Solid Rocket Motor Combustion Instability Modeling in COMSOL Multiphysics

Sean R. Fischbach[1]
[1]Qualis Corp. / Jacobs ESSSA Group / NASA Marshall Spce Flight Center
*MSFC Huntsville 4203/3118, AL 35803, sean.r.fischbach@nasa.gov

Abstract: Combustion instability modeling of Solid Rocket Motors (SRM) remains a topic of active research. Many rockets display violent fluctuations in pressure, velocity, and temperature originating from the complex interactions between the combustion process, acoustics, and steady-state gas dynamics. Recent advances in defining the energy transport of disturbances within steady flow-fields have been applied by combustion stability modelers to improve the analysis framework [1, 2, 3]. Employing this more accurate global energy balance requires a higher fidelity model of the SRM flow-field and acoustic mode shapes. The current industry standard analysis tool utilizes a one dimensional analysis of the time dependent fluid dynamics along with a quasi-three dimensional propellant grain regression model to determine the SRM ballistics. The code then couples with another application that calculates the eigenvalues of the one dimensional homogenous wave equation. The mean flow parameters and acoustic normal modes are coupled to evaluate the stability theory developed and popularized by Culick [4, 5]. The assumption of a linear, non-dissipative wave in a quiescent fluid remains valid while acoustic amplitudes are small and local gas velocities stay below Mach 0.2. The current study employs the COMSOL multiphysics finite element framework to model the steady flow-field parameters and acoustic normal modes of a generic SRM. The study requires one way coupling of the CFD High Mach Number Flow (HMNF) and mathematics module. The HMNF module evaluates the gas flow inside of a SRM using St. Robert's law to model the solid propellant burn rate, no slip boundary conditions, and the hybrid outflow condition. Results from the HMNF model are verified by comparing the pertinent ballistics parameters with the industry standard code outputs (i.e. pressure drop, thrust, ect.). These results are then used by the coefficient form of the mathematics module to determine the complex eigenvalues of the Acoustic Velocity Potential Equation (AVPE). The mathematics model is truncated at the nozzle sonic line, where a zero flux boundary condition is self-satisfying. The remaining boundaries are modeled with a zero flux boundary condition, assuming zero acoustic absorption on all surfaces. The results of the steady-state CFD and AVPE analyses are used to calculate the linear acoustic growth rate as is defined by Flandro and Jacob [2, 3]. In order to verify the process implemented within COMSOL we first employ the Culick theory and compare the results with the industry standard. After the process is verified, the Flandro/Jacob energy balance theory is employed and results displayed.

Keywords: Combustion Instability, Solid Rocket Motor.

1. Introduction

Combustion instability modeling within the Solid Rocket Motor (SRM) industry has been largely limited to the JANNAF Standard Stability Prediction code (SSP) embedded within the Solid Propellant Performance program (SPP'04) [5]. The SPP'04 software is the industry standard for modeling SRM internal ballistics and contains and extensive, and ever growing, array of modules for predicting various mechanisms including erosive burning, boundary layer loss, grain regression, etc. The SSP code, originally developed solely for axial instabilities, is based on the Culick [4] acoustic formulation for combustion instability, and utilizes internal ballistics parameters derived by the SPP'04 analysis. Recent advances in combustion stability modeling by Flandro *et al.* [6] employ an energy based approach, allowing for the inclusion of rotational and thermal oscillations. Independently, Myers [1] developed a general unsteady energy corollary for the transport of disturbances in steady flows. Application of the Myers corollary to combustion instability modeling was developed and derived in detail by Jacob [3]. With the increase in computational resources and a more detailed theory for stability modeling the utilization of a high fidelity analysis tool, like COMSOL mutiphysics, is now practical and tractable for the SRM industry. The present study investigates the feasibility and advantage of employing COMSOL in the prediction of combustion instability in SRMs.

1.1 Acoustic Wave Equation Stability Model

Beginning with a linearized form of the governing equations, a wave equation can be constructed in a manner similar to classical acoustic theory. The assumption is made that the steady portion of the flow parameters do not change over the time period of a single acoustic oscillation. This allows for the flow parameters to be written as a sum of mean and oscillating parts, $Q = \bar{Q} + q'$, respectively. After neglecting the terms of second order in the oscillatory parameter, the linear inhomogeneous wave equation becomes,

$$\nabla^2 p' - \frac{1}{\bar{a}^2} p'_{tt} = h \tag{1}$$

with,

$$h = -q\nabla \cdot (\bar{\boldsymbol{u}} \cdot \nabla \mathbf{u}' + \mathbf{u}' \cdot \nabla \bar{\boldsymbol{u}}) + \frac{1}{\bar{a}^2} \bar{\boldsymbol{u}} \cdot \nabla p'_t + \frac{\gamma}{\bar{a}^2} p'_t \nabla \cdot \bar{\boldsymbol{u}} \tag{2}$$

The traditional approach to calculating linear stability starts by first assuming the classical acoustic formulation, $h = 0$, and solving for the normal modes. The difference of the homogenous and inhomogeneous solution are compared through a spatial average. Further detail can be found in the works of Culick [4]. Assuming that the oscillatory parameter can be modeled as a damped oscillator,

$$p' = p_n e^{i\bar{a}K_n t}, \tag{3}$$

with,

$$K_n = (\Omega_n - i\alpha_n)/\bar{a}, \tag{4}$$

and,

$$\Omega_n = (\omega_n - \delta\omega_n), \tag{5}$$

the shift in frequency, ω_n, and growth constant, α_n , can be determined. An alpha greater than zero represents a linearly unstable system and a growth constant less than zero depicts a stable system. The equation for alpha represents a list of sources and sinks of acoustic energy within the combustion chamber. Written in the SSP nomenclature, the main contributors are,

$$\alpha_n = \frac{-\gamma}{2E_n} \iint \boldsymbol{n} \cdot R_s \bar{\boldsymbol{u}} p_n^2 dS + \frac{-1}{2E_n} \iint \frac{1}{K_n^2} \left(\frac{dp_n}{dz}\right)^2 \bar{u}_b - r\frac{\rho_p}{\rho_g} (p')^2 dS_b, \tag{6}$$

with

$$E_n = \iiint (p')^2 \, dV. \tag{7}$$

1.2 Energy Corollary Model

Investigating combustion instability through the lens of an energy equation allows for a more complete accounting of all flow disturbances. The historical focus on acoustic oscillations due to the often audible tones from unstable combustors captures a majority of flow unsteadiness, but miss contributions made by vorticity and entropy oscillations. These contributions, which may be small in some configurations, become important when unsteady combustion and complex geometries are considered. Myers [1] defines an energy corollary by employing the continuity, momentum and entropy equations and expanding the flow variables into slow changing and fast changing time scales.

$$\frac{\partial E_2}{\partial t} = D_2 - \nabla \cdot \boldsymbol{W}_2 \tag{8}$$

$$E_2 = \frac{p_1^2}{2\rho_0 a_0^2} + \rho_1 \boldsymbol{u}_0 \cdot \boldsymbol{u}_1 + \frac{1}{2}\rho_0 \boldsymbol{u}_1^2 + \frac{\rho_0 \rho T_0 s_1^2}{2C_p} \tag{9}$$

$$D_2 = -\rho_0 \boldsymbol{u}_0 \cdot (\boldsymbol{u}_1 \times \boldsymbol{\Omega}_1) - \rho_1 \boldsymbol{u}_1 \cdot (\boldsymbol{u}_0 \times \boldsymbol{\Omega}_0) - \rho_0 T_1 \boldsymbol{u}_0 \cdot \nabla s_1 - \rho_0 s_1 \boldsymbol{u}_1 \cdot \nabla T_0 - \rho_1 s_1 \boldsymbol{u}_0 \cdot \nabla T_0 + \boldsymbol{m_1 \psi_1} \tag{10}$$

$$\boldsymbol{W}_2 = \boldsymbol{u}_1 p_1 + \frac{\boldsymbol{u}_0}{\rho_0} p_1 \rho_1 + \rho_0 \boldsymbol{u}_1 (\boldsymbol{u}_0 \cdot \boldsymbol{u}_1) + \rho_1 \boldsymbol{u}_0 (\boldsymbol{u}_0 \cdot \boldsymbol{u}_1) \tag{11}$$

Jacob [3] applied the Myer framework to SRM combustion instability modeling, recasting the formulation into the traditional alpha notation.

$$\alpha_n = \frac{\alpha'_n}{2E_n^2} \tag{12}$$

$$E_n^2 = \iiint \frac{p_n^2}{2\rho_0 a_0^2} + \rho_n \boldsymbol{u}_0 \cdot \boldsymbol{u}_n + \frac{1}{2}\rho_0 \boldsymbol{u}_n^2 \tag{13}$$

$$\alpha'_n = \iiint -\nabla \cdot \Big[\rho_n \boldsymbol{u}_n + \frac{\boldsymbol{u}_0}{\rho_0} p_n \rho_n + \rho_0 \boldsymbol{u}_n (\boldsymbol{u}_0 \cdot \boldsymbol{u}_n) + \rho_n \boldsymbol{u}_0 (\boldsymbol{u}_0 \cdot \boldsymbol{u}_n)\Big] - \rho_0 \boldsymbol{u}_0 \cdot (\boldsymbol{u}_n \times \boldsymbol{\Omega}_n) + \boldsymbol{m_1 \psi_{1n}} - \rho_n \boldsymbol{u}_n \cdot (\boldsymbol{u}_0 \times \boldsymbol{\Omega}_0) dV \tag{14}$$

In this formulation it is assumed that unsteady entropy is equal to zero. Both combustion stability models depend on an established quasi-steady and unsteady flow-field. These parameters are established using the COMSOL multiphysics finite element software.

1.3 Unsteady Flow Calculations

The above combustion stabilityframework do not attempt to model the entire time dependent flow-field of a SRM. This would require a costly and time consuming time accurate CFD model along with a yet to be defined acoustic admittance boundary conditions for the motor propellant. Traditionally the quasi-steady and unsteady flow-fields are solved separately, with the unsteady variables modeled via the homogenous wave equation.

$$\nabla \cdot \left(-\frac{1}{\bar{\rho}} (\nabla p') \right) + \frac{\omega^2}{\bar{\rho} \bar{a}^2} p'_{tt} = 0 \qquad (15)$$

This form of the wave equation assumes no mean flow contributions. These assumptions are valid for the majority of the rocket chamber, but break down within the converging section of the nozzle. The expulsion of unsteady energy through the nozzle of a rocket is identified as the predominate source of acoustic damping for most rocket systems. Recently, an approach to address nozzle damping with mean flow effects was implemented by French [7]. This new approach solves the acoustic velocity potential equation (AVPE) formulated by perturbing the Euler equations.

$$\begin{aligned} \nabla^2 \psi - (\lambda / c)^2 \psi - \boldsymbol{M} \cdot [\boldsymbol{M} \cdot \nabla(\nabla \psi)] \\ -2(\lambda \boldsymbol{M}/c + \boldsymbol{M} \cdot \nabla \boldsymbol{M}) \cdot \nabla \psi \\ -2\lambda \psi [\boldsymbol{M} \cdot \nabla(1/c)] = 0 \end{aligned} \qquad (16)$$

The AVPE includes effects of the mean flow variables on the unsteady acoustic mode shape and can be applied up to the sonic line. The present study utilizes both formulations as a way of verifying the COMSOL implementation against SSP and displaying the benefits of the more accurate AVPE.

3. Use of COMSOL Multiphysics

Implementing the above combustion instability models in COMSOL multiphysics requires the use of High Mach Number Flow (HMNF) CFD module, the Coefficient Form PDE module, and the Pressure Acoustics Module. First the HMNF module is used to model the SRM internal ballistics. The CFD results are post processed and the Mach equal to one surface is extracted. The Mach plane is used to generate a new geometry for the proceeding acoustic analyses, which do not model the nozzle exit cone. The results of the HMNF are used to define the mean flow terms in the AVPE. Results from the CFD and acoustic analyses are combined in post processing to calculate the stability alpha.

3.1 High Mach Number Flow Analysis

The present study employs COMSOL multiphysics finite element framework to model the steady flow-field parameters of a generic SRM using the Spalart-Allmaras turbulent flow equations within the HMNF module. A steady state analysis was performed with the initialization necessary for the Spalart-Allmara model. Equations 17-19 provide an overview of the conservation of mass, momentum and energy, respectively.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0 \qquad (17)$$

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho(\boldsymbol{u} \cdot \nabla)\mathbf{u} = \nabla \cdot [-p\mathrm{I} + \tau] + \mathbf{F} \qquad (18)$$

$$\begin{aligned} \rho C_p \left(\frac{\partial T}{\partial t} + (\boldsymbol{u} \cdot \nabla) T \right) = -(\nabla \cdot \mathrm{q}) \\ + \boldsymbol{\tau} \colon \mathbf{S} - \frac{T}{\rho} \frac{\partial \rho}{\partial T}\bigg|_p \left(\frac{\partial \rho}{\partial t} + (\boldsymbol{u} \cdot \nabla) p \right) + \mathbf{Q} \end{aligned} \qquad (19)$$

The HMNF module makes use of these fully compressible Navier-Stokes equation for an ideal gas, which are detailed in the COMSOL documentation. In order to model the injection of hot gas due to the burning propellant St. Robert's law to model the solid propellant burn rate defined as,

$$\dot{r} = a p^n \qquad (20)$$

with $\dot{r}$ being the burn rate, p is the pressure at the wall, n is the burn rate exponent and a is an empirical factor. The normal velocity of the gas is then modeled as,

$$v_g = \dot{r} \frac{\rho_p}{\rho_g} \qquad (21)$$

with ρ_p as the density of the propellant, and ρ_g as the density of the combustion products. All other solid boundaries are modeled with the no-slip boundary condition, and the exit plane is modeled with the hybrid outflow condition. Figure 1 displays the HMNF geometry with the flow boundary conditions identified.

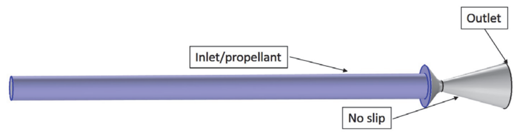

Figure 1. SRM geometry with boundary conditions.

3.2 Pressure Acoustics Analysis

The Pressure Acoustics (PA) module is used to provide the unsteady flow parameters for combustion stability modeling. The PA module performs and Eigenvalue study on equation 14 using the results from the HMNF analysis to supply the mean flow speed of sound and density. An acoustic hard wall boundary condition is used on all surfaces including the sonic (Mach = 1) line. Results from the Eigenvalue analysis and steady state analysis are combined in eq. (6) to calculate the motor's growth alpha. The results are compared with the industry standard, SSP, as a verification of the COMSOL implementation. Figure 2 displays the geometry utilized in the unsteady Eigenvalue analyses.

3.2 Coefficient Form Partial Differential Equation Analysis

The Coefficient Form Partial Differential Equation module is used to determine the complex eigenvalues of the AVPE. Similarly to the PA analysis, the mean flow terms in eq. (16) are derived from the HMNF results. The mathematics model is truncated at the nozzle sonic line, where a zero flux boundary condition is self-satisfying. The remaining boundaries are modeled with a zero flux boundary condition, assuming zero acoustic absorption on all surfaces. The results of the steady-state CFD and AVPE analyses are used to calculate the linear acoustic growth rate as is defined by Flandro and Jacob [2, 3].

Figure 2. SRM geometry truncated at the sonic line.

4. Results

The pertinent SRM ballistics parameters derived from the HMNF analysis are compared to the SPP results in table 2. The HMNF model shows a higher pressure at the head and aft end of the motor along with an elevated mass flow rate. These differences will have little effect on the stability analysis and comparison with SSP. It is expected that the differences arise due to how the initial guess used in the COMSOL analysis interacts with the inlet B.C. For instance, if the initial pressure was 814 psi for the entire chamber, then the burn rate, injection Mach number, and mass flow rate would be higher than if the initial guess had a linear pressure drop down the bore of the motor. But, the results indicate that, with some future work, the burning propellant of a SRM can be successfully modeled using the HMNF module in COMSOL.

Table 2: Comparison of Ballistics Parameters

	P_h (psi)	P_a (psi)	$\dot{m}$ (lb/s)	Thrust (lb)
HMNF	834	777	98.84	27130
SPP	818	757	95.15	26690
% diff	1.95	2.64	3.88	1.65

A frequency comparison between the PA module and SSP is presented in table 3. Since the SSP analysis is strictly one dimensional, some differences are expected due to sudden area changes. Also, the geometry in the SSP model is slightly shorter than the COMSOL model due to its inability to model the sonic line. Figure 3 displays the normalized acoustic mode shapes of the first six longitudinal modes derived by the PA analysis.

Table 3: Comparison of Acoustic Frequencies

Freq. (Hz)	1L	2L	3L	4L	5L	6L
PA	115	231	346	462	578	695
SSP	116	233	350	467	584	701
% diff	0.86	0.86	1.14	1.07	1.03	0.86

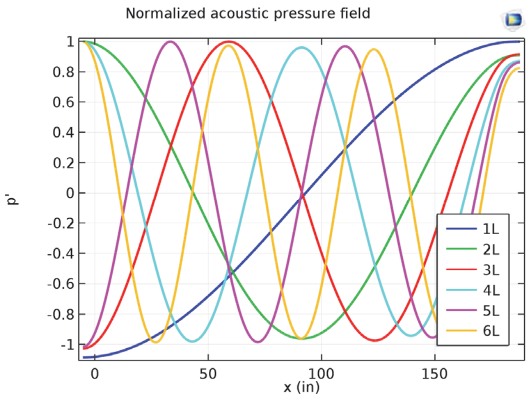

Figure 3. Acoustic mode shapes derived from PA Eigenvalue analysis.

Using the mode shapes from fig. 3 and the results from the HMNF analysis the right hand side of eq. (6) are evaluated and presented in fig. 4. For each of the six longitudinal modes the growth rate is divided simplified into three terms; Pressure Coupling (PC), Flow Turning (FT), and Rotational Flow Correction (RFC). These terms represent the major contributors that can be compared in this analysis. Another major contributor that is absent is the Nozzle Damping (ND) term. The comparison would not be credible do to the differences between the two models. Figure 4 display a good comparison between the two approaches. The COMSOL results are consistently smaller than the SSP model, but do not diverge greatly. It should be noted that the FT alpha is actually a negative value, but is presented as a magnitude for ease of comparison.

With a good comparison between two codes using the same stability model, focus is turned to a comparison of the homogeneous wave equation and AVPE. The inclusion of the mean flow terms on the right hand side of the wave equation act to shift the frequency and mode shape of the longitudinal acoustic modes. Figure 5 displays the normalized real component of the acoustic mode shapes for the first six longitudinal modes derived by the AVPE analysis. The results from the AVPE analysis have a real and imaginary component due to the acoustic/mean flow interactions.

Comparing the mode shapes in figures 3 and 5 little difference is observed in the first three acoustic modes, with noticeable variances in the latter three mode shapes. Focusing on the converging section of the nozzle on the right hand side of fig. 5 the effect of compressibility on the acoustic mode shapes is clearly observed. Figure 6 displays the normalized imaginary part of the AVPE mode shapes. The imaginary portion of the mode shape represent a phase shift derived from mean flow effects.

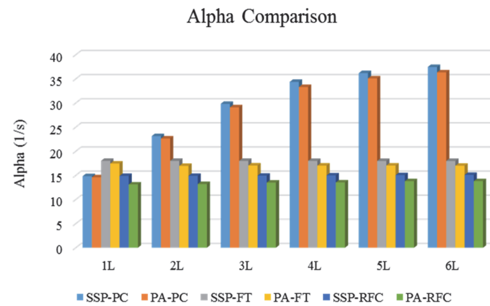

Figure 4. Alpha comparison between SSP and COMSOL.

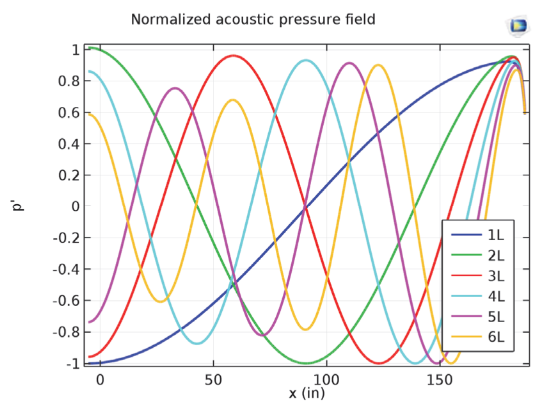

Figure 5. Real Acoustic mode shapes derived from AVPE Eigenvalue analysis.

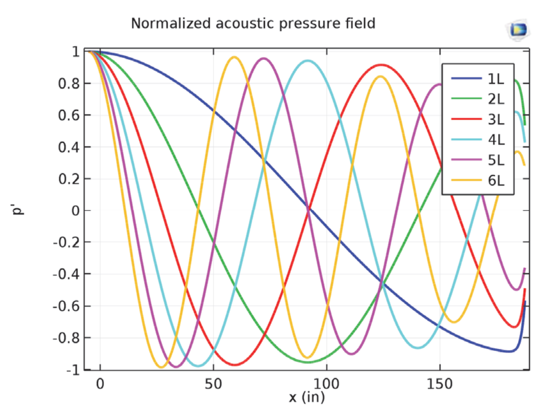

Figure 6. Imaginary Acoustic mode shapes derived from AVPE Eigenvalue analysis.

Along with a shift in the acoustic phasing, the inclusion of mean flow terms also shift the frequency of oscillation. A comparison of the PA frequencies and the AVPE frequencies is presented in table 4. Very little frequency shift is noticed in the low frequencies, with an increase in effect at higher frequencies. The subtle changes to the acoustic mode shape and frequency can have a large impact on the stability modeling. Acoustic results from PA and AVPE analyses are used to calculate PC and ND alpha terms using the energy based formulation. The bar graph in figure 7 displays a comparison of the results.

Table 4: Comparison of Acoustic Frequencies

Freq. (Hz)	1L	2L	3L	4L	5L	6L
PA	115	231	346	462	578	695
AVPE	115	230	345	460	576	692
% diff	0.0	0.43	0.29	0.43	0.35	0.43

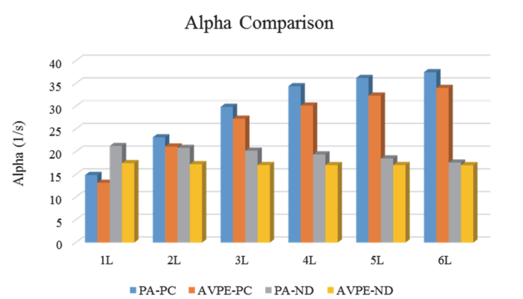

Figure 7. Energy Corollary alpha comparison between PA and AVPE.

5. Conclusions

COMSOL multiphysics finite element software was used to model the combustion instability of a simplified SRM. The internal ballistics of the SRM were modeled with the HMNF CFD module and verified against an industry standard 1-D analysis tool. Two acoustic models were created using the Pressure Acoustics module and the Coefficient Form Partial Differential Equation module. The results from the PA module were used in conjunction with the CFD results to model the SRM stability using the Culick framework. These results were compared with the industry standard SSP for verification. Finally, the results from the AVPE were combined with the CFD results to model the SRM stability using the energy corollary framework. To demonstrate the advantage to using the AVPE, the stability alphas were calculated with the PA and math module solutions.

This study has succeeded in establishing a proof of concept for using COMSOL multiphysics as a SRM combustion instability modeling tool. Future work will be focused on refining the inlet boundary condition modeled using the propellant burn rate. Also, further model will need to be developed to include vortical and entropy waves into the energy corollary.

9. References

1 M. K. Myers, "Transport of Energy by Disturbances in Arbitrary Steady Flows," *Journal of Fluid Mechanics,* vol. 226, pp. 383-400, 1991.

2 E. J. Jacob, G. A. Flandro, P. W. Gloyer and J. C. French, "Nonlinear Liquid Rocket Combustion Instability Behavior using UCDS Process," in *AIAA/ASME/SAE/ASEE Joint Prpulsion Conference & Exhibit*, Nashville, TN, 2010.

3 E. J. Jacob, "A Study of Nonlinear Combustion Instability," *Ph.D. Dissertation,* 2009.

4 F. E. Culick, Unsteady Motions in Combustion Chambers for Propulsion Systems, Cambridge University Press, 2007.

5 S. Dunn, D. Coats and J. French, "Standard Stability Prediction Method For Solid Rocket Motors: Axial Mode Computer Profram User's Manual," Software & Engineering Associates, Carson City, 2005.

6 G. A. Flandro, S. R. Fischbach, J. Majdalani and J. French, "Nonlinear Rocket Motor Stability Prediction: Limit Amplitude, Triggering, and Mean Pressure Shift," *Physics of Fluids,* vol. 19, 2007.

7 J. C. French, "Nozzle Acoustic Dynamics and Stability Modeling," *Journal of Propulsion and Power,* vol. 27, 2011.