Understanding Radiation Thermometry – Part II
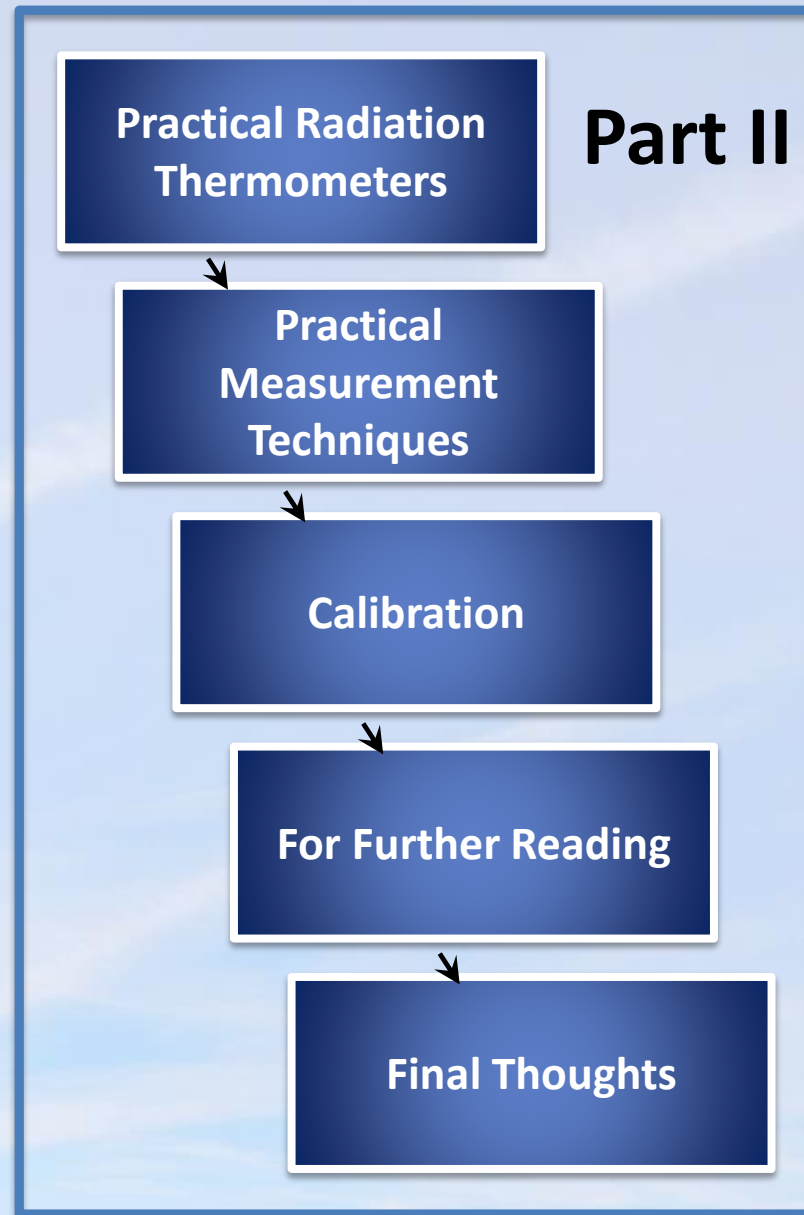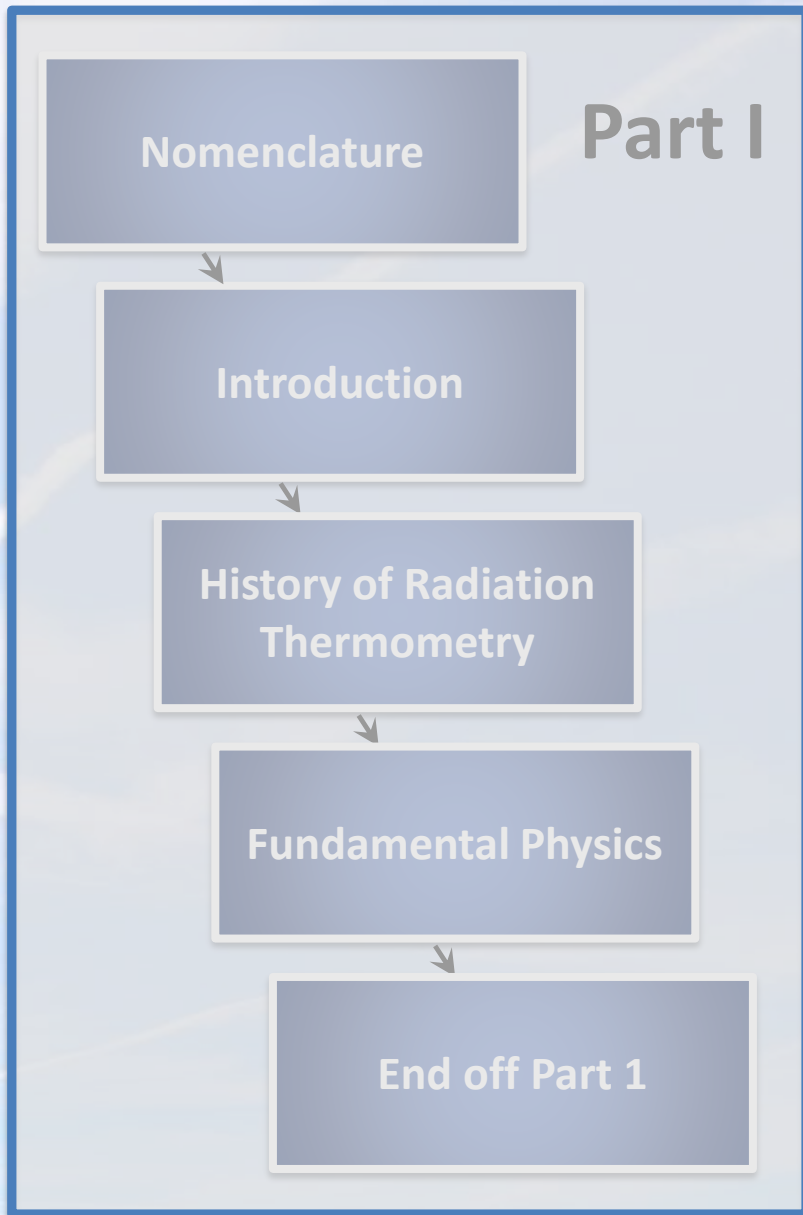
Timothy K. Risch

NASA Armstrong Flight Research Center

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Lesson Plan

Part I

- Nomenclature
- Introduction
- History of Radiation Thermometry
- Fundamental Physics
- End off Part 1

Part II

- Practical Radiation Thermometers
- Practical Measurement Techniques
- Calibration
- For Further Reading
- Final Thoughts

Preliminaries

An Excel® worksheet containing solutions to all examples are available from Tim Risch at Timothy.K.Risch@NASA.gov

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Nomenclature - I

c speed of light, 2.99792458×10^8 m/s

C_1 Planck's first constant, $2\bar{h}c^2 = 1.191043 \times 10^8$ W-µm⁴/m²-sr

C_2 Planck's second constant, $\bar{h}c/k = 14{,}387.75$ µm-K

C_3 Constant in Wien's displacement law

C_4 Constant in equation for maximum blackbody intensity

$D_i(\lambda)$ detector spectral response function for detector i

$\bar{h}$ Planck's constant, 6.626068×10^{-34} J-s

k Boltzmann constant, $1.3806503 \times 10^{-23}$ J/K

$i_{b,\lambda}(\lambda, T)$ spectral emissive radiance of a perfect blackbody at wavelength λ and temperature T, W/m²-sr-µm

$i_\lambda(\lambda, T)$ spectral radiant intensity of a non-blackbody at wavelength λ and temperature T, W/m²-sr-µm

Nomenclature - II

$i_b(T)$ total radiant intensity of a blackbody at temperature T, W/m²-sr-μm

$i(T)$ total radiant intensity non-blackbody at temperature T, W/m²-sr-μm

T actual surface temperature, K

T_r ratio temperature, K

T_λ measured surface temperature at wavelength λ assuming a perfect emitter, K

Nomenclature - III

$\varepsilon(T)$ total emissivity of a non-blackbody at temperature T

$\overline{\varepsilon_i}$ wavelength-averaged emissivity for detector i

$\overline{\varepsilon_r}$ wavelength averaged emissivity ratio for detector 1 and 2, $\overline{\varepsilon_2}/\overline{\varepsilon_1}$

ε_λ monochromatic emissivity of a non-blackbody at wavelength λ and temperature T

$\overline{\varepsilon}(\overline{T})$ inferred total emissivity of a non-blackbody at temperature $\overline{T}$

ε_r emissivity ratio at two wavelengths λ_1 and λ_2, $\varepsilon_{\lambda_1}/\varepsilon_{\lambda_2}$

$\Delta\lambda_i$ bandwidth of narrow-band detector i, μm

Λ equivalent wavelength, $\lambda_1\lambda_2/(\lambda_2 - \lambda_1)$, μm

λ_i wavelength of detector i, μm

Nomenclature - IV

λ_l lower wavelength on wide-band radiative thermometer, µm

λ_u upper wavelength on wide-band radiative thermometer, µm

π ratio of perimeter to diameter for a circle, 3.14159265358979

σ Stefan-Boltzmann constant, $\frac{\pi C_1}{15}\left(\frac{\pi}{C_2}\right)^4$ = 5.670401 × 10^{-8}

      $W/m^2\text{-}K^4$

Ω solid angle, sr

Note that the nomenclature or symbology for radiation is not standard across all disciplines or sources. For consistency, we have adopted the naming and symbolic convention used by Howell, J. R. , R. Siegel, and M. P. Mengüç, *Thermal Radiation Heat Transfer,* Fifth Edition, Taylor & Francis, New York, 2010.

Practical Measurement Techniques

"To point out of the advantages that would arise from ascertaining the heat of a body at a very high temperature would be unnecessary, the importance of the subject is allowed."
- **J. M'Sweeny, M.D**. - 1829

Radiation Temperature Measurement - I

- For a blackbody, there are three primary ways to determine the temperature by measuring:
  - › The total emitted radiation
  - › The distribution across wavelengths or the total radiation across a wavelength band
  - › The emitted radiation at one wavelength
- Measuring any of these should uniquely determine the temperature

Radiation Temperature Measurement - II

- As discussed before, real surfaces with emissivities less than one do not behave like a blackbody.

- Instead, they emit radiation at rates less than a blackbody

- This introduces a new problem, for any given measurement of a non-ideal surface, we now have two unknowns:

  › Temperature

  › Emissivity

- Without assuming either one or the other, the state of the surface cannot be uniquely determined.

Three Possible Solutions:

1. For a single measurement, assume the emissivity and calculate the temperature (spectral method)

2. Make two measurements at different wavelengths and assume a relationship between the emissivities at each wavelength and calculate a single temperature (ratio method)

3. Make multiple measurements at different wavelengths and assume some functional form of the emissivity and find the best fit to the temperature and emissivity (multi-spectral method)

For now, all of the examples assume narrow-band detectors, but later we show how to accommodate wide-band detectors

Some Temperature Definitions

- "True" Surface Temperature - The actual temperature of the surface; what we would measure with a thermometer not affected by the surface emissivity. This is the temperature we would measure if, for example, we used a contact thermometer such as a thermocouple.

- "Equivalent Blackbody" Surface Temperature - The temperature we would infer from a radiation thermometer if we assumed that the surface was a perfect emitter. Again, because all real surfaces emit less than a blackbody, the equivalent blackbody surface temperature will always be lower than the true surface temperature.

Method 1 – Spectral Method - I

From a single equivalent blackbody temperature measurement T_λ, assume the spectral emissivity ε_λ and calculate the "true" surface temperature T. This can be achieved by using the definitions of black and gray emissive intensity:

$$i_\lambda(T) = \varepsilon_\lambda \cdot i_{b,\lambda}(T) = i_{b,\lambda}(T_\lambda) \qquad (11)$$

And then solving for the true surface temperature:

$$\varepsilon_\lambda \cdot \frac{C_1}{\lambda^5(e^{C_2/\lambda T} - 1)} = \frac{C_1}{\lambda^5(e^{C_2/\lambda T_\lambda} - 1)} \qquad (12)$$

Method 1 – Spectral Method - II
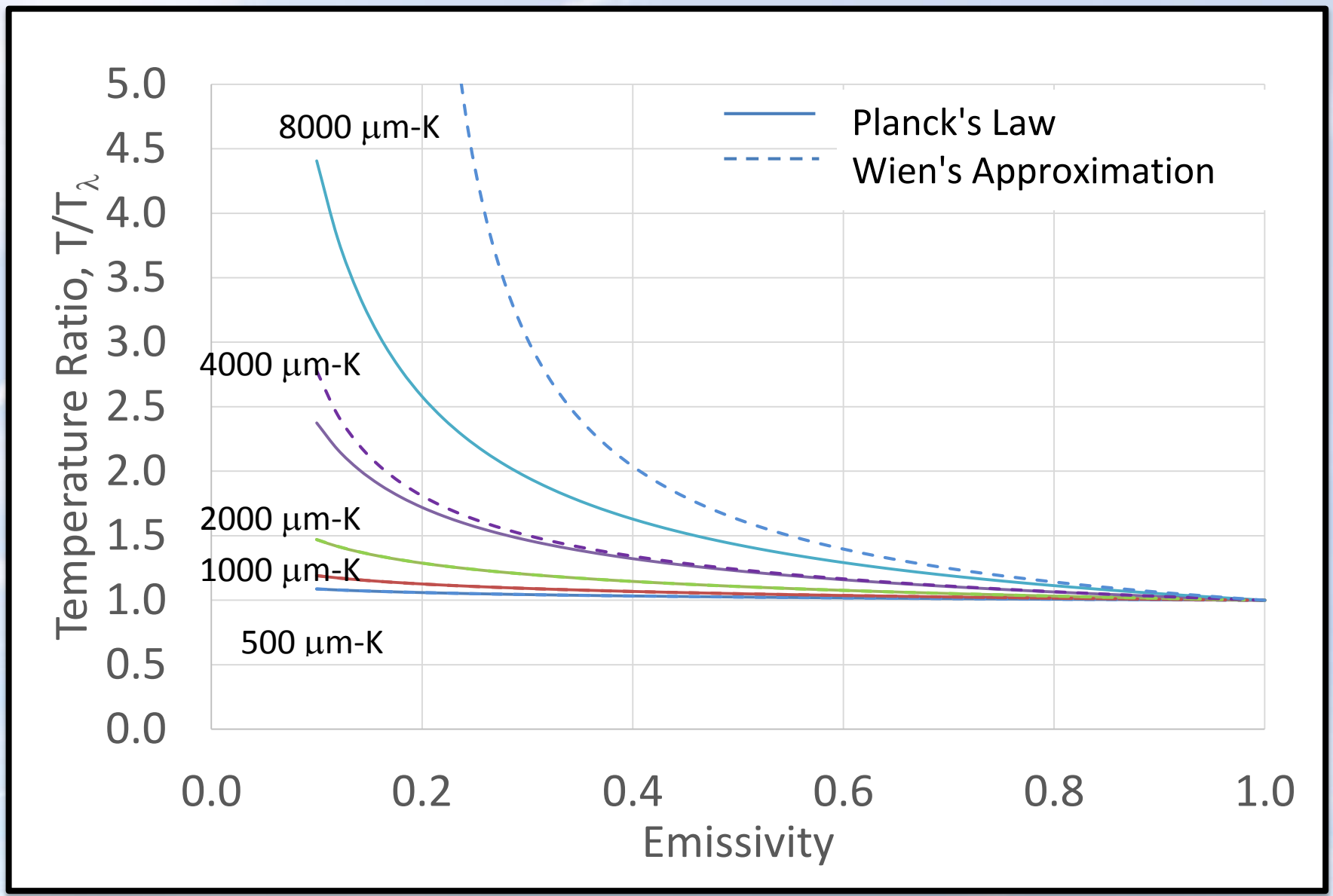
Solving for true surface temperature this is:

$$\frac{1}{T} = \frac{\lambda}{C_2} \ln\left(\varepsilon_\lambda \cdot \left(e^{C_2/\lambda T_\lambda} - 1\right) + 1\right) \qquad (13)$$

Equation 13 can be simplified under the special case when $C_2/\lambda T_\lambda \gg 1$ (Wien's approximation) then:

$$\frac{1}{T} = \frac{1}{T_\lambda} + \frac{\lambda}{C_2} \ln \varepsilon_\lambda \qquad (14)$$

Method 1 – Spectral Method - IV

Example 4 - I

E4: What is the true surface temperature when a surface at an equivalent blackbody temperature of 3,820 K is measured with a 0.5-μm wavelength detector and an emissivity of 0.8 is assumed?

Example 4 - II

A4: The wavelength temperature product is 3,820 K times 0.5 μm or 1,910 μm-K. This is much less than C_2 or 14,388 μm-K. So, Wien's approximation is valid and routine bb_tstw can be used to give:

$$\frac{1}{T} \doteq \frac{1}{T_\lambda} + \frac{\lambda}{C_2} \ln \varepsilon_\lambda = \frac{1}{3,820\ K} + \frac{0.5\ \mu\text{m}}{14,388\ \mu\text{m}-\text{K}} \ln(0.8)$$

$$T = 3,937\ \text{K}$$

Note that this value is within about 0.1 K of the value calculated with the full Planck equation.

Example 5 - I

E5: Repeat Example 4, except with an 8-μm detector. What is the true surface temperature?

Example 5 - II

A5: The wavelength temperature product is 3,820 K times 8 μm or 30,560 μm-K. This is greater than C_2 or 14,388 μm-K. So, the full Planck equation (Equation 1) must be used using routine bb_tst:

$$T = \frac{C_2/\lambda}{\ln\left(\varepsilon_\lambda \cdot \left(e^{C_2/\lambda T_\lambda} - 1\right) + 1\right)}$$

$$T = \frac{14{,}388 \ \mu m{-}K/8 \ \mu m}{\ln\left(0.8 \cdot \left(e^{14{,}388 \ \mu m{-}K/8 \ \mu m \ 3820 \ K} - 1\right) + 1\right)} = 4{,}579 \ K$$

If indeed we had used Wien's approximation, the calculated true surface temperature would have been over 7,200 K and off by over 2,600 K.

Method 1 – Spectral Method - V

- We can see by looking at Equation 13, that the correction to the measured surface temperature is dependent on the wavelength and when Wien's approximation applies in Equation 14, the correction is proportional to the wavelength. This suggests that using a short wavelength detector will minimize the effect of emissivity on our temperature measurement.

Spectral Method Error Estimation - I

By differentiating Equation 13 with respect to emissivity, we can determine the sensitivity of the computed "true" surface temperature to the assumed emissivity for a detector i with wavelength λ_i. The result is:

$$\frac{d \ln T}{d \ln \varepsilon_{\lambda_i}} = \frac{\varepsilon_{\lambda_i}}{T} \frac{dT}{d\varepsilon_{\lambda_i}} = -\frac{\lambda_i T}{C_2} \cdot \frac{\left(e^{C_2/\lambda_i T} - 1\right)}{e^{C_2/\lambda_i T}} \tag{15}$$

applying Wien's approximation, this becomes:

$$\frac{d \ln T}{d \ln \varepsilon_{\lambda_i}} = -\frac{\lambda_i T}{C_2} \tag{16}$$
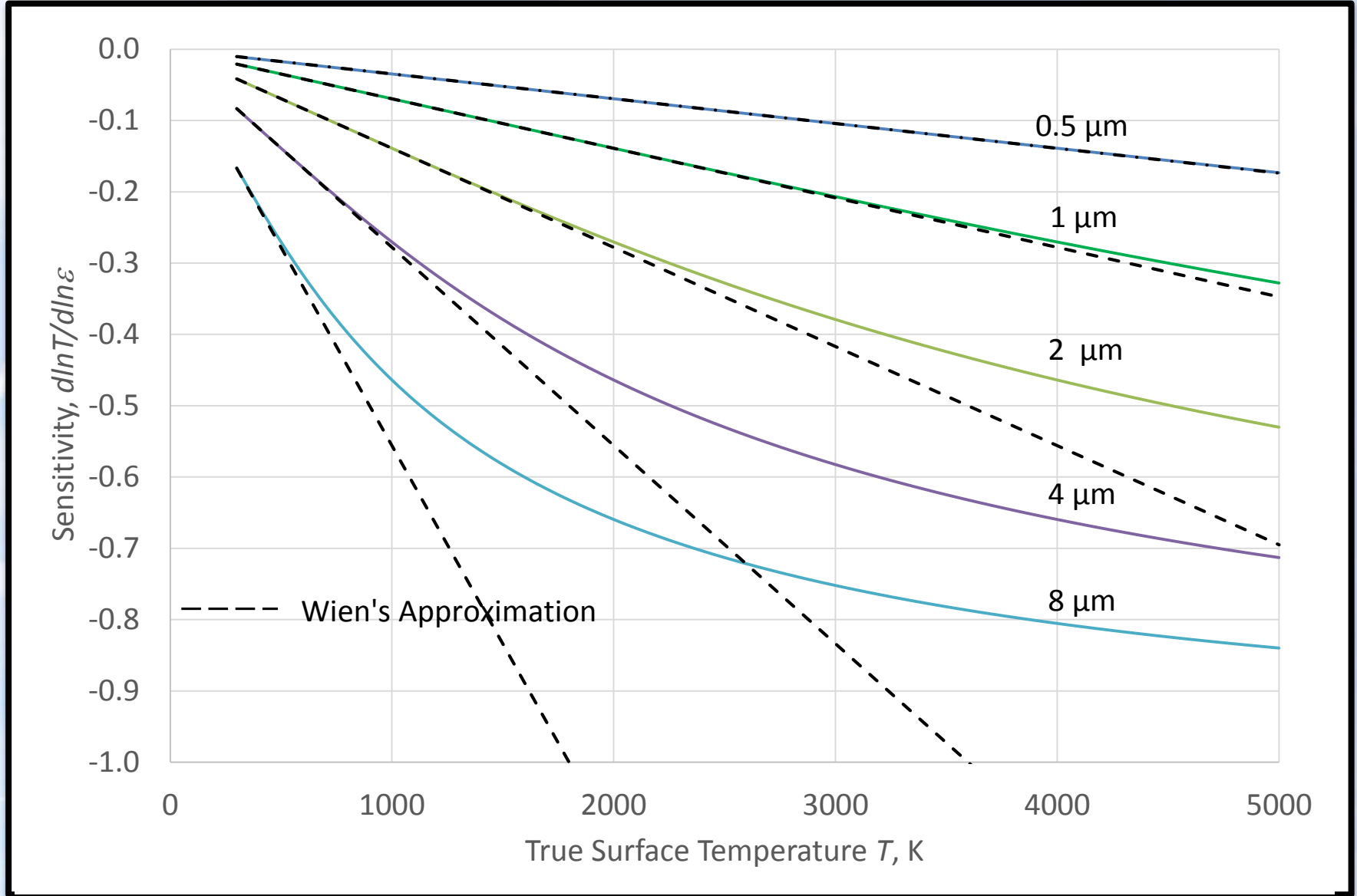
Spectral Method Error Estimation - II

Using the sensitivity and an estimate of the uncertainty in the emissivity produces an estimate of the uncertainty in the surface temperature:

$$\frac{\Delta T}{T} = \frac{d \ln T}{d \ln \varepsilon_{\lambda_i}} \cdot \frac{\Delta \varepsilon_{\lambda_i}}{\varepsilon_{\lambda_i}} \qquad (17)$$

Spectral Method Error Estimation - III

Example 6 - I

E6: If the spectral emissivity of graphite at 0.53 μm is estimated to be 0.8 with an estimated uncertainty of 20%, what is the estimated uncertainty in the temperature if a detector at this wavelength indicates an equivalent blackbody temperature of 2,950 K?

Example 6 - II

A6: The wavelength temperature product is 2,950 K times 0.53 μm or 1,563.5 μm-K. This is much less than C_2 or 14,388 µm-K. So, Wien's approximation is valid and Equation 14 and routine bb_tstw can be used:

$$\frac{1}{T} \doteq \frac{1}{T_\lambda} + \frac{\lambda}{C_2} \ln \varepsilon_\lambda = \frac{1}{2{,}950 \text{ K}} + \frac{0.52 \text{ µm}}{14{,}388 \text{ µm}-\text{K}} \ln(0.8) = 3.038 \times 10^{-3} \text{ 1/K}$$

$$T = 3{,}023 \text{ K}$$

$$\frac{d \ln T}{d \ln \varepsilon_{\lambda_i}} = -\frac{0.53\mu\text{m} \cdot 3{,}023 \text{ K}}{14{,}388 \text{ µm}-\text{K}} = -11\%$$

Therefore, since the emissivity uncertainty is estimated to be 20%, the resulting uncertainty in surface temperature is 20% × 11% or 2.2%. For a temperature of 3,023 K, the absolute uncertainty is 67 K.

Example 7 - I

E7: Repeat example 6 except for a detector with a wavelength of 5.8 μm where the emissivity is estimated to be 0.8 with an estimated uncertainty of 20% at a temperature of 2,950 K.

Example 7 - II

A7: The wavelength temperature product is 2,950 K times 5.8 μm or 17,110 μm-K. This is greater than C_2 or 14,388 μm-K, so, Wien's approximation is not valid and the full Planck equation (Equation 1) and routine bb_tst must be used.

$$T = \frac{14{,}388 \text{ μm–K}/5.8\ \mu m}{\ln\left(0.8 \cdot \left(e^{14{,}388 \text{ μm–K}/5.8 \text{ μm} \cdot 2{,}950 \text{ K}} - 1\right) + 1\right)} = 3{,}445 \text{ K}$$

$$\frac{d \ln T}{d \ln \varepsilon_{\lambda_i}} = \frac{\lambda_i T}{C_2} \cdot \frac{e^{C_2/\lambda_i T_{\lambda_i}} - 1}{e^{C_2/\lambda_i T_{\lambda_i}}} = \frac{5.8 \text{μm} \cdot 3{,}445 \text{ K}}{14{,}388 \text{ μm–K}} \cdot \frac{e^{\frac{14{,}388 \text{ μm–K}}{5.8 \text{μm}} \cdot 3{,}445 \text{ K}} - 1}{e^{\frac{14{,}388 \text{ μm–K}}{5.8 \text{μm}} \cdot 3{,}445 \text{ K}}} = -71\%$$

Therefore, since the emissivity uncertainty is estimated to be 20%, the resulting uncertainty in temperature is 20% × 71% or 14%. For a temperature of 3,445 K, the absolute uncertainty is about 491 K!

Rule 1

- When the emissivity is unknown and must be estimated, the most accurate surface temperature measurement is made when a detector with a wavelength as short as possible is used.

Determination of Emissivities at Other Wavelengths

Once the true surface temperature has been determined from a detector at one wavelength, the emissivity at other detector wavelengths can be determined from:

$$\varepsilon_\lambda = \frac{\left(e^{C_2/\lambda T} - 1\right)}{\left(e^{C_2/\lambda T_\lambda} - 1\right)} \tag{18}$$

Of course, using a measurement at the same wavelength will return our initial guess for ε_λ.

Emissivity Uncertainty Estimation - I

The uncertainty in the emissivity at a second wavelength ε_{λ_i} based on the estimated uncertainty in a given wavelength ε_{λ_1} is, by differentiation:

$$\frac{d \ln \varepsilon_{\lambda_i}}{d \ln \varepsilon_{\lambda_1}} = \frac{\varepsilon_{\lambda_1}}{\varepsilon_{\lambda_i}} \cdot \frac{d \varepsilon_{\lambda_i}}{d \varepsilon_{\lambda_1}} = \frac{\lambda_1}{\lambda_i} \cdot \frac{\left(e^{-C_2/\lambda_1 T} - 1 \right)}{\left(e^{-C_2/\lambda_i T} - 1 \right)} \qquad (19)$$

If Wien's approximation is valid at both wavelengths then:

$$\frac{d \ln \varepsilon_{\lambda_i}}{d \ln \varepsilon_{\lambda_1}} = \frac{\lambda_1}{\lambda_i} \qquad (20)$$
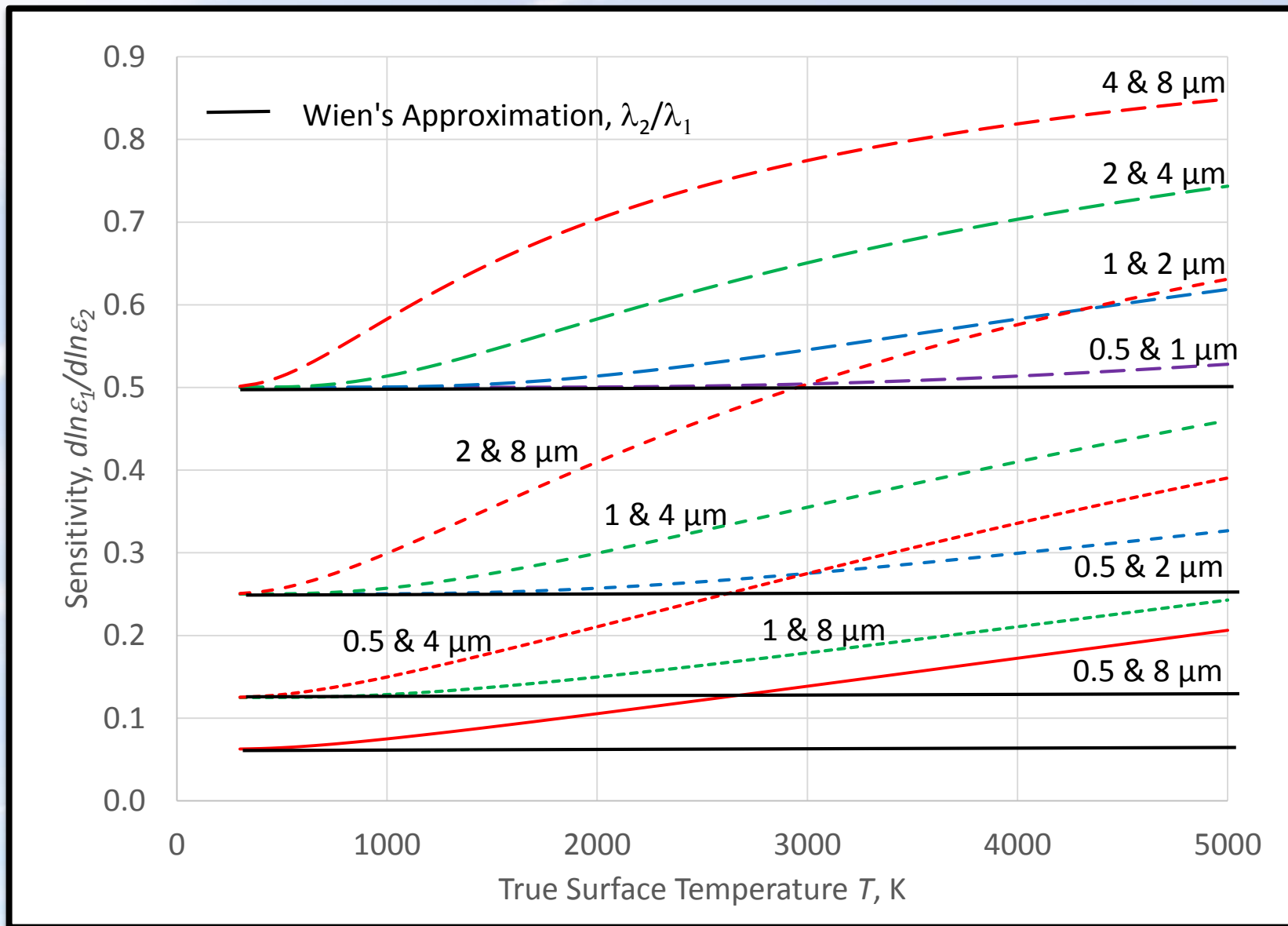
Emissivity Uncertainty Estimation - II

- Unlike with temperature, the uncertainty in the computed emissivity decreases with increasing wavelength.

- The error is approximately inversely proportional to the ratio of the two wavelengths.

- Therefore emissivities derived from longer wavelength detectors have less uncertainty. However, since the emissivity is usually a strong function of wavelength, values at long wavelengths may or may not be useful if emissivities over the entire wavelength band are needed.

Emissivity Uncertainty Estimation - III

Example 8 - I

E8: A 0.53-μm wavelength detector is used to determine the surface temperature. The measured equivalent blackbody temperature from this detector is 1,950 K and the emissivity is estimated to be 0.6 with an estimated uncertainty of 25%. What are the inferred emissivities and uncertainties at 1.0 μm and 5.8 μm if detectors at both wavelengths indicate equivalent blackbody temperatures of 1,740 K?

Example 8 - II

A8: The true surface temperature using Equation 13 and routine bb_tst is:

$$T = \frac{14{,}388\ \mu\text{m–K}/0.53\ \mu m}{\ln\left(0.8 \cdot \left(e^{14{,}388\ \mu\text{m–K}/0.53\ \mu\text{m}\cdot\ 1{,}950\ \text{K}} - 1\right) + 1\right)} = 2{,}024\ \text{K}$$

The emissivity at 1.0 μm using Equation 15 and routine bb_emiss is:

$$\varepsilon_{\lambda_2} = \frac{e^{14{,}388\ \mu\text{m–K}/1.0\ \mu m - 2{,}024\ \text{K}} - 1}{e^{14{,}388\ \mu\text{m–K}/1.0\ \mu m - 1{,}740\ \text{K}} - 1} = 0.313$$

Example 8 - II

The sensitivity to the emissivity at wavelength 2 based on the assumed emissivity at wavelength 1 using Equation 19 and routine bb_dlne2dlne1 is:

$$\frac{d \ln \varepsilon_{\lambda_2}}{d \ln \varepsilon_{\lambda_1}} = \frac{0.53 \ \mu\text{m}}{1.0 \ \mu\text{m}} \cdot \frac{e^{14,388 \ \mu\text{m}-\text{K}/0.53 \ \mu\text{m} \cdot 2,024 \ \text{K}} - 1}{e^{14,388 \ \mu\text{m}-\text{K}/1.0 \ \mu\text{m} \cdot 2,024 \ \text{K}} - 1} = 0.530$$

And the relative uncertainty in the emissivity is:

$$\frac{\Delta \varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = \frac{d \ln \varepsilon_{\lambda_2}}{d \ln \varepsilon_{\lambda_1}} \cdot \frac{\Delta \varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = 0.530 \cdot 25\% = 13.3\%$$

The emissivity at 5.8 μm using routine bb_emiss is:

$$\varepsilon_{\lambda_2} = \frac{e^{14,388 \ \mu\text{m}-\text{K}/5.8 \ \mu\text{m}-2,024 \ \text{K}} - 1}{e^{14,388 \ \mu\text{m}-\text{K}/5.8 \ \mu\text{m}-1,740 \ \text{K}} - 1} = 0.761$$

Example 8 - III

The sensitivity to the emissivity at wavelength 2 based on the assumed emissivity at wavelength 1 using routine bb_dlne2dlne1 is:

$$\frac{d \ln \varepsilon_{\lambda_2}}{d \ln \varepsilon_{\lambda_1}} = \frac{0.53 \mu m}{5.8 \mu m} \cdot \frac{e^{14,388 \, \mu m - K/0.53 \mu m \cdot 2,024 \, K} - 1}{e^{14,388 \, \mu m - K/5.8 \mu m \cdot 2,024 \, K} - 1} = 0.129$$

And the relative uncertainty in the emissivity is:

$$\frac{\Delta \varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = \frac{d \ln \varepsilon_{\lambda_2}}{d \ln \varepsilon_{\lambda_1}} \cdot \frac{\Delta \varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = 0.129 \cdot 25\% = 3.2\%$$

Note that the emissivity uncertainty at 5.8 μm is four times less than at 1μm.

Rule 2

- The uncertainty in the spectral emissivity at a given wavelength decreases approximately inversely with the wavelength that is used.

Method 2 – Ratio method - I

Make measurements T_{λ_1} and T_{λ_2} with detectors at two different wavelengths λ_1 and λ_2. Assume a ratio for the spectral emissivities at the two wavelengths ε_r (could be 1) and then calculate the true temperature T by solving the following:

$$\varepsilon_r = \frac{\varepsilon_{\lambda_1}}{\varepsilon_{\lambda_2}} = \frac{i_{b,\lambda_1}(T_{\lambda_1})/i_{b,\lambda_1}(T)}{i_{b,\lambda_2}(T_{\lambda_2})/i_{b,\lambda_2}(T)} \qquad (21)$$

and then calculate the individual emissivities from:

$$\varepsilon_{\lambda_1} = i_{b,\lambda_1}(T_{\lambda_1})/i_{b,\lambda_1}(T)$$
$$\varepsilon_{\lambda_2} = i_{b,\lambda_2}(T_{\lambda_2})/i_{b,\lambda_2}(T) \qquad (22)$$

Method 2 – Ratio method - II

If Wien's approximation is valid, the approximate solution for the true temperature T can be calculated directly by:

$$\frac{1}{T} = \frac{1}{T_r} - \frac{\Lambda}{C_2} \ln \varepsilon_r \qquad (23)$$

where Λ is an equivalent wavelength

$$\Lambda = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} \qquad (24)$$

and T_r is the ratio temperature, the temperature of an equivalent blackbody having the same ratio of spectral radiances at the two specified wavelengths as that of the target.

$$\frac{1}{T_r} = \frac{\Lambda}{\lambda_1 T_{\lambda_1}} + \frac{\Lambda}{\lambda_2 T_{\lambda_2}} \qquad (25)$$

Method 2 – Ratio method - III

Once the true surface temperature is known, the emissivities can then be determined by:

$$\varepsilon_{\lambda_i} = \exp\left(\frac{C_2}{\lambda_i}\left(\frac{1}{T_{\lambda_i}} - \frac{1}{T}\right)\right)$$

(26)

Ratio Method Observations

- For an emissivity ratio of 1, the calculated ratio temperature of the instrument is the true surface temperature.

- The equivalent wavelength can be much greater than either of the two individual wavelengths when the difference between the two wavelengths is small.

- As the difference between the two wavelengths increases, the value of the equivalent wavelength approaches 1.

- For Wien's approximation to be valid, the wavelength temperature product of the two individual detectors need to be small, not the product of the equivalent temperature and the effective wavelength.

- Equation 22 for the true surface temperature is identical to the equation for surface temperature in the spectral method (Equation 14) with the effective wavelength replacing the single wavelength and the ratio temperature replacing the single detector temperature.

Example 9 - I

E9: For a two-band radiation thermometer with detector wavelengths at 0.5 and 0.6 μm, what are the true surface temperature and the spectral emissivities when the measured equivalent blackbody temperatures at the two wavelengths are 2,800 and 2,750 K, respectively and an emissivity ratio of 0.9 is assumed?

Example 9 - II

A9: In this case, Wien's approximation is valid so that using the approximate method from Equation 23 and calculating the effective wavelength Λ from Equation 24 using routine bb_elam is:

$$\Lambda = \frac{0.5 \text{ μm} \cdot 0.6 \text{ μm}}{0.6 \text{ μm} - 0.5 \text{ μm}} = 3.0 \text{ μm}$$

Using Equation 25, the ratio temperature T_r from routine bb_etemp is:

$$T_r = \left(\frac{3.0 \text{μm}}{0.5 \text{ μ}m \cdot 2{,}800 \text{ K}} + \frac{3.0 \text{ μ}m}{0.6 \text{ μ}m \cdot 2{,}750 \text{ K}} \right)^{-1} = 3{,}080 \text{ K}$$

and using Equation 22 and routine bb_tstw, the true surface temperature is:

$$T = \left(\frac{1}{3{,}080 \text{ K}} - \frac{3.0 \text{ μ}m}{14{,}388 \text{ μm-K}} \cdot \ln 0.9 \right)^{-1} = 3{,}304 \text{ K}$$

Example 9 - III

Knowing the individual measured detector temperatures and the surface temperature, the emissivities are determined using Equation 18 and routine bb_emiss:

$$\varepsilon_{\lambda_1} = \frac{\left(e^{14,388\ \mu m-K/0.5\ \mu m \cdot 3,304\ K} - 1\right)}{\left(e^{14,388\ \mu m-K/0.5\ \mu m \cdot 2,800\ K} - 1\right)} = 0.209$$

$$\varepsilon_{\lambda_2} = \frac{\left(e^{14,388\ \mu m-K/0.6\ \mu m \cdot 3,304\ K} - 1\right)}{\left(e^{14,388\ \mu m-K/0.6\ \mu m \cdot 2,750\ K} - 1\right)} = 0.232$$

So that $\varepsilon_{\lambda_1}/\varepsilon_{\lambda_2} = 0.9$, consistent with our initial assumption.

Ratio Method Sensitivities

The sensitivity of the true surface temperature to the emissivity ratio can be calculated for the case where Planck's law is valid by differentiating equation 17. This is:

$$\frac{d \ln T}{d \ln \varepsilon_r} = \frac{\left(e^{C_2/(\lambda_1 T)} - 1 \right) \cdot \left(e^{C_2/(\lambda_2 T)} - 1 \right)}{\left(\frac{C_2 e^{C_2/(\lambda_2 T)}}{\lambda_2 T} \cdot \left(e^{-C_2/(\lambda_1 T)} - 1 \right) - \frac{C_2 e^{C_2/(\lambda_1 T)}}{\lambda_1 T} \cdot \left(e^{C_2/(\lambda_2 T)} - 1 \right) \right)} \quad (27)$$

When Wien's approximation is valid, then the sensitivity can be easily determined utilizing the similarity to the spectral method as discussed before. The sensitivity then becomes:

$$\frac{d \ln T}{d \ln \varepsilon_{\lambda_i}} = \frac{\Lambda T}{C_2} \quad (28)$$

Example 10 - I

E10: For the same conditions as Example 9, what would the error in the true surface temperature be if an assumed emissivity ratio of 1.0 was used but the actual ratio of the surface emissivities was 0.9?

Example 10 - II

A10: This problem can be solved two ways: directly or using sensitivities. We will use sensitivities first.

In this case Wien's approximation is valid since both the wavelength-temperature products are smaller than C_2. The sensitivity from Equation 26 is:

$$\frac{d \ln T}{d \ln \varepsilon_{\lambda_i}} = \frac{3.0 \ \mu m \cdot 3{,}304 \ K}{14{,}388 \ \mu m - K} = 0.688$$

So that the estimated uncertainty in temperature is:

$$0.688 \cdot 0.1 = 6.9\% \text{ or } 228 \ K$$

Example 10 - III

Rather than using sensitivities, we can calculate the difference directly:

For an emissivity of 1.0, the true surface temperature is just the ratio temperature, or 3080 K. Therefore, the error is:

$$3080 \text{ K} - 3{,}304 \text{ K} = 224 \text{ K}$$

almost exactly equal to the linear approximation using sensitivities.

Example 11 - I

E11: For a two-band radiation thermometer with detector wavelengths at 4.0 and 8.0 μm, what is the error introduced when an emissivity ratio of 1.0 is assumed, when in fact the emissivity ratio of the sample has a ratio of 0.9? Assume the measured spectral temperatures at the two wavelengths are 2,800 and 2,750 K, the same as in Problem 9.

Example 11 - II

A11: Since the products of T_λ and λ for both wavelengths are not less than C_2 or 14,388, μm-K, the full Planck Equation must be used to solve for the true surface temperature (Equation 31 and routine bb_tratio).

$$\frac{\left(e^{-C_2/\lambda_2 T_{\lambda_2}} - 1 \right)}{\left(e^{-C_2/\lambda_2 T} - 1 \right)} \cdot \frac{\left(e^{-C_2/\lambda_1 T} - 1 \right)}{\left(e^{-C_2/\lambda_1 T_{\lambda_1}} - 1 \right)} - \varepsilon_r = 0$$

Example 11 - III

The calculated values for true temperatures are:

$$\varepsilon_r = 0.9, T = 4{,}118 \text{ K}$$

$$\varepsilon_r = 1.0, T = 2{,}974 \text{ K}$$

The difference is 1,144 K!

This example points out the large error using long wavelengths and the need to use short wavelengths whenever possible.

Example 11 - IV

The sensitivity of the emissivity ratio to surface temperature is (Equation 27):

$$\frac{d \ln \varepsilon_r}{d \ln T} = \frac{1}{T} \cdot \frac{(e^{C_2/\lambda_2 T} - 1)}{(e^{C_2/\lambda_1 T} - 1)} \cdot \left[\left(\frac{C_2}{\lambda_2 T}\right) e^{\frac{C_2}{\lambda_2 T}} \frac{(e^{C_2/\lambda_1 T} - 1)}{(e^{C_2/\lambda_2 T} - 1)^2} - \left(\frac{C_2}{\lambda_1 T}\right) \frac{e^{C_2/\lambda_1 T}}{(e^{C_2/\lambda_2 T} - 1)} \right]$$

Since the temperature change is so great, linear sensitivities extrapolated from one point will not be accurate.

However, the calculated sensitivities are:

$$\varepsilon_r = 0.9, \frac{d \ln T}{d \ln \varepsilon_r} = -3.77$$

$$\varepsilon_r = 1.0, \frac{d \ln T}{d \ln \varepsilon_r} = -2.56$$

Example 11 - V

The temperature error can be estimated using a centered average of the temperature and sensitivity:

$$\Delta T = -(4{,}118\ \text{K} + 2{,}974\ \text{K}) \cdot \frac{(3.77 + 2.56)}{4} \cdot 0.1 = 1{,}122\ \text{K}$$

almost exactly the same as the direct method.

Further Observations

- The selection of the optimal detector wavelengths for the ratio method is based upon two competing factors. As difference in the two detector wavelengths approaches zero the value of the effective wavelength goes to infinity and therefore tends to drive the temperature uncertainty higher. However, at the same time the smaller the difference in wavelengths makes the estimation of the emissivity ratio more accurate and therefore drives the temperature uncertainty lower.

Rule 3

- The optimal detector wavelengths for the ratio method are based on the two competing factors: 1) the need to keep the wavelength difference small to ensure an accurate assumed ratio, but 2) not too small that the effective wavelength increases and becomes too large.

Multispectral Methods - I

- Multispectral methods are an extension of the ratio method where a larger number of detectors are used.

- Multiple measurements are then used to perform a best fit to single temperature and wavelength relationship.

- The emissivity relationship can be a constant value or some other more complex function such as a polynomial.

Multispectral Methods - II

- Advantages
  - › Large number of detectors reduces noise and averages errors inherent in measurements

- Disadvantages
  - › Requires more complex hardware
  - › Multiple detectors increases data collection requirements and requires increased processing
  - › Data may not provide an identifiably unique solution unless a large number of measurements are made

Multispectral Hardware Utilizing Dispersive Spectrometer and Silicon Array Detector

Focusing Mirrors

Entrance Slit

Grating

32-Channel Array Silicon Detector
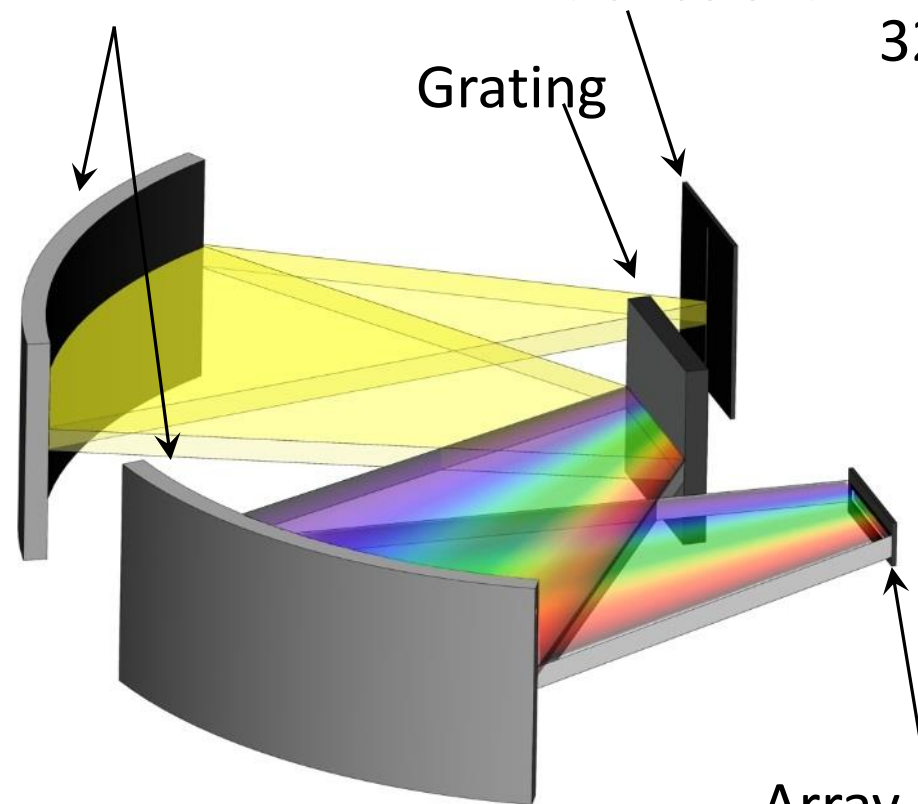
Array Detector
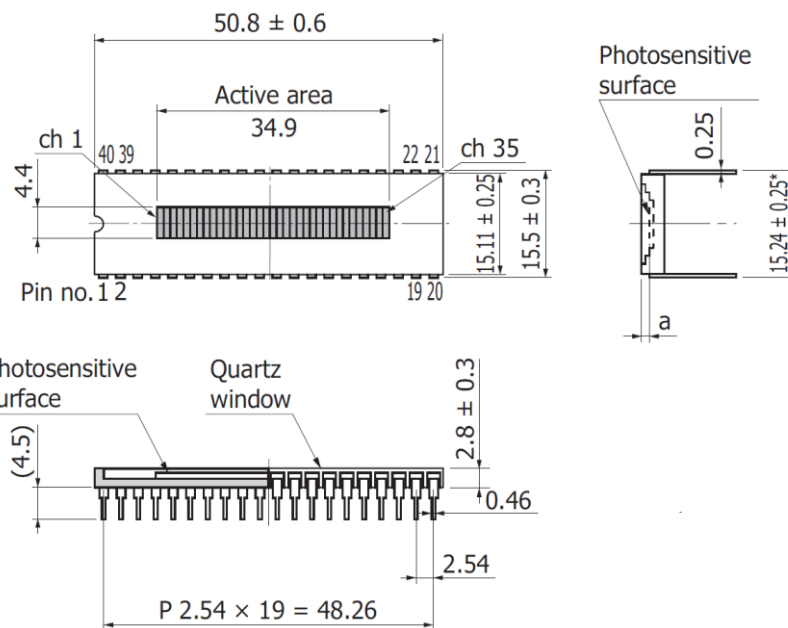
Example 12 - I

E12: Given data to the right from a 32-channel silicon array detector, calculate the spectral emissivity and true surface temperature if the spectral emissivity is assumed to follow a second-order polynomial with wavelength.
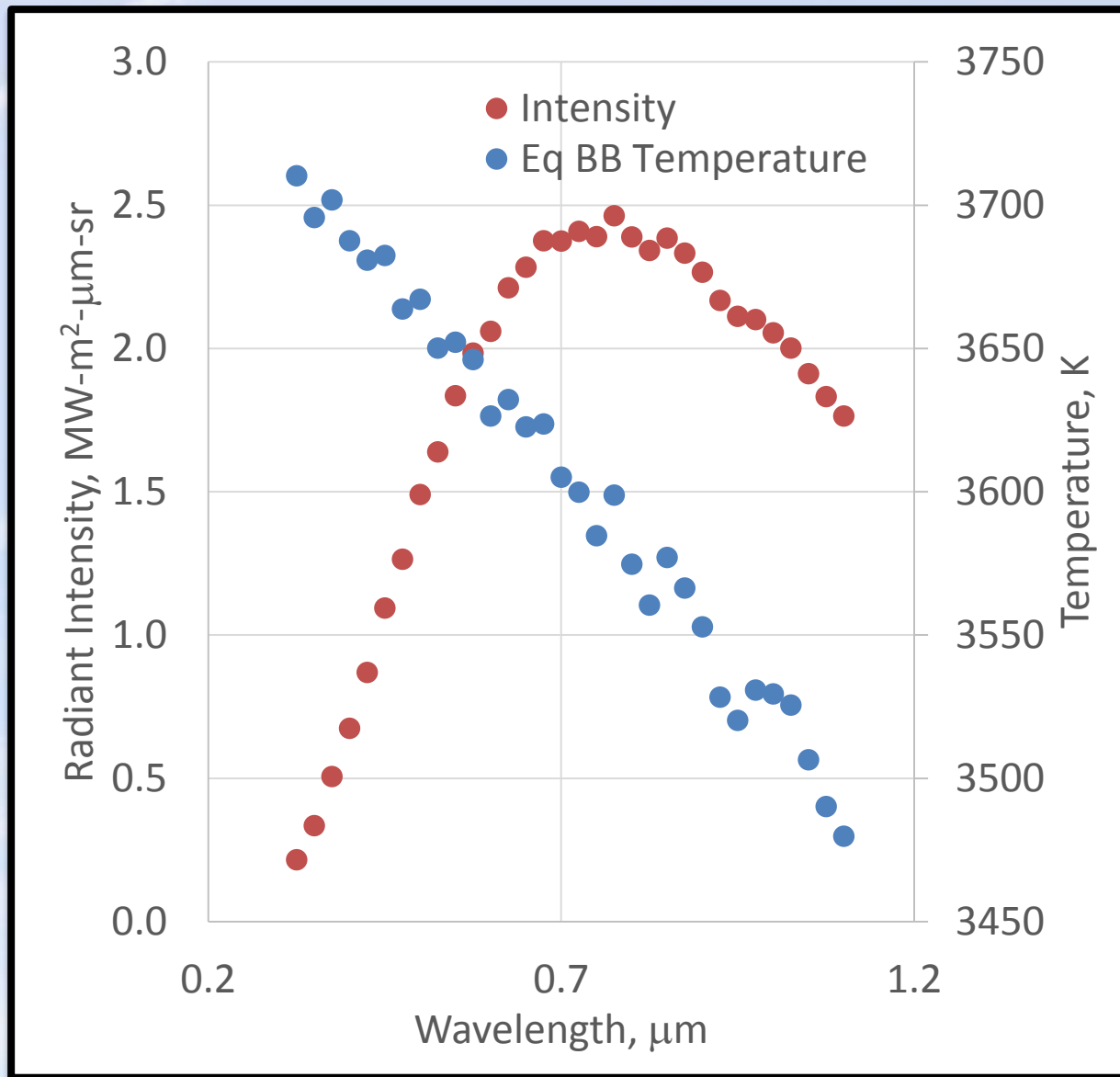
Example 12 - II

A12: The solution requires that we find values for T, a_0, a_1, and a_2 that minimize the function f:

$$f = \sum_{i}^{n} \left(\varepsilon_{\lambda_i} \cdot i_{b,\lambda_i}(T) - i_{\lambda_i}(T_{\lambda_i}) \right)^2 \qquad (29)$$

where

$$\varepsilon = a_2 \lambda^2 + a_1 \lambda + a_0 \qquad (30)$$

Example 12 - III

Results:

$T = 3{,}802$ K

$a_2 = -0.06222$

$a_1 = 0.0610$

$a_0 = 0.7347$
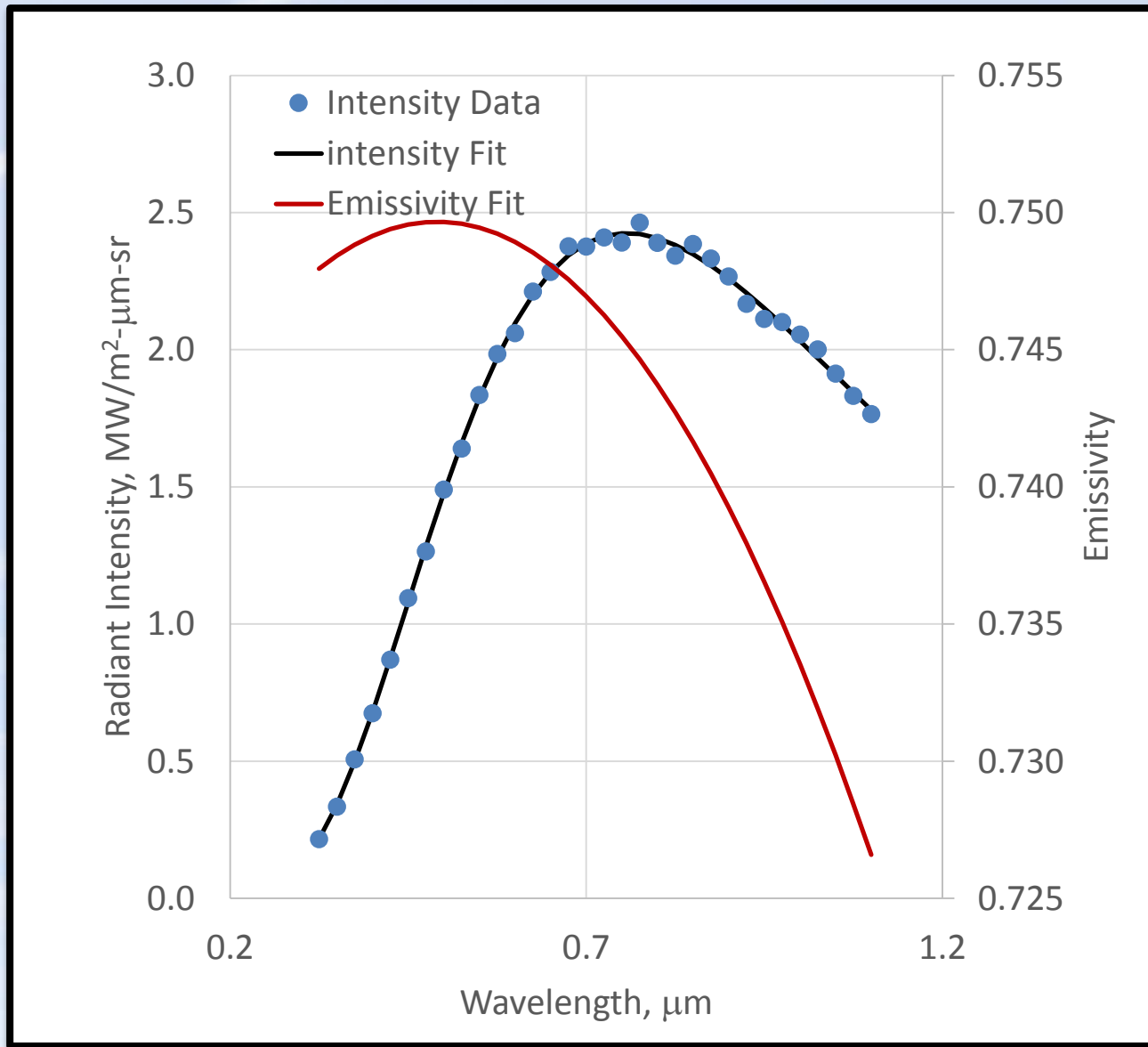
Wide-Bandwidth Detectors

"The laws of light and of heat translate each other;—so do the laws of sound and colour; and so galvanism, electricity and magnetism are varied forms of this selfsame energy."
— Ralph Waldo Emerson

Wide-Bandwidth Detectors - I

- Up until now, we have considered only narrow-band detectors, ones in which the wavelength band has been limited

- Sometime, using wide-band detectors prove to be an advantage

- However, using wide-band detectors is more difficult and requires additional understanding to properly interpret the measured data

Wide-Bandwidth Detectors - II

- Advantages

  › Provide a higher signal since the signal is measured over a wider band

  › Require less hardware since the wavelength limiting device is eliminated

- Disadvantages

  › Introduces non-linearities in the calibration because the blackbody intensity varies with wavelength and the response of the detector is not uniform across all wavelengths

  › Makes data reduction and analysis more complex

Wide-Bandwidth Detectors - III

- The spectral response curves as a function of wavelength $D(\lambda)$ are available for many detectors from manufacturers.
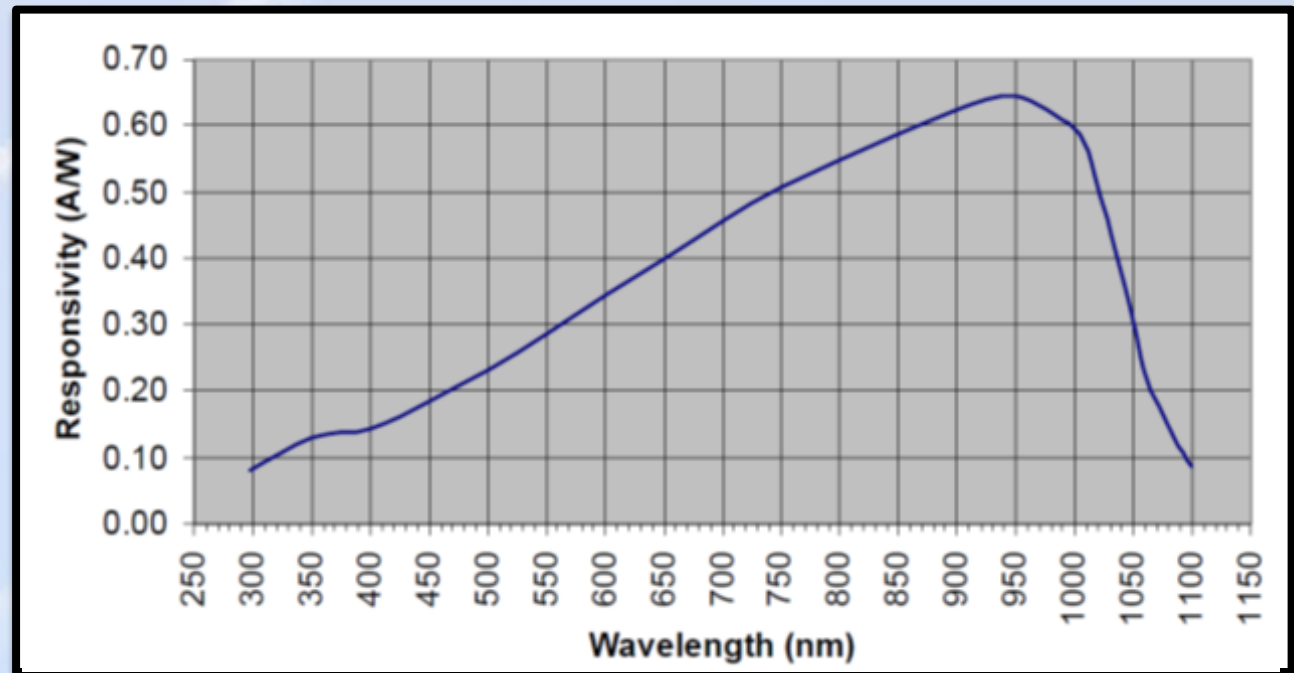
- Often, though, these are only nominal specifications and can vary with the actual detector and can change over time for some detectors.

- Measuring the response function requires more specialized equipment and methods not available in most laboratories, so that the exact response is often not available.

- However, only estimates of spectral response are needed since many times the difference between the actual and nominal response is sufficient.

Wide-Bandwidth Detectors - III
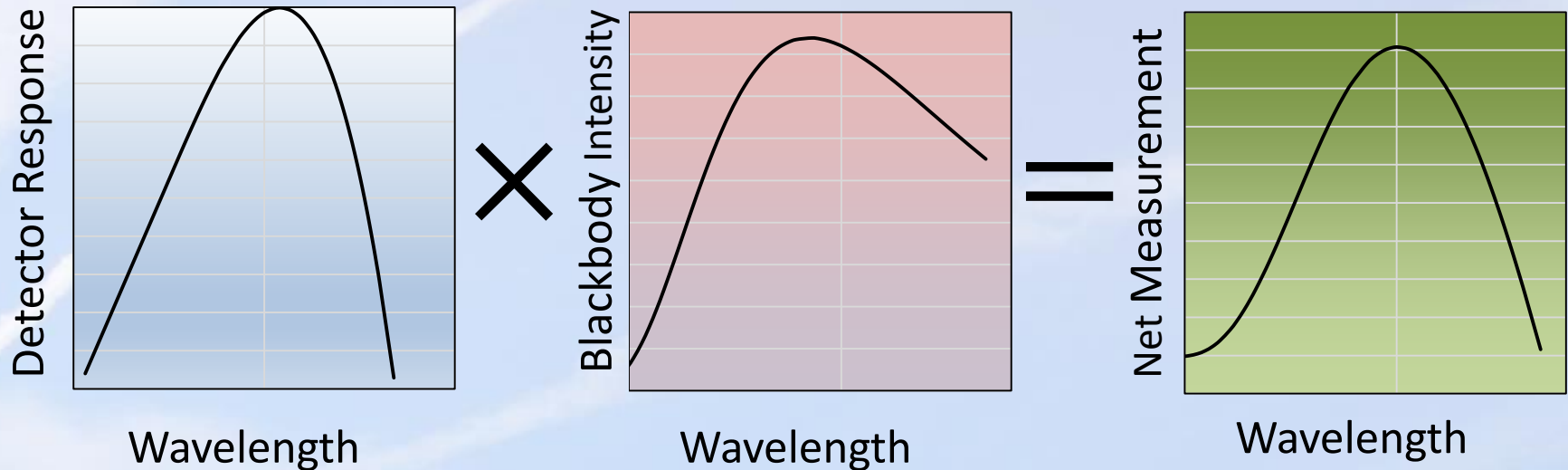
Non-linearities in the calibration are introduced because the measured signal is the net product of the detector response multiplied by the blackbody emission intensity.

Wide-Bandwidth Detectors - IV

- Using a wide-band detector can then require relating the integral of the blackbody function $i_{b,\lambda}(\lambda, T)$ multiplied by the detector response $D(\lambda)$ across the bandwidth

- When measuring materials with emissivities that vary over wavelength, the data analysis becomes even more complex and requires the inclusion of the emissivity $\varepsilon_\lambda(\lambda)$ with wavelength

- So that the measured signal becomes a function of the:

$$\int_{\lambda_l}^{\lambda_u} D(\lambda) \cdot \varepsilon_\lambda(\lambda) \cdot i_{b,\lambda}(\lambda, T) d\lambda \tag{31}$$

And not just $i_{b,\lambda}(\lambda, T)$.

Wide-Bandwidth Detectors - V

Often, the analysis will use the average wavelength in the center of the band to calculate the blackbody intensity and assume a uniform intensity across the band, so that:

$$I(\lambda_l, \lambda_u, T) = \int_{\lambda_l}^{\lambda_u} i_{b,\lambda}(\lambda, T) d\lambda \cong i_{b,\lambda}(\bar{\lambda}, T)\Delta\lambda \qquad (32)$$

where:

$$\bar{\lambda} = \frac{1}{2}(\lambda_l + \lambda_u) \text{ and } \Delta\lambda = \lambda_u - \lambda_l \qquad (33)$$

Example 13 - I

E13: What is the non-linearity introduced by averaging the response of a detector covering the range of 1 to 4 μm at temperatures between 600 to 1100 K? Assume a uniform spectral response function.

Example 13 - II

A13: The problem requires that we compare:

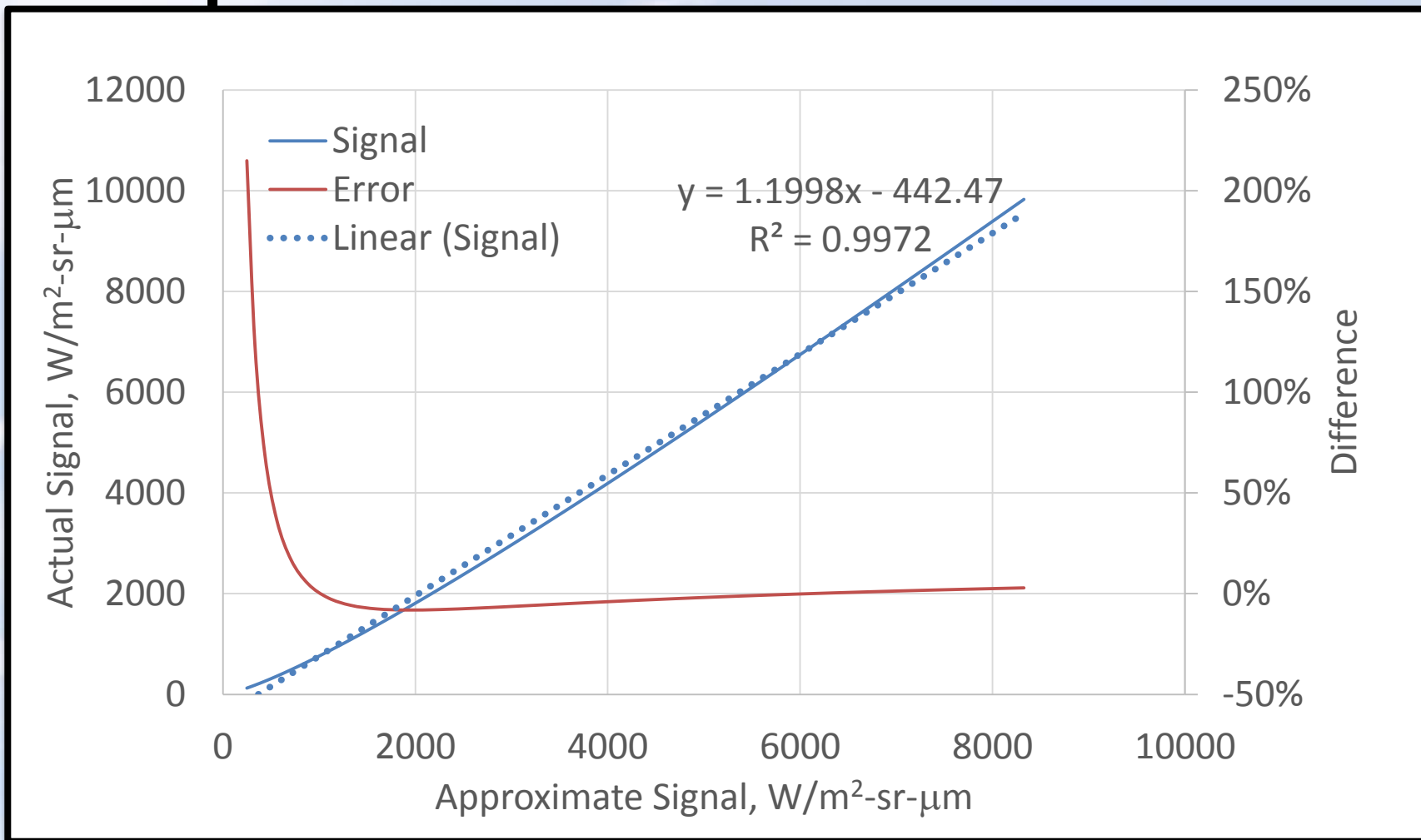$$\int_{\lambda_l}^{\lambda_u} i_{b,\lambda}(\lambda, T)\, d\lambda$$

to:

$$i_{b,\lambda}\left(\bar{\lambda}, T\right)\Delta\lambda$$

where

$$\bar{\lambda} = \frac{1}{2}\left(\lambda_l + \lambda_u\right) \text{ and } \Delta\lambda = \lambda_u - \lambda_l$$

So $\bar{\lambda}$ = 2.5 μm and $\Delta\lambda$ = 3.0 μm. The comparison is shown on the following page.

Example 13 - III

Note the non-linearity in the calibration and the high error at low intensities.

Example 14

E14: What is the non-linearity introduced by averaging the response of a detector covering the range of 0.25 to 1.05 μm at temperatures between 2000 to 3000 K? Assume a linearly increasing detector spectral response function as shown below:
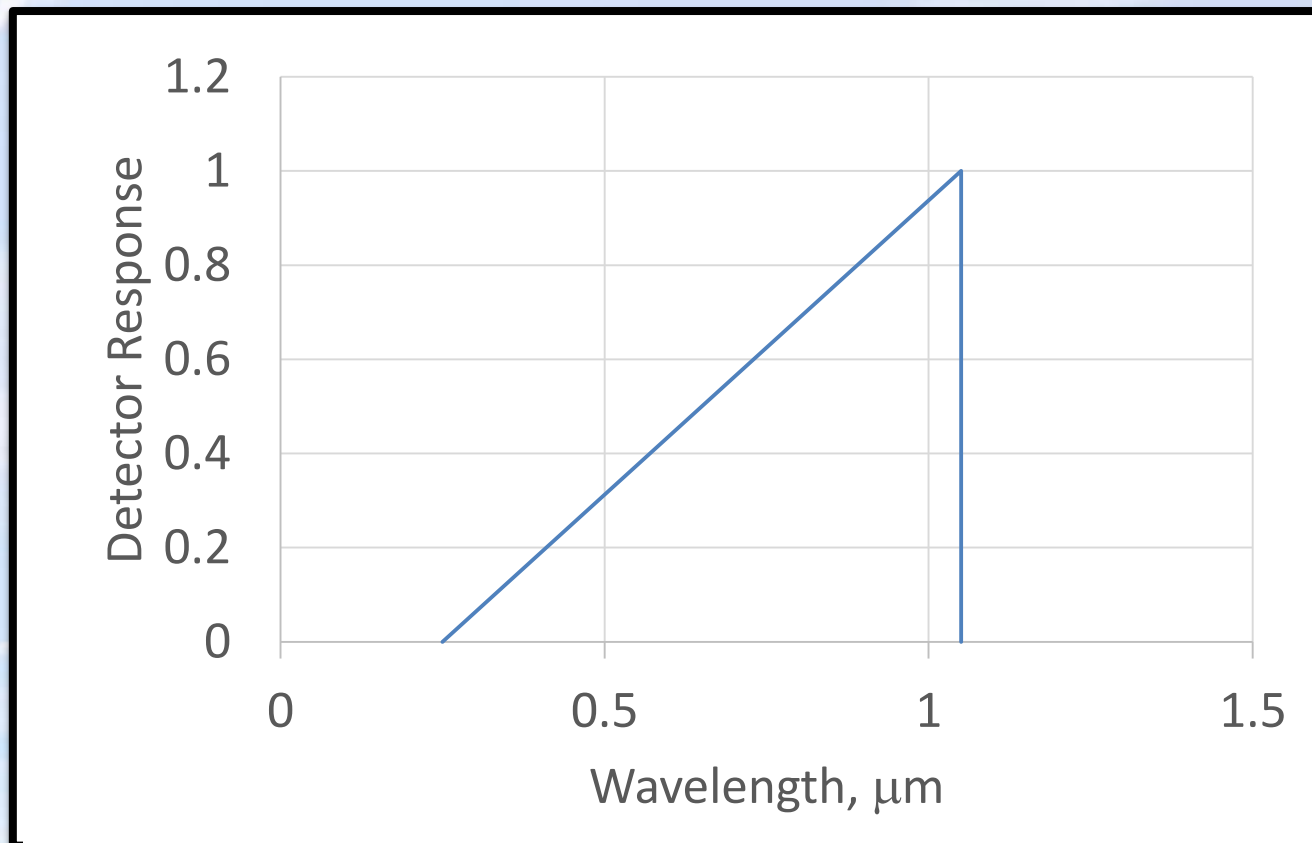
Example 14 - II

A14: The problem requires that we compare:

$$\int_{\lambda_l}^{\lambda_u} D(\lambda) \cdot i_{b,\lambda}(\lambda, T) d\lambda$$

to:

$$D(\bar{\lambda}) \cdot i_{b,\lambda}(\bar{\lambda}, T) \Delta\lambda$$

where

$$\bar{\lambda} = \frac{1}{2}(\lambda_l + \lambda_u) \text{ and } \Delta\lambda = \lambda_{ul} - \lambda_1$$

So $\bar{\lambda}$ = 0.65 μm and $\Delta\lambda$ = 0.8 μm. The comparison is shown on the following page.

Example 14 -III
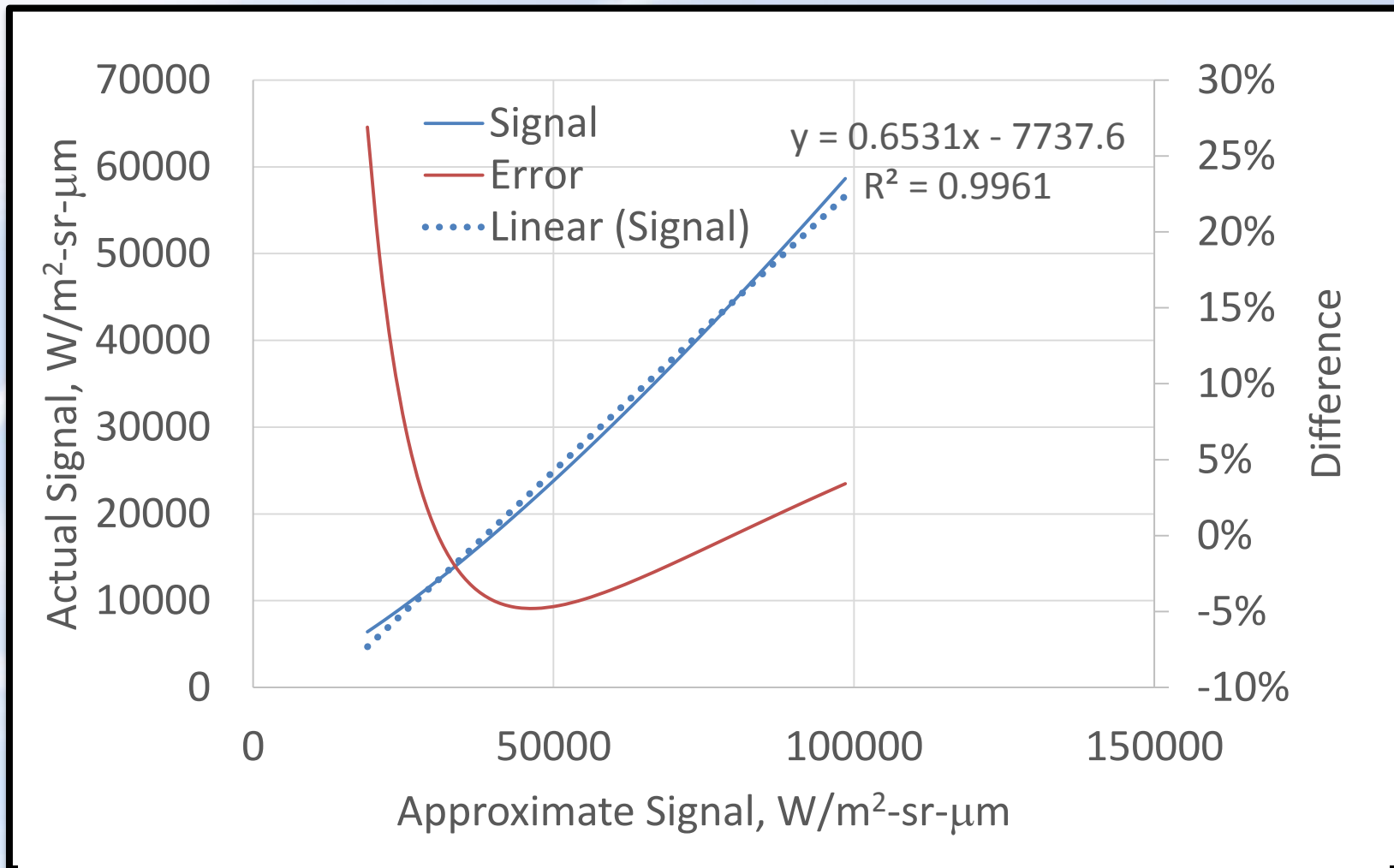
The result is similar to problem 13. Again note the non-linearity in the calibration and the high error at low intensities.

Wide-Bandwidth Detectors - V

- When measurements are made of materials with emissivities that vary over wavelength, the data analysis becomes even more complex and requires the inclusion of the emissivity with wavelength:

$$\int_{\lambda_l}^{\lambda_u} D(\lambda) \cdot \varepsilon(\lambda) \cdot (i_{b,\lambda}(\lambda, T) d\lambda = i(\lambda_l, \lambda_u, T) \quad (33)$$

Wide-Bandwidth Detectors - VI

- When the two or more detectors are used, the relationships for the unknown temperatures and spectral emissivities involve integrals over the wavelength band.

- Similar to the narrow-band ratio method, we can devise an analogous way to determine temperature and a wavelength-averaged emissivity.

Wide-Bandwidth Detectors - VII

For two, wide-band detectors measuring equivalent blackbody temperatures of T_1 and T_2, we can define an equivalent ratio method where we solve the following:

$$\overline{\varepsilon_r} = \frac{\overline{\varepsilon_1}}{\overline{\varepsilon_2}} = \frac{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)(i_{b,\lambda}(T_1)\,d\lambda}{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)(i_{b,\lambda}(T)\,d\lambda} \cdot \frac{\int_{\lambda_{2l}}^{\lambda_{2u}} D_2(\lambda)(i_{b,\lambda}(T)\,d\lambda}{\int_{\lambda_{2l}}^{\lambda_{2u}} D_2(\lambda)(i_{b,\lambda}(T_2)\,d\lambda} \quad (34)$$

Where $\overline{\varepsilon_1}$ and $\overline{\varepsilon_2}$ are wavelength average emissivities. The next example will demonstrate this method.

Wide-Bandwidth Detectors - VIII

Alternatively, we can approximate the wide-band signal using the average value across the band so that:

$$\overline{\varepsilon_r} = \frac{\overline{\varepsilon_1}}{\overline{\varepsilon_2}} = \frac{i_{b,\lambda}\left(T_1, \overline{\lambda_1}\right)}{i_{b,\lambda}\left(T, \overline{\lambda_1}\right)} \cdot \frac{i_{b,\lambda}\left(T, \overline{\lambda_2}\right)}{i_{b,\lambda}\left(T_2, \overline{\lambda_2}\right)} \qquad (35)$$

Given T_1 and T_2 we calculate T that satisfies either Equation 34 or 35.

Example 15 - I

E15: A commercial Silicon/InGas sandwich detector has the detector response below. If the Silicon detector measures 3000 K and the InGas measures 2800 K equivalent blackbody temperatures, what are the average spectral emissivities for the two wavelengths and the true surface temperature assuming an emissivity ratio of 0.9?
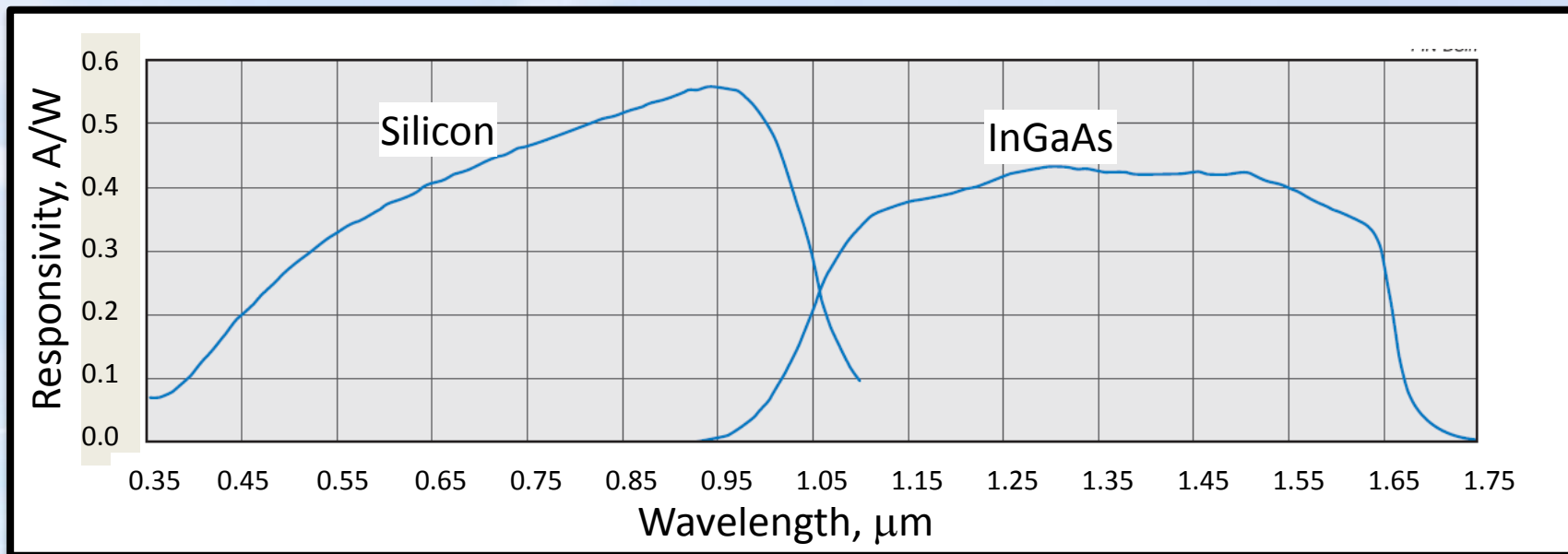
Example 15 - II

A15: We model the detector response function by the line segments below. The problem requires that we solve Equation 34 or 35 with $\overline{\varepsilon_r} = 0.9$ and $T_1 = 3000$ K and $T_2 = 2800$ K.
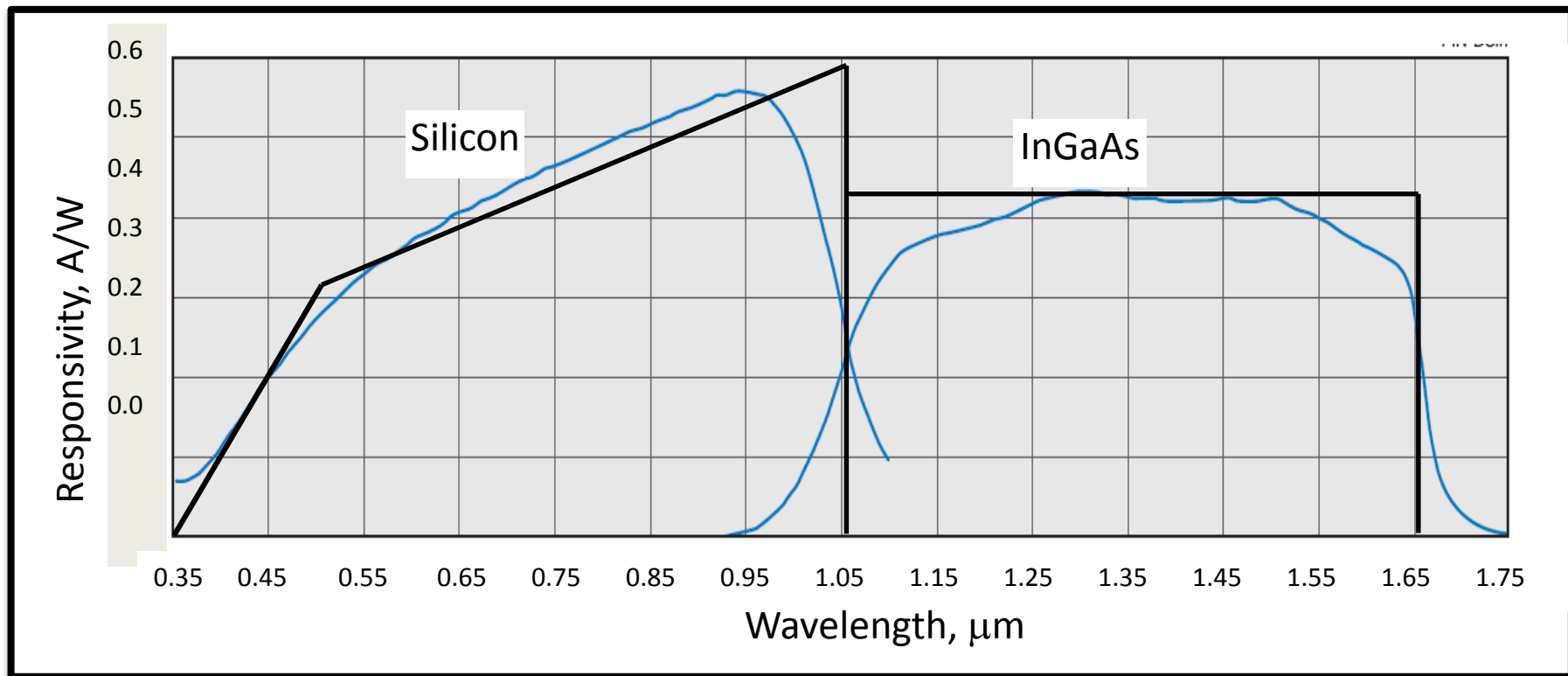
Example 15 - III

$$\overline{\varepsilon_r} = 0.9$$

Parameter	Integrated Band	Averaged Band
T, K	3,236	3,166
$\overline{\varepsilon}_1$	0.64	0.70
$\overline{\varepsilon}_2$	0.71	0.78

Note that while the temperature is reasonably close between the two methods (70 K or 2%) the emissivities are less accurate, differing by about 8%.
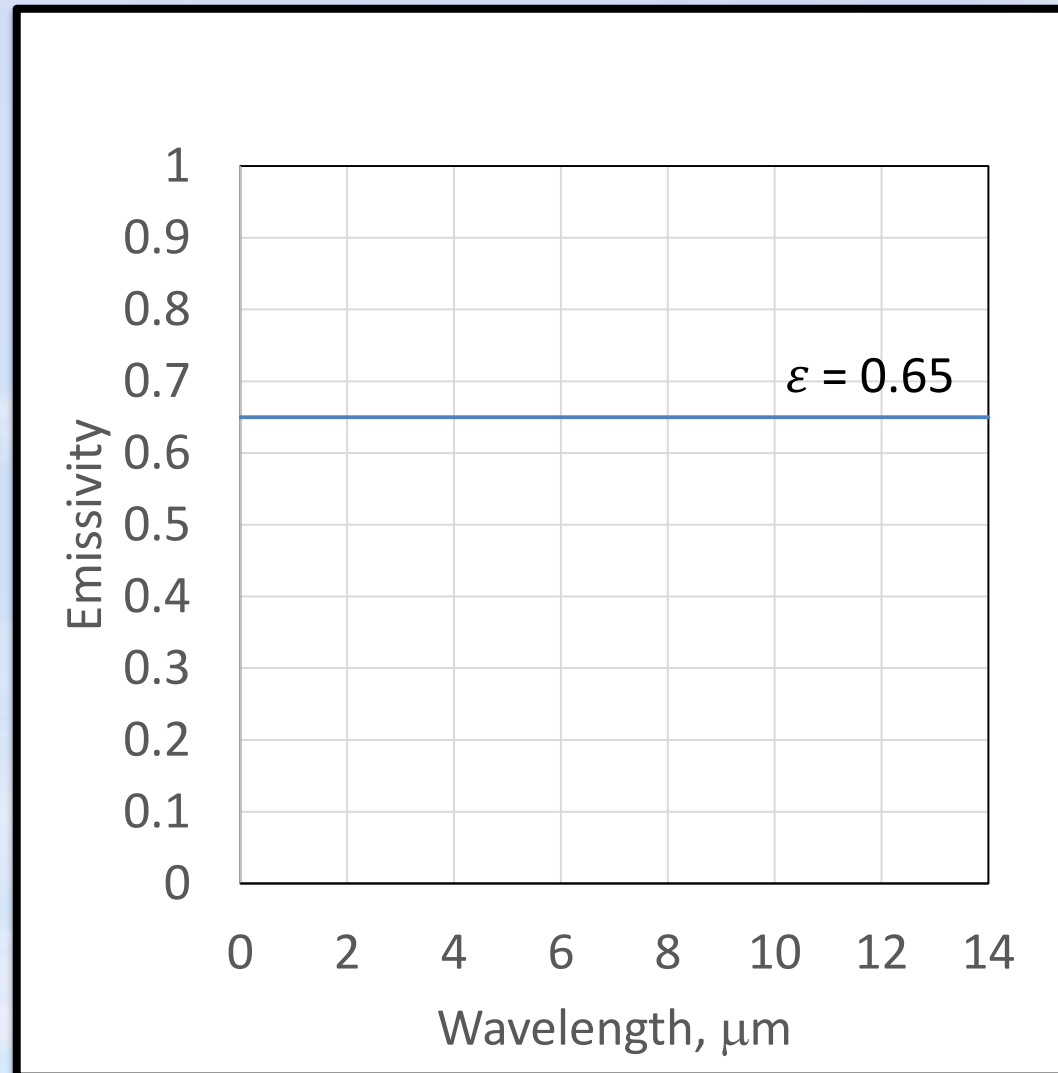
Example 16 - I

E16: This examples demonstrates the method of reducing data from wide-band detectors using effective, average wavelengths.

Consider a dual-band instrument with spectral bands covering 1 to 4.5 μm and 2 to 13.5 μm. The detector response functions are shown. The average detector wavelengths are 2.75 and 7 μm. Using the average detector wavelengths, what is the surface temperature and what is the inferred emissivity assuming an emissivity ratio of 1?
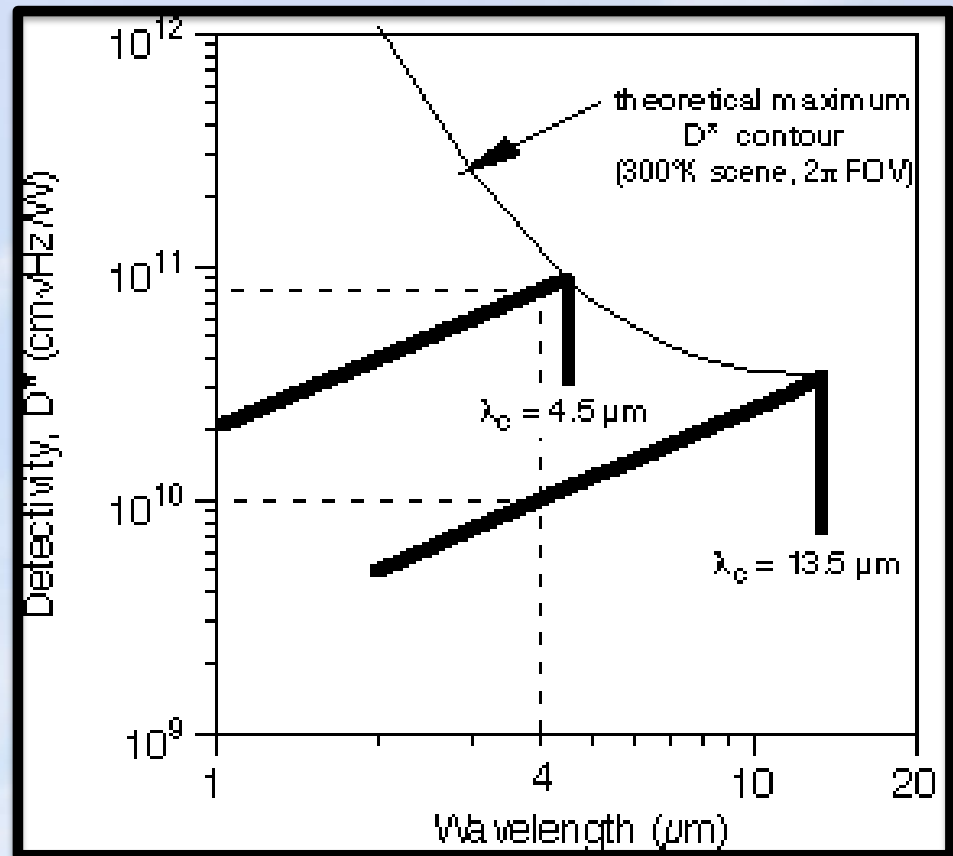
Example 16 - II

Our example assumes a gray material which means that the spectral emissivity is independent of wavelength. We assume the emissivity is equal to 0.65 and the true surface temperature is 3,000 K.

Chart with y-axis "Emissivity" (0 to 1) and x-axis "Wavelength, μm" (0 to 14), showing a horizontal line at $\varepsilon = 0.65$.

Example 16 - III

Using the function bb_tiibl, we can calculate what the measured equivalent blackbody temperatures T_λ would be by solving the following equation given the detector response functions $D(\lambda)$, the emissivity ε (independent of wavelength), and the spectral blackbody radiant intensity:

$$\varepsilon \int_{\lambda_l}^{\lambda_u} D(\lambda)\, i_{b,\lambda}(T)\, d\lambda = \int_{\lambda_l}^{\lambda_u} D(\lambda)\, i_{b,\lambda}(T_\lambda)\, d\lambda$$

The calculated blackbody temperatures for detectors 1 and 2 are 2,589 and 2,426 K, respectively.

Example 16 - IV

Using the average wavelengths, we can calculate the equivalent blackbody temperature T and emissivity for an emissivity ratio of 1.0 from the two detector equivalent blackbody temperatures T_1 and T_2. This solved using the function bb_tratio which solves the following equation:

$$\frac{\left(e^{-C_2/\lambda_2 T \lambda_2}-1\right)}{\left(e^{-C_2/\lambda_2 T}-1\right)} \cdot \frac{\left(e^{-C_2/\lambda_1 T}-1\right)}{\left(e^{-C_2/\lambda_1 T \lambda_1}-1\right)} - \varepsilon_r = 0$$

For ε_r=1, the result is:

$$T\text{=2,879 K and } \varepsilon = 0.79$$

Versus the correct values of T=3,000 K and ε = 0.65

This demonstrates the uncertainty introduced using wide-bandwidth detectors when assuming a single, average wavelength for each detector.
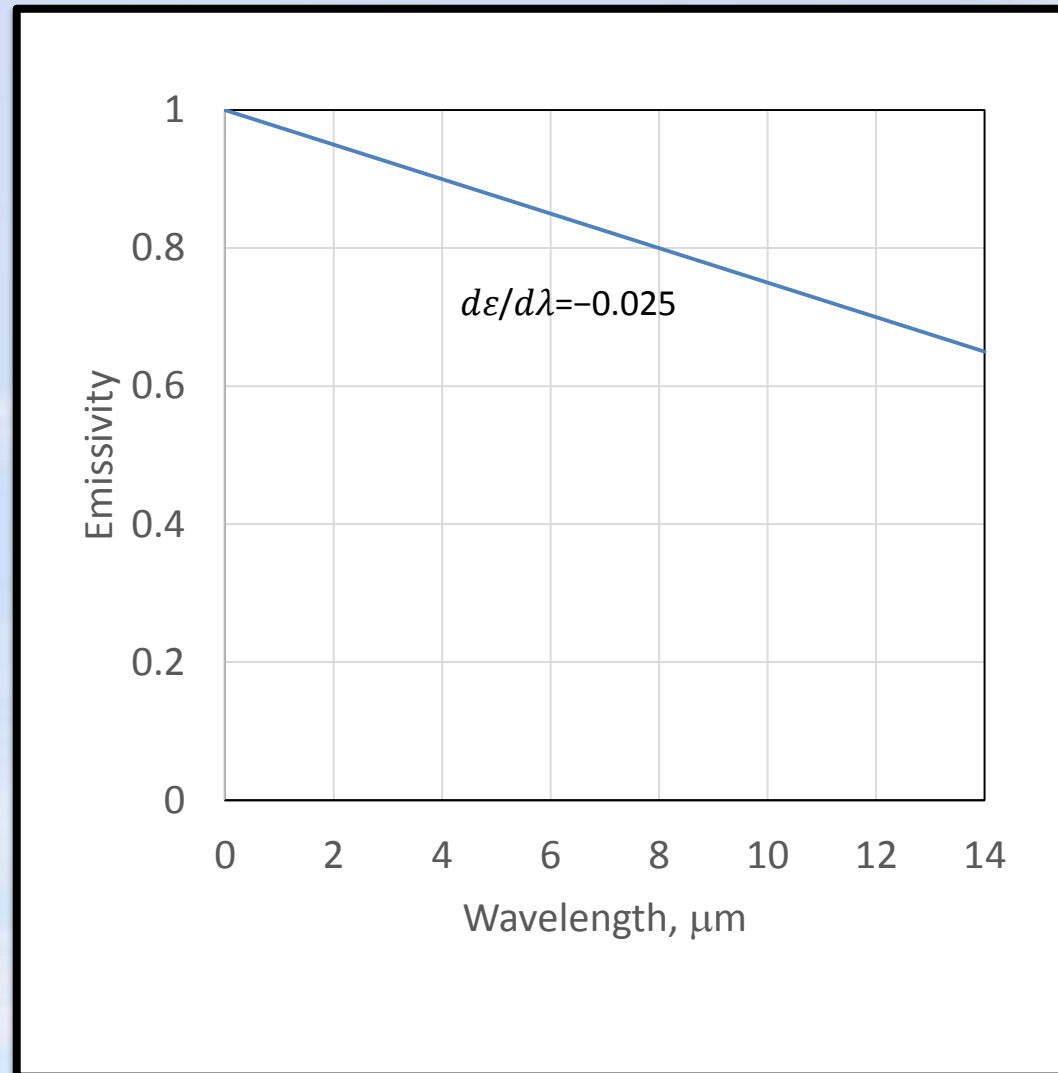
Example 17 - I

E17: For the final problem, we construct a problem that demonstrates not only the complexity in reducing data from wide-band detectors, but also the difficulty in interpreting the data.

Consider the same dual-band instrument with spectral bands covering 1 to 4.5 μm and 2 to 13.5 μm as used in the Example 16. The detector response functions are shown.
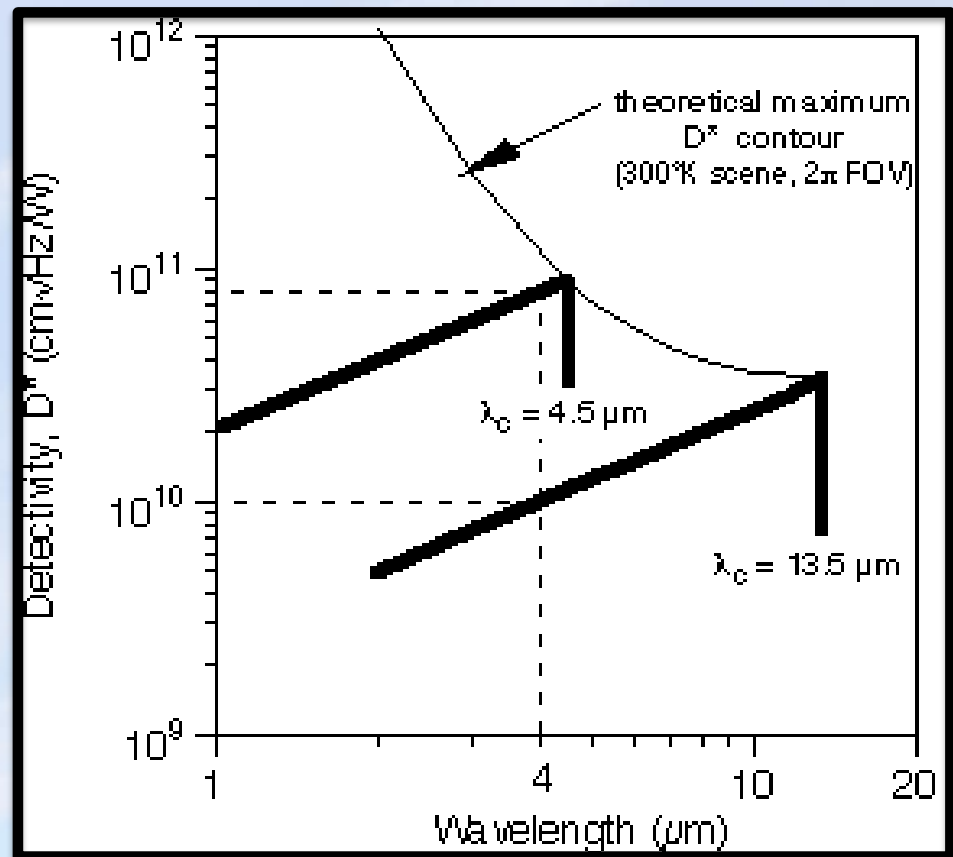
Example 17 - II

The material has a spectral emissivity as a function of wavelength as shown. The true surface temperature is 3,000 K. Based on this, the calculated equivalent blackbody temperatures for the two detectors are 2,949 and 2,719 K respectively. If an emissivity ratio of 1 is assumed for the two detectors, what are the calculated true surface temperature and spectral emissivities?

Plot: Emissivity vs Wavelength, μm. $d\varepsilon/d\lambda = -0.025$

Example 17 - III

For the given true surface temperature T, the calculated equivalent blackbody temperatures T_{λ_i} for each detector can be obtained by solving the following:

$$\int_{\lambda_{il}}^{\lambda_{iu}} \epsilon_{\lambda_i}(\lambda)\, D_i(\lambda)\, i_{b,\lambda}(T)\, d\lambda = \int_{\lambda_{il}}^{\lambda_{iu}} D_i(\lambda)\, i_{b,\lambda}\left(T_{\lambda_i}\right) d\lambda$$

and as previously stated for detectors 1 and 2 are 2,941 and 2,841 K, respectively.

Example 17 - IV

A17: Using the wide-band method with Equation 34 assuming an emissivity ratio $\overline{\varepsilon_r}$ = 1, the calculated true surface temperature is 3,175 K with emissivities for the two detectors equal to 0.81 (routine bb_itratio). This is a 179-K error. If an emissivity ratio of 1.13 is assumed (equal to the ratio of the emissivities at the mean detector wavelengths), then a surface temperature of 2,707 K is calculated and the error in surface temperature increases to 294 K. The corresponding emissivities are calculated to be 1.27 and 1.10 for detector 1 and 2, respectively. These are not physically realistic.

Example 17 - V

If instead the equivalent narrow band assumption using Equation 34 with an emissivity ratio of 1.13 (routine bb_tratio) is used, the calculated true surface temperature is 2,622 K with emissivities for detector 1 and detector 2 being 1.29 and 1.12, respectively. Using an emissivity ratio of 1 results in a calculated temperature of 3,127 K and emissivities for both detectors equal to 0.88.

While the assumption of unity emissivity ratio comes closer to the correct surface temperature in both the wide-band and equivalent narrow-band methods, the result is fortuitous since the actual emissivity ratio across the two bands is not actually one.

The difficulty is that over such a wide range of wavelengths, it is difficult to choose the proper ratio since the wavelength dependence on spectral emissivity generally is not known.

Example 17 - VI

Parameter	$\bar{\varepsilon}_r = 1.0$		$\bar{\varepsilon}_r = 1.16$	
	Integrated Band	Averaged Band	Integrated Band	Averaged Band
T, K	3,175	3,127	2,707	2,623
$\bar{\varepsilon}_1$	0.81	0.88	1.27	1.29
$\bar{\varepsilon}_2$	0.81	0.88	1.10	1.12

Rule 4

- Unless other considerations dictate, avoid using wide-band detectors especially when accurate emissivities are needed.

Calibration

"Until you can measure something and express it in numbers, you have only the beginning of understanding."
– **Lord Kelvin**

Calibration

- Detectors are calibrated against a source providing a known spectral intensity at a given wavelength.

- However, it is often more convenient to express known calibration conditions in terms of an equivalent blackbody temperature, rather than a spectral intensity.

- The calibrated spectral intensity and blackbody temperature are equivalent and can be used to determine the system response accounting for detector nonlinearities, transmission losses, and electrical gains.

Calibration Sources - I

- Broadband Sources

  › Blackbody Thermal Cavity – Near UV to IR

  › Incandescent Lamp – Visible to Near IR

  › Deuterium Arc Lamp – UV to Visible

- Discrete Wavelength Sources

  › Mercury Lamp

  › Noble Gas (Neon, Argon, Krypton, Xenon) Lamp
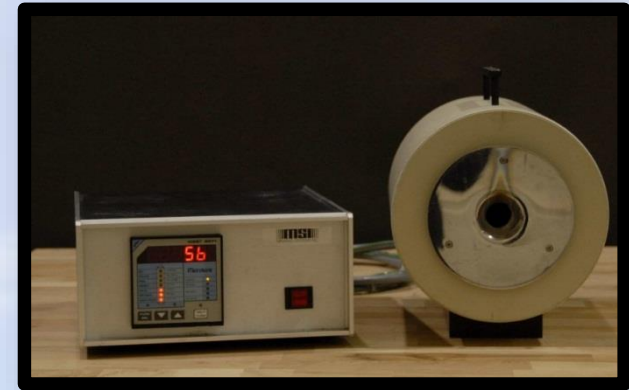
Portable Blackbody Source

Deuterium Arc Lamp
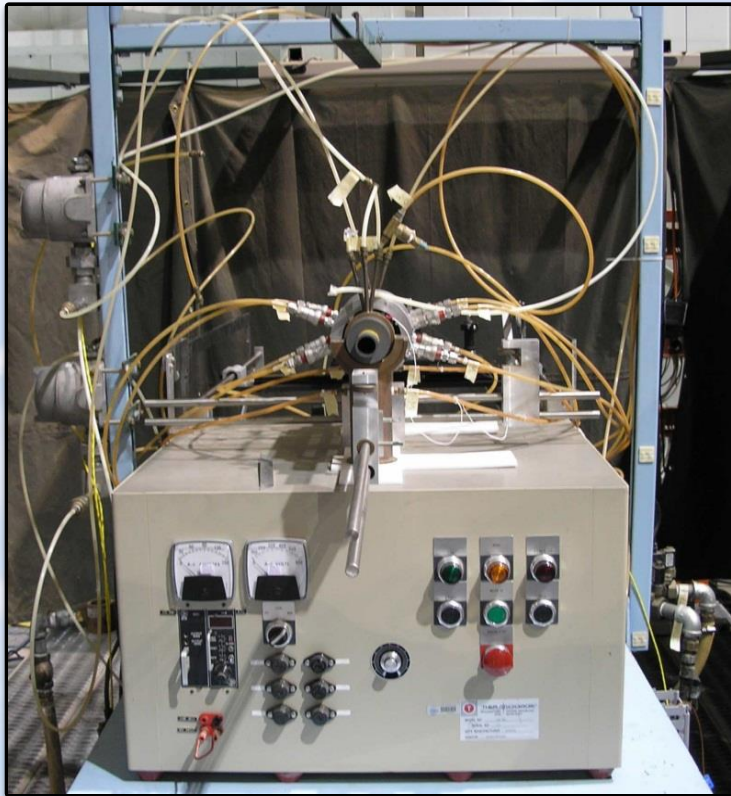
Blackbody Source

- A blackbody calibration source is an especially convenient and accurate calibration source.

- Recall, a perfect blackbody is an ideal emitter and absorbs all incident radiation regardless of the spectral character of directionality of the incident radiation.

- This behavior is simulated using a heated cavity designed to have an effective near unity emissivity.

- The specified calibration conditions can be determined by measuring the temperature of the cavity, rather than the absolute spectral radiance, like other sources.
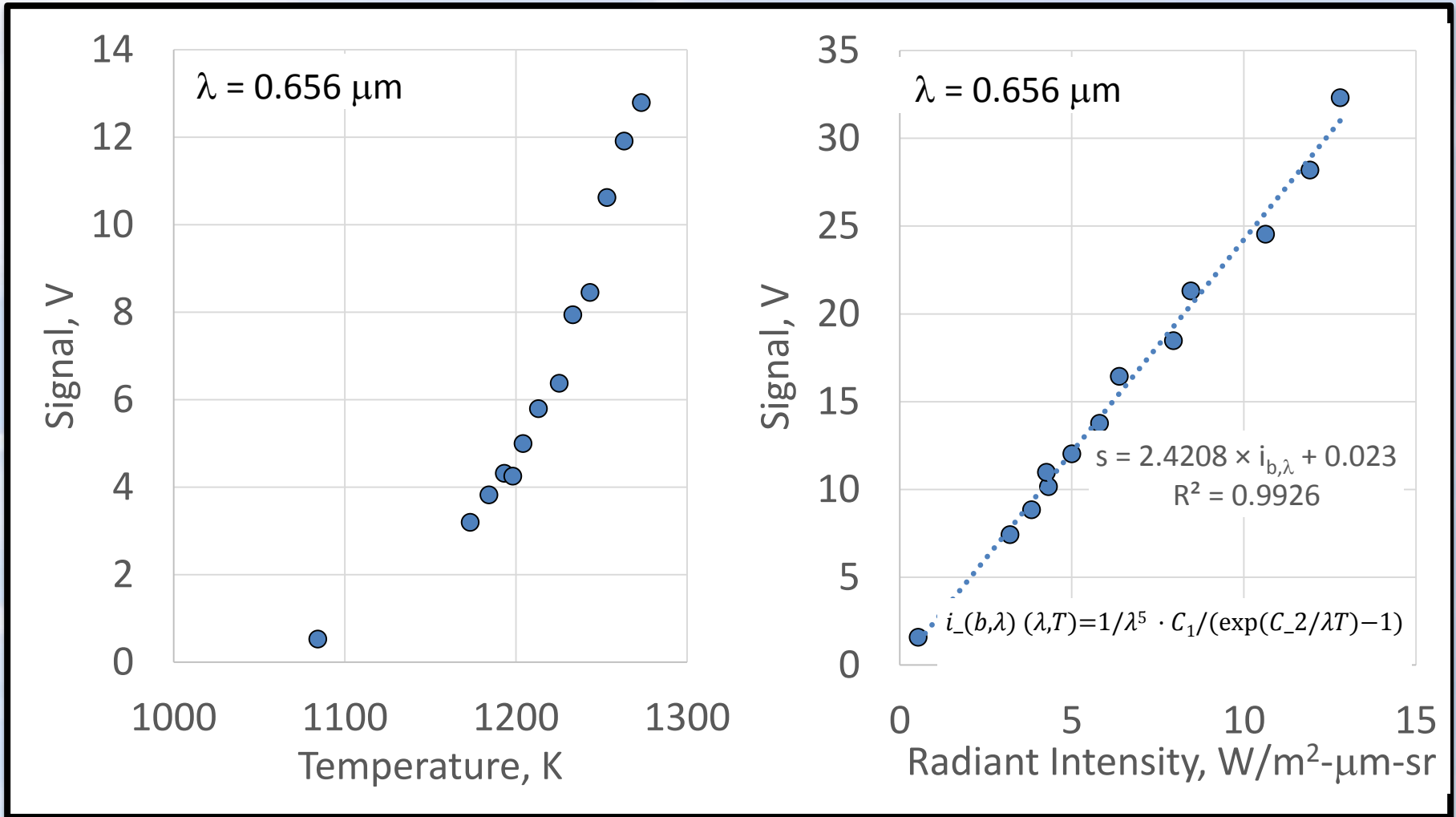
Diagram of Blackbody Calibration Source
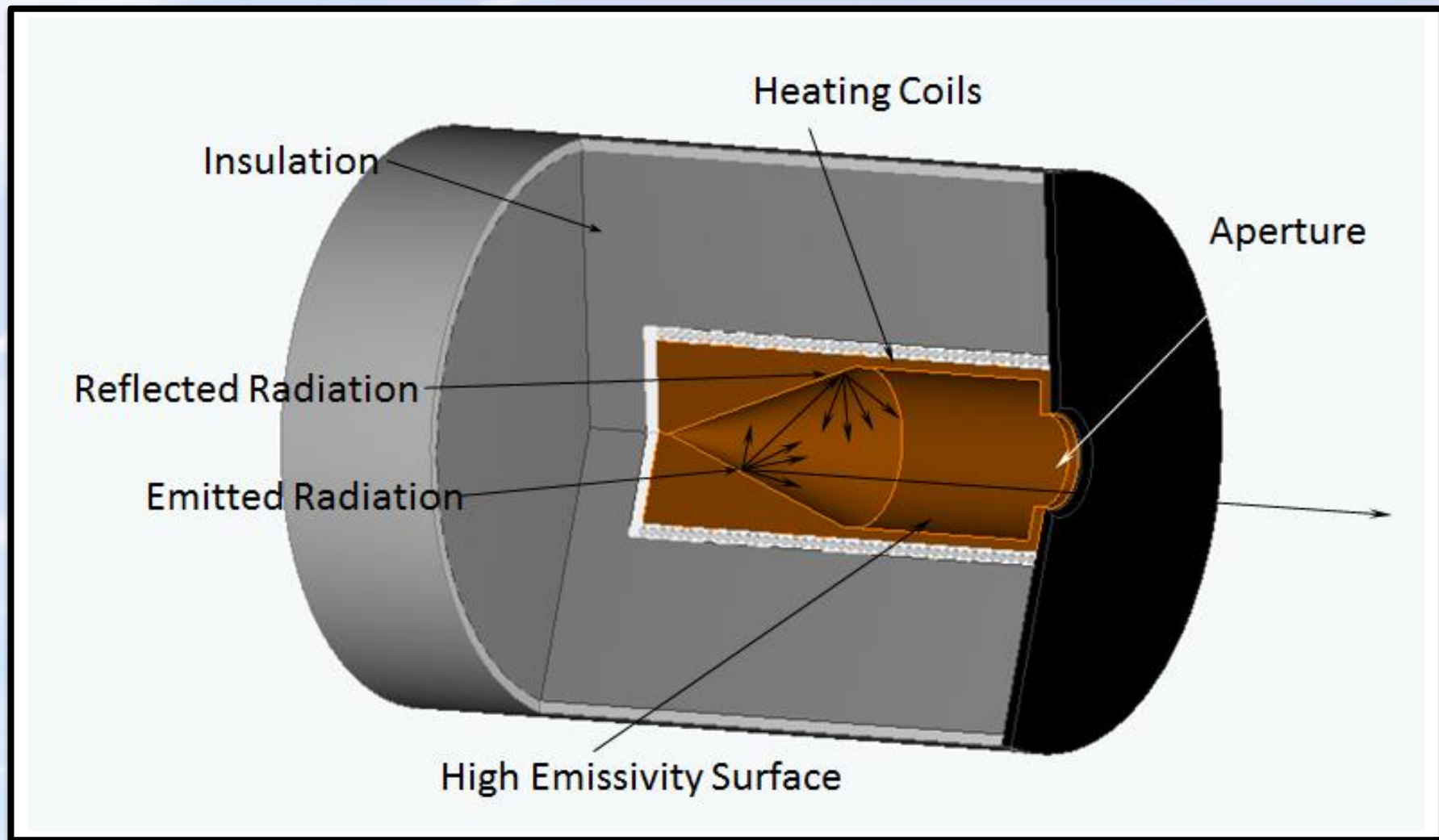
Calibration Sources - II
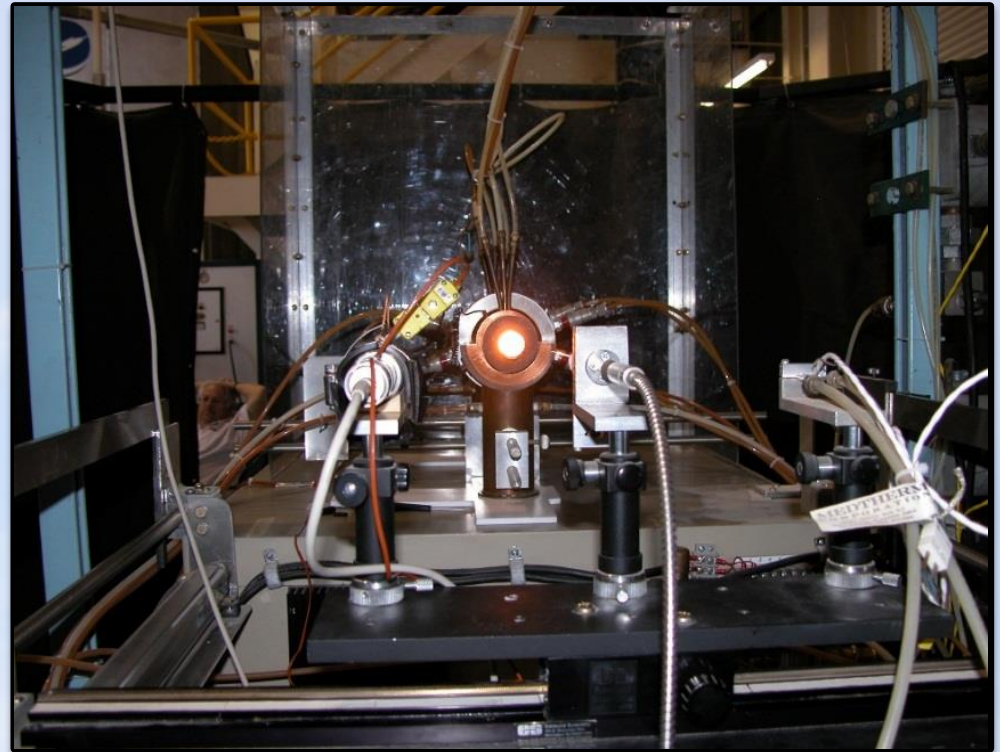
Resistively Heated, High-temperature Blackbody Furnace – 500 to 3000 °C

Blackbody calibration data for 0.656-μm detector. The blackbody temperature was preset and the signal voltage measured. Plotting this versus the calculated radiant intensity produces a straight line of signal versus calculated intensity.

Calibration Uncertainty

- All calibration sources have inherent error in the spectral intensity output.

- For blackbody sources, this error manifests itself in the uncertainty in the cavity temperature measurement

- Uncertainty in the cavity intensity propagates error just like the emissivity.

- Uncertainty in cavity temperature can be calculated from the quantity $dT/d\varepsilon$.

Example 18 - I

E18: A blackbody source is estimated to have an uncertainty of 10 K and is used to perform a single-point calibration on two detectors, 1) a short-wavelength, 0.5-μm detector and 2) a longer-wavelength 3 μm detector. For an unknown material, the equivalent blackbody temperatures are measured to be 1,600 and 1,500 K, respectively. What is the value and the uncertainty in the derived emissivity at 3 μm assuming the material has an emissivity of 0.8 ±0.1 at 0.5 μm?

Example 18 - II

A17: The equivalent blackbody temperature from the measurement of the 0.5-μm detector for T_{λ_1} = 1,600 K and ε_{λ_1} = 0.8 is:

$$T = \frac{C_2}{\lambda_1} \cdot \frac{1}{\ln\left[\varepsilon_{\lambda_1}\left(e^{C_2/\lambda T_{\lambda_1}} - 1\right) + 1\right]} = 1,604 \ K$$

The calculated emissivity at 3 μm is:

$$\varepsilon_\lambda = \frac{\left(e^{C_2/\lambda T_\lambda} - 1\right)}{\left(e^{C_2/\lambda T} - 1\right)} = 0.78$$

Assuming the temperature errors in the two calibration measurements are uncorrelated and random, the total uncertainty in the emissivity will be:

$$\Delta\varepsilon_{\lambda_2} = \sqrt{\left(\left(\frac{\partial\varepsilon_{\lambda_2}}{\partial T_{\lambda_1}}\right)\Delta T_{\lambda_1}\right)^2 + \left(\left(\frac{\partial\varepsilon_{\lambda_2}}{\partial T_{\lambda_2}}\right)\Delta T_{\lambda_2}\right)^2 + \left(\left(\frac{\partial\varepsilon_{\lambda_2}}{\partial\varepsilon_{\lambda_1}}\right)\Delta\varepsilon_{\lambda_1}\right)^2}$$

where $\Delta T_{\lambda_1} = 10\text{K}$ and $\varepsilon_{\lambda_1} = 0.1$.

Example 18 - III

or, in terms of sensitivities:

$$\frac{\Delta \varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = \sqrt{\left(\left(\frac{\partial \ln \varepsilon_{\lambda_2}}{\partial \ln \varepsilon_{\lambda_1}}\right)\left(\frac{\partial \ln \varepsilon_{\lambda_1}}{\partial \ln T_{\lambda_1}}\right)\frac{\Delta T_{\lambda_1}}{T_{\lambda_1}}\right)^2 + \left(\left(\frac{\partial \ln \varepsilon_{\lambda_2}}{\partial \ln T_{\lambda_2}}\right)\frac{\Delta T_{\lambda_2}}{T_{\lambda_2}}\right)^2 + \left(\left(\frac{\partial \ln \varepsilon_{\lambda_2}}{\partial \ln \varepsilon_{\lambda_1}}\right)\frac{\Delta \varepsilon_{\lambda_1}}{\varepsilon_{\lambda_1}}\right)^2}$$

evaluating terms gives:

$$\frac{d \ln \varepsilon_{\lambda_1}}{d \ln T_{\lambda_1}} = \frac{C_2}{\lambda T_{\lambda_1}} \cdot \frac{e^{C_2/\lambda T_{\lambda_1}}}{\left(e^{C_2/\lambda T_{\lambda_1}} - 1\right)} = 18.0$$

$$\frac{d \ln \varepsilon_{\lambda_2}}{d \ln T_{\lambda_2}} = \frac{C_2}{\lambda T_{\lambda_2}} \cdot \frac{e^{C_2/\lambda T_{\lambda_2}}}{\left(e^{C_2/\lambda T_{\lambda_2}} - 1\right)} = 3.33$$

$$\frac{d \ln \varepsilon_{\lambda_2}}{d \ln \varepsilon_{\lambda_1}} = \frac{\lambda_1}{\lambda_2} \cdot \frac{\left(e^{-C_2/\lambda_1 T} - 1\right)}{\left(e^{-C_2/\lambda_2 T} - 1\right)} = 0.176$$

Example 18 - IV

So the estimated relative uncertainty is:

$$\frac{\Delta \varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = \sqrt{\left(0.176 \cdot 18.0 \cdot \frac{10}{1{,}600}\right)^2 + \left(3.33 \cdot \frac{10}{1{,}500}\right)^2 + \left(0.176 \cdot \frac{0.1}{0.8}\right)^2}$$

$$\frac{\Delta \varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = \sqrt{\underbrace{3.94 \times 10^{-4}}_{\text{Detector 1 calibration}} + \underbrace{4.94 \times 10^{-4}}_{\text{Detector 2 calibration}} + \underbrace{4.83 \times 10^{-4}}_{\text{Emissivity}}} = 3.7\%$$

and therefore:

$$\Delta \varepsilon_{\lambda_2} = 0.78 \cdot 0.037 = 0.029$$

$$\varepsilon_{\lambda_2} = 0.78 \pm 0.029$$

Note the uncertainty resulting from the calibration is on the same order as the estimation of the emissivity at 0.5 μm.

Closing

"If this fire determined by the sun, be received on the blackest known bodies, its heat will be long retain'd therein; and hence such bodies are the soonest and the strongest heated by the flame fire…"
– **Hermann Boerhaave**
A New Method of Chemistry, 2nd edition (1741), 262

Closing - I

- Radiation thermometry is a useful technique for measuring the temperature of bodies that cannot be readily measured by contact sensors.

- It is important to understand the uncertainty in radiation thermometry measurements and choose the characteristics of the instrumentation and the measurement method accordingly.

- The most accurate temperature measurements are obtained when using short wavelength detectors.

- The uncertainty in the derived spectral emissivity at a given wavelength decreases approximately inversely with the wavelength that is used, and therefore longer wavelength detectors provide more accurate emissivity measurements.

Closing - II

- Combining measurements of two or more detectors can allow the simultaneous determination of both the temperature and the emissivity.

- However, the uncertainty in both the temperature and emissivity increase when the measurements of two detectors are combined.

- Multi-spectral methods using a large number of detectors can often provide more accurate temperature and emissivity values over a wider range of wavelengths.

- Prefer the use of narrow-band detectors over wide-band detectors unless other factors dictate.

Summary of Radiative Thermometry Rules

- **Rule 1-** When the emissivity is unknown and must be estimated, the most accurate surface temperature measurement is made when a detector with a wavelength as short as possible is used.

- **Rule 2 -** The uncertainty in the spectral emissivity at a given wavelength decreases approximately inversely with the wavelength that is used.

- **Rule 3 -** The optimal detector wavelengths for the ratio method are based on the two competing factors: 1) the need to keep the wavelength difference small to ensure an accurate assumed ratio, but 2) not too small that the effective wavelength increases and becomes too large.

- **Rule 4 -** Unless other considerations dictate, avoid using wide-band detectors especially when accurate emissivities are needed.

For Further Reading

1. Howell, J. R. , R. Siegel, and M. P. Mengüç, *Thermal Radiation Heat Transfer,* Fifth Edition, Taylor & Francis, New York, 2010.

2. DeWitt, D. P. and G. D. Nutter, *Theory and Practice of Radiation Thermometry*, John Wiley and Sons, New York, 1988.

3. Rogalski, A., "History of Infrared Detectors", *Opto-Electron. Rev.*, 20 no. 3, pp 279-308, 2012.

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