Consideration of Dynamical Balances

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A presentation about concepts rather than techniques

Richardson's Forecast

Table 5: Six-hour Changes in Pressure Thickness

Richardson's Values: No Initialization

Layer	$(\partial \Delta p/\partial t)\Delta t$	Horizontal Convergence	Vertical Convergence
I	48.5	65.9	-17.4
II	28.4	-23.7	52.1
III	25.3	47.6	-22.3
IV	22.3	7.5	14.8
V	20.8	48.0	-27.2
Sum	145.4	145.4	0.0

Lynch, Peter, 1994a: **Richardson's Marvellous Forecast.** *Proceedings of the International Symposium on the Life Cycles of Extratropical Cyclones*, Bergen, Norway, 27 June--1 July, 1994, 38--48.

Geostrophic Adjustment

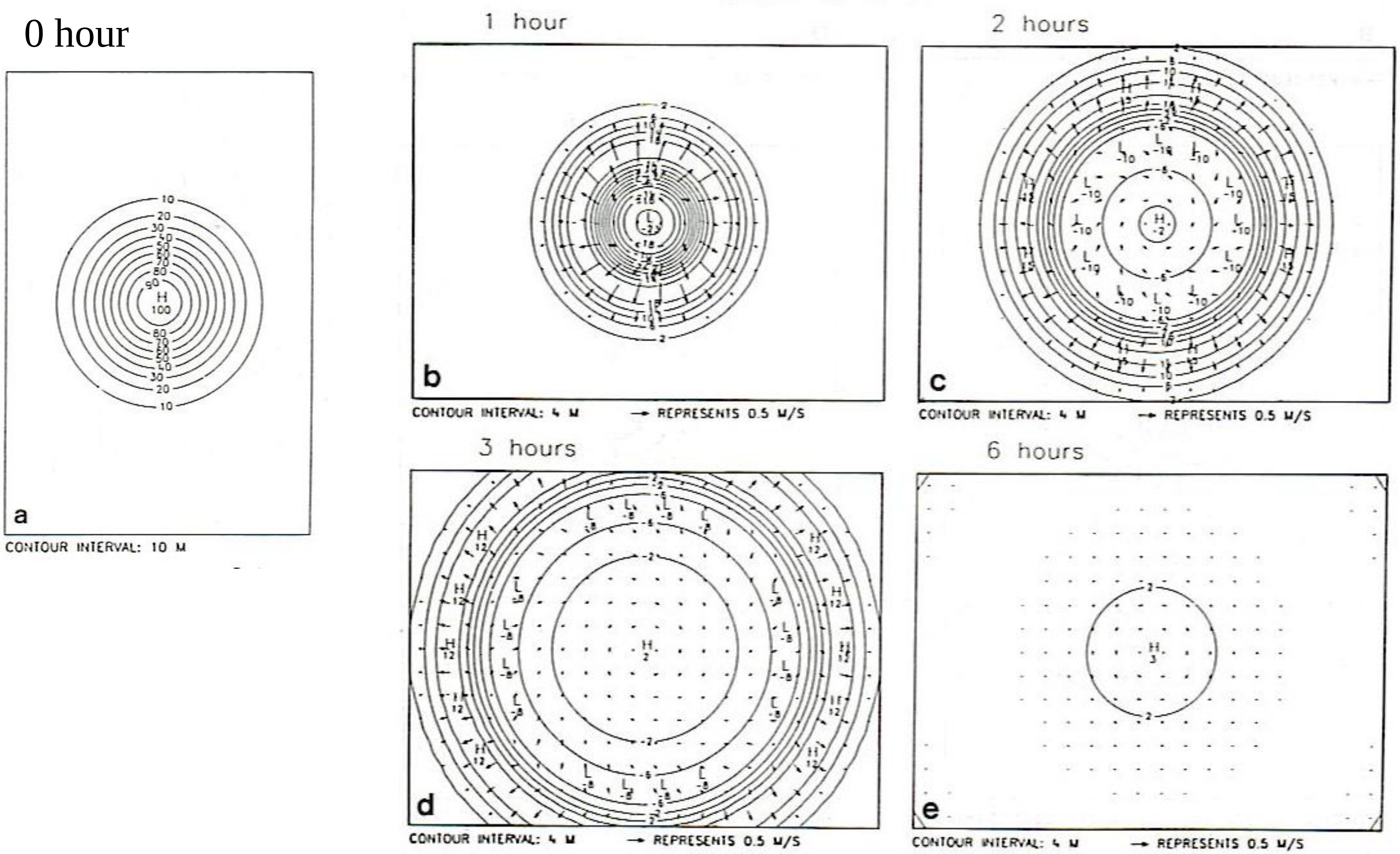


Figure 6.3 (a) Geostrophic adjustment of initial geopotential perturbation. (b–e) Solutions at 1, 2, 3, and 6 hours. Contoured field is geopotential, and wind arrows indicate speed and direction of windfield. (After Barwell and Bromley, 1988)

Daley 1992

Filtered Equations

1st-order balance

e.g., quasi-geostrophic equations

2nd-order balance

e.g., nonlinear balance equation and
quasi-geostrophic omega equation

Dynamical Initialization

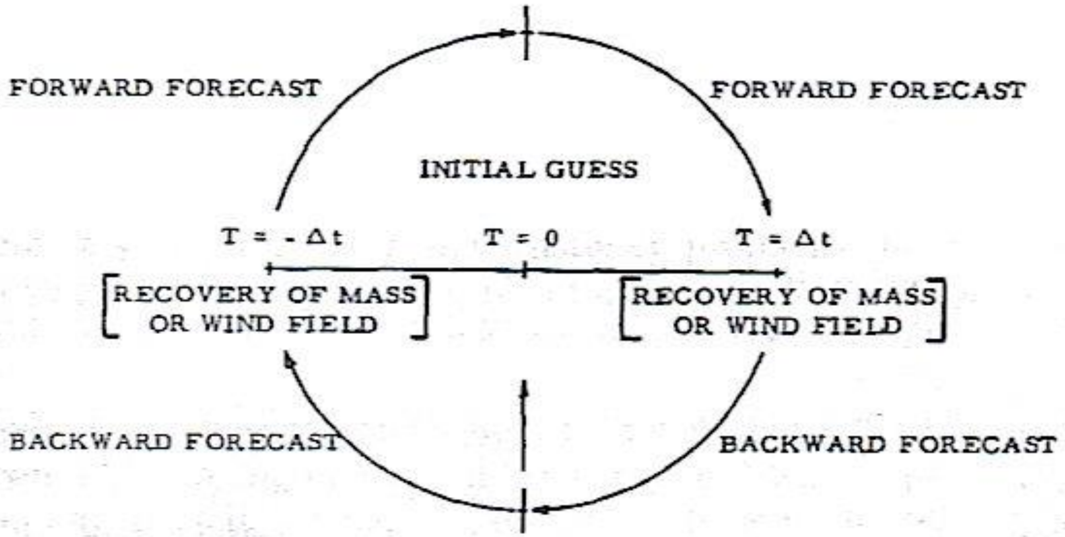


FIGURE 1.—Schematic representation of iteration methods for initialization with the primitive forecast equations.

Nitta and Hovermale 1969 *MWR*

Normal Modes of the Linearized PE

Linearize P. E. about $u = 0$, $v = 0$, $T = T_r$, $p_s = p_{sr}$, $z_s = 0$:

$$\frac{\partial u}{\partial t} = fv - \frac{\partial \phi}{\partial x}$$
$$\frac{\partial v}{\partial t} = -fu - \frac{\partial \phi}{\partial y}$$
$$\frac{\partial \phi}{\partial t} = -\tau_p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$$

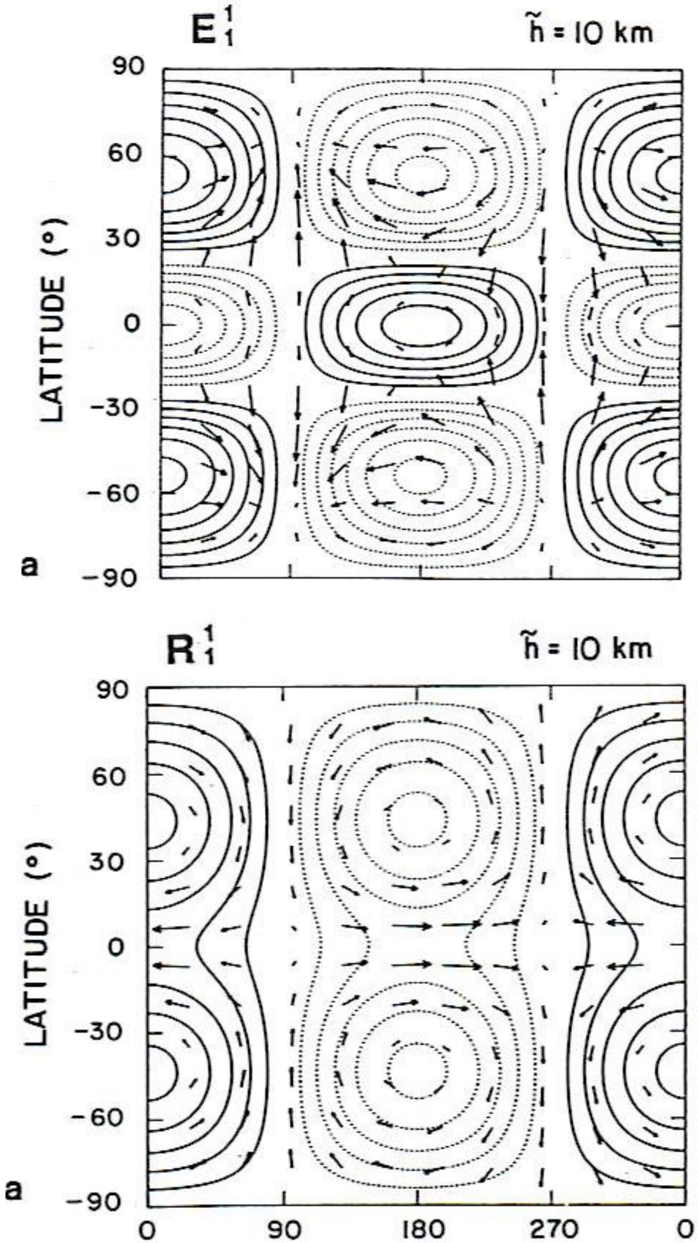
Solutions:

$$\begin{pmatrix} u \\ v \\ \phi \end{pmatrix} = \sum_{m,n,k,j} c_{m.n.k,j} \; \Phi_k(p) \; \exp(imx + i\lambda_{m.n.k,j} t) \begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{\phi} \end{pmatrix} (y)_{m,n,k,j}$$

Daley 1991; Errico 1986 NCAR/TN-277

Linear Normal-Mode Initialization

g(t=0) = 0



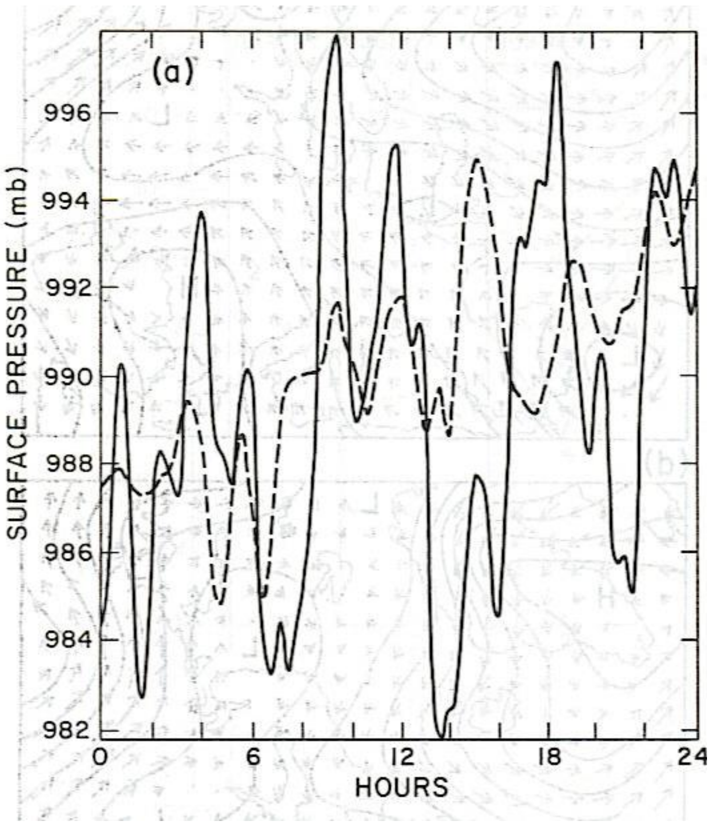
Daley 1991

Structures

of two

normal modes

Temperton and Williamson 1979



Nonlinear Normal Mode Initialization

dg/dt (t=0) = 0

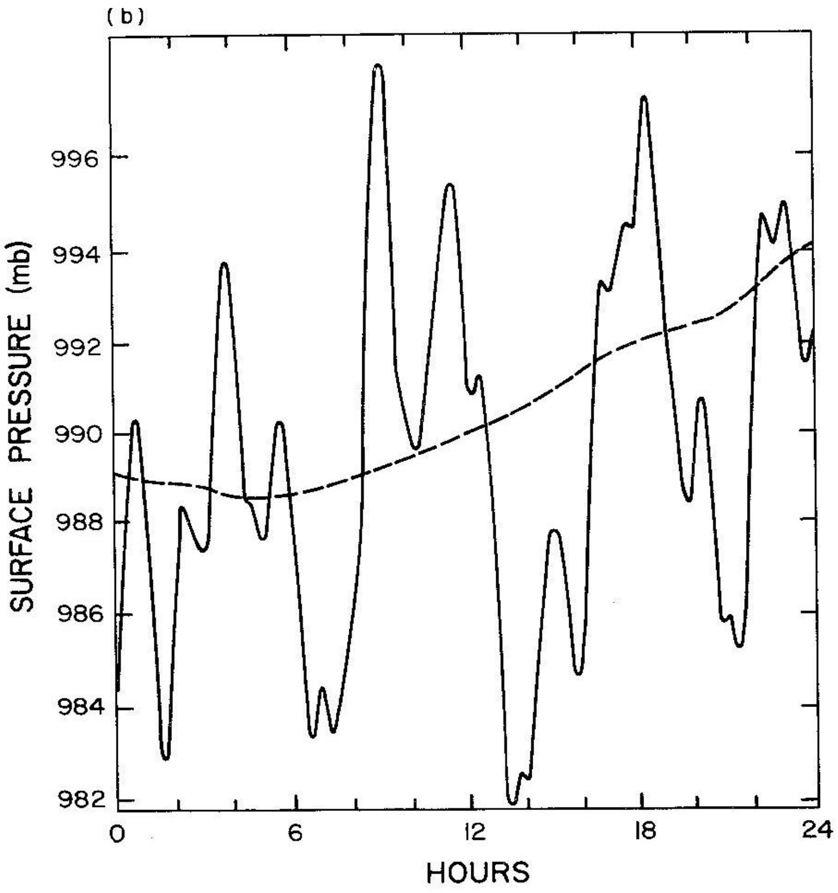


Figure 10.1 Time evolution of surface pressure during a 24 hour model integration for (a) linear and (b) nonlinear normal mode initialization. Solid curves, uninitialized; dashed curves, initialized. (After Williamson and Temperton, *Mon. Wea. Rev.* **109**: 745, 1981. The American Meteorological Society.)

Machenhauer 1997 Cont. Atmos. Phys.; Baer and Tribbia 1997 MWR

Why is the extra-tropical atmosphere quasi-balanced?

Charney: 1955 *Tellus* (a paraphrase)

The observed extra-tropical motions are dominantly quasi-balanced because:

1. The principal atmospheric forcing is large scale and long period.

2. The quasi-balanced motion must be relatively stable with respect to gravity-wave perturbations. (*by inference; also see Errico 1981*)

3. Dissipation must be sufficient to remove what energy is otherwise leaked into gravity waves. (*added by R. Errico*)

Lorenz 1980 *JAS* Atmospheric dynamics lies on a slow-manifold.

Gravity Waves as Forced and Damped harmonic Oscillators

The amplitude of a gravity wave structure is governed by an equation of the form:

$$\frac{dg}{dt} = -i\lambda g + F(t) - \nu g$$

Consider harmonic forcing $F(t) = F(0)\exp(-i\mu t)$. Then

$$g(t) = [g(0) - \frac{F(0)}{i\lambda - i\mu + \nu}]\exp(-(i\lambda + \nu)(t)) \quad + \quad \frac{F(t)}{i\lambda - i\mu + \nu}$$

QG Theory and NNMI

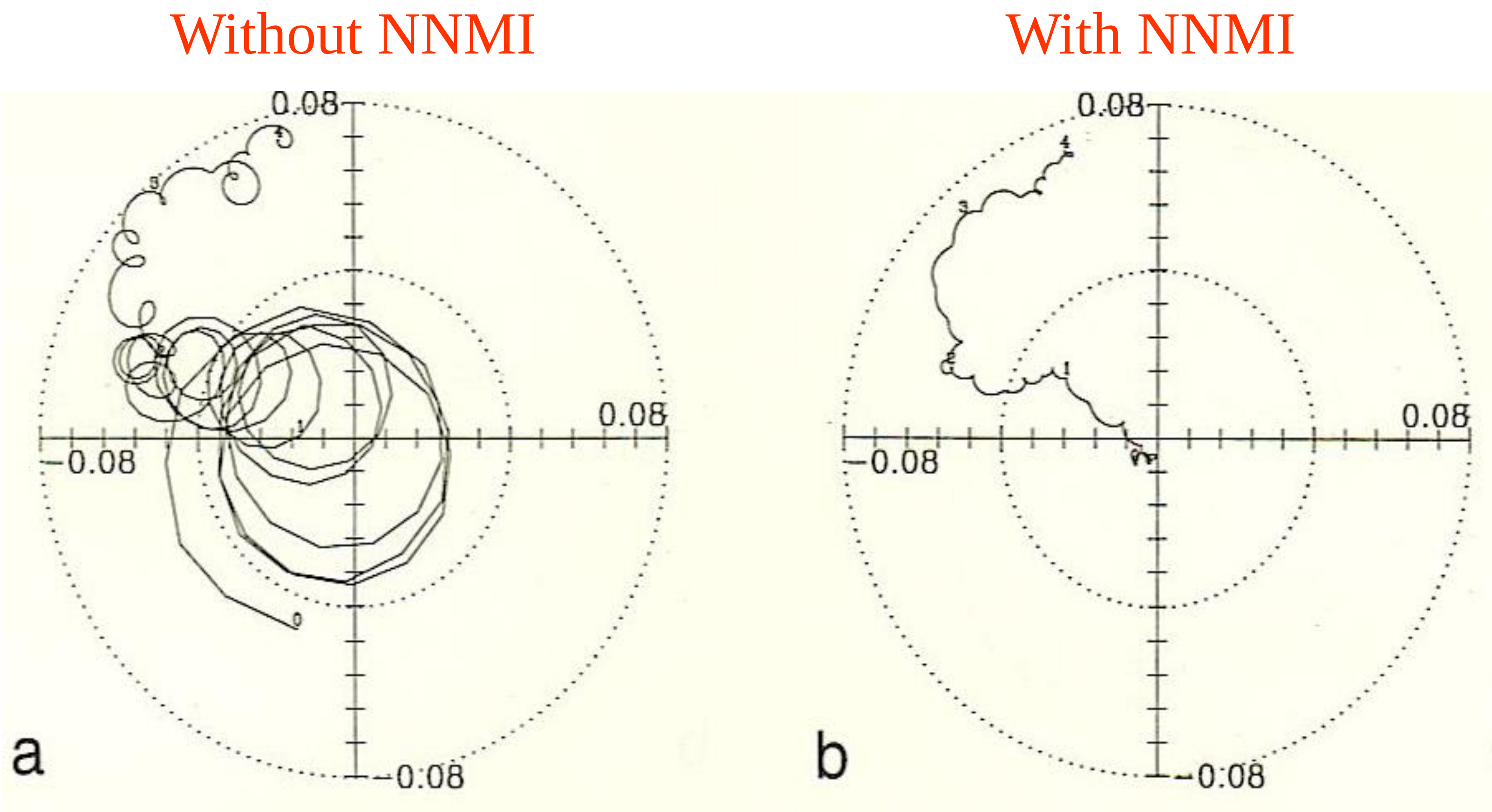
In the extra-tropics, the NNMI balance condition dg/dt =0 is equivalent to

(1) The nonlinear balance equation relating mass and vorticity fields, with some additional small terms;
(2) The QG-omega equation defining the wind divergence, with some additional small terms;

(3) Solved with the constraint that a form of linearized potential vorticity is specified;
(4) And applied only to large vertical but small horizontal scales for which the resonant frequency is large.

The choice of constraint and scale selectivity matter!!

Harmonic Dial for External m=4 Mode, Period=3.7h

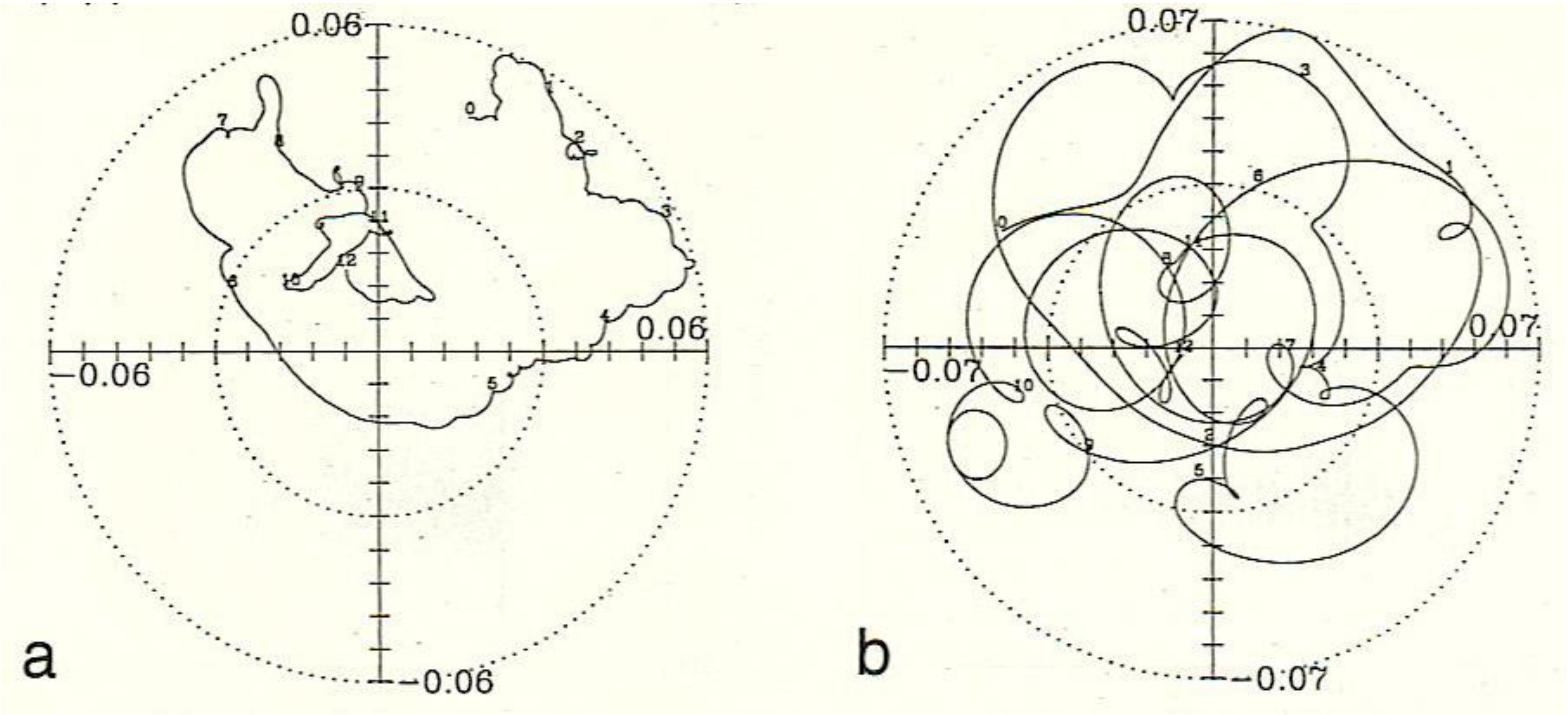


Errico 1997 J Japan Met Soc

Harmonic Dials from a Climate Simulation

External Mode P=3.7h

Internal Mode P=11.6h

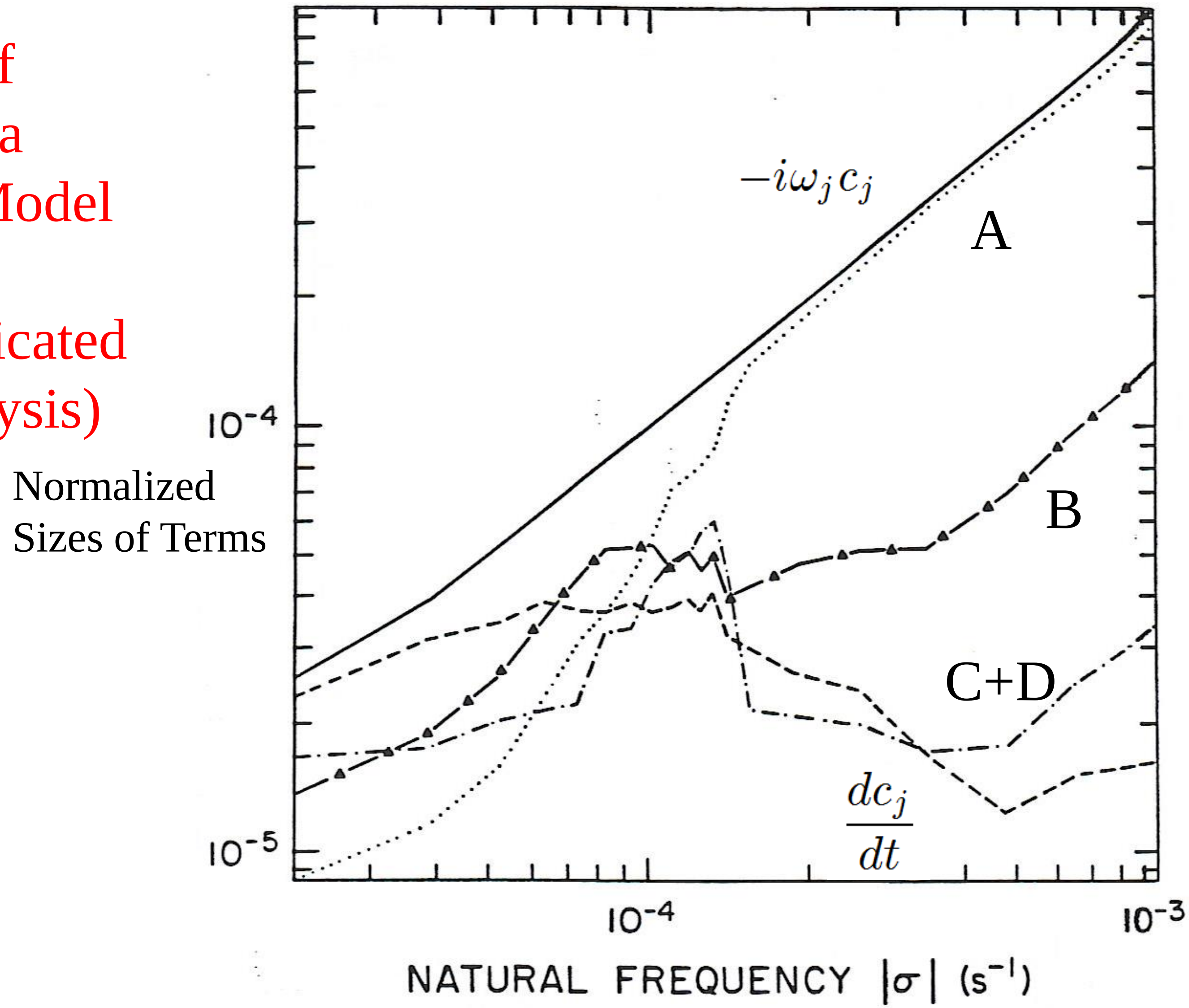


Errico 1997 J Japan Met Soc

$$\frac{dc_j}{dt} = -i\omega_j c_j + A(r,r) + B(r,g) + C(g,g) + D$$

Balance of Modes in a Climate Model

(a sophisticated scale analysis)



Errico 1984, 1990 MWR; Errico et al. 1988 MWR

Why does balance matter in data assimilation?

1. Unrealistic initial imbalances will create unrealistic forecasts

2. Unrealistic imbalances can be accentuated through moist diabatic processes.

3. Large initial imbalances will tend to create less accurate backgrounds

4. Balance can be exploited to relate u, v, T, ps (esp. in extra-tropics)

5. Errors in balanced initial conditions will tend to create balanced background errors, so the error statistics should reflect that; i.e., background errors of u, v, T, ps tend to be correlated, esp. in extra-tropics.

Consistency between analysis and initialization

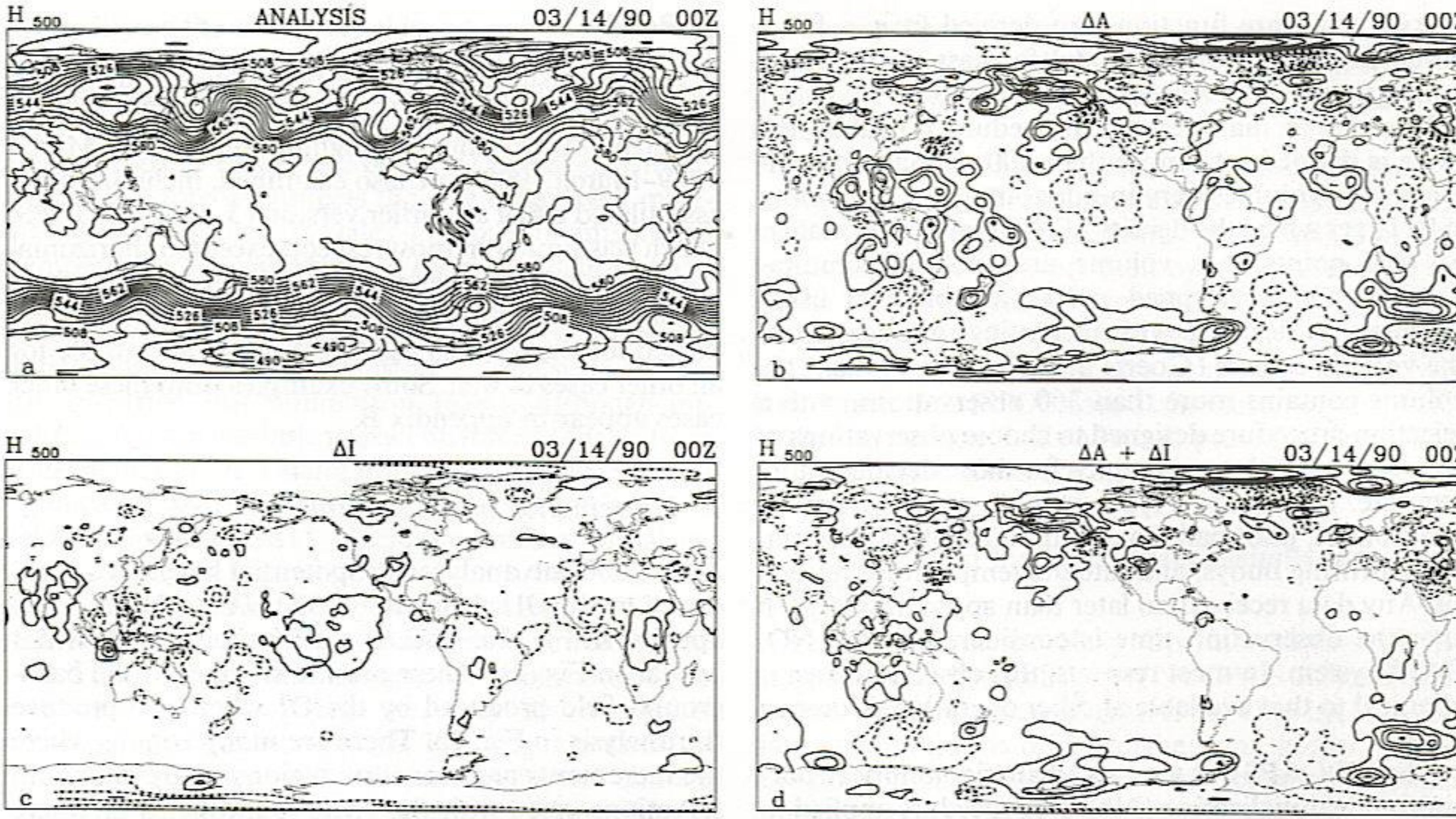
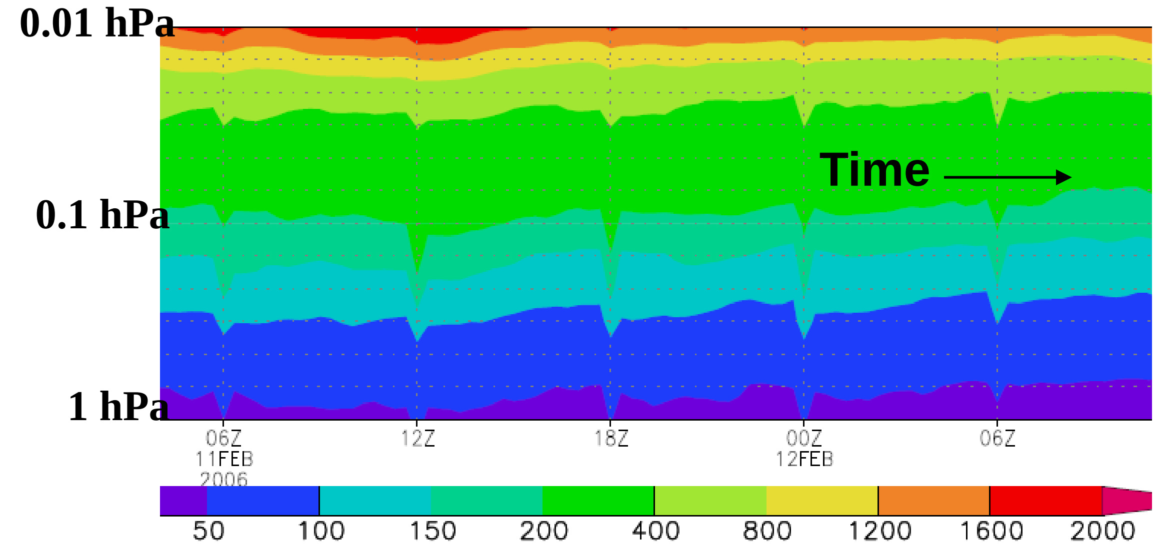
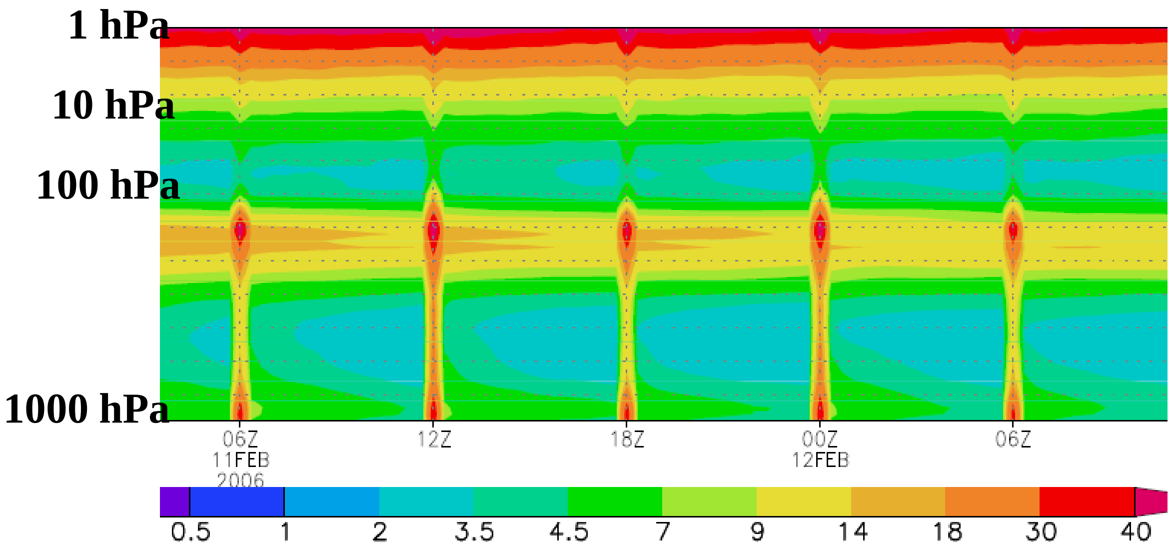


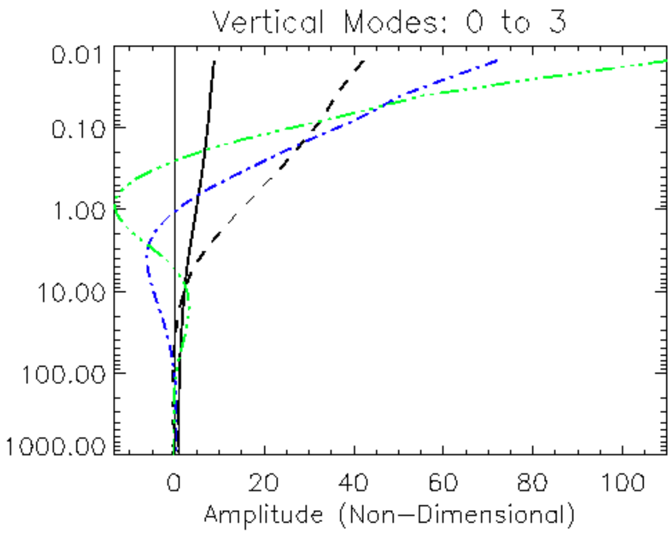
FIG. 2. The 500-mb height field on 14 March 1990 (a) as analyzed by NOGAPS, (b) analysis increments, (c) initialization increments, and (d) the sum of analysis and initialization increments. Contour interval is 60 m in (a) and 10 m in (b)–(d). Zero contours are omitted; negative contours are dashed; and labels in (a) are dekameters.

Errico, Rosmond, Goerss 1993 MWR

Global mean squared divergence tendency





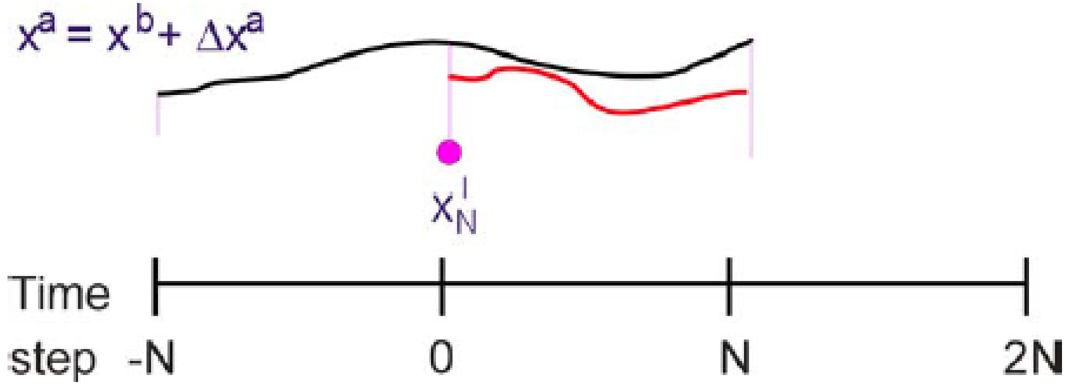


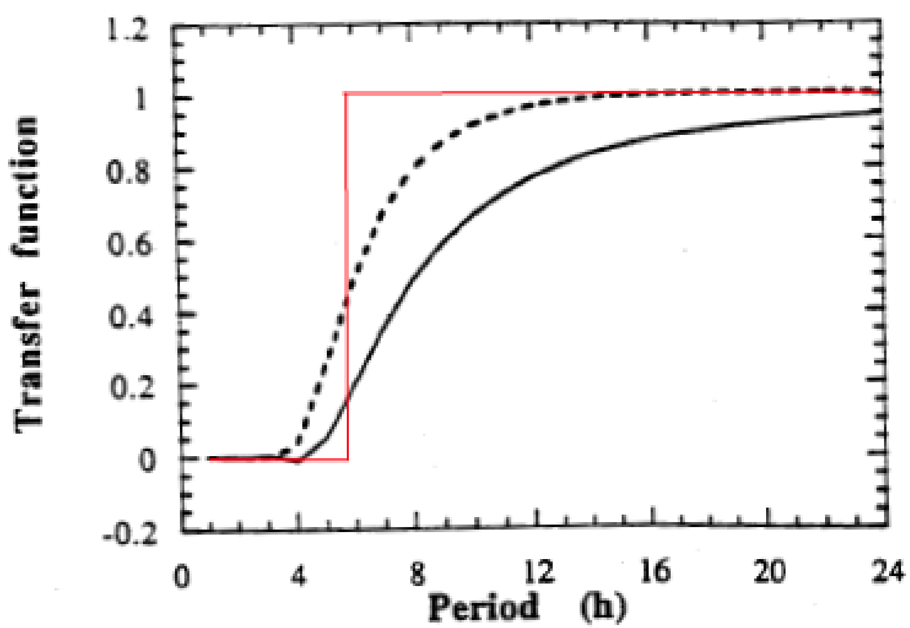
Structures of 3 largest scale vertical normal modes

GMAO-GSI 3DVAR

Digital Filter

$$x_0^I = \sum_{k=-N}^{N} h_k x_k^u$$





Lynch and Huang 1992 MWR
Fillion et al 1995 Tellus

IAU (Incremental Analysis Update)

For $n = -N, \ldots, N-1$

$$f(t_0 + (n+1)\Delta t) = f(t_0 + n\Delta t) \ + \ \Delta f_M(t_0 + n\Delta t) \ + \ \frac{1}{2N}\Delta f_A(t_0)$$

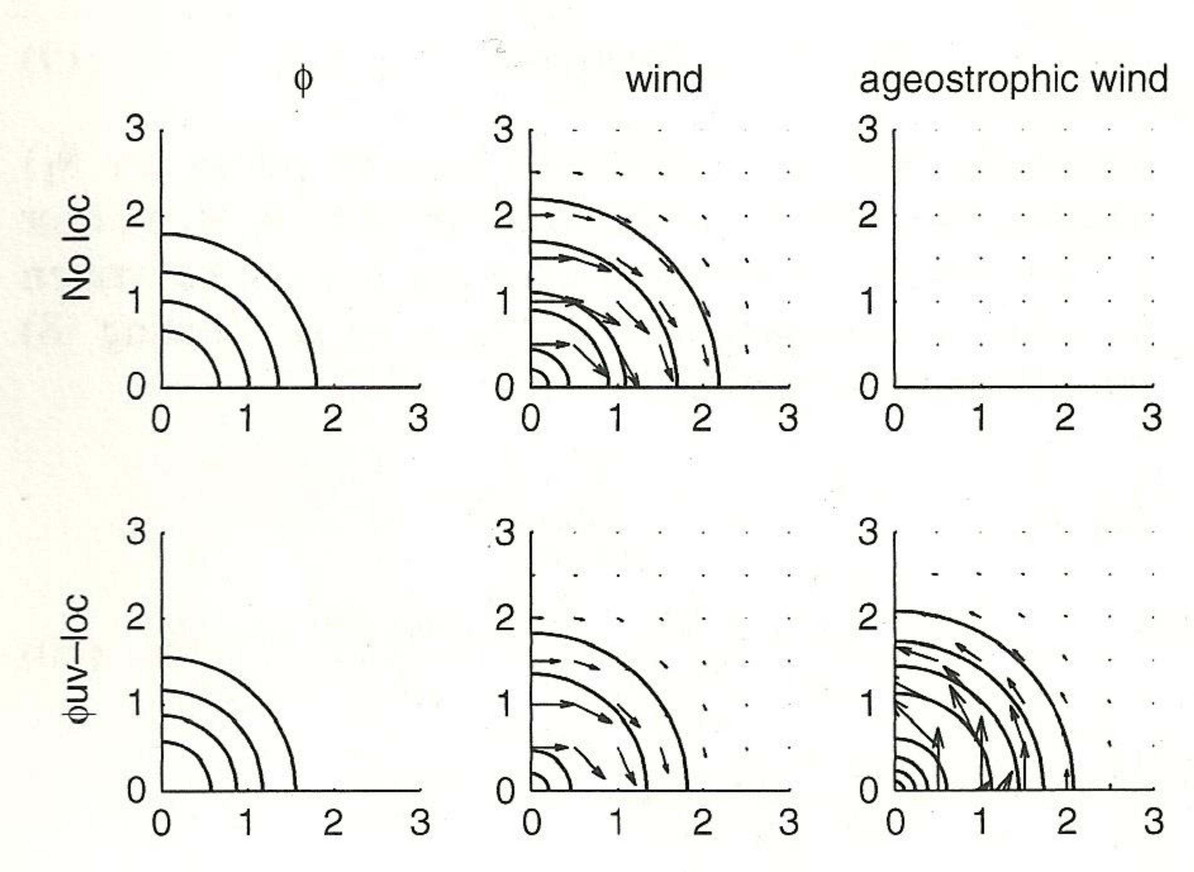
Bloom et al. 1996 MWR

Localization in Ensemble DA

Example response to a single observation of ϕ

Balanced Bkg Error Covariances

Same, with Schur product Localization



Kepert 2009 QJRMS

Lessons Learned

1. There are many ways to balance models, each with varying degrees of success.

2. Most balance schemes have some undesirable consequences.

3. Balance should not be applied everywhere, at all scale, in the same way, to the same degree.

4. Balance should be considered when performing an analysis.

5. Details matter.

Common Misconceptions About Balance

F: Small scales are not balanced.

T: Balance depends on both vertical and horizontal scales.

T: Deep modes are likely balanced even on the mesoscale.

F: Atmospheric fields are on a "slow manifold."

T: Some atmospheric forcing has short time scales.

T: In realistic models, freely propagating gravity waves are present to some degree.

Common Misconceptions about Initialization

F: Initialization is inappropriate when gravity waves are important.

T: It is necessary when gravity waves may affect forecasts.

T: It removes waves which are not really there.

T: It is unnecessary when gravity waves are unimportant.

Techniques may come and go,
but fundamentals remain (almost) forever.

(Unless, of course, they are neglected.)

Recommended References

References cited in the lecture or in the papers listed below.

General concepts about balancing in general and nonlinear normal mode initialization in particular:
Daley, R., 1991: *Atmospheric Data Analysis*, Cambridge University Press. 420pp.

Presentation of some basic NNMI concepts using a simple, periodic f-plane model.
Errico, R. M., 1989: *Theory and application of nonlinear normal mode initialization*. NCAR Technical Note, NCAR/TN--344+IA, 145 pp.

Short presentation of some fundamental issues regarding balance and initialization:
Errico, R.M., 1997: On the removal of gravitational noise in numerical forecasts. *J. Meteor. Soc. Japan*, **75**, 219-227.

Revelation of some peculiar effects of nudging methods for assimilation:
Bao, J.W. and R. M. Errico, 1997: An adjoint analysis of the nudging method for data assimilation. *Mon. Wea. Rev.*, **125**, 1355-1373.

Incorporation of balance within (rather than after) the assimilation problem, including for the mesoscale:
Papers by Luc Fillion, Andrew Lorenc