective case study on the execution of MRISK in actual, risk-management practice. ERA was created in 2008 and is part of NASA's Aeronautics Research Mission Directorate's (ARMD's) Integrated Systems Research Program (ISRP). The ERA project was created to explore and document the feasibility, benefits, and technical risk of vehicle concepts and enabling technologies. The project invests in technologies with the potential to neutralize or reduce negative environmental impacts. The goal is to select vehicle concepts and technologies that improve fuel efficiencies, reduce harmful emissions, and lower noise levels of aircrafts. The ERA project uses several Integrated Technology Demonstrations (ITDs) to advance aircraft concepts and technologies that will reduce the impact of aviation on the environment. These demonstrations will focus on aircraft drag, weight, fuel usage, noise, and emission reduction. Engineers assessed environmentally friendly aircraft technologies and then matured the most promising ones to the point that they can be tested in a real world environment.

For an ITD to test in a real world environment, several risks have to be reviewed such as design challenges, fabrication challenges, integration challenges, testing challenges, availability of personnel and facilities, etc. To determine how severe the risk is on the ITD, each risk is given a risk score. Each risk includes Cost, Schedule, and Technical dimensions scored in accordance with a Likert scale [1,5]. Risks are ultimately binned into green, yellow, and red categories as shown in Table 2 based on the combination of their multidimensional consequence score with the likelihood of the risk (one to five Likert scale). For the ultimate risk score green corresponds to a low impact risk, yellow a medium impact risk, and red a high impact risk.

Note that the risk scores will fluctuate over the course of the project. Many factors are time dependent. As certain phases of the project end risks associated with that phase end as well. Furthermore, new risks arise throughout the project, creating a very fluid and changing risk environment. For this paper a snapshot of the project timeline was taken and MRISK calculations were performed for that particular timestamp. The data used for those calculations are included in the appendix of this paper.

Risk, in the context of mission execution for the NASA ERA project, is operationally defined as (1) The scenario(s) leading to degraded performance with respect to one or more performance measures (e.g., scenarios leading to destruction of key assets; scenarios leading to exceeding the mass limits; scenarios leading to cost overruns; scenarios leading to schedule slippage). (2) The likelihood(s) (qualitative or quantitative) of those scenarios. (3) The consequence(s) (qualitative or quantitative) that would result if those scenarios were to occur (NASA, 2008).

ERA uses NASA's Continuous Risk Management (CRM) process to manage risks continuously. The CRM process, as described in the *Agency Risk Management Procedural Requirements*, defines consequence as the worst credible potential result(s) of a risk (NASA, 2008). Consequence scores range from one to five and ERA confines the risks to three major categories; Technical, Cost, and Schedule. Each of the consequence categories should be individually scored when making a determination of risk consequence; that is, each risk should have a Technical consequence, a Cost consequence, and a Schedule consequence, etc.

Naturally, risk managers may use the Maximization method based on the assumption that the method ensures the risk is not underestimated. This approach does create the potential to overestimate as argued by its advocates. However, overestimation may limit the application of mitigation in risk management. Mitigation refers to the active de-escalation of risks (NASA, 2008). The limiting factor for mitigation is usually budget, i.e. a finite-mitigation budget forces the risk manager to limit the scope or number of risks mitigated by the project. Therefore, estimation may de-prioritize dangerous risks and over-prioritize low risks. The Averaging method, has the disproportional tendency to rate risks at a medium severity. If the majority of risks are in the middle of the risk scale, risk managers may have a difficult time distinguishing the criticality of risks. The Euclidean method assumes co-planar dimensions, which may not be accurate, and thus could rate risks incorrectly. Also, this approach does not take into account the weights of the other risk factors and the correlation between Technical, Cost, and Schedule risks. For example, higher costs tend to follow schedule increases, schedule decreases may create more technical risk, tighter schedules tend to have greater cost risk, and difficult technical challenges tend to take longer and cost more to execute. Without the correlation between the different consequence categories, a less severe risk may be prioritized above a risk of higher severity, resulting in unnecessary allocations of resources for risk mitigations strategies. For example, the standard approach would imply a risk with Technical, Cost, and Schedule consequences of (4,4,4) would be considered an equal priority to a risk with consequences of (1,1,4).

In addition to the intuitive notion that consequences are correlated, several NASA sources acknowledge that the consequence components of Cost and Schedule in particular are interdependent and that risk assessments should account for covariance. For example, the

2008 Goddard Space Flight Center (GSFC) Symposium presentation *Perspectives on NASA Mission Cost and Schedule Performance Trends* by David Bearden stated “While Significant Variability is Evident, for Every 10% of Schedule Growth, there is a Corresponding 12% Increase in Cost”. David Bearden presents physical evidence in his presentation that shows there is a correlation between schedule growth and increases in cost.

Consequently, NASA leadership developed the NASA *Schedule Management Handbook* which expressly states “NPR 7120.5 requires that the project’s schedule baseline be integrated with the budget and technical baselines to form an overall project integrated baseline. This correlation is essential to ensure that adequate resources are available to accomplish the work when it is scheduled. Without this correlation and validation the project IMS [Integrated Master Schedule] loses credibility” (NASA, January 2010).

Additionally, NASA developed the Joint Cost and Schedule Confidence Level (JCL) in order to encourage more effective project planning “earlier in the life cycle and to support development of more accurate estimates”. NPR 7120.5, *NASA Space Flight Program and Project Management Handbook* defines “Joint Cost and Schedule Confidence Level. (1) The probability that the program/project cost will be equal to or less than the targeted cost and that schedule will be equal to or less than the targeted schedule date. (2) A process and product that helps inform management of the likelihood of a project’s programmatic success. (3) A process that combines a project’s cost, schedule, and risk into a complete picture” (NASA, 2014).

MRISK was conceived for a multidimensional risk analysis and thereby follows the underlying intent of the JCL by combining Cost and Schedule dimensions as well as other dimensions into a single metric that properly accounts for correlation between dimensions. The procedure accounts for covariance amongst multiple risk dimensions. Additionally, the application problems associated with Averaging, Maximization, and Euclidean analysis made the case for MRISK to the ERA project management team. That is to say, the ERA project converted from the Maximization method to MRISK after coming to understand the benefits of MRISK.

ERA Examples

In the following examples, the MRISK formula result is rounded and combined with the likelihood using Table 2 to determine the risk rating. Table 2 is not a necessary step for MRISK. MRISK has the ability to measure likelihood along with all the other consequence dimensions. However, Table 2 is an example of a customary figure in the practice of risk management (Conrow 2003) and this paper shows that MRISK is adaptable to this traditional practice used in ERA.

Furthermore, to determine the overall severity rating of a risk, likelihood score and the consequence score are taken into consideration. Thus far, we have discussed different methods of combining the consequence scores. In particular, we presented the Maximization method, averaging, Euclidean distance, and MRISK. Once the consequence and likelihood scores are selected, Table 2 is used to determine the overall risk rating. For example, with a consequence score of 3 and a likelihood score of 2, the risk would be given a score of 18, which is of medium severity.

Risk		Consequence				
		1	2	3	4	5
Likelihood	5	10	16	20	23	25
	4	7	13	18	22	24
	3	4	9	15	19	21
	2	2	6	11	14	17
	1	1	3	5	8	12

Table 3. ERA Risk Scoring Matrix

Example

Likelihood=2; Consequence=(2,1,5)

$$\text{variance-covariance matrix, } S = \begin{bmatrix} 0.73181 & 0.21235 & 0.54867 \\ 0.21235 & 1.67585 & 0.62334 \\ 0.54867 & 0.62334 & 1.75407 \end{bmatrix}$$

$$d_{min} = [(2,1,5) - (1,1,1)] \begin{bmatrix} 0.73181 & 0.21235 & 0.54867 \\ 0.21235 & 1.67585 & 0.62334 \\ 0.54867 & 0.62334 & 1.75407 \end{bmatrix}^{-1} [(2,1,5) - (1,1,1)]^T = 10.60895$$

$$d_{max} = [(2,1,5) - (5,5,5)] \begin{bmatrix} 0.73181 & 0.21235 & 0.54867 \\ 0.21235 & 1.67585 & 0.62334 \\ 0.54867 & 0.62334 & 1.75407 \end{bmatrix}^{-1} [(2,1,5) - (5,5,5)]^T = 26.56579$$

$$MRisk = 4 \left(\frac{10.60895}{10.60895 + 26.56579} \right) + 1 = 2.14152$$

$$Risk(2,2) = 6$$

Risk		Consequence				
		1	2	3	4	5
Likelihood	5	Green	Yellow	Red	Red	Red
	4	Green	Yellow	Yellow	Red	Red
	3	Green	Green	Yellow	Yellow	Red
	2	Green	X	Yellow	Yellow	Yellow
	1	Green	Green	Green	Green	Yellow

Table 4. Mapping of (2,2) to 6

Example of MRISK Without Correlation

In the event that correlation does not exist, the variance-covariance matrix will be a matrix with only the diagonal elements, the variances. Subsequently,

$$d(\mathbf{x}, \mathbf{y}) = \sqrt{(\mathbf{x} - \mathbf{y}) \begin{bmatrix} s_{11}^2 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & s_{nn}^2 \end{bmatrix}^{-1} (\mathbf{x} - \mathbf{y})^T}$$

$$= \sqrt{[(x_1 - y_1), (x_2 - y_2), \dots, (x_n - y_n)] \begin{bmatrix} \frac{1}{s_{11}^2} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \frac{1}{s_{nn}^2} \end{bmatrix} [(x_1 - y_1), (x_2 - y_2), \dots, (x_n - y_n)]^T}$$

Rewriting $d(\mathbf{x}, \mathbf{y})$ we have,

$$d(\mathbf{x}, \mathbf{y}) = \sqrt{\frac{1}{s_{11}^2}(x_1 - y_1)^2 + \frac{1}{s_{22}^2}(x_2 - y_2)^2 + \dots + \frac{1}{s_{nn}^2}(x_n - y_n)^2}$$

So, this is a normalized Euclidean distance between \mathbf{x} and \mathbf{y} . In this case, the Euclidean distance is normalized with the variances. Furthermore, if all of the variances equal one, this will be Euclidean distance. This shows the flexibility of MRISK, which can account for both Mahalanobis and Euclidean distance.

Example

Likelihood = 2; Consequence = (2,1,5)

$S = \begin{bmatrix} 0.73181 & 0 & 0 \\ 0 & 1.67585 & 0 \\ 0 & 0 & 1.75407 \end{bmatrix}$, the variance-covariance matrix.

$$d_{min} = [(2,1,5) - (1,1,1)] \begin{bmatrix} 0.73181 & 0 & 0 \\ 0 & 1.67585 & 0 \\ 0 & 0 & 1.75407 \end{bmatrix}^{-1} [(2,1,5) - (1,1,1)]^T = 10.48812$$

$$d_{max} = [(2,1,5) - (5,5,5)] \begin{bmatrix} 0.73181 & 0 & 0 \\ 0 & 1.67585 & 0 \\ 0 & 0 & 1.75407 \end{bmatrix}^{-1} [(2,1,5) - (5,5,5)]^T = 21.84567$$

$$MRisk = 4 \left(\frac{10.48812}{10.48812 + 21.84567} \right) + 1 = 2.29748$$

$$Risk(2, 2) = 6$$

Likelihood	Risk				
	1	2	3	4	5
5	Green	Yellow	Red	Red	Red
4	Green	Yellow	Yellow	Red	Red
3	Green	Green	Yellow	Yellow	Red
2	Green	Green	Yellow	Yellow	Yellow
1	Green	Green	Green	Green	Yellow

Table 5. Mapping of (2,2) to 6

Notice that the result is the same as MRISK with covariance. The results are the same, because the ERA covariance matrix was used, and the ERA risks have relatively low covariance.

Dispersion Mapping of ERA Data

The ERA project implements continuous risk management, which means the number and types of risks change throughout the life of the project. As risks are mitigated, become obsolete, or as new risks are created the list of risks will change. Therefore, the covariance between Cost, Schedule, and Technical will change throughout the life of the project. The appendix contains the summary of 179 risks on the ERA project at the time of writing this paper. Several of these risks will be highlighted due to their particular importance for comparing MRISK to alternative methods, Maximization, Euclidean, and Averaging.

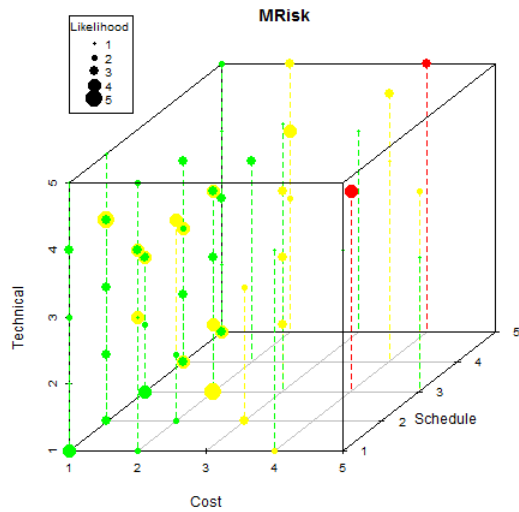


Figure 1. 3-D plot of risks rated with MRISK method

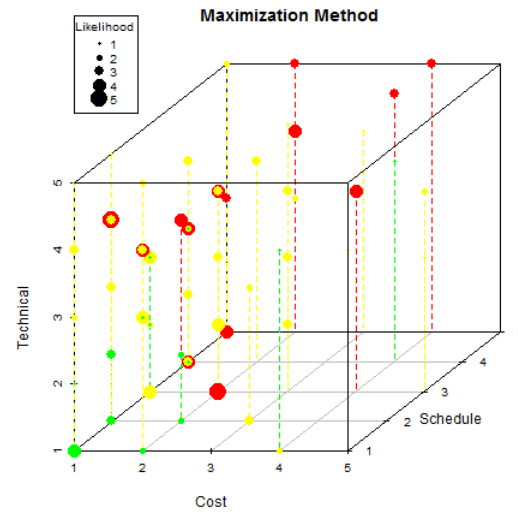


Figure 2. 3-D plot of risks rated with Maximization method

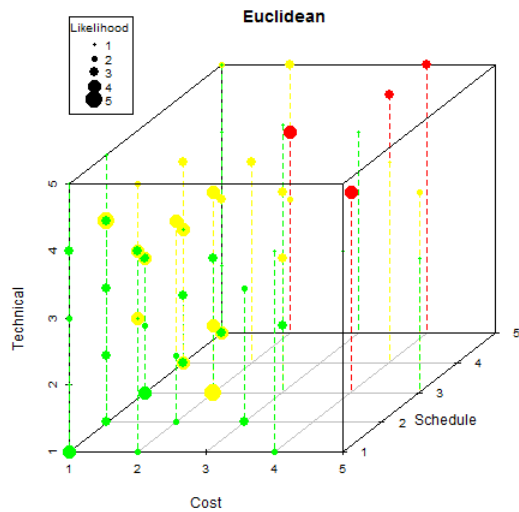


Figure 3. 3-D plot of risks rated with Euclidean method

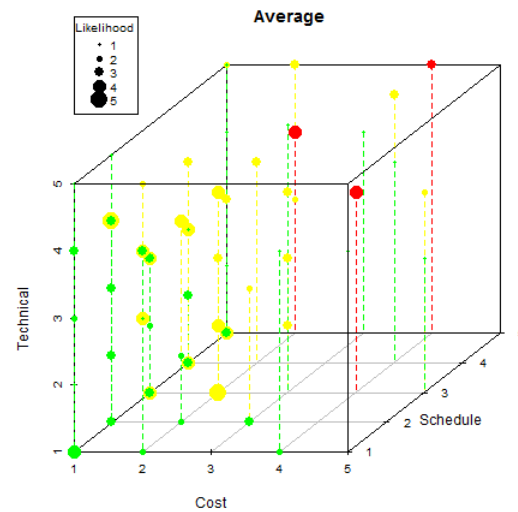


Figure 4. 3-D plot of risks rated with Averaging method

Figure 1 through 4 show three dimensional plots with axes of Cost, Schedule, and Technical for each method of determining the combined consequence score. The size of the point reflects

the likelihood score with a score of one being the smallest point (and lowest likelihood) and five being the largest point (and highest likelihood). The colors demonstrate the overall risk rating, which combines the likelihood and the consequence score. Green symbolizes a low risk, yellow symbolizes a medium risk, and red symbolizes a high risk. From these plots, we can see that using the Maximization method results in a large proportion of medium and high risks. When dealing with risks, being more conservative may seem like the right answer; however, when mitigation resources are scarce, taking an overly conservative approach may exhaust resources before mitigation can buy down risks. The Euclidean distance values are calculated just as the MRISK values are, except the covariance matrix is the identity matrix. Additionally, Euclidean distance and MRISK are very similar. This is due to the weak correlation between the risk factors (i.e., MRISK behaves in a similar fashion to Euclidean distance for the ERA project).

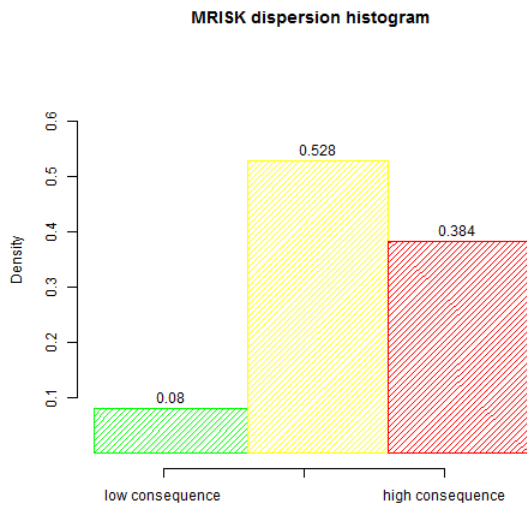


Figure 5. Histogram showing dispersion of risks rated with MRISK method

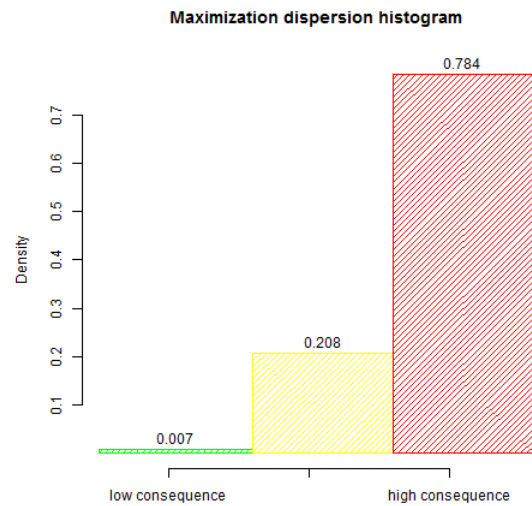


Figure 6. Histogram showing dispersion of risks rated with Maximization method

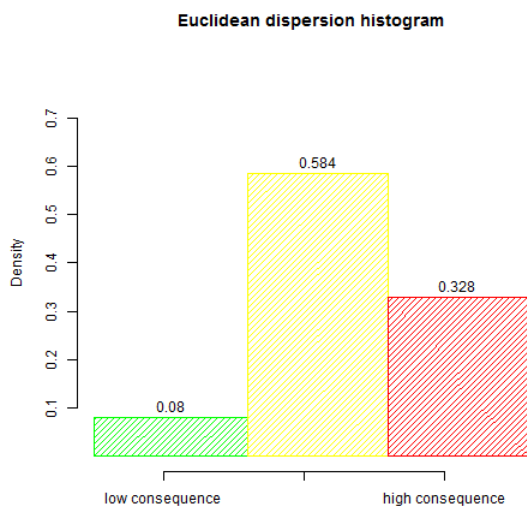


Figure 7. Histogram showing dispersion of risks rated with Euclidean method

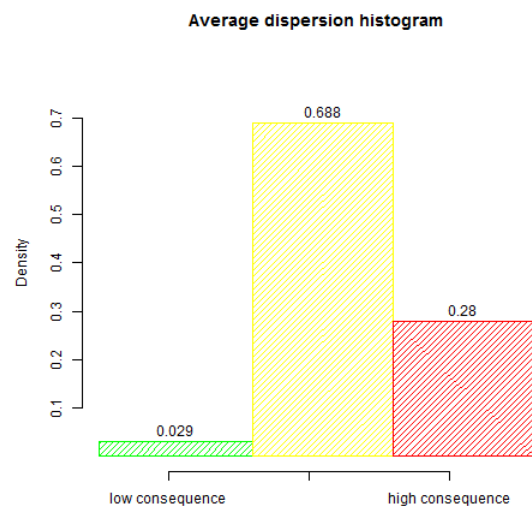


Figure 8. Histogram showing dispersion of risks rated with Maximization method

In Figure 5 through 8, the dispersion of consequence scores is shown. The Maximization method has disproportional tendencies towards the high extreme. The other three methods disproportionately tend to fall in the middle, but MRISK has the most even dispersion among the methods. This brings us to the point, that MRISK has the least systematic tendencies. Because of this dispersion, using this method more clearly defines the cutoffs between green, yellow, and red. That is to say, MRISK allows one to better distinguish between a low risk and a medium risk or a medium and a high risk. Therefore, one can better allocate scarce resources towards right risks.

Next, let us consider an example of MRISK calculations and take a deeper look at the differences between the various methods which may not be readily apparent in the charts above.

Example: Risk ID 236

Likelihood = 1; *Consequence* = (*Cost, Schedule, Technical*) = (4,4,4)

Maximization Method: (4,4,4) → 4; *Risk*(1,4) = 8

MRISK:

$$d_{min} = [(4,4,4) - (1,1,1)] \begin{bmatrix} 0.73181 & 0.21235 & 0.54867 \\ 0.21235 & 1.67585 & 0.62334 \\ 0.54867 & 0.62334 & 1.75407 \end{bmatrix}^{-1} [(4,4,4) - (1,1,1)]^T = 15.12334$$

$$d_{max} = [(4,4,4) - (5,5,5)] \begin{bmatrix} 0.73181 & 0.21235 & 0.54867 \\ 0.21235 & 1.67585 & 0.62334 \\ 0.54867 & 0.62334 & 1.75407 \end{bmatrix}^{-1} [(4,4,4) - (5,5,5)]^T = 1.68037$$

$$MRisk\ Consequence = 4 \left(\frac{15.12334}{15.12334 + 1.68037} \right) + 1 = 4.60000$$

$$Risk(1,5) = 12$$

Here the Maximization method returns a risk score of 8 or low risk, but both MRISK and Euclidean distance return risk score of 12 or medium risk. This example is important to highlight, because it is a counter example to the argument that the Maximization method always returns a higher risk score and therefore, is more conservative approach in all situations.

Conclusions

An accurate portrayal of risks is a key step in informed decision-making. A key challenge of risk assessment in the modern era is multidimensionality. Prior to 1990, multidimensional risk was not an issue, but in the modern era it has become an issue that all risk managers must address. MRISK is an alternative multidimensional approach to dealing with risk management utilizing Mahalanobis distance that lacks the disproportional tendencies of legacy methods. Maximization, Averaging, and Euclidean methods are alternative, legacy methods used in risk management with deficiencies in the presence of multidimensionality. Specifically, the Maximization method does not properly account for covariance and has disproportional tendencies towards the extreme of the risk scale. Averaging is not a multidimensional metric and has disproportional tendencies towards the center of the risk scale. Euclidean is a multidimensional metric, but does not account for covariance among risk dimensions.

In contrast, MRISK is a sound methodology that accounts for covariance among several dimensions and does not exhibit disproportional tendencies apparent in the other legacy methods. MRISK is scalable to any number of risk dimensions. MRISK is effective whether the risk dimensions are highly correlated, weakly correlated, or have no correlation at all, because it calculates the covariance between the risk dimensions and uses the covariance in its calculations. As a result, MRISK is fully compliant with organizational mandates regarding covariance between dimensions. As cited in this paper, organizations such as NASA often require covariance integration within risk analysis, e.g. Covariance(Cost, Schedule). MRISK represents the best multidimensional method available, because of its reliance on Mahalanobis distance. MRISK accounts for covariance when present, via a covariance matrix. MRISK also accounts for lack of covariance by reducing to the Euclidean method when covariance is not present.

MRISK was successfully used as part of the risk management process on NASA's ERA project. MRISK allowed ERA risk managers to more accurately allocate resources on the project. The MRISK method integrated seamlessly with traditional risk management practices, such as 5x5 view graphs and risk dispersion maps. MRISK has been an effective improvement on the risk management process for NASA and could be applied in any organization with similar challenges.

Appendix: Risk Table

The following table shows the observed combinations of consequence scores and their counts. L = Likelihood, C = Cost, S = Schedule, and T = Technical under Risk Dim (Risk Dimensions).

Risk Dim (L,C,S,T)	Count
(1,1,1,1)	64
(1,1,1,2)	1
(1,1,1,3)	4
(1,1,1,5)	1
(1,1,2,1)	4
(1,1,2,2)	1
(1,1,2,3)	1
(1,1,2,5)	2
(1,1,3,1)	2
(1,1,3,2)	1
(1,1,3,3)	1
(1,1,4,1)	2
(1,1,4,3)	1
(1,2,1,3)	1
(1,3,3,5)	1
(1,3,5,4)	1
(1,4,1,4)	1
(1,4,4,4)	1
(1,5,1,4)	1
(1,5,3,3)	1
(2,1,1,1)	19
(2,1,1,3)	3
(2,1,2,1)	5
(4,2,1,3)	1
(4,2,1,4)	1
(4,2,2,4)	2
(4,2,3,2)	1

Risk Dim (L,C,S,T)	Count
(2,1,2,2)	3
(2,1,2,3)	2
(2,1,3,1)	11
(2,1,3,2)	1
(2,1,4,3)	1
(2,1,5,1)	1
(2,1,5,5)	1
(2,2,1,1)	1
(2,2,1,5)	1
(2,2,2,1)	1
(2,2,2,2)	2
(2,2,3,3)	1
(2,2,5,3)	1
(2,3,2,3)	1
(2,4,1,1)	1
(2,5,3,4)	1
(3,1,1,1)	5
(3,1,1,4)	1
(3,1,2,1)	2
(3,1,2,2)	1
(3,1,2,3)	1
(3,1,2,4)	1
(3,1,3,1)	5
(4,2,3,4)	2
(4,2,5,4)	1
(4,4,3,4)	1
(5,1,2,4)	1

Risk Dim (L,C,S,T)	Count
(3,1,3,3)	2
(3,1,4,1)	1
(3,1,4,2)	2
(3,1,4,4)	1
(3,1,5,1)	1
(3,1,5,3)	1
(3,2,1,3)	1
(3,2,1,4)	1
(3,2,3,1)	1
(3,2,3,3)	1
(3,2,3,4)	1
(3,2,4,4)	1
(3,2,5,4)	1
(3,2,5,5)	1
(3,3,2,1)	1
(3,3,3,2)	1
(3,3,3,3)	2
(3,3,3,4)	1
(3,4,4,5)	1
(3,4,5,5)	1
(4,1,1,1)	1
(4,1,3,1)	5
(4,1,4,1)	1
(4,1,4,3)	1
(4,1,5,1)	1
(5,2,3,1)	1

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