COMSOL CONFERENCE
2015 BOSTON

JACOBS
ESSSA Group

Solid Rocket Motor Combustion Instability Modeling in COMSOL Multiphysics

Dr. Sean Fischbach

Jacobs ESSSA Group/ Qualis Corp. / Marshall Space Flight Ctr.

*MSFC Huntsville sean.r.fischbach@nasa.gov

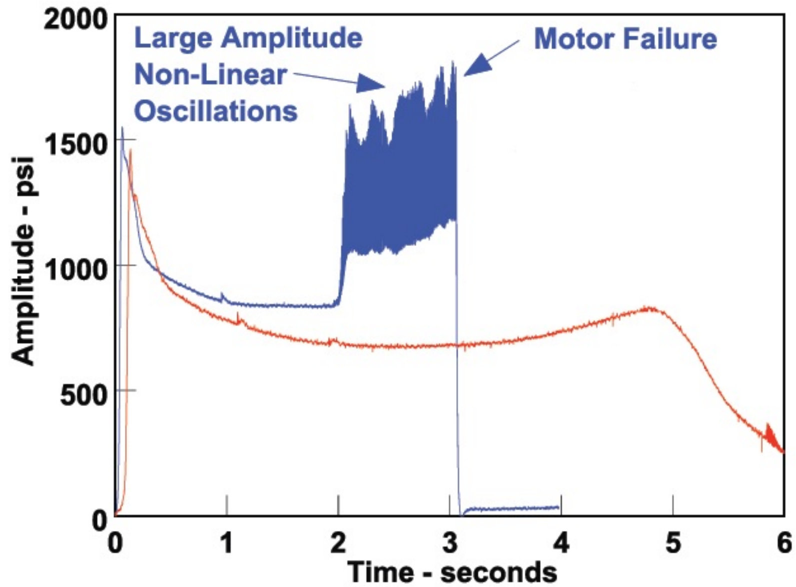
Outline

- Introduction and problem statement
- Overview of Combustion Instability (CI) modeling
  - Industry standard approach and software
  - Acoustic wave equation model
  - Energy balance model
- Use of COMSOL
  - High Mach Number Flow (HMNF) module
  - Pressure Acoustics (PA) module
  - Coefficient Form Partial Differential Equation (PDE) module
- Results

JACOBS
ESSSA Group

Introduction

- CI in Solid Rocket Motors (SRM) is characterized by undesirable fluctuations of pressure, velocity, and temperature
  - Unsteady energy release from propellant surface
  - Internal fluid dynamics i.e. vortex shedding, turbulence, etc.
  - Chamber and grain geometry
- Modeling CI in SRMs requires accurate representation of the steady and unsteady flow parameters
- The present study investigates the feasibility and advantage of employing COMSOL in the prediction of CI in SRMs



JACOBS ESSSA Group

Combustion Instability Modeling

- Solid Propellant Performance (SPP) '04 program is the industry standard SRM ballistics prediction software.
  - One Dimensional fluid dynamics
  - Three dimensional grain geometry and regression
  - Includes various ballistics mechanisms (i.e. erosive burning, nozzle boundary layer loss…)
- Standard Stability Prediction (SSP) code uses outputs from SPP '04 to evaluate the Culick stability model.
- Culick/wave equation stability model
  - Flow parameters split into steady and unsteady terms
  - Inhomogenous wave equation including mean flow terms on the right hand side.
  - Unsteady terms modeled using 1-D homogenous wave equation

$$\nabla^2 p' - \frac{1}{\bar{a}^2} p'_{tt} = -q\nabla \cdot (\bar{\boldsymbol{u}} \cdot \nabla \mathbf{u}' + \mathbf{u}' \cdot \nabla \bar{\boldsymbol{u}}) + \frac{1}{\bar{a}^2} \bar{\boldsymbol{u}} \cdot \nabla p'_t + \frac{\gamma}{\bar{a}^2} p'_t \nabla \cdot \bar{\boldsymbol{u}}$$

$$P = \overline{P} + p' e^{\alpha_{\mathrm{motor}} t} \qquad \overline{P} = \text{mean chamber pressure}$$

$$\alpha_{\mathrm{motor}} = \alpha_{\mathrm{pc}} + \alpha_{\mathrm{ft}} + \alpha_{\mathrm{nd}} + \alpha_{\mathrm{pd}} + \alpha_{\mathrm{blp}} + \ldots \qquad p' = \text{unsteady pressure}$$

JACOBS
ESSSA Group

Combustion Instability Modeling cont.

- Flandro/Jacob energy corollary model
  - Myers unsteady energy corollary used to model flow disturbances in the presence of mean flow
  - Flow parameters split into steady and unsteady parts
  - Model can account for acoustic, vortical, and thermal (entropy) oscillations

$$\frac{\partial E_2}{\partial t} = D_2 - \nabla \cdot \boldsymbol{W}_2 \qquad E_2 = \frac{p_1^2}{2\rho_0 a_0^2} + \rho_1 \boldsymbol{u}_0 \cdot \boldsymbol{u}_1 + \frac{1}{2}\rho_0 \boldsymbol{u}_1^2 + \frac{\rho_0 \rho T_0 s_1^2}{2C_p}$$

$$D_2 = -\rho_0 \boldsymbol{u}_0 \cdot (\boldsymbol{u}_1 \times \boldsymbol{\Omega}_1) - \rho_1 \boldsymbol{u}_1 \cdot (\boldsymbol{u}_0 \times \boldsymbol{\Omega}_0) - \rho_0 T_1 \boldsymbol{u}_0 \cdot \nabla s_1 - \rho_0 s_1 \boldsymbol{u}_1 \cdot \nabla T_0 - \rho_1 s_1 \boldsymbol{u}_0 \cdot \nabla T_0 + \boldsymbol{m_1 \psi_1}$$

$$\boldsymbol{W}_2 = \boldsymbol{u}_1 p_1 + \frac{\boldsymbol{u}_0}{\rho_0} p_1 \rho_1 + \rho_0 \boldsymbol{u}_1 (\boldsymbol{u}_0 \cdot \boldsymbol{u}_1) + \rho_1 \boldsymbol{u}_0 (\boldsymbol{u}_0 \cdot \boldsymbol{u}_1)$$

- Jacob recast the Myers energy model into the traditional alpha notation

W: $$\alpha_n = \frac{-\gamma}{2E_n} \iint \boldsymbol{n} \cdot R_s \bar{\boldsymbol{u}}\, p_n^2 dS - \frac{1}{2E_n} \iint \frac{1}{K_n^2} \left(\frac{dp_n}{dz}\right)^2 \bar{u}_b - r \frac{\rho_p}{\rho_g} (p')^2 dS_b$$

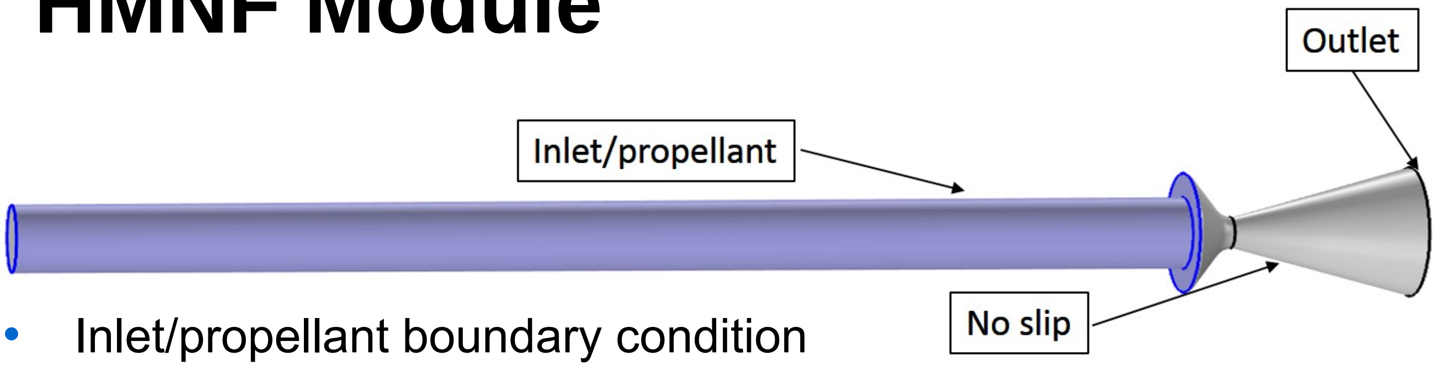
E: $$\alpha_n' = \iiint -\nabla \cdot \left[\rho_n \boldsymbol{u}_n + \frac{\boldsymbol{u}_0}{\rho_0} p_n \rho_n + \rho_0 \boldsymbol{u}_n (\boldsymbol{u}_0 \cdot \boldsymbol{u}_n) + \rho_n \boldsymbol{u}_0 (\boldsymbol{u}_0 \cdot \boldsymbol{u}_n)\right] - \rho_0 \boldsymbol{u}_0 \cdot (\boldsymbol{u}_n \times \boldsymbol{\Omega}_n) - \rho_n \boldsymbol{u}_n \cdot (\boldsymbol{u}_0 \times \boldsymbol{\Omega}_0) dV$$

COMSOL Implementation of CI Theory

- A CI analysis of a simplified SRM was conducted using multiple modules of COMSOL multiphysics
- The HMNF module was used to model the SRM internal ballistics
  - Spalart-Allmaras turbulent flow model
  - Slip boundary condition on all chamber and nozzle walls
  - Gas injection modeled using St. Robert's Law
- PA module was used to model the unsteady field variables
  - Geometry truncated at the Mach = 1 plane
  - Hard wall boundary used on all boundaries
- Acoustic Velocity Potential Equation (AVPE) modeled using the Coefficient Form PDE module.
  - AVPE is generated by combining the linearized conservation of mass and momentum equations
  - Retain mean flow effects on the acoustics as Mach numbers exceed 0.2.
- Results from the PA module and the AVPE are post processed in conjunction with the HMNF results to calculate alpha for both CI models
  - Alpha terms using the PA results are compared with SSP
  - Alpha terms using the AVPE are compared with the PA results to measure improvement

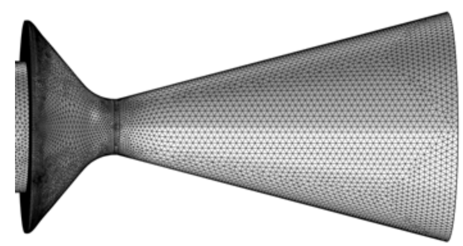
JACOBS ESSSA Group

HMNF Module



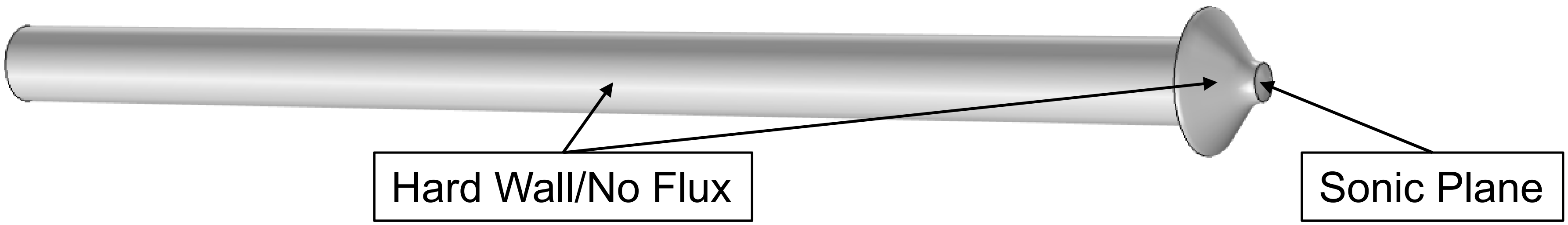
- Inlet/propellant boundary condition
  - Regression rate of the solid propellant was modeled using, $\dot{r} = ap^n$
  - Conservation of mass at the propellant/flame surface provides the injection velocity, $v_g = \dot{r}\frac{\rho_p}{\rho_g}$
  - The assumption is made that the flame temperature is independent of burning pressure
- The velocity is allowed to slip on the nozzle closure and cone walls
  - Assists in extracting the M=1 plane
  - Acoustics are insensitive to near wall mean flow velocities
- Mesh consists of 1,316,965 Tetrahedral, 61,233 Triangular, 855 Edge, and 68 Vertex elements with focus applied to the nozzle
- Stationary analysis with the wall distance initializer

Fluid Property	k	M_n	γ	μ
Value	0.005315415 [lbf/(s*R)]	0.02775 [kg/mol]	1.1752	3.892E-6 [lbf*s/ft^2]



JACOBS ESSSA Group

Pressure Acoustics and AVPE



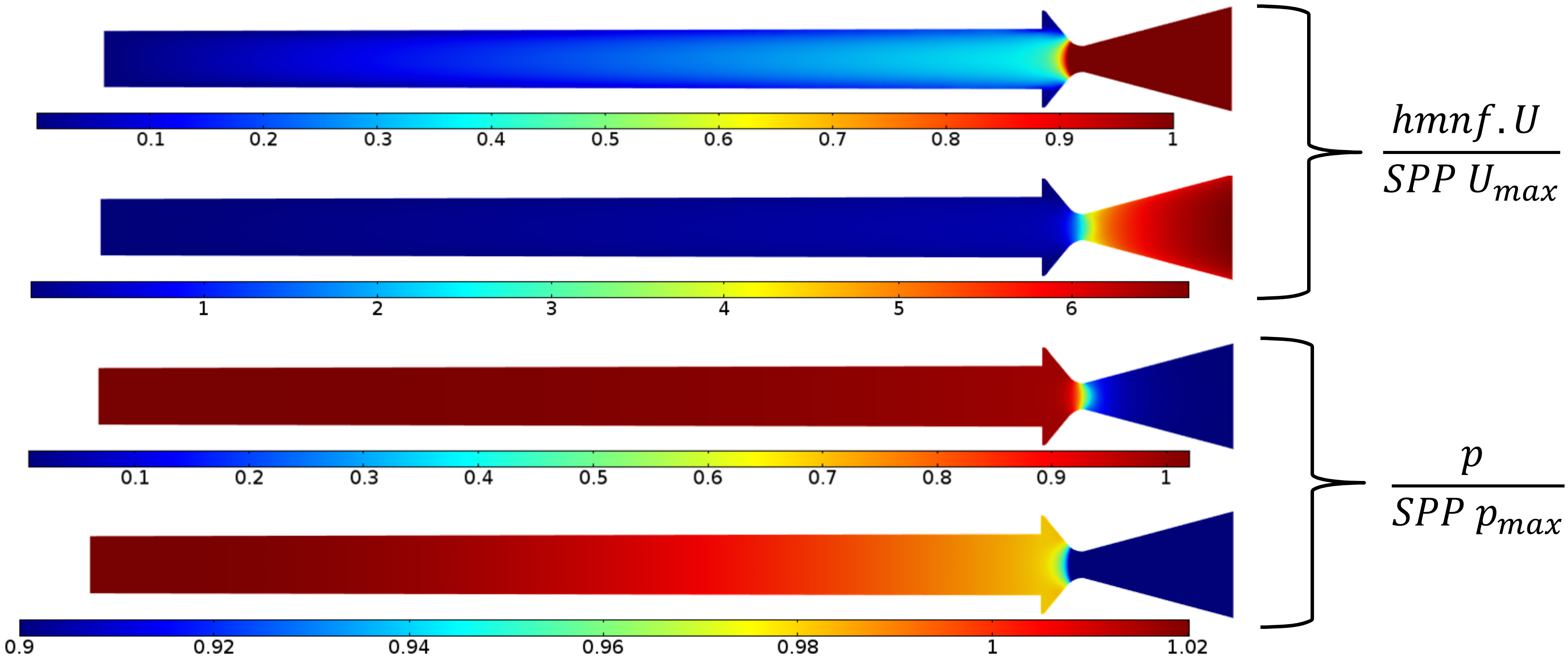
- Sound Hard Wall / No Flux boundary conditions were applied to all boundaries
  - Assumes zero acoustic absorption or excitation at boundaries
- For the PA and AVPE models the required mean flow and material properties were supplied by the HMNF analysis
- AVPE allows for mean flow terms to affect the acoustics,

$$\nabla^2\psi - (\lambda/c)^2\psi - \boldsymbol{M}\cdot[\boldsymbol{M}\cdot\nabla(\nabla\psi)] - 2(\lambda\boldsymbol{M}/c + \boldsymbol{M}\cdot\nabla\boldsymbol{M})\cdot\nabla\psi - 2\lambda\psi[\boldsymbol{M}\cdot\nabla(1/c)] = 0$$

- In the Coefficient Form PDE module the terms of the AVPE containing mean flow parameters were incorporated using domain source terms
- Mesh consists of 1,144,440 Tetrahedral, 67,286 Triangular, 818 Edge, and 60 Vertex elements with focus applied to the sonic line
- Eignvalue studies were conducted for both modules

JACOBS
ESSSA Group

HMNF Results and SPP Comparison

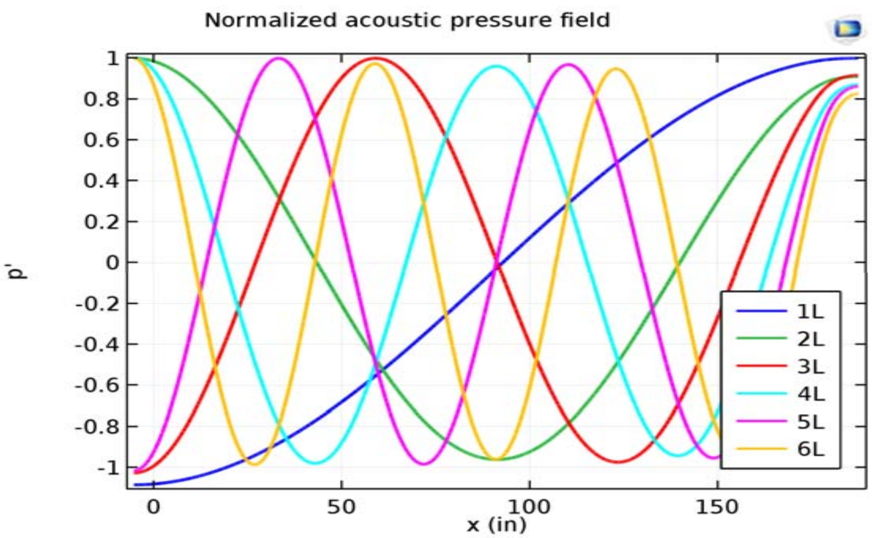


	P_h (psi)	P_a (psi)	$\dot{m}$ (lb/s)	Thrust (lb)
HMNF	1.02	1.03	1.04	1.02
% diff	1.95	2.64	3.88	1.65

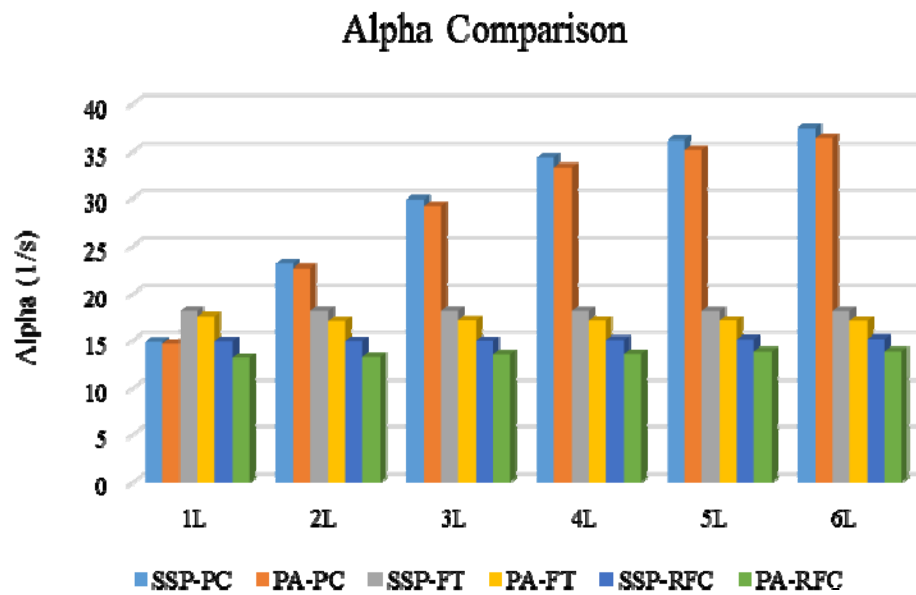
- HMNF results normalized by the SSP value.

JACOBS ESSSA Group

PA Results and SSP Comparison



Freq. (Hz)	1L	2L	3L	4L	5L	6L
PA	115	231	346	462	578	695
SSP	116	233	350	467	584	701
% diff	0.86	0.86	1.14	1.07	1.03	0.86



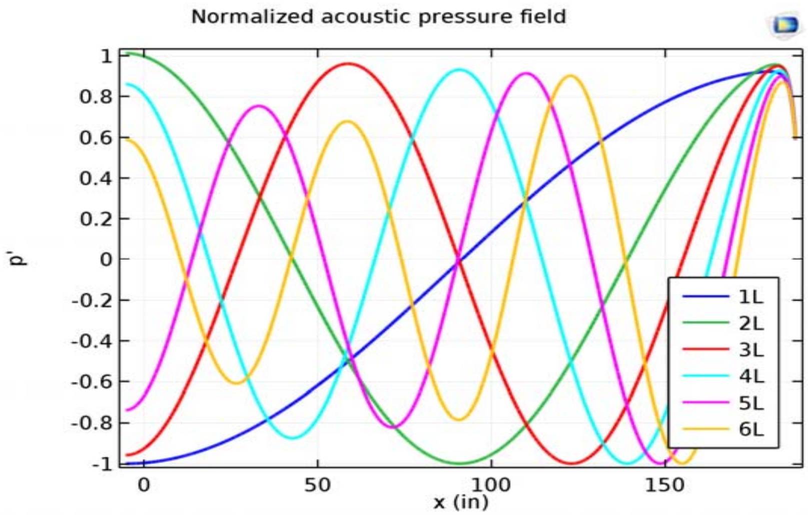
$$\alpha_{PC} = \frac{-\gamma}{2E_n} \iint \boldsymbol{n} \cdot R_s \bar{\boldsymbol{u}}\, p_n^2 dS$$

$$\alpha_{FT} - \frac{1}{2E_n} \iint \frac{1}{K_n^2} \left(\frac{dp_n}{dx}\right)^2 \bar{u}_b dS_b$$

$$\alpha_{FT} = \frac{1}{2E_n} \iint r \frac{\rho_p}{\rho_g} (p')^2 dS_b \qquad E_n = \iiint (p')^2 dV$$

JACOBS
ESSSA Group

AVPE Results and PA Comparison

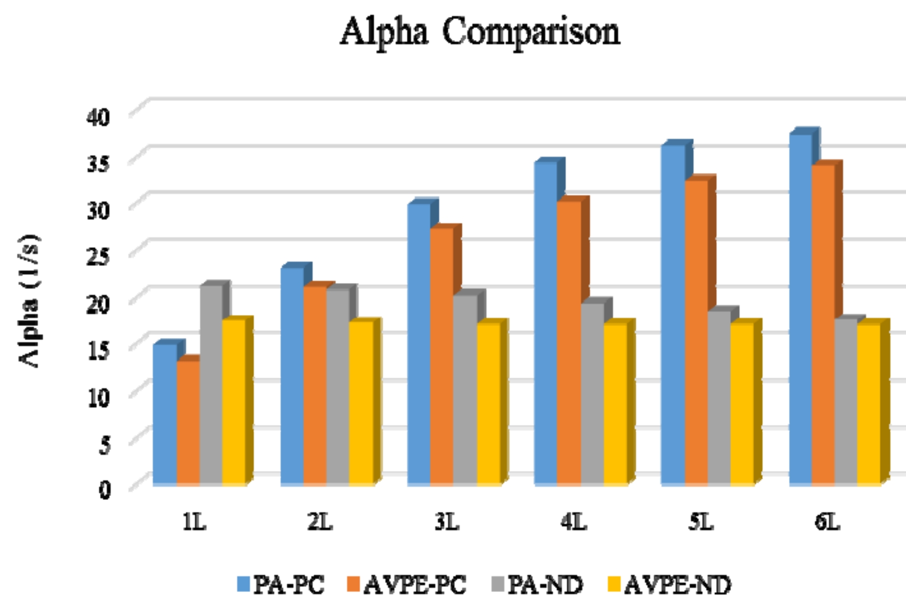


Freq. (Hz)	1L	2L	3L	4L	5L	6L
PA	115	231	346	462	578	695
AVPE	115	230	345	460	576	692
% diff	0.0	0.43	0.29	0.43	0.35	0.43

$$\alpha_{PC} = \frac{1}{2E_n^2} \iint \boldsymbol{n} \cdot \left(\rho_n \boldsymbol{u}_n + \frac{\boldsymbol{u}_0}{\rho_0} p_n \rho_n \right) S_b$$

$$\alpha_{ND} = \frac{1}{2E_n^2} \iint \boldsymbol{n} \cdot \left(\rho_n \boldsymbol{u}_n + \frac{\boldsymbol{u}_0}{\rho_0} p_n \rho_n \right) S_N$$

$$E_n^2 = \iiint \frac{p_n^2}{2\rho_0 a_0^2} + \rho_n \boldsymbol{u}_0 \cdot \boldsymbol{u}_n + \frac{1}{2} \rho_0 \boldsymbol{u}_n^2 \, dV$$



JACOBS ESSSA Group

Conclusions

- A simplified SRM was modeled using the COMSOL multiphysics finite element software
  - HMNF CFD was used to model mean flow parameters
  - PA and Coefficient PDE modules were used to model flow unsteadiness
- Pertinent ballistics parameters from the HMNF analysis compared well with the industry standard SPP
- Acoustic frequencies and CI alpha terms from the PA module compare well with the industry standard SSP
- Coefficient PDE results compare well with the PA results with the calculated CI terms showing the effect of a more accurate mode shape definition.
- The present study demonstrates that COMSOL multiphysics can be used as a CI modeling tool and that the increased fidelity will result in improved results.