A Model for Jet-Surface Interaction Noise Using Physically Realizable Upstream Turbulence Conditions

Mohammed Z. Afsar

Imperial College London, 180 Queen's Gate, London, SW7, UK

S.J. Leib

Ohio Aerospace Institute, 22800 Cedar Point Road, Cleveland, Ohio 44142, USA

and

Richard F. Bozak

National Aeronautics and Space Administration, Glenn Research Center, Cleveland, Ohio 44135, USA Research Fellow,

Motivations and Objectives

- Jet flows of technological interest are often close enough to solid boundaries so that the surface plays a direct role in the generation of sound as well as its propagation
- Proposed next-generation aircraft configurations may have exhaust systems tightly integrated with the airframe
- One problem of interest:
  - Exhaust jet interacting with wing or other nearby edge
- Experiments show that the presence of an external surface enhances the noise produced by the jet alone
- The aim of this paper is to further develop a prediction method for the noise generated by the interaction of a turbulent jet with the trailing-edge of a flat plate

Technical Approach

- The prediction method is based on application of the non-homogeneous Rapid-distortion Theory (RDT) introduced recently by Goldstein, Afsar and Leib (2013) (GAL)
  - Initial application to prediction of noise from interaction of large-aspect ratio rectangular jets with flat plate
- Improved source model for transverse velocity correlations and relation to 'gust' spectrum
- Use results from Reynolds-averaged Navier-Stokes (RANS) solutions to obtain the mean flow and inform the source model

Outline

- Brief review of GAL formulation
- New source model and effect on low-frequency roll-off
- RANS solutions
- RANS-based edge-noise predictions
- Conclusions

Review of GAL Formulation

- Rapid Distortion Theory
  - Linear analysis to study the interaction of turbulence with solid surfaces
- Assumptions and Approximations:
  - The turbulence intensity is small
  - The time scale for interaction is short compared with those over which non-linearity and viscous dissipation take place
- Problem is governed by the compressible Rayleigh equation

$$\frac{D_0 u_i}{Dt} + \delta_{1i} u_j \frac{\partial U}{\partial y_j} + \frac{\partial p'}{\partial y_i} = 0 \qquad \frac{D_0 p'}{Dt} + \frac{\partial c^2 u_j}{\partial y_j} = 0$$

$$U = U(\boldsymbol{y}_T) \quad ; \quad c^2 = c^2(\boldsymbol{y}_T) \quad ; \quad \boldsymbol{y} = (y_1, y_2, y_3) = (y_1, \boldsymbol{y}_T) \quad ; \quad \frac{D_0}{Dt} = \frac{\partial}{\partial t} + U(\boldsymbol{y}_T)\frac{\partial}{\partial y_1}$$

Review of GAL Formulation: General Solution

Pressure in terms of scalar function $$p\text{¢}\left(\boldsymbol{y},t\right) = - \frac{D_0^3 f}{Dt^3}\left(\boldsymbol{y},t\right),$$

Momentum flux in terms of scalar and arbitrary convected quantity

$$u_i\left(\boldsymbol{y},\mathrm{t}\right) = \left(\mathrm{d}_{ij}\frac{D_0}{D\mathrm{t}} - \mathrm{d}_{i1}\frac{\partial U}{\partial y_j}\right)\left\{\frac{\partial}{\partial y_j}\frac{D_0\mathrm{f}}{D\mathrm{t}} + 2\frac{\partial U}{\partial y_j}\frac{\partial \mathrm{f}}{\partial y_1}\right\} + \mathrm{e}_{ijk}\frac{1}{c^2}\frac{\partial U}{\partial y_j}\frac{\partial}{\partial y_k}\mathrm{J}\left(\mathrm{t} - \frac{y_1}{U\left(\boldsymbol{y}_T\right)}, \boldsymbol{y}_T\right),$$

Scalar satisfies inhomogeneous adjoint Rayleigh equation

$$L_a\phi \equiv \left\{\frac{D_0^3}{D\boldsymbol{\tau}^3} - \frac{\partial}{\partial y_i}c^2\left(\frac{\partial}{\partial y_i}\frac{D_0}{D\tau} + 2\frac{\partial U}{\partial y_i}\frac{\partial}{\partial y_1}\right)\right\}\phi = -\tilde{\omega}_c\left(\boldsymbol{\tau} - y_1/U\left(\boldsymbol{y}_T\right), \boldsymbol{y}_T\right),$$

Solution in terms of Rayleigh equation Green's function $$L\, g\left(\boldsymbol{y},t\,|\,\boldsymbol{x},t\right) = d\left(\boldsymbol{y} - \boldsymbol{x}\right)d\left(t - t\right)$$

$$p'\left(\boldsymbol{x},t\right) = \int_{-T}^{T}\int_V \frac{D_0^3 g\left(\boldsymbol{y},\tau\,|\,\boldsymbol{x},t\right)}{Dt^3}\tilde{\omega}_c\left(\tau - y_1/U\left(\boldsymbol{y}_T\right), \boldsymbol{y}_T\right)d\boldsymbol{y}d\boldsymbol{\tau}$$

$$\rho v'_\perp\left(\boldsymbol{x},t\right) \equiv u_i\left(\boldsymbol{x},t\right)\frac{\partial U}{\partial x_i}/\left|\nabla U\right| = -\frac{\partial U/\partial x_i}{\left|\nabla U\right|}\int_{-T}^{T}\int_V g_i\left(\boldsymbol{y},\tau\,|\,\boldsymbol{x},t\right)\tilde{\omega}_c\left(\tau - y_1/U\left(\boldsymbol{y}_T\right), \boldsymbol{y}_T\right)\, d\boldsymbol{y}d\tau$$

$\tilde{\omega}_c\left(\tau - y_1/U\left(\boldsymbol{y}_T\right), \boldsymbol{y}_T\right)$ Arbitrary, purely convected quantity

Review of GAL Formulation The 'Gust' Solution

- Split Rayleigh equation Green's function into two components:

$$g\left(\boldsymbol{y},\mathrm{t}\mid\boldsymbol{x},t\right)=g^{(0)}\left(\boldsymbol{y},\mathrm{t}\mid\boldsymbol{x},t\right)+g^{(s)}\left(\boldsymbol{y},\mathrm{t}\mid\boldsymbol{x},t\right)$$

- $g^{(0)}\left(\boldsymbol{y},\mathrm{t}\mid\boldsymbol{x},t\right)$ satisfies the inhomogeneous Rayleigh equation with

$$\hat{n}_i\,\partial\left[D_0^3 g^{(0)}\left(\boldsymbol{y},t\mid\boldsymbol{x},t\right)\big/Dt^3\right]\big/\partial y_i=0 \qquad \text{for } \boldsymbol{y}_T\in S,\ \ -\infty<y_1<\infty$$

- $g^{(s)}\left(\boldsymbol{y},\mathrm{t}\mid\boldsymbol{x},t\right)$ is the 'scattered' solution which satisfies the homogeneous Rayleigh equation subject to:
  - $\hat{n}_i\,\partial\left[D_0^3 g^{(s)}\left(\boldsymbol{y},t\mid\boldsymbol{x},t\right)\big/Dt^3\right]\big/\partial y_i=0 \qquad \text{for } \boldsymbol{y}_T\in S,\ \ -\infty<y_1<0$
  - Jump conditions across plate's downstream extension $0<y_1<¥$
- The streamwise-homogeneous solution, $g^{(0)}\left(\boldsymbol{y},\mathrm{t}\mid\boldsymbol{x},t\right)$, is referred to as the 'gust' solution and is the input to the interaction problem

$$\left[\rho v'_\perp\left(\boldsymbol{x},t\right)\right]^{(0)}=-\frac{\partial U/\partial x_i}{\left|\nabla U\right|}\int_{-T}^{T}\int_V g_i^{(0)}\left(\boldsymbol{y},\tau\mid\boldsymbol{x},t\right)\tilde{\omega}_c\left(\tau-y_1\big/U\left(\boldsymbol{y}_T\right),\boldsymbol{y}_T\right)d\boldsymbol{y}\,d\tau$$

Review of GAL Formulation
Relation between $\tilde{\omega}_c$ Spectrum and Measurable Turbulence Statistics

- Assume: Relation between $\tilde{\omega}_c$ and $r v_{\wedge}$ ¢is the same as that in a streamwise-homogeneous flow where surface is doubly infinite
- Invert relation between $\tilde{\omega}_c$ and $r v_{\wedge}$ ¢in the 'gust' solution for a two-dimensional mean flow (spanwise homogeneous)
- Relate:
  - Spectrum of $\tilde{\omega}_c$:

$$S\left(y_2,\tilde{y}_2;k_3,\omega\right)\equiv\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{i\left(\omega\tau-k_3\eta_3\right)}\left\langle\tilde{\omega}_c\left(t,y_2,y_3\right)\tilde{\omega}_c\left(t+\tau,\tilde{y}_2,y_3+\eta_3\right)\right\rangle d\tau d\eta_3$$

To

  - Measurable turbulence quantities:

$$\left\langle v'_\perp\left(\boldsymbol{x},t\right)v'_\perp\left(x_1,\tilde{x}_2,x_3+\eta_3,t+\tau\right)\right\rangle\equiv\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\int_{-\infty}^{\infty}v'_\perp\left(\boldsymbol{x},t\right)v'_\perp\left(x_1,\tilde{x}_2,x_3+\eta_3,t+\tau\right)dtdx_3$$

Review of GAL Formulation

Relation between $\tilde{\omega}_c$ Spectrum and Measurable Turbulence Statistics

$$S\left(y_2,\tilde{y}_2;k_3,\omega\right)=\frac{\left|U'\left(y_2\right)U'\left(\tilde{y}_2\right)\right|}{\left[U\left(y_2\right)U\left(\tilde{y}_2\right)\right]}\frac{\left(1+\frac{y_2-y_d}{y_d}b_0\right)\left(1+\frac{\tilde{y}_2-y_d}{y_d}b_0\right)}{E\left(y_2;k_3,\omega\right)\left[E\left(\tilde{y}_2;k_3,\omega\right)\right]^*}F_\perp\left(y_d,y_d\middle|y_2,\tilde{y}_2,\omega,k_3\right)$$

where

$$E\left(y_2;k_3,\omega\right)\equiv\frac{\left[U\left(y_d\right)-U\left(y_2\right)\right]}{c^2\left(y_d\right)}\left(1-\frac{\left|y_2-y_d\right|}{y_d}b_0\right)+\frac{\left[\omega^2/U^2\left(y_2\right)+k_3^2\right]}{\sqrt{\omega^2/U^2\left(y_2\right)+k_3^2-k_\infty^2}}\frac{i\pi U^2\left(y_2\right)b_0}{c_\infty^2U''y_d}$$

$$f_\perp\left(x_2,\tilde{x}_2\middle|k_1,\tilde{k}_1,\eta_3,\tau\right)\equiv\frac{1}{\left(2\pi\right)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-i\left(x_1k_1-\tilde{x}_1\tilde{k}_1\right)}\left\langle\rho v_\perp'^{(0)}\left(\boldsymbol{x},t\right)\rho v_\perp'^{(0)}\left(\tilde{x}_1,\tilde{x}_2,x_3+\eta_3,t+\tau\right)\right\rangle dx_1d\tilde{x}_1,$$

$$F_\perp\left(x_2,\tilde{x}_2\middle|y_2,\tilde{y}_2,\omega,k_3\right)=\frac{1}{\left(2\pi\right)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-i\left(\omega\tau-k_3\eta_3\right)}f_\perp\left(x_2,\tilde{x}_2\middle|\omega/U\left(y_2\right),\omega/U\left(\tilde{y}_2\right),\eta_3,\tau\right)d\eta_3d\tau$$

- ‘Gust’ solution provides the input (upstream boundary condition) to the jet-surface interaction problem in terms of measureable turbulence quantities
- Model needed for transverse momentum fluctuation space-time correlations.

Review of GAL Formulation
Scattered Solution

- The 'scattered' solution, $g^{(s)}(\boldsymbol{y},\mathrm{t} \mid \boldsymbol{x},t)$, represents the effects of the presence of the trailing edge
- Satisfies homogeneous Rayleigh equation
- Streamwise-discontinuous boundary conditions on the plate and its downstream extension
- Solve by Wiener-Hopf technique
- Low-frequency asymptotic solution

Acoustic Spectrum for Jet-Surface Interaction in Planar Flow

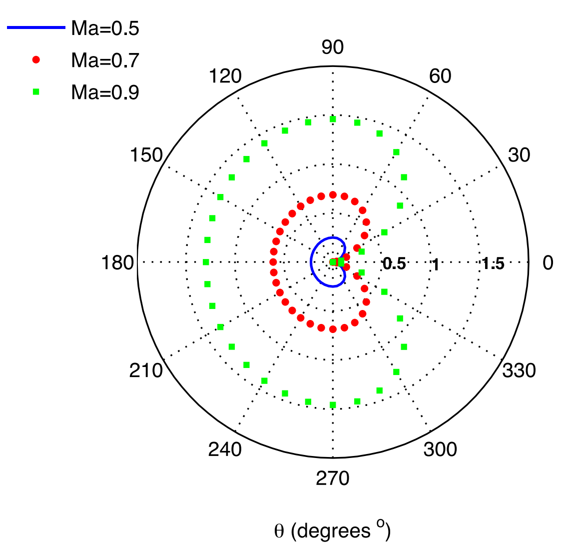
$$I_\omega(\boldsymbol{x}) \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega\tau}\,\overline{p^s(\boldsymbol{x},t)\,p^s(\boldsymbol{x},t+\tau)}\,d\tau \approx \left(\frac{k_\infty}{4\pi|\boldsymbol{x}|}\right)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} D\big(\theta,\psi,M(y_2)\big)S\left(y_2,\tilde{y}_2;k_3^{(s)},\omega\right)dy_2\,d\tilde{y}_2$$

Directivity Factor

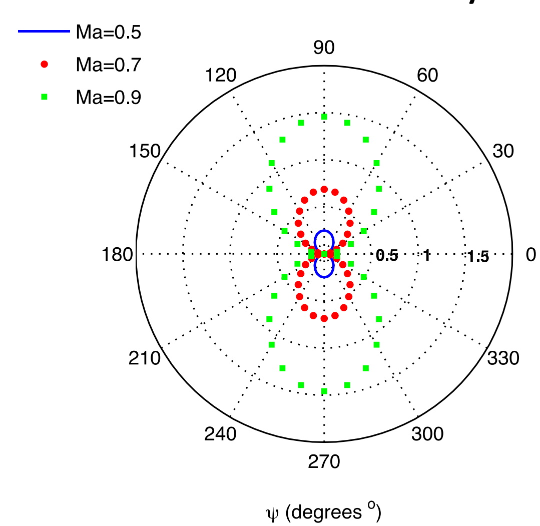
$$D\big(\theta,\psi,M(y_2)\big)=\frac{\left[M(y_2)M(\tilde{y}_2)\right]^{3/2}(\beta-\cos\theta)}{\left[1-M(y_2)\cos\theta\right]\left[1-M(\tilde{y}_2)\cos\theta\right]\sqrt{\left[1-\beta M(y_2)\right]\left[1-\beta M(\tilde{y}_2)\right]}}$$

$$M(y_2)=U(y_2)\big/c_\infty \qquad \beta \equiv \left(1-\sin^2\theta\cos^2\psi\right)^{1/2} \qquad k_3^{(s)}=\frac{\omega}{c_\infty}\sin\theta\cos\psi$$

Polar Directivity



Azimuthal Directivity

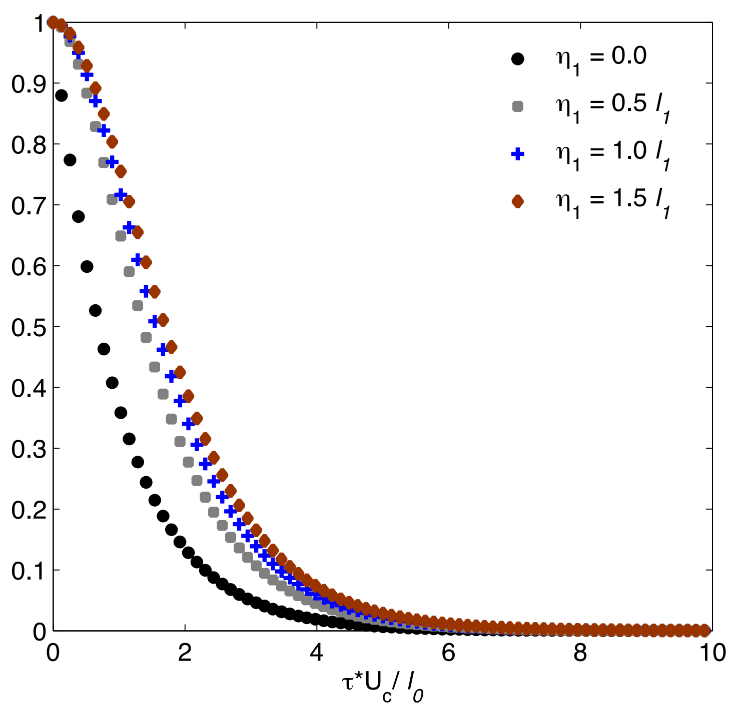


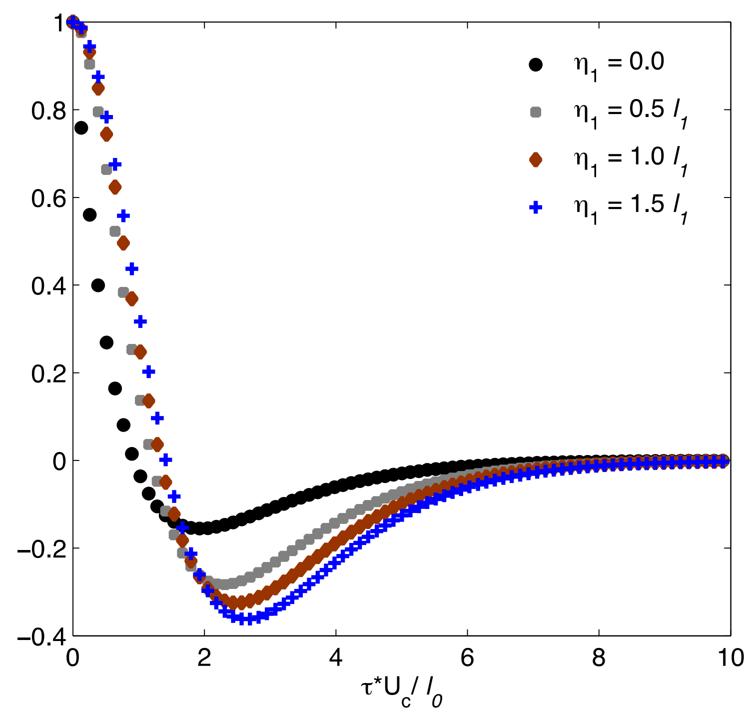
Upstream Turbulence Model

$$\left\langle v'_\perp(\boldsymbol{x},t)\, v'_\perp(x_1,\tilde{x}_2,x_3+\eta_3,t+\tau)\right\rangle \equiv \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\int_{-\infty}^{\infty} v'_\perp(\boldsymbol{x},t)\, v'_\perp(x_1,\tilde{x}_2,x_3+\eta_3,t+\tau)\,dt\,dx_3$$

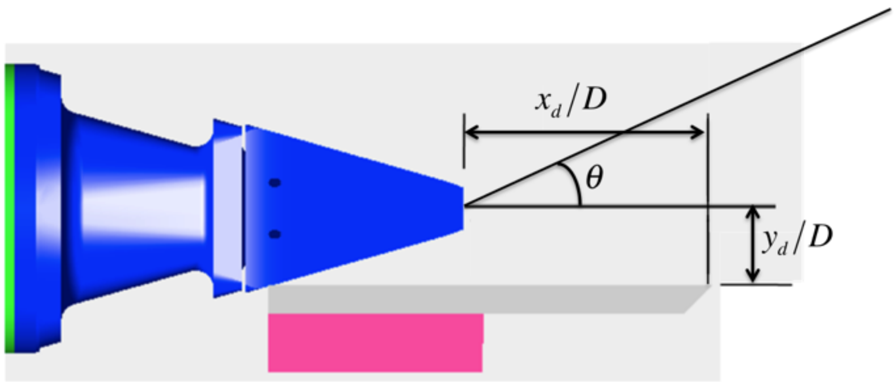
$$\left\langle \rho v'_\perp(x_1,y_d,x_3,t)\,\rho v'_\perp(\tilde{x}_1,y_d,x_3+\eta_3,t+\tau)\right\rangle = L_3\Psi(\bar{x}_1)\left[a_0+a_1\tau\frac{\partial}{\partial\tau}+a_2\eta_1\frac{\partial}{\partial\eta_1}+\dots\right]e^{-X(\tau,\eta_1,\eta_3)}$$

$$X(h_1,h_3,t)=\sqrt{(h_1/l_1)^2+(h_1-U_c t)^2/l_0^2+(h_3/l_3)^2}$$





Application to Interaction of a Large-Aspect Ratio Rectangular Jet with a Semi-Infinite Flat Plate



Cases:

Trailing edge distance from nozzle exit plane: xd/D = 5.7

Standoff distance: yd/D = 1.2

Aspect Ratios: AR =4 and AR =8

Jet exit acoustic Mach numbers: Ma = 0.7 and 0.9

Effect of Source Model on Low-Frequency Roll-off of Edge-Noise Spectrum

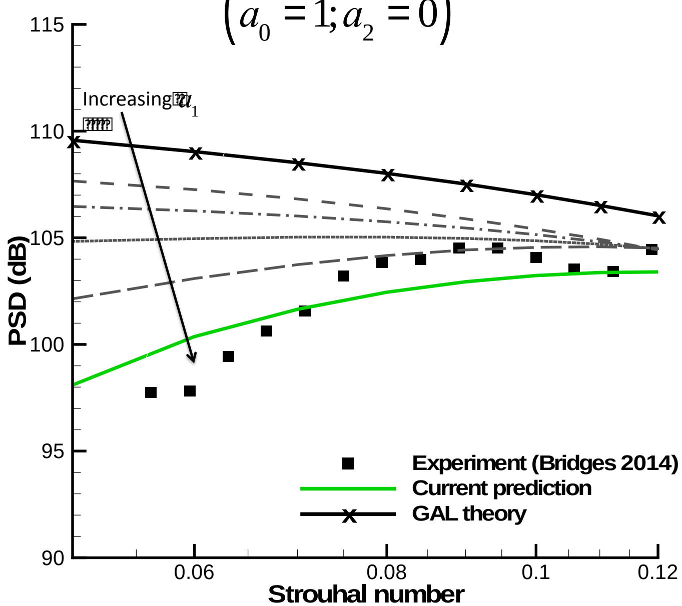
Analytic Mean Profile

$$U(y_2) = U_d\left[e^{-a^2(y_2-y_d)^2} - e^{-a^2(t_d/2)^2}\right] \Big/ \left[1 - e^{-a^2(t_d/2)^2}\right]$$

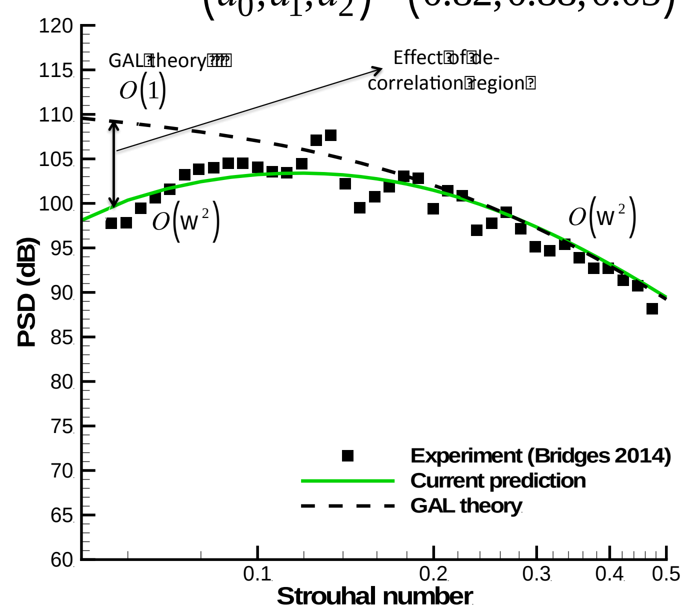
$$\Upsilon_0 = 0.04\left(r_\infty U_d\right)^2; \left(l_0, l_1, l_3\right)/D_J = \left(0.53, 0.01, 0.01\right); \left(L_2, L_3\right)/D_J = \left(0.5, 20\right)$$

$$U_c = 0.68U_d; b_0 = 0.52$$

$$\left(a_0 = 1; a_2 = 0\right)$$



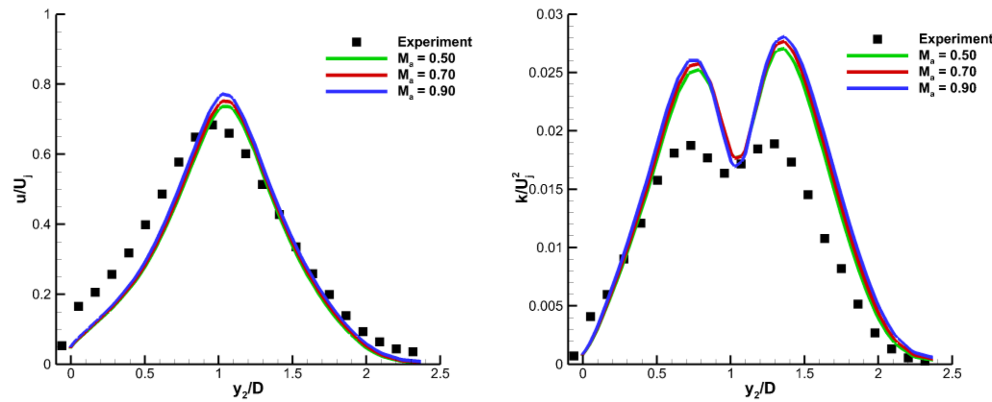
$$\left(a_0, a_1, a_2\right) = \left(0.82, 0.88, 0.05\right)$$



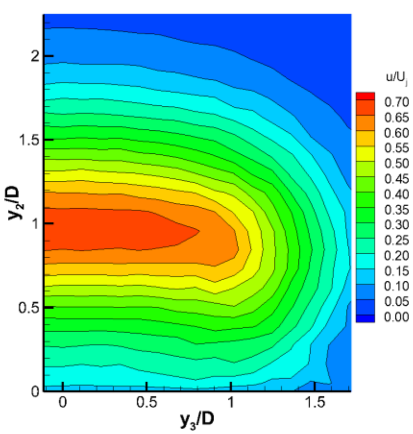
RANS Solutions
Comparisons with Experiment

- SolidWorks® Flow Simulation
  - Cartesian meshing
  - Immersed boundary approach
  - Solution-adaptive refinement
  - Two-scale wall functions
  - Modified k-e (Lam-Bremhorst)

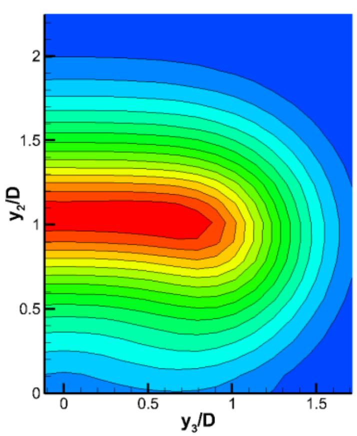
Transverse Profiles of Normalized Mean Velocity and Turbulent kinetic energy at edge of plate



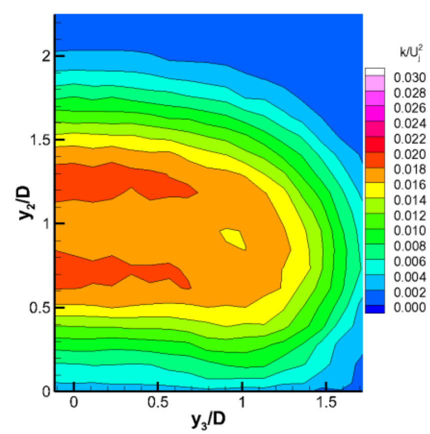
Contours of Normalized Mean Velocity and Turbulent kinetic energy at edge of plate



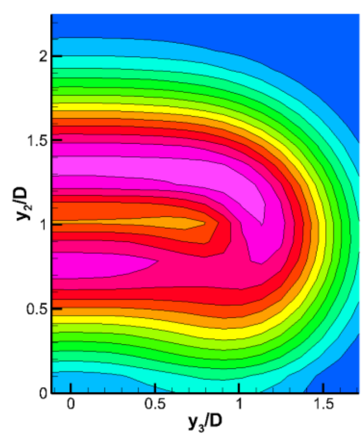
Experiment
Zaman et al.



SolidWorks®



Experiment
Zaman et al.



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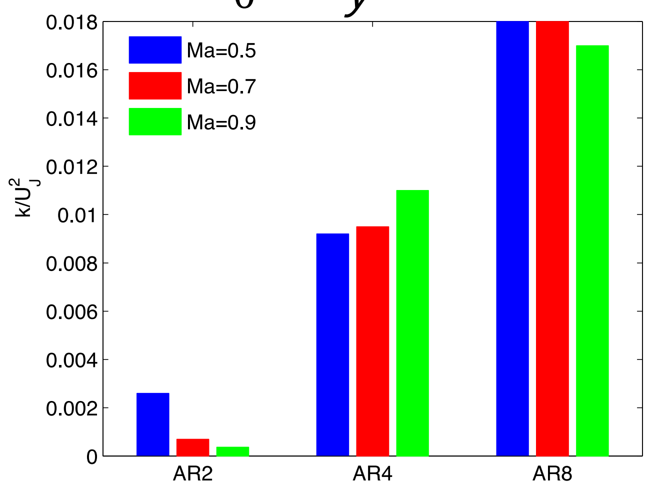
RANS Solutions
Turbulence Quantities for Noise Predictions

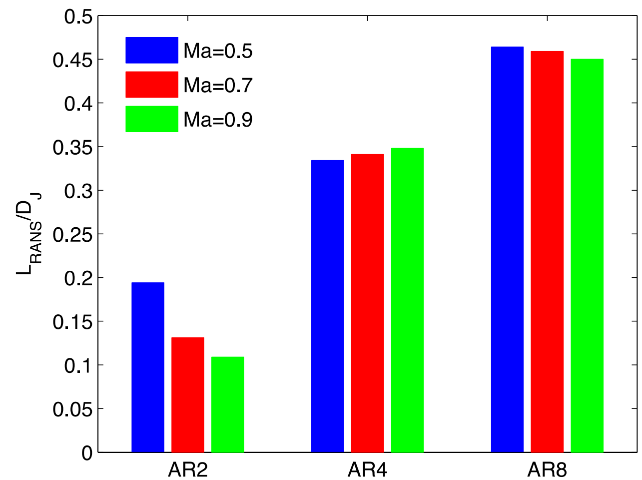
- Variation of Tke and Turbulent Length Scales at the edge of the plate with Jet Exit Velocity and Nozzle Aspect Ratio

Source Amplitude

$$Y_0 = C_y k$$

$$L_{RANS} = k^{3/2} / e$$



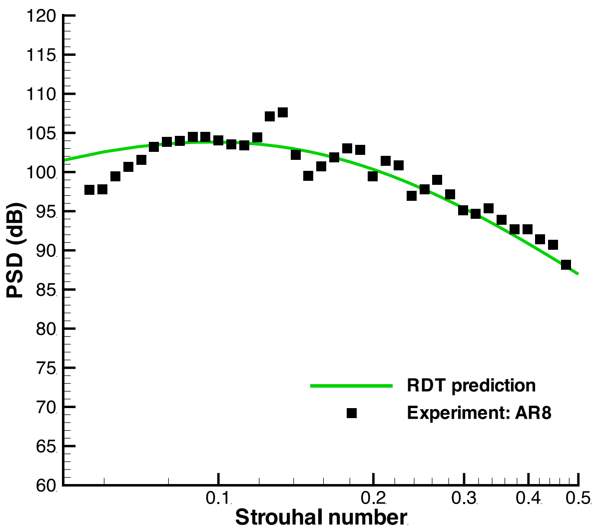


- Little variation with exit Mach number
- Significant variation with aspect ratio

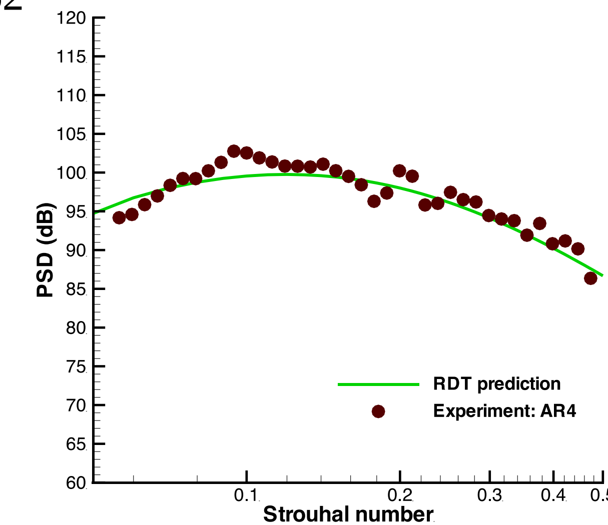
RANS-Based Edge-Noise Predictions

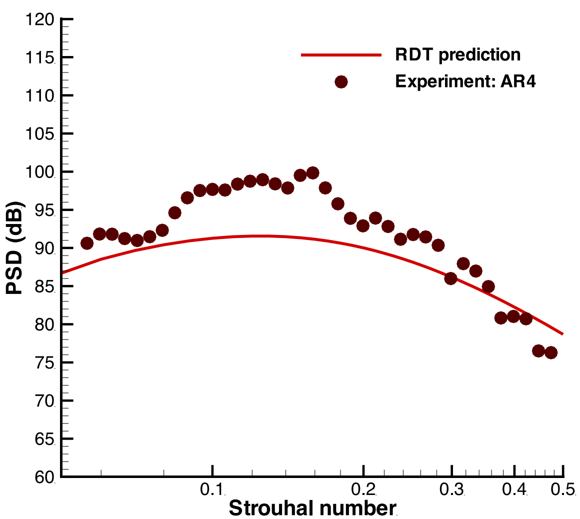
$$Y_0 = 0.04\left(r_{¥}U_d\right)^2;\left(l_0,l_1,l_3\right)/D_J = \left(0.53,0.01,0.01\right);\left(L_2,L_3\right)/D_J = \left(0.5,20\right)$$

$$U_c = 0.68U_d;b_0 = 0.52$$

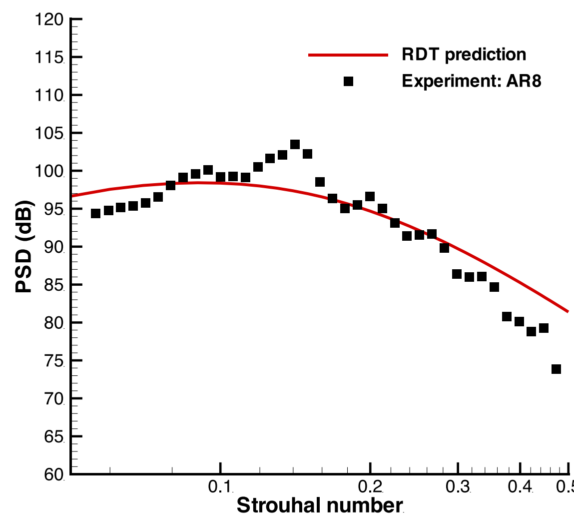


$Ma = 0.9$





$Ma = 0.7$



Summary and Conclusions

- Extended the GAL model to include a finite de-correlation region in the upstream turbulence correlation function
- Showed that the presence of a de-correlation region directly affects the low-frequency algebraic decay of the jet-surface interaction noise spectrum
- Implemented a RANS-based RDT prediction method that takes into account the reduction in length scales and turbulent kinetic energy with nozzle aspect ratio predicted by these flow solutions.
- This approach generally gives reasonably good predictions even for moderate aspect ratio jets.